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Polarized deep-inelastic scattering from nuclei: A relativistic approach

G. Piller,[1] W. Melnitchouk,[2] and A. W. Thomas[3]
[1]Physik Department, Technische Universität München, D-85747 Garching, Germany
[2]Department of Physics, University of Maryland, College Park, Maryland 20742
[3]Department of Physics and Mathematical Physics and Institute for Theoretical Physics, University of Adelaide, Adelaide 5005, Australia

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We discuss spin-dependent, deep-inelastic scattering from nuclei within a covariant framework. In the relativistic impulse approximation this is described in terms of the amplitude for forward, virtual-photon scattering from an off-mass-shell nucleon. The general structure of the off-shell nucleon hadronic tensor is derived, and the leading behavior of the off-shell nucleon structure functions computed in the Bjorken limit. The formalism, which is valid for nucleons bound inside nuclei with spin 1/2 or 1, is applied to the case of the deuteron. [S0556-2813(96)05908-0]

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I. INTRODUCTION

The role of special relativity in nuclei has been an important consideration in recent years in the pursuit of consistent descriptions of nuclear electromagnetic processes at high energies [1]. Evidence suggests, for instance, that nonrelativistic models are inadequate to account for elastic form factors of few-nucleon systems at large momentum transfers, or to provide a quantitative description of proton-nucleus cross sections over the full energy range. While meson degrees of freedom almost certainly play some role in these discrepancies, for a full and consistent theoretical description that does not manifestly break Lorentz covariance, one must also incorporate effects that arise from the off-mass-shell nature of the bound nucleons.

A process in which effects associated with the off-mass-shell deformation of the nucleon structure function may also have an impact is deep-inelastic scattering (DIS) of leptons from nuclei. Until recently, the issue of nucleon off-shellness has largely been ignored in this problem, even in calculations based on the so-called relativistic impulse approximation. Often the only relativistic corrections made are kinematic, without consideration of the dynamics that may be affected when the nucleon in a nuclear medium is off shell.

The in-medium modification of the nucleon structure in unpolarized scattering was discussed in Refs. [2,3], and, as the only example for which relativistic nuclear wave functions have been calculated, a quantitative study of the effects for a deuterium nucleus made in Ref. [4]. The deuterium analysis was extended to polarized processes in Ref. [5], where it was found that, while small for moderate values of Bjorken-x, the off-shell effects can become sizable in the region of relativistic kinematics, $x \gtrsim 0.8$ (see also Ref. [6] for a nonrelativistic treatment). Apart from the appreciation of the importance of relativity in nuclei per se, there is also a practical need to understand the role of off-shell effects in light nuclei such as deuterium or helium. In the absence of free neutron targets, these nuclei are the only sources of information on the spin structure of the neutron in DIS, which is essential for testing fundamental QCD sum rules, such as the Bjorken sum rule.

In this paper we present a full derivation of the modification of the nucleon structure in the off-mass-shell region for spin-dependent DIS. A brief outline for the special case of massless quarks was given previously in Ref. [5]. Here we extend the discussion in [5] to the most general case, including mass terms. In Sec. II we analyze the most general form of the antisymmetric part of the off-shell hadronic tensor which enters the calculation of the spin-dependent cross section. Any relativistic model of polarized DIS from nuclei within the impulse approximation must be consistent with the symmetry constraints on the truncated hadronic tensor derived here. Furthermore, we calculate the scaling behavior of the expansion coefficients of the off-shell tensor in leading twist. This enables us to present for the first time a model-independent proof of gauge invariance of the off-shell, spin-dependent hadronic tensor.

Working within the nuclear impulse approximation (i.e., neglecting effects due to final state interactions), in Sec. III we present a model-independent result for the nuclear structure function g_1^A in terms of the off-shell nucleon tensor and the off-shell nucleon-nucleus scattering amplitude. The impulse approximation assumes that inelastic scattering from nuclei proceeds via incoherent scattering from individual nucleons, which is believed to be a good approximation if one is sufficiently far away from the small-x region. The formal results are valid for spin 1/2 and spin 1 nuclei. (They can also be applied to the case of DIS from a nucleon dressed by a meson cloud [7–10].) Furthermore, we investigate the conditions for the validity of the usual convolution model [11–16], which involves factorization of subprocesses at the cross section (rather than at the amplitude) level.

Using these results, in Sec. IV we illustrate the application of the formalism to the special case of the deuteron. Some of the numerical results, obtained in the massless quark limit, have been presented in Ref. [5]. For completeness, we compare here the results for the proton and deuteron structure functions with the latest available data on g_1^p and g_1^D, as well as with unpolarized quark distributions. Finally, in Sec. VI we make some concluding remarks.

