Nontrivial aspects of the onset of nuclear collectivity: Static moments

L. Zamick
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855

D. C. Zheng
W. K. Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125

(Received 27 February 1996)

We consider several topics concerning static magnetic dipole and electric quadrupole moments (μ and Q) as signatures of the onset of nuclear collectivity. Having previously noted that in ^{50}Cr there is an abrupt change of sign in Q of yrast states with $J^\pi = 10^+, 12^+$, and 14^+ relative to lower J states, we discuss whether these states are oblate or prolate. We next show that configuration mixing leads to much larger changes in Q than in μ. We then look for other bands of interest in ^{50}Cr. Finally we discuss the Jolos–von Brentano relationship which relates Q of 2_1^+ states to $B(E2)$'s for transitions from and to the 2_1^+ states. [S0556-2813(96)02408-9]

PACS number(s): 21.10.Ky, 21.10.Re, 21.60.Ev, 27.40.+z

In a recent publication [1], the current authors noted that in shell model calculations for ^{50}Cr, in which up to three nucleons were allowed to be excited from the $f_{7/2}$ shell to the rest of the f-p shell, the static quadrupole moments of the yrast states with $J^\pi = 2^+, 4^+, 6^+$, and 8^+ were negative but those for $J^\pi = 10^+, 12^+$, and 14^+ were positive. However, the question of whether the latter three states were oblate or prolate was not answered definitively. We here address this issue.

With the FPD6 interaction [2], and allowing up to three nucleons to be excited from the $f_{7/2}$ shell ($t=3$), the static quadrupole moments (in e fm^2) were -27.5, -34.8, -8.1, and -20.7 for $J_n^\pi = 2_1^+, 4_1^+, 6_1^+$, and 8_1^+ respectively and were $+45.7$, $+18.6$, and $+11.4$ for $J_n^\pi = 10_1^+, 12_1^+$, and 14_1^+. With the KB3 interaction [3], the corresponding values are -24.8, -30.0, -15.6, -14.7 for $J_n^\pi = 2_1^+, 4_1^+, 6_1^+$, and 8_1^+ and $+26.5$, $+13.0$, and $+8.2$ for $J_n^\pi = 10_1^+, 12_1^+$, and 14_1^+. Note that there is not a smooth transition in going from $J_n^\pi = 8_1^+$ to $J_n^\pi = 10_1^+$ The value of Q for 8_1^+ is fairly large and negative while the value for 10_1^+ is large and positive.

If K were a good quantum number, we could use the rotational formula

$$Q(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_K, \tag{1}$$

where Q_K is the intrinsic quadrupole moment, to determine K. For $J^\pi = 10^+, 12^+$, and 14^+, if K is small ($K \leqslant 6$), then $Q(J)$ and Q_K have opposite signs. But if K is sufficiently large, $Q(J)$ and Q_K will have the same sign.

We expect considerable band mixing. Nevertheless, we feel that a crude analysis using the above formula would be helpful in determining in which ballpark we are. To reduce the ambiguity of the effective charges, we take ratios. Thus,

$$\frac{Q(10^+)}{Q(12^+)} = 1.387 \frac{3K^2 - 110}{3K^2 - 156}, \tag{2}$$

$$\frac{Q(12^+)}{Q(14^+)} = 1.325 \frac{3K^2 - 156}{3K^2 - 210}. \tag{3}$$

For the FPD6 interaction [2], the first equation in the above gives $K = 8.5$ while the second one gives $K = 12.1$. With the KB3 interaction [3], the corresponding numbers are similar: $K = 9.2$ and $K = 12.8$. Thus this admittedly crude analysis favors a ''high-K prolate'' interpretation for these states.

It should be noted that the lower spin states, especially $J^\pi = 2^+$ and 4^+, are best described as low-K prolate states. Thus all the states are prolate but the nature of the $J^\pi = 10^+$, 12^+, and 14^+ ''band'' is quite different from that of $J^\pi = 2^+$, 4^+, 6^+, and 8^+. We clearly have a band crossing phenomenon and it is interesting to note that one shell model configuration $(f_{7/2})^{10}$ contains in some sense both of the two bands.

In a recent experimental work, Pakou *et al.* [4] measured g ($g = \mu/J$) factors of states in ^{50}Cr with the following results:

J^π	g
2_1^+	0.54(11)
4_1^+	0.43(9)
6_1^+	0.54(16)
8_1^+	0.54(9)

For 4_1^+, 6_1^+, and 8_1^+, these are much smaller than the g factors calculated in the single j shell model [5]. The suggestion was made in the 1994 paper [4] that the onset of nuclear collectivity brought the g factors close to the rotational result for a $K=0$ band of $g \simeq g_R = Z/A$.

