# Magnetic moments of C isotopes studied with antisymmetrized molecular dynamics

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We studied the magnetic dipole moments $\mu$ of even-odd C isotopes, ranging from proton-rich to neutron-rich nuclei, with antisymmetrized molecular dynamics (AMD). The results are in good agreement with the experimental data. In the $^9$C ground state the total intrinsic spin of the protons is found to be nonzero ($S_p \neq 0$), which is unusual in even-odd nuclei. The interesting point is that the spin-orbit force breaks slightly the coupling off of intrinsic spins of the even nucleon group in isospin $T=3/2$ nuclei. This result is consistent with the newly measured $\mu$ data that, when combined with $^9$Li data, indicate an unusual $\langle\sigma\rangle$ value larger than unity. A $\mu$ moment $-1.05\mu_N$ of $^{17}$C is theoretically predicted. We also show a good reproduction of $E2$ transition data. [S0556-2813(96)50408-5]

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Owing to the radioactive beams numerous experimental measurements of the electric and magnetic moments have been performed and are proposed to reveal the properties of unstable nuclei [1–3]. Systematic investigations of the structure of these unfamiliar nuclei are now being developed also by theoretical studies. The antisymmetrized molecular dynamics (AMD) approach [4–7] has already proved to be useful for the systematic study of the structure of light unstable nuclei. In previous works [6,7] on Li, Be, and B isotopes we have succeeded in reproducing the magnetic dipole moments $\mu$, electric quadrupole moments $Q$, and $E2$ transition strength for many odd-even nuclei ranging from the stability line to the drip line. The results are also in good agreement with the recently measured $\mu$ and $Q$ moments of $^{15}$B [2]. Furthermore the theoretical prediction for $\mu$ of $^{17}$B has been found to be supported by the recent experimental measurement [2]. There also exist recent calculations with other models such as the traditional shell model of [8], which give some reproductions and predictions of properties of light unstable nuclei. However, the unique point of AMD calculations is that electric $Q$ and magnetic $\mu$ moments and electric quadrupole transition rates $B(E2)$ are reproduced adopting neither effective charge nor effective gyromagnetic ratio but using only bare values in the model. It is due to the flexibility of AMD wave functions that we can describe drastic changes of structure as a function of neutron or proton number. In previous works [6,7] with AMD analysis, we discussed the neutron number dependence of the electric and magnetic properties in relation to the change of intrinsic structure in series of isotopes. In the discussion of $\mu$ moments, we estimated important contributions from nonzero total orbital angular momentum of the even nucleon group, which means core excitation in a simple independent-particle model.

Since there exist many isotopes in the case of C, we expect to achieve a more systematic study in the wide mass number region. Another attractive point of C is found in the proton-rich nuclei $^9$C, $^{10}$C, and $^{11}$C, which have rather stable mirror nuclei $^9$Li, $^{10}$Be, and $^{11}$B, respectively. For example the $\mu$ moment of the $^9$C nucleus on the proton drip-line has been measured recently with polarized radioactive beams at RIKEN [3]. Assuming mirror symmetry, the isoscalar-moment analysis of $\mu$ moments was made for the first time in the case of $T=3/2$, and the spin expectation value $\langle\sigma\rangle=1.44$ was deduced from the experimental data of $\mu$ moments of $^9$Li and $^9$C [3]. This unusually large value is in contrast with the case of $T=1/2$ mirror nuclei, where a value $\langle\sigma\rangle\leqslant 1.0$ is generally accepted [9]. In the present paper we perform an AMD calculation for the magnetic dipole moments of even-odd C isotopes and show that the calculated results are in good agreement with the data. In the case of $^9$C we will show that actually the even number of protons have nonzero total intrinsic spin $S_p\neq 0$, which is not seen in other C isotopes. In order to indicate the reliability of our AMD study we also show good reproduction of $E2$ transitions and binding energies.