II. POLARIZED NUCLEON STRUCTURE FUNCTIONS

Here we analyze the general structure of the amplitude for the forward scattering of a virtual photon from a polarized, off-mass-shell nucleon. (In fact, the formalism is valid for any spin-1/2 fermion with substructure.) Recall that the antisymmetric part of the hadronic tensor for an on-shell nucleon in terms of the structure functions g_1^N and g_2^N is written as

$$MW_{\mu\nu}^N(p,s,q)$$
$$=i\frac{M}{p\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\left(s^\beta g_1^N(p,q)+\left(s^\beta-\frac{s\cdot q}{p\cdot q}p^\beta\right)g_2^N(p,q)\right), \tag{1}$$

where p and q are the four-momenta of the nucleon and photon, respectively. Here M stands for the nucleon mass, and s is the nucleon polarization vector. Since we will be interested in the leading twist components of the structure functions only, we will not discuss the structure function g_2, which contains both twist-2 and twist-3 contributions.

Our aim will be to generalize the tensor structure in $W_{\mu\nu}^N$ to describe deep-inelastic scattering from an off-shell nucleon, namely one with $p^2\neq M^2$. We start with the observation that the antisymmetric nucleon tensor can be written as

$$MW_{\mu\nu}^N(p,s,q)=\overline{u(p,s)}\hat{G}_{\mu\nu}(p,q)u(p,s), \tag{2}$$

where $\hat{G}_{\mu\nu}(p,q)$ is the "truncated" nucleon tensor, whose Dirac structure represents deep-inelastic scattering from a generally off-shell nucleon, and $u(p,s)$ is the free Dirac spinor for a nucleon with momentum p and spin s. In the following we give the general expression for the truncated tensor $\hat{G}_{\mu\nu}$, which satisfies the discrete symmetries, in terms of a number of "truncated coefficient functions," or off-shell nucleon structure functions. Following this, we derive expressions for the off-shell structure functions in the Bjorken limit.

A. Truncated nucleon structure functions

In analyzing the off-shell nucleon structure, it will be convenient to expand the truncated nucleon tensor $\hat{G}_{\mu\nu}$ in terms of independent basis tensors, such that $\hat{G}_{\mu\nu}$ is invariant under parity and time reversal. These constraints can be summarized by the following conditions:

$$\hat{G}_{\mu\nu}(p,q)=\mathcal{P}\hat{G}^{\mu\nu}(\tilde{p},\tilde{q})\mathcal{P}^\dagger, \tag{3a}$$

$$\hat{G}_{\mu\nu}(p,q)=(\mathcal{T}\hat{G}^{\mu\nu}(\tilde{p},\tilde{q})\mathcal{T}^\dagger)^*, \tag{3b}$$

where $\mathcal{P}$ and $\mathcal{T}$ are the parity and time reversal operators, respectively. (In the Dirac representation they are expressed in terms of the Dirac matrices as: $\mathcal{P}=\gamma_0$ and $\mathcal{T}=-i\gamma_5\mathcal{C}$, where $\mathcal{C}=i\gamma^2\gamma_0$ is the charge conjugation operator.) We also use the notation $\tilde{p}_\mu=p^\mu$ and $\tilde{q}_\mu=q^\mu$ to distinguish covariant and contravariant four-vectors. The truncated tensor must also be Hermitian, which requires that (note that $\hat{G}_{\mu\nu}=-\hat{G}_{\nu\mu}$)

$$\hat{G}_{\mu\nu}(p,q)=\gamma_0\hat{G}^\dagger_{\nu\mu}(p,q)\gamma_0. \tag{3c}$$

According to the constraints in Eqs. (3) the most general form for the truncated tensor must be [5,6]

$$\begin{aligned}\hat{G}_{\mu\nu}(p,q)=i\epsilon_{\mu\nu\alpha\beta}q^\alpha(&p^\beta \not{p}\gamma_5G_{(p)}+p^\beta\not{q}\gamma_5G_{(q)}+\gamma^\beta\gamma_5G_{(\gamma)}\\&+i\sigma^{\beta\lambda}p_\lambda\gamma_5G_{(\sigma p)}+i\sigma^{\beta\lambda}q_\lambda\gamma_5G_{(\sigma q)}\\&+ip^\beta\sigma^{\lambda\rho}p_\lambda q_\rho\gamma_5G_{(\sigma pq)}),\end{aligned} \tag{4}$$

where the coefficients $G_{(i)}$ are scalar functions of p and q. In Eq. (4) we have listed only those structures which lead to a gauge-invariant truncated tensor, i.e., $q^\mu\hat{G}_{\mu\nu}=q^\nu\hat{G}_{\mu\nu}=0$. In Sec. II B we will show that, at least in the Bjorken limit, terms such as $\epsilon_{\mu\nu\alpha\beta}p^\alpha\gamma^\beta$ that do not satisfy this condition, are absent.