This result has motivated us here to calculate the g factors in larger shell model spaces. We allow up to t nucleons to be excited from the $f_{7/2}$ shell to the rest of the f-p shell and show results for $t = 0,1,2$, and 3 for the g factors in Table I.

TABLE I. g factors ($g=\mu/J$) in ^{50}Cr for the KB3 interaction as a function of t, the maximum number of nucleons allowed to be excited from the $f_{7/2}$ shell to the rest of the f-p shell.

J	$t=0$ [a]	$t=1$ [a]	$t=2$ [a]	$t=3$ [a]	$t=3$ [b]
2	0.707	0.679	0.578	0.579	0.540
4	0.949	0.894	0.831	0.804	0.756
6	0.885	0.858	0.816	0.792	0.745
8	0.769	0.822	0.841	0.828	0.779
10	0.486	0.519	0.515	0.509	0.474
12	0.609	0.609	0.591	0.588	0.550
14	0.712	0.698	0.687	0.678	0.633

[a]For free g values: $g_{l,\pi}=1$, $g_{l,\nu}=0$, $g_{s,\pi}=5.586$, $g_{s,\nu}=-3.826$.
[b]For renormalized g values: $g_{l,\pi}=1.1$, $g_{l,\nu}=-0.1$, $g_{s,\pi}=3.910$, $g_{s,\nu}=-2.678$.

We should first remark that from our previous work on static quadrupole moments Q [1], we agree that there is an onset of nuclear collectivity, in the sense that the $B(E2)$'s become bigger as t increases, the energy levels look more rotational, and Q for J up to 8 become more negative relative to $t=0$. In that work, the FPD6 interaction [2] was used. In this work, we show the behavior of Q using the KB3 interaction [3]. This also shows the increase in magnitude of Q for $J^\pi=2^+,4^+,6^+$, and 8^+ (more negative).

However, when we look at the g factors, the change is not so drastic. Even for $t=3$ one still gets large g factors. The values for 2_1^+, 4_1^+, 6_1^+, and 8_1^+ using free $g_{l,\pi}$, $g_{l,\nu}$, $g_{s,\pi}$, and $g_{s,\nu}$ values are 0.58, 0.80, 0.79, and 0.83, respectively. These are considerably larger than the experimental values.

If we use quenched spin g factors $g_{s,\pi/\nu}=0.7g_{s,\pi/\nu}$, along with $g_{l,\pi}=1.1$ and $g_{l,\nu}=-0.1$, the corresponding results for g decrease somewhat to 0.54, 0.76, 0.74, and 0.78. But they are still substantially larger than experiment.

Thus the *calculated* onset of nuclear collectivity consists of large changes in the $B(E2)$'s and Q, but much smaller changes in magnetic g factors. With the bare g_l and g_s values, the percent change for the g factors in going from $t=0$ to $t=3$ for 2_1^+, 4_1^+, 6_1^+, and 8_1^+ is 18.1%, 15.3%, 10.4%, and 7.6%, respectively. As can be seen from Table II, there are more than a factor of 2 changes for Q.

It should be mentioned that our $t=3$ results for the quadrupole moments have been found to be ''qualitatively equivalent to the full fp space results'' obtained by Martínez-Pinedo *et al.* [6].

TABLE II. Static quadrupole moments Q (in units of $e$fm^2) in ^{50}Cr for the KB3 interaction as a function of t, the maximum number of nucleons allowed to be excited from the $f_{7/2}$ shell to the rest of the f-p shell.

J	$t=0$	$t=1$	$t=2$	$t=3$
2	-12.240	-20.392	-20.824	-24.665
4	-12.148	-22.792	-24.950	-29.810
6	-4.415	-14.459	-9.661	-15.531
8	0.478	-8.490	-10.454	-14.698
10	19.118	23.481	24.494	26.461
12	6.546	10.488	11.591	12.998
14	6.810	8.759	8.208	8.232

TABLE III. Other possible positive-parity bands in ^{50}Cr in the $t=3$ calculation with the KB3 interaction.

J_n^π	E_x (MeV)	μ (μ_N)	Q ($e$fm^2)
4_2^+	3.003	5.680	31.853
6_2^+	3.595	-0.414	40.262
8_2^+	5.611	2.172	19.469
10_1^+	5.993	5.095	26.461
10_2^+	6.500	6.137	12.468
12_1^+	7.435	7.058	12.998
14_1^+	9.949	9.490	8.232

It would be nice in the near future to bring about a reconciliation between theory and experiment.