In AMD the wave function of the $A$-nucleon system is written by a parity projected Slater determinant,

$$|\Phi^{\pm}(\mathbf{Z})\rangle=(1\pm P)\frac{1}{\sqrt{A!}}\det[\varphi_j(i)],\quad \varphi_j=\phi_{\mathbf{Z}_j}\chi_{\alpha_j},\quad (1)$$

where $\chi_{\alpha_j}$ is the spin isospin wave function labeled with $\alpha_j=$p↑,p↓,n↑, or n↓ and $\phi_{\mathbf{Z}_j}$ is the $i$th spatial wave function with a Gaussian form,

$$\langle\mathbf{r}|\phi_{\mathbf{Z}_j}\rangle=\left(\frac{2\nu}{\pi}\right)^{3/4}\exp\left[-\nu\left(\mathbf{r}-\frac{\mathbf{Z}_j}{\sqrt{\nu}}\right)^2+\frac{1}{2}\mathbf{Z}_j^2\right].\quad (2)$$

In order to construct the ground state, we make the energy variational calculation for the trial function $\Phi^{\pm}(\mathbf{Z})$ with complex parameters $\{\mathbf{Z}\}=\{\mathbf{Z}_1,\mathbf{Z}_2,\ldots,\mathbf{Z}_A\}$ by introducing the frictional cooling equations as follows:

$$\frac{d\mathbf{Z}_j}{dt}=(\lambda+i\mu)\frac{1}{i\hbar}\frac{\partial}{\partial\mathbf{Z}_j^*}\frac{\langle\Phi^{\pm}(\mathbf{Z})|H|\Phi^{\pm}(\mathbf{Z})\rangle}{\langle\Phi^{\pm}(\mathbf{Z})|\Phi^{\pm}(\mathbf{Z})\rangle}$$

$$\text{and c.c.}(\mu<0).\quad (3)$$

The energy minimum is obtained after long enough cooling time and the intrinsic states are projected on total angular-momentum eigenstates $P^J_{KM}\Phi^{\pm}$. For each spin $J$, we choose

 

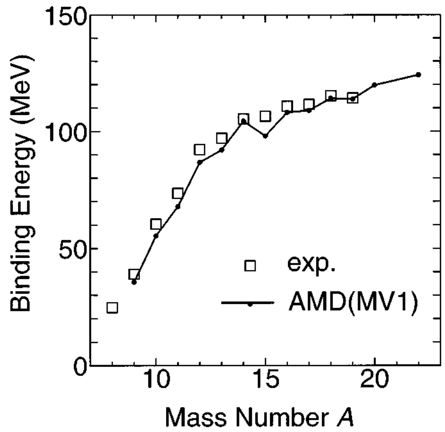


FIG. 1. Binding energies of C isotopes. Solid lines show the AMD calculations and square points indicate the experimental data.

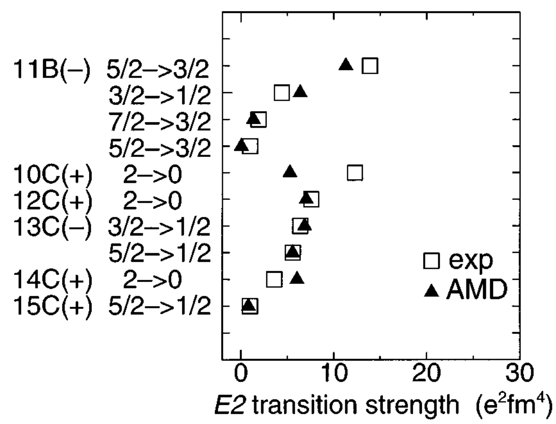


FIG. 2. $E2$ transition strength $B(E2)$. The theoretical values are obtained with the interaction parameters (i) given in the text, only $^{15}$C is calculated with parameters (ii).

the $K$ quantum number so as to make the projected energy a minimum. For a more precise explanation the reader is referred to Refs. [5,6].

The adopted central interaction is the MV1 force [10] with density dependent term. We also include the two-body spin-orbit force G3RS [11] and Coulomb force.

Except for positive parity states of $^{13}$C and $^{15}$C we adopted the interaction parameter set (i) where for the MV1 force $m=0.576$, $b=0.0$, $h=0.0$, and for the G3RS force $u=u_I=-u_{II}=900$ MeV. In case of positive parity states of $^{13}$C and $^{15}$C, we adopted the interaction parameter set (ii) where $m=0.576$, $b=0.0$, $h=0.0$, and $u=1500$ MeV to pull down the energy of the lowest $1/2^+$ state. However, even with the interaction set (ii) we could not make the $1/2^+$ states the lowest positive parity states of $^{13}$C and $^{15}$C. This situation is similar to the AMD calculation of the $1/2^+$ ground state of $^{11}$Be in Ref. [6]. The calculated binding energies of C isotopes are shown in Fig. 1 and seen to reproduce the systematic properties of the data very well. The $B(E2)$ values in C isotopes and the associated nuclei are presented in Fig. 2. The AMD results of $B(E2)$ (triangles) agree well with the data (squares) except for $^{10}$C $(2^+\to 0)$.