The structure function g_1^N of an on-shell nucleon is obtained by multiplying $\hat{G}_{\mu\nu}$ with the nucleon Dirac spinors [see Eq. (2)] and with the projection operator:

$$P_{\mu\nu}=i\epsilon_{\mu\nu\alpha\beta}q^\alpha\frac{p\cdot q}{s\cdot q}\frac{s\cdot qM^2s^\beta-p\cdot qp^\beta}{2M\{M^2[q^2+(s\cdot q)^2]-(p\cdot q)^2\}}. \tag{5}$$

As a result we get $g_1^N(x)$ as the on-shell limit ($p^2\to M^2, y\to 1$) of the off-shell structure function $\tilde{g}_1^N(x/y,p^2)$, defined as

$$\begin{aligned}\tilde{g}_1^N\left(\frac{x}{y},p^2\right)=2p\cdot q(&p\cdot qG_{(q)}+G_{(\gamma)}+MG_{(\sigma p)}\\&-Mp\cdot qG_{(\sigma pq)}),\end{aligned} \tag{6}$$

where $x/y=Q^2/2p\cdot q$, and the Q^2 dependence in g_1^N has been suppressed. The definition of the generalized structure function, $\tilde{g}_1^N$ in Eq. (6), will turn out to be useful when we discuss the nuclear spin-structure function, g_1^A, in Sec. III.

B. Truncated functions in the Bjorken limit

In this section we discuss the scaling properties of the coefficient functions $G_{(i)}$, and the question of the gauge-transformation properties of the truncated tensor. We work throughout in the Bjorken limit ($Q^2,p\cdot q\to\infty$, $Q^2/p\cdot q$ fixed), and consider only the leading twist contributions to the hadronic tensor. In this limit, the tensor $\hat{G}_{\mu\nu}$ can be written as

$$\hat{G}_{\mu\nu}(p,q)=\int d\tilde{k}\,\mathrm{Tr}[\mathcal{H}(p,k)r_{\mu\nu}(k,q)], \tag{7}$$

where the trace is over quark indices. In Eq. (7) the antisymmetric tensor $r_{\mu\nu}$ describes the hard, photon-quark interaction, and is given by

$$r_{\mu\nu}(k,q)=(\not{k}+m)\gamma_\mu(\not{k}+\not{q}+m)\gamma_\nu(\not{k}+m)-(\mu\leftrightarrow\nu), \tag{8a}$$

$$\equiv A_{\mu\nu\alpha}(k,q)\gamma_5\gamma^\alpha+A_{\mu\nu\alpha\beta}(k,q)\sigma^{\alpha\beta}, \tag{8b}$$

where

$$A_{\mu\nu\alpha}(k,q)=i(q^2\epsilon_{\mu\nu k\alpha}+(k^2-m^2)\epsilon_{\mu\nu q\alpha} -2(k_\mu\epsilon_{kq\alpha\nu}-k_\nu\epsilon_{kq\alpha\mu})), \tag{8c}$$

$$A_{\mu\nu\alpha\beta}(k,q)=-im(q^2 g_{\mu\alpha}g_{\nu\beta}+2q_\alpha(k_\mu g_{\nu\beta}-k_\nu g_{\mu\beta})). \tag{8d}$$

Here k is the interacting quark four-momentum, and m is its mass. We use the notation $\epsilon_{\mu\nu kq}\equiv\epsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta$. (The complete forward scattering amplitude would also contain a crossed photon process which we do not consider here, since in the subsequent model calculations we focus on valence quark distributions.) The function $\mathcal{H}(k,p)$ represents the soft quark-nucleon interaction. Since one is calculating the imaginary part of the forward scattering amplitude, the integration over the quark momentum k is constrained by δ functions which put both the scattered quark and the noninteracting spectator system on-mass-shell:

$$d\tilde{k}\equiv\frac{d^4k}{(2\pi)^4}\frac{2\pi\delta[(k+q)^2-m^2]2\pi\delta[(p-k)^2-m_S^2]}{(k^2-m^2)^2}, \tag{9}$$

where $m_S^2=(p-k)^2$ is the invariant mass squared of the spectator system.