In our previous work [1], we focused on yrast states in ^{50}Cr and showed that whereas the 2_1^+, 4_1^+, 6_1^+, and 8_1^+ states have negative static quadrupole moments Q, the 10_1^+, 12_1^+, and 14_1^+ have positive Q's. There is a band crossing and, to some extent, even the simplest configuration $(f_{7/2})^{10}$ has in it both the ground-state band and the second band which overtakes the ground-state band at $J^\pi=10^+$.

In Table III we show for $t=3$ a common feature of the states 2_2^+, 4_2^+, 6_2^+, and 8_2^+. They have rather large, positive quadrupole moments. This result contradicts the yrast band calculation for which the Q's are comparable in magnitude but are negative.

We also show in Table III the values of Q for the 10_1^+, 10_2^+, 12_1^+, and 14_1^+ states. They are also positive. It is not clear how to extend the band 8_2^+—whether to include the 10_1^+ or 10_2^+ state. Since the two 10^+ states are rather close in energy, it could be that some admixture of these looks most like a member of the band.

There have been measurements in other parts of the periodic table where the g factors for even-even nuclei differ substantially from Z/A. For example, for ^{150}Sm, Vass *et al.* [7] reported that $g(4+)/g(2+)=1.60(12)$ while $g(6+)/g(2+)=1.14(34)$. Of course, since in this calculation, we are dealing with ^{50}Cr we cannot say that their measurement supports our calculation or vice versa. But at least it suggests that one should be on the lookout for the types of behaviors that both works seem to find.

Recently Jolos and von Brentano (hereinafter referred to as J-vB) [8] have presented a formula which relates quadrupole moments of the 2_1^+ states to various $B(E2)$ values. This connection is of great interest because it is much more difficult to measure static quadrupole moments than it is to measure $B(E2)$'s. They feel that the formula should be extremely accurate (better than 1.5%) for deformed nuclei for which $E^*(4_1^+)/E^*(2_1^+)\geq 2.9$, where $E^*(4_1^+)$ and $E^*(2_1^+)$ are the excitation energies of the 4_1^+ and 2_1^+ states relative to the ground state. Also for ''realistic cases'' the predictions given by the formula agree with IBM-1 results to better than 2% for $N=12$ and 6% for $N=6$. Their relationship can be written as

$$\frac{|Q(2_1^+)|}{\sqrt{B(E2:2_1^+\rightarrow 0_1^+)}}=\frac{8}{7}\sqrt{\pi G(1+R_1-W)}, \qquad (4)$$

TABLE IV. The experimental (expt) values [10–12] and the results of shell-model (SM), Jolos–von Brentano (J-vB), and rotational (rot) formulas for static quadrupole moments [$Q(2_1^+)$ for ''expt'' and ''SM''; $|Q(2_1^+)|$ for ''J-vB'' and ''rot''] (in $e\text{fm}^2$) of 2_1^+ states in selected s-d and f-p shell nuclei. The predictions of the J-vB and rotational formulas based on the shell-model $B(E2)$ values should be compared with the shell model results. In the parentheses we give the percentage deviations of the ''J-vB'' and ''rot'' results from the shell model. Effective charges $e_p=1.5$ and $e_n=0.5$ are assumed.

Nucleus	$\frac{E^*(4_1^+)}{E^*(2_1^+)^{\text{expt}}}$	$\frac{E^*(4_1^+)}{E^*(2_1^+)^{\text{SM}}}$	$Q(2_1^+)_{\text{expt}}$	$Q(2_1^+)_{\text{SM}}$	$\lvert Q(2_1^+)_{\text{J-vB}}\rvert$	$\lvert Q(2_1^+)_{\text{rot}}\rvert$
^{20}Ne	2.61	2.37	-23 ± 3	-15.83	13.96(−11.8%)	15.78(−0.3%)
^{22}Ne	2.65	2.47	-19 ± 4	-15.67	15.92(+1.6%)	15.89(+1.4%)
^{24}Mg	3.01	2.90	-18 ± 2	-19.25	18.46(−4.1%)	19.90(+3.3%)
^{28}Si	2.60	2.34	-16 ± 3	20.75	19.18(−7.6%)	20.25(−2.4%)
^{46}Ti	2.26	1.90	-21 ± 6	-17.30	17.72(+2.4%)	23.21(+34.2%)
^{48}Ti	2.33	2.25	-13.5 ± 8.8	-14.72	20.20(+37.2%)	20.17(+37.0%)
^{50}Cr	2.40	2.35	-36 ± 7	-24.82	27.31(+10.0%)	26.63(+7.3%)