In Table I theoretical results of magnetic dipole moments for even-odd C isotopes are shown together with the experimental data. They are in good agreement with the experimental data. In order to analyze the magnetic moments in relation with the structure of the states, we should recall that magnetic moments consist of three terms: $\mu_{sp}$ caused by the intrinsic spin of protons, $\mu_{sn}$ by the intrinsic spin of neutrons, and $\mu_l$ due to the orbital angular momentum of protons

$$\mu_{sp}=5.586\langle S_{pz}\rangle, \quad \mu_{sn}=-3.826\langle S_{nz}\rangle, \quad \mu_l=\langle L_{pz}\rangle, \tag{4}$$

where the notation $\langle\rangle$ indicates the expectation value with the highest $M$ states $|JM\rangle=|JJ\rangle$ and units are in $\mu_N$. The contributions due to these terms are also shown in Table I. One of the remarkable points is that the intrinsic spins of the even number of protons do not couple to $S_p=0$ in $^9$C. In most even-odd nuclei, the proton intrinsic spins couple off to almost zero and therefore the term $\mu_{sp}$ gives little contribution

TABLE I. The magnetic dipole moments of even-odd C isotopes. The contributions of the neutron spin, proton spin, and the orbital angular momentum in the $\mu$ moments are also shown. The Schmidt values are also shown in the table.

| | | | AMD ($\mu_N$) | | | |
|---|---|---|---|---|---|---|
| | Expt. ($\mu_N$) | Schmidt ($\mu_N$) | $\mu$ | $\mu_{sn}$ | $\mu_{sp}$ | $\mu_l$ |
| $^9$C($3/2^-$) | $-1.39$ [a] | $-1.91$ | $-1.53$ | $-1.91$ | 0.11 | 0.27 |
| $^{11}$C($3/2^-$) | $-0.96$ [b] | $-1.91$ | $-0.90$ | $-1.32$ | 0.02 | 0.40 |
| $^{13}$C($1/2^-$) | 0.70 [b] | 0.64 | 0.99 | 0.64 | 0.01 | 0.34 |
| $^{13}$C($1/2^+$) | – | $-1.91$ | $-1.90$ | $-1.87$ | 0.05 | $-0.08$ |
| $^{13}$C($5/2^+$) | $\|1.40\|$ [b] | $-1.91$ | $-1.52$ | $-1.74$ | 0.01 | 0.21 |
| $^{15}$C($1/2^+$) | $\|1.32\|$ [b] | $-1.91$ | $-1.26$ | $-1.09$ | 0.01 | $-0.19$ |
| $^{17}$C($3/2^+$) | – | 1.15 | $-1.05$ | $-1.35$ | 0.03 | $-0.27$ |

[a]Experimental data from Ref. [3].
[b]Experimental data from Ref. [13].

TABLE II. Dependence of the magnetic dipole moments of $^9$C and $^9$Li on spin-orbit force strength. The contributions from the neutron spin, proton spin, and the orbital angular momentum are presented individually. The Schmidt values and the results in Ref. [12] are also shown.

| LS force | $^9$C(3/2$^-$) $\mu_{exp}=-1.39\mu_N$ | | | | $^9$Li(3/2$^-$) $\mu_{exp}=3.44\mu_N$ | | | |
|---|---|---|---|---|---|---|---|---|
| $u$ | $\mu$ | $\mu_{sn}$ | $\mu_{sp}$ ($S_{pz}$) | $\mu_l$ | $\mu$ | $\mu_{sn}$ ($S_{nz}$) | $\mu_{sp}$ | $\mu_l$ |
| 0 MeV | −1.65 | −1.91 | 0.00 (0.00) | 0.26 | 3.51 | 0.00 (0.00) | 2.79 | 0.72 |
| 900 MeV | −1.53 | −1.91 | 0.11 (0.02) | 0.27 | 3.44 | −0.07 (0.02) | 2.79 | 0.71 |
| 1200 MeV | −1.44 | −1.91 | 0.20 (0.04) | 0.27 | 3.33 | −0.15 (0.04) | 2.79 | 0.69 |
| 1500 MeV | −1.31 | −1.91 | 0.32 (0.06) | 0.28 | 3.21 | −0.24 (0.06) | 2.79 | 0.66 |
| Schmidt | −1.91 | −1.91 | 0.00 (0.00) | 0.00 | 3.79 | 0.00 (0.00) | 2.79 | 1.00 |
| Ref. [12] | −1.50 | | | | 3.57 | | | |