Taking the trace over the quark spin indices we find

$$\mathrm{Tr}[\mathcal{H}r_{\mu\nu}]=A_{\mu\nu\alpha}H^\alpha+A_{\mu\nu\alpha\beta}H^{\alpha\beta}, \tag{10}$$

where H_α and $H_{\alpha\beta}$ are vector and tensor coefficients, respectively. The general structure of H_α and $H_{\alpha\beta}$ can be deduced from the transformation properties of the truncated nucleon tensor $\hat{G}_{\mu\nu}$ and the tensors $A_{\mu\nu\alpha}$ and $A_{\mu\nu\alpha\beta}$. Namely, from $A^*_{\mu\nu\alpha}(k,q)=A_{\nu\mu\alpha}(k,q)$ and $A^{\mu\nu\alpha}(\tilde{k},\tilde{q})=-A_{\mu\nu\alpha}(k,q)$, we have

$$H^\alpha(p,k)=-\mathcal{P}H_\alpha(\tilde{p},\tilde{k})\mathcal{P}^\dagger, \tag{11a}$$

$$H^\alpha(p,k)=(\mathcal{T}H_\alpha(\tilde{p},\tilde{k})\mathcal{T}^\dagger)^*, \tag{11b}$$

$$H^\alpha(p,k)=\gamma_0 H^{\alpha\dagger}(p,k)\gamma_0. \tag{11c}$$

Similarly, since $A^*_{\mu\nu\alpha\beta}(k,q)=A_{\nu\mu\alpha\beta}(k,q)$ and $A^{\mu\nu\alpha\beta}(\tilde{k},\tilde{q})=A_{\mu\nu\alpha\beta}(k,q)$, one finds

$$H^{\alpha\beta}(p,k)=\mathcal{P}H_{\alpha\beta}(\tilde{p},\tilde{k})\mathcal{P}^\dagger, \tag{12a}$$

$$H^{\alpha\beta}(p,k)=-(\mathcal{T}H_{\alpha\beta}(\tilde{p},\tilde{k})\mathcal{T}^\dagger)^*, \tag{12b}$$

$$H^{\alpha\beta}(p,k)=\gamma_0 H^{\alpha\beta\dagger}(p,k)\gamma_0. \tag{12c}$$

With these constraints, the tensors H_α and $H_{\alpha\beta}$ can be projected onto Dirac and Lorentz bases as follows:

$$H_\alpha=p_\alpha\gamma_5(\not{p}g_1+\not{k}g_2)+k_\alpha\gamma_5(\not{p}g_3+\not{k}g_4) +i\gamma_5\sigma_{\lambda\rho}p^\lambda k^\rho(p_\alpha g_5+k_\alpha g_6)+\gamma_\alpha\gamma_5 g_7 +i\gamma_5\sigma_{\lambda\alpha}(p^\lambda g_8+k^\lambda g_9), \tag{13a}$$

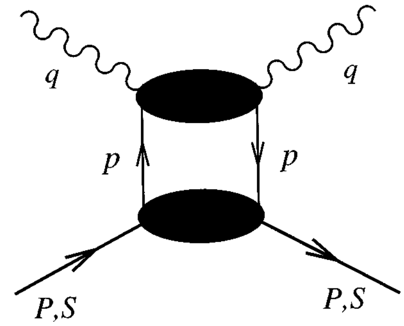

FIG. 1. DIS from a polarized nucleus in the impulse approximation. The nucleus, virtual nucleon, and photon momenta are denoted by P, p, and q, respectively, and S stands for the nuclear spin vector. The upper blob represents the truncated antisymmetric nucleon tensor $\hat{G}_{\mu\nu}$, while the lower one corresponds to the polarized nucleon-nucleus amplitude $\hat{A}$.

$$H_{\alpha\beta}=(p_\alpha k_\beta-p_\beta k_\alpha)\sigma_{\lambda\rho}p^\lambda k^\rho f_1+(p_\alpha\sigma_{\lambda\beta}-p_\beta\sigma_{\lambda\alpha}) \times(p^\lambda f_2+k^\lambda f_3)+(k_\alpha\sigma_{\lambda\beta}-k_\beta\sigma_{\lambda\alpha})(p^\lambda f_4+k^\lambda f_5) +\sigma_{\alpha\beta}f_6+\epsilon_{\lambda\rho\alpha\beta}p^\lambda k^\rho\gamma_5(\not{p}f_7+\not{k}f_8) +\epsilon_{\lambda\rho\alpha\beta}\gamma_5\gamma^\rho(p^\lambda f_9+k^\lambda f_{10}), \tag{13b}$$

where the functions $g_{1\ldots9}$ and $f_{1\ldots10}$ are scalar functions of p and k.

Performing the integration over k in Eq. (7) and using Eqs. (13), we obtain expressions for the truncated structure functions $G_{(i)}$ in terms of the nonperturbative coefficient functions f_i and g_i. The explicit forms of these are given in Appendix I. From Eq. (4) we then obtain the leading twist contributions to the truncated nucleon tensor $\hat{G}_{\mu\nu}$. It is important to note that at leading twist the non-gauge-invariant contributions to $\hat{G}_{\mu\nu}$ vanish, so that the expansion in Eq. (4) is the most general one which is consistent with the gauge invariance of the hadronic tensor.