where

$$G=\left(\frac{7}{10}\right)\frac{B(E2:4_1^+\rightarrow 2_1^+)}{B(E2:2_1^+\rightarrow 0_1^+)}, \tag{5}$$

$$R_1=\frac{B(E2:2_2^+\rightarrow 0_1^+)}{B(E2:2_1^+\rightarrow 0_1^+)}, \tag{6}$$

and

$$W=\frac{B(E2:2_2^+\rightarrow 2_1^+)}{B(E2:4_1^+\rightarrow 2_1^+)}. \tag{7}$$

Of course the rotational formulas of Bohr and Mottelson can also be combined to give a relationship between $B(E2)$ and $Q(2_1^+)$. These are Eq. (1) and

$$B(E2:KJ_1\rightarrow KJ_2)=\frac{5}{16\pi}e^2Q_0^2\langle J_1K20|J_2K\rangle^2, \tag{8}$$

where $\langle J_1K20|J_2K\rangle$ is the Clebsch-Gordan coefficient. For a $K=0$ band, one gets

$$|Q(2_1^+)|=\sqrt{\frac{64\pi}{49}B(E2:2_1^+\rightarrow 0_1^+)}. \tag{9}$$

Note that in the rotational limit, $G=1$. If one also takes $R_1=W=0$, i.e., if one neglects interband transitions, one then recovers the above rotational formula from the J-vB equation (4). It is interesting to find out if, for a nonperfect rotor, the J-vB relation would yield a more accurate $Q(2_1^+)$. To this end, we conduct a theoretical experiment by performing shell-model calculations for the B(E2) values that go into Eqs. (4) and (9) and comparing the predictions of these two formulas for $|Q(2_1^+)|$ to the ''exact'' values obtained in the shell-model calculations. We do this calculation for selected deformed nuclei in the s-d (^{20}Ne, ^{22}Ne, ^{24}Mg, and ^{28}Si) and f-p (^{46}Ti, ^{48}Ti, and ^{50}Cr) region. For the s-d shell, we use the Brown-Wildenthal interaction [9]; for the f-p shell, we use the modified Kuo-Brown interaction KB3 [3]. For the s-d shell nuclei, the calculations are carried out in the full one-major-shell space. For the f-p shell nuclei, the full space calculation is only done for ^{46}Ti. For the other two nuclei, a maximum number of t nucleons is allowed to leave the $f_{7/2}$ orbital and occupy the rest of the f-p shell with $t=4$ for ^{48}Ti and $t=3$ for ^{50}Cr.

Our results are listed in Table IV where we also list the experimental values for the ratio $E^*(4_1^+)/E^*(2_1^+)$ and $Q(2_1^+)$. The calculated values for various $B(E2)$'s that go into the J-vB formula (4) and the rotational formula (9) are listed in Table V where the experimental $B(E2:2_1^+\rightarrow 0_1^+)$ values are also shown. With one notable exception, the J-vB

TABLE V. Input from shell-model calculations into the J-vB and rotational formulas, obtained for the Wildenthal interaction for the s-d shell and the KB3 interaction for the f-p shell. The $B(E2)$ values listed are in units of $e^2\text{fm}^4$. The ratios G, R_1, and W are defined in the text. We also give the experimental $B(E2:2_1^+\rightarrow 0_1^+)$ values (in the parentheses).

Nucleus	$B(E2:2_1^+\rightarrow 0_1^+)$	(expt)	$B(E2:2_2^+\rightarrow 0_1^+)$	$B(E2:2_2^+\rightarrow 2_1^+)$	$B(E2:4_1^+\rightarrow 2_1^+)$	G	R_1	W
^{20}Ne	60.67	(68)	0.03	4.41	72.20	0.83	0.001	0.061
^{22}Ne	61.53	(46)	4.57	0.55	82.62	0.94	0.074	0.0066
^{24}Mg	96.48	(86.4)	8.63	21.07	128.23	0.93	0.089	0.164
^{28}Si	99.92	(65.2)	0.36	13.86	141.41	0.99	0.004	0.098
^{46}Ti	131.32	(201)	4.48	70.25	173.59	0.93	0.034	0.405
^{48}Ti	99.10	(144)	23.44	40.99	148.02	1.05	0.236	0.277
^{50}Cr	172.82	(216)	11.27	2.13	245.72	1.00	0.065	0.0087

TABLE VI. Same as Table V but using experimental input. Only the results for the f-p shell nuclei are listed.