to the total magnetic moment, even with the stronger spin-orbit force. However in the case of $^9$C, the proton spin term $\mu_{sp}$ is not zero and is very sensitive to the strength of the LS interaction. In Table II we present the dependence of $\mu(^9\mathrm{C})$ on the strength of the LS force, compared with the results of the mirror nucleus $^9$Li. In the AMD results the pairing off of the proton intrinsic spins in $^9$C is broken by an LS force stronger than 900 MeV. Adopting the strength parameter $u=1200$ MeV, the breaking is estimated as $\langle S_{pz}\rangle=0.04$, which is slight but gives a significant contribution of $\mu_{sp}=0.2\ \mu_N$ to the magnetic moment. The theoretical magnetic moments with $u=1200$ MeV, $-1.44\ \mu_N$ for $^9$C, and 3.33 $\mu_N$ for $^9$Li, are in reasonable agreement with the data. The reason for $S_p\neq 0$ with the increase of the LS force strength is probably that the $^9$C (and $^9$Li) ground states are soft towards the alignment of proton intrinsic spins. The breaking of the coupling off of proton spins gives the expectation value $\langle\sigma\rangle=1.07$, greater than unity.

The nonzero value of $\mu_l$ in Table I means that the orbital angular momenta of protons couple to $L_p\neq 0$ in most even-odd C isotopes. The mixing of the components with the nonzero total orbital angular momentum of protons plays an important role in estimating the shift of the observed data from the Schmidt values, except for $^{13}$C(1/2$^-$). In a previous paper, the $N$ dependence of $\mu$ moments of odd-even Li and B isotopes has been explained quantitatively by the components with the nonzero total orbital angular momentum of even neutrons.

In the present calculation, the spin parity of the ground states of $^{17}$C is found to be $\frac{3}{2}^+$. The predicted value of $\mu(^{17}\mathrm{C})$ is $-1.05\ \mu_N$, which is quite different from the Schmidt value $+1.15\ \mu_N$ for a neutron in a $d_{3/2}$ orbit. This is because the total spin $J=3/2$ does not consist only of the angular momentum of single valence neutron in the ground state. The dominant component is a $d_{5/2}$ state of a valence neutron which couples with the orbital angular momentum $L_p=2$ of the even protons and $L_n=2$ of the residual even neutrons so as to give total $J=3/2$. Therefore the negative sign of the calculated result seems natural, since it is reasonable to compare the $\mu(^{17}\mathrm{C})$ with the Schmidt value $-1.91\ \mu_N$ for $d_{5/2}$ neutron. In the analysis of $\mu$ moments, we have obtained the conclusion that the significant contribution to the magnetic dipole moments is due to the even nucleons, except for nucleons forming a closed shell. The contribution from the angular momentum of the even nucleons should not be ignored, but all the valence protons and neutrons must be taken into consideration.

In summary, we have studied the magnetic dipole moments of even-odd C isotopes systematically with the method of antisymmetrized molecular dynamics. The theoretical results are in reasonable agreement with the measured data of $\mu$ moments as well as $B(E2)$ and binding energies. One of the unique points in AMD calculations is that electric and magnetic moments are reproduced without effective charge or effective gyromagnetic ratio, using bare values in the model. We have found an abnormal property of $^9$C that the total intrinsic spin of the even number of protons is not zero in the ground state, because the spin-orbit force breaks the coupling off of proton intrinsic spins. The breaking due to the spin-orbit force, though it is slight, gives a significant contribution to the $\mu$ moment. The effect of the spin-orbit force to the $\mu(^9\mathrm{C})$ and $\mu(^9\mathrm{Li})$ has been shown to be significant while it does not greatly affect the magnetic moments of other C isotopes. In many even-odd C isotopes the nonzero total orbital angular momentum of protons has been found to give rise to a significant contribution $\mu_l$ to the magnetic moments. The value of $\mu(^{17}\mathrm{C})$ was predicted to be $-1.05\ \mu_N$.

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