III. NUCLEAR STRUCTURE FUNCTIONS

Our discussion of polarized deep-inelastic scattering from nuclei is restricted to the nuclear impulse approximation, illustrated in Fig. 1. Nuclear effects which go beyond the impulse approximation include final state interactions between the nuclear debris of the struck nucleon [17], corrections due to meson exchange currents [18–20] and nuclear shadowing (see [21–24] and references therein). Since we are interested in the medium- and large-x regions, coherent multiple scattering effects, which lead to nuclear shadowing for $x\lesssim0.1$, will not be relevant. In addition, it has been argued [6] that meson exchange currents are less important in polarized deep-inelastic scattering than in the unpolarized case since their main contribution comes from pions.

Within the impulse approximation, deep-inelastic scattering from a polarized nucleus with spin 1/2 or 1 is then described as a two-step process, in terms of the virtual photon-nucleon interaction, parametrized by the truncated antisymmetric nucleon tensor $\hat{G}_{\mu\nu}(p,q)$, and the polarized nucleon-nucleus scattering amplitude $\hat{A}(p,P,S)$. The antisymmetric part of the nuclear hadronic tensor can then be written as

$$M_A W^A_{\mu\nu}(P,S,q) \equiv i\frac{M_A}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\left[S^\beta g_1^A(P,q) + \left(S^\beta - \frac{S\cdot q}{P\cdot q}P^\beta\right)g_2^A(P,q)\right] \quad (14)$$

$$= \int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\hat{A}(p,P,S)\hat{G}_{\mu\nu}(p,q)]. \quad (15)$$

Here P, p, and q are the nucleus, off-shell nucleon, and photon momenta, respectively. For a spin 1/2 nucleus, such as ^{3}He, the vector S^α $(S^2=-1, P\cdot S=0)$ is the nuclear spin vector, while for spin 1 targets, such as deuterium, S^α is defined in terms of polarization vectors ε^m_α [25]:

$$S^\alpha(m) \equiv -i\epsilon^{\alpha\beta\lambda\rho}\varepsilon^{m*}_\beta\varepsilon^m_\lambda P_\rho/M_A, \quad (16)$$

where $m=0,\pm 1$ is the spin projection. Using the fact that the nuclear recoil states are on shell, one finds for the nucleon-nucleus amplitude

$$\hat{A}(p,P,S) = \sum_R \langle P-p,R|N(0)|P,S\rangle\langle P,S|\overline{N(0)}|P-p,R\rangle \times 2\pi\delta((P-p)^2-M_R^2). \quad (17)$$

The sum is taken over all possible nuclear recoil states R with momentum $P-p$ and invariant mass M_R, and $N(0)$ stands for the nucleon field operator at the origin. The amplitude $\hat{A}(p,P,S)$ can furthermore be expanded in the nucleon Dirac space as

$$\hat{A}(p,P,S) = \sum_R 2\pi\delta((P-p)^2-M_R^2)(\gamma_5\gamma_\alpha\mathcal{A}^\alpha_R + \sigma_{\alpha\beta}\mathcal{B}^{\alpha\beta}_R), \quad (18)$$

where the pseudovector and tensor structure $\mathcal{A}^\alpha_R$ and $\mathcal{B}^{\alpha\beta}_R$ are functions of p, P, and S. Other structures do not contribute to $W^A_{\mu\nu}$ and are omitted. For example, a pseudoscalar term $(\sim\gamma_5)$ is forbidden by hermiticity and time reversal invariance of $W^A_{\mu\nu}$. From Eqs. (4), (15), and (18) we obtain, after the appropriate projection,

$$g_1^A(x) = \frac{P\cdot q}{4\pi^2 M_A S\cdot q}\sum_R \int dy\,dp^2[q\cdot\mathcal{A}_R(p\cdot qG_{(q)} + G_{(\gamma)}) + p\cdot\mathcal{A}_R p\cdot qG_{(p)} + \mathcal{T}\cdot\mathcal{B}_R M(G_{(\sigma p)} - p\cdot qG_{(\sigma pq)})], \quad (19)$$

with the tensor

$$\mathcal{T}_{\alpha\beta} = \frac{1}{(S\cdot q)^2M_A^2 - (P\cdot q)^2}(p\cdot qP\cdot q\epsilon_{\alpha\beta Pq} + M_A^2 p\cdot qS\cdot q\epsilon_{\alpha\beta qS} + P\cdot qq_\alpha\epsilon_{\beta pPq} - P\cdot qq_\beta\epsilon_{\alpha pPq} + M_A^2 S\cdot qq_\alpha\epsilon_{\beta pqS} - M_A^2 S\cdot qq_\beta\epsilon_{\alpha pqS}). \quad (20)$$