Nucleus	$B(E2{:}2_1^+\rightarrow 0_1^+)$	G	R_1	W	$\lvert Q(2_1^+)_{\text{J-vB}}\rvert$	$\lvert Q(2_1^+)_{\text{rot}}\rvert$
^{46}Ti	201	0.686	0.003	0.260	20.5	28.7
^{48}Ti	144	0.564	0.060	0.580	12.6	24.3
^{50}Cr	216	0.516	0.106	0.000	22.5 [a]	29.8
				0.219	20.1 [b]	29.8

[a]We consider only the $2_2^+\rightarrow 0_1^+$ transition to determine W.
[b]We add $2_2^+\rightarrow 0_1^+$ and $2_3^+\rightarrow 0_1^+$ transitions. The states are close together: $E(2_2^+)=2.924$MeV and $E(2_3^+)=3.161$MeV.

predictions agree with the shell-model results to better than 12%. However, for all the nuclei that are considered here, only for one nucleus (^{46}Ti) has the J-vB formula done a better job than the rotational formula. This is surprising because one would expect that there is more physics put into the J-vB formula.

The biggest disagreement between the rotational formula and the shell-model results occurs in ^{46}Ti and ^{48}Ti, where the discrepancies are 34% and 37%, respectively. The J-vB formula seems to cure the problem for ^{46}Ti but not for ^{48}Ti. The problem in the latter case is that R_1 and W are almost the same and so cancel each other out.

In Table VI we apply the J-vB relation to experimental inputs in the f-p shell, which are obtained from the Nuclear Data Sheets [13]. Note that the experimental $B(E2)$'s are somewhat larger than those calculated with the KB3 interaction with effective charges of $e_p=1.5$ and $e_n=0.5$. The experimental values of G are considerably smaller than the calculated values. In other words, $B(E2{:}4_1^+\rightarrow 2_1^+)_{\text{expt}}$ is smaller than $B(E2{:}4_1^+\rightarrow 2_1^+)_{\text{theory}}$. There are considerable differences in the values of R_1 and W as well.

Using the J-vB relation with experimental data the values of $Q(2_1^+)$ are significantly smaller than those using the rotational model. For ^{46}Ti, and ^{48}Ti, and ^{50}Cr, the J-vB (rotational) values of $Q(2_1^+)$ are, respectively, 20.5 (28.7), 12.7 (24.3), and 22.5 (29.8). For ^{46}Ti and ^{48}Ti, the J-vB analysis gives an improved fit. For ^{50}Cr, the J-vB analysis gives too small a value of $Q(2^+)$ compared with experiment.

It is difficult to give a definite assessment of the J-vB relation in the regions that we have considered, which in some cases are beyond what the authors envisioned. However, we greatly admire the spirit of this work and support that ideas along these lines continue to be pursued.

This work was supported in part by a U. S. Department of Energy Grant No. DE-FG05-86ER-40299 (L.Z.) and a National Science Foundation Grant No. PHY94-12818 (D.C.Z.). We thank Noemie Koller and Alfredo Poves for useful communications.

[1] L. Zamick, M. Fayache, and D.C. Zheng, Phys. Rev. C **53**, 188 (1996).
[2] W.A. Richter, M.G. Van Der Merwe, R.E. Julies, and B.A. Brown, Nucl. Phys. **A523**, 325 (1991).
[3] A. Poves and A.P. Zuker, Phys. Rep. **70**, 235 (1981).
[4] A. Pakou, J.B. Billows, A.W. Monteford, and D.D. Warner, Phys. Rev. C **50**, 2608 (1994).
[5] A. Pakou, R. Tanczyn, D. Turner, W. Jan, G. Kumbartzki, N. Benczer-Koller, Xiao-li Li, Huan Liu, and L. Zamick, Phys. Rev. C **36**, 2088 (1987).
[6] A. Poves (private communications). In this work the author concludes that the lowest $J=10^+$, 12^+, and 14^+ states in ^{50}Cr are oblate.
[7] T. Vass, A.W. Mountford, G. Kumbartzki, N. Benczer-Koller, and R. Tanczyn, Phys. Rev. C **48**, 2640 (1993).
[8] R.V. Jolos and P. von Brentano, Kóln Laboratory report, 1995.
[9] B.A. Brown and B.H. Wildenthal, Annu. Rev. Nucl. Part. Sci. **38**, 29 (1988).
[10] R.H. Spear, Phys. Rep. C **73**, 369 (1981).
[11] C.W. Towsley, D. Cline, and R.N. Horoshko, Nucl. Phys. **A250**, 381 (1975).
[12] R.M.S. Lesser, D. Cline, P. Goode, and R.N. Horoshko, Nucl. Phys. **A190**, 579 (1972).
[13] Nucl. Data Sheets **68**, 1 (1993); **68**, 271 (1993); **75**, 1 (1995).