Here $x=Q^2/2P\cdot q$ is the Bjorken scaling variable and $y=p\cdot q/P\cdot q$ is the light-cone fraction of the deuteron momentum carried by the struck nucleon. Equation (19) shows that factorization of the nuclear structure function g_1^A into nuclear $(\mathcal{A}^\alpha_R, \mathcal{B}^{\alpha\beta}_R)$ and nucleon $(G_{(i)})$ parts, as would be required for convolution, is not possible. In addition, the presence of the structure $G_{(p)}$ in g_1^A, but not in g_1^N [see Eq. (6)], leads to explicit convolution breaking. Convolution can be recovered, however, in the nonrelativistic limit [6], as will be shown explicitly in the next section for the case of the deuteron.

IV. DEUTERON

The general results of the previous section can now be applied to the case of a deuteron target. Consistent with the standard treatments of the relativistic deuteron bound state problem, the deuteron recoil state is taken to be a single nucleon. From Eq. (17) one therefore obtains the nucleon-deuteron scattering amplitude:

$$\hat{A}(p,P,S) = \sum_s \langle P-p,s|N(0)|P,S\rangle\langle P,S|\overline{N(0)}|P-p,s\rangle \times 2\pi\delta((P-p)^2-M^2), \quad (21)$$

where the sum is taken over the spin s of the recoil nucleon. The deuteron-nucleon matrix elements in Eq. (21) are related to the deuteron-nucleon vertex function Γ_D via [26] (see also Ref. [27]):

$$\langle P-p,s|N(0)|P,S\rangle = \frac{1}{\not{p}-M}\Gamma^\alpha_D(P,p)\varepsilon^m_\alpha\mathcal{C}\overline{u}^T(P-p,s). \quad (22)$$

The deuteron polarization vector, ε^m_α, is related to the spin vector S by Eq. (16). $\mathcal{C}$ is the charge conjugation operator, and $u(P-p,s)$ is the Dirac spinor of the recoil nucleon with momentum $P-p$ and spin s. In terms of the deuteron-nucleon vertex function, Eqs. (21) and (22) give

$$\hat{A}(p,P,S) = \varepsilon_\alpha^m \epsilon_\beta^{m*} (\not{p} - M)^{-1} \Gamma_D^\alpha(P,p)(\not{P} - \not{p} - M) \times \overline{\Gamma}_D^\beta(P,p)(\not{p} - M)^{-1} 2\pi\delta((P-p)^2 - M^2), \tag{23}$$

with $\overline{\Gamma}_D = \gamma_0 \Gamma_D^\dagger \gamma_0$.

To be specific, we will use in our numerical calculations the relativistic vertex function from Ref. [26]. In this work Γ_D is represented through a relativistic deuteron wave function which contains S- and D-state components u and w, and also triplet and singlet P-state contributions v_t and v_s. After choosing the polarization of the deuteron one can express the pseudovector and tensor components, $\mathcal{A}^\alpha$ and $\mathcal{B}^{\alpha\beta}$, of the deuteron-nucleon amplitude (18) in terms of the deuteron-nucleon vertex function — or equivalently through the deuteron wave function components [see Eq. (24) below]. Then Eq. (19) yields the deuteron spin structure function g_1^D.

We will follow this procedure in the deuteron rest frame where the photon momentum is chosen along the $-\hat{z}$ direction, $q = (q_0; \mathbf{0}_T, -|\mathbf{q}|)$. Furthermore, we are free to fix the deuteron spin projection to be $m = +1$. As in the general case in Eq. (19), one does not obtain a result compatible with exact convolution. However, by writing $q\cdot\mathcal{A}$, $p\cdot\mathcal{A}$, and $\mathcal{T}\cdot\mathcal{B}$ in terms of the relativistic deuteron wave function, we find that all nonfactorizable corrections to convolution are at least of order $(v/c)^2$, or involve relativistic P-state wave functions. This is easily seen by separating $q\cdot\mathcal{A}$ and $\mathcal{T}\cdot\mathcal{B}$ into an ''on-shell'' part, which is proportional to the nonrelativistic deuteron wave function [see Eq. (29) below], and an off-shell component: $q\cdot\mathcal{A} \equiv q\cdot\mathcal{A}_{\rm on} + q\cdot\mathcal{A}_{\rm off}$, and $\mathcal{T}\cdot\mathcal{B} \equiv \mathcal{T}\cdot\mathcal{B}_{\rm on} + \mathcal{T}\cdot\mathcal{B}_{\rm off}$, where

$$q\cdot\mathcal{A}_{\rm on} = \mathcal{T}\cdot\mathcal{B}_{\rm on} = 2\pi^2 P\cdot qM\left[u^2 + (1 - 3\cos^2\theta)\frac{uw}{\sqrt{2}} - \left(1 - \frac{3}{2}\cos^2\theta\right)w^2 + \frac{p_z}{M}\left(u - \frac{w}{\sqrt{2}}\right)^2\right], \tag{24}$$

with $p_z = |\mathbf{p}|\cos\theta$, $E_p = \sqrt{M^2 + \mathbf{p}^2}$ and $\cos\theta = (yM_D - p_0)/|\mathbf{p}|$. The ''off-shell'' components $q\cdot\mathcal{A}_{\rm off}$ and $\mathcal{T}\cdot\mathcal{B}_{\rm off}$, and also $p\cdot\mathcal{A}$ are given in Appendix B. They are either of higher order in (v/c) compared with the leading ''on-shell'' contribution in Eq. (24), or they involve relativistic P states.

Using Eqs. (6), (19), and (24) we can decompose g_1^D into a convolution component plus an off-shell correction:

$$g_1^D(x) = \int_x^1 \frac{dy}{y} \Delta f(y) g_1^N\left(\frac{x}{y}\right) + \delta^{\rm (off)} g_1^D(x). \tag{25}$$

Here we can identify

$$\Delta f(y) = \int d^4p\, \Delta S(p)\, \delta\left(y - \frac{p^+}{M_D}\right) \tag{26}$$

with the difference of probabilities to find a nucleon in the deuteron with light-cone momentum fraction y and spin parallel and antiparallel to that of the deuteron. For a deuteron with polarization $m = +1$, $\Delta S(p)$ corresponds to the spectral function:

$$\Delta S(p) = \Psi_{+1}^\dagger(p)(\hat{\mathcal{S}}_0 + \hat{\mathcal{S}}_z)\Psi_{+1}(p)\, \delta(p_0 - M_D + E_p), \tag{27}$$

where $\Psi_m(p)$ is the usual (normalized) nonrelativistic deuteron wave function (see, e.g., Ref. [28]). $\hat{\mathcal{S}}_0$ and $\hat{\mathcal{S}}_z$ are the zero and z components of the nucleon spin operator, defined as [6]

$$\hat{\mathcal{S}}_0 = \frac{\mathbf{S}\cdot\mathbf{p}}{M} = \frac{1}{2M}(\boldsymbol{\sigma}^p + \boldsymbol{\sigma}^n)\cdot\mathbf{p}, \tag{28a}$$

$$\hat{\mathcal{S}}_z = \frac{1}{2}(\sigma_z^p + \sigma_z^n), \tag{28b}$$

with $\boldsymbol{\sigma}^{p,n}$ the SU(2) Pauli spin matrices acting on the proton and neutron spin wave function, respectively. In terms of the deuteron wave function $\Psi_m(p)$, one has

$$q\cdot\mathcal{A}_{\rm on} = \mathcal{T}\cdot\mathcal{B}_{\rm on} = 8\pi^3 P\cdot qM\mathcal{N}\Psi_{+1}^\dagger(p)(\hat{\mathcal{S}}_0 + \hat{\mathcal{S}}_z)\Psi_{+1}(p). \tag{29}$$

The factor $\mathcal{N} = \int d|\mathbf{p}|\mathbf{p}^2(u^2 + w^2)$ ensures that the normalization agrees with that of the relativistic calculation. The function $\Delta f(y)$ then satisfies $\int_0^1 dy \Delta f(y) = 1 - 3/2\omega_D$, where $\omega_D = \int d|\mathbf{p}|\mathbf{p}^2 w^2/\mathcal{N}$ is the nonrelativistic D-state probability. In the NN potential model of Ref. [26] with a pseudovector πNN interaction, the D-state probability is $\omega_D = 4.7\%$. The convolution term in Eq. (25) completely agrees with the nonrelativistic limit up to order (v/c) if contributions from relativistic P states are neglected, and the off-shell structure function $\tilde{g}_1^N(x/y, p^2)$ from Eq. (6) is replaced by the on-shell one. The relativistic, convolution breaking, ''off-shell'' contribution $\delta^{\rm (off)} g_1^D$ in Eq. (25) is given explicitly in Appendix B.

We should also make a note about comparing calculations which use relativistic and nonrelativistic wave functions. While convolution itself is valid to order $(v/c)^2$ [6], the renormalization of the relativistic wave function itself introduces corrections of order $(v/c)^2$, since the P-state wave functions $v_{s,t}$ are of order v/c compared with the S- and D-state functions. Therefore the correct nonrelativistic limit can be obtained directly from the relativistic calculation only to order v/c [29].

V. NUMERICAL RESULTS

Using the results of the preceeding sections, we present here the numerical results for the polarized nucleon and deuteron structure functions. In our relativistic treatment, the modeling of the virtual photon-off-shell nucleon interaction is most naturally done in terms of relativistic quark-nucleon vertex functions.

A. Relativistic quark-nucleon vertex functions

While the scaling behavior of $G_{(i)}$ can be derived from the parton model, their complete evaluation requires model-dependent input for the nonperturbative parton-nucleon physics, which in our case is parametrized by the functions $f_{1\ldots10}$ and $g_{1\ldots9}$. Because the nucleon recoil state that remains a spectator to the hard collision is on-mass shell, the functions $f_{1\ldots10}, g_{1\ldots9}$ can be simply written in terms of relativistic quark-nucleon vertex functions, $\mathcal{V}$ [2,4,5,30–32]. Expressed through $\mathcal{V}$ and the propagator of the spectator (''diquark'') system, $S_D(p-k)$, one obtains for the matrix $\mathcal{H}(k,p)$ from Eq. (7):

$$\mathcal{H}(k,p)=\mathrm{Im}[\mathcal{V}(k,p)S_D(p-k)\overline{\mathcal{V}}(k,p)], \tag{30}$$

where $\overline{\mathcal{V}}=\gamma_0\mathcal{V}^\dagger\gamma_0$. Because both the quark and nucleon have spin 1/2, the spectator system can be in either a spin 0 or spin 1 state. Therefore the only vertices that need to be considered (for quarks in the ground state) are those which transform as scalars or pseudovectors under Lorentz transformations. We approximate the ''diquark'' propagators by the form $S_D(p-k)=[(p-k)^2-m_0^2]^{-1}$ for $S=0$, and $S_D^{\alpha\beta}(p-k)=[-g^{\alpha\beta}+(p-k)^\alpha(p-k)^\beta/m_1^2]/[(p-k)^2-m_1^2]$ for $S=1$, where m_0 and m_1 are the masses associated with the scalar and pseudovector spectator states.

Following earlier work [5], we will use the ansatz

$$\mathcal{V}_0(k,p)=I\phi_0^{(a)}(k,p)+\beta\not{k}\phi_0^{(b)}(k,p), \tag{31a}$$

$$\mathcal{V}_1^\alpha(k,p)=\gamma^\alpha\gamma_5\phi_1(k,p), \tag{31b}$$

for the $S=0$ and $S=1$ vertices, respectively. The parameter β determines the relative contributions from the two scalar vertices. Although a more general approach is possible, in which one could include all possible Lorentz and Dirac structures, in practice since the vertices will be constrained phenomenologically, the above set will suffice. Without loss of generality, to simplify the numerical analysis we will also work in the massless quark limit, $m\to 0$.

Inserting the above vertex functions into Eq. (7) and comparing with Eq. (4), we can determine the functions $f_{1\ldots 10}$, $g_{1\ldots 9}$ appearing in Eqs. (13). For the scalar vertices we find

$$g_4=-2\beta^2(\phi_0^{(b)})^2,$$

$$g_7=-(\phi_0^{(a)})^2-k^2\beta^2(\phi_0^{(b)})^2, \tag{32a}$$

$$g_9=2\beta\phi_0^{(a)}\phi_0^{(b)},$$

while for the pseudovector vertex:

$$g_1=-\frac{2}{m_1^2}(\phi_1)^2=-g_2=-g_3=g_4=-\frac{2}{m_1^2}g_7, \tag{32b}$$

with all other functions being zero.

The momentum dependence in the vertices is parametrized by the multipole form:

$$\Phi_S(p,k)=N_S(p^2)k^2/(k^2-\Lambda_S^2)^{n_S}$$

($p^2=M^2$ for the free nucleon). The cutoff parameters Λ_S and exponents n_S are fixed by fitting the unpolarized up (u_V) and down (d_V) valence quark distributions, as discussed in Refs. [2,5]. The normalization constants N_S are determined through baryon number conservation. A best fit to the experimental u_V+d_V and d_V/u_V data [33] at $Q^2=10$ GeV^2 [when evolved from the renormalization scale $\mu^2\simeq$ (0.32 GeV)^2 using leading order evolution[1]], is shown in Fig. 2. The cutoffs used for the scalar vertices are $\Lambda_0^{(a,b)}=(1.0,1.1)$ GeV, and the exponents $n_0^{(a,b)}=(2.0,2.8)$, with the mixing parameter $\beta=2.73$. The parameters for the pseudovector vertex are $\Lambda_1=1.8$ GeV and $n_1=3.2$. The mass parameters associated with the intermediate spectator states are taken to be $m_{0(1)}=(p-k)^2=0.9(1.6)$ GeV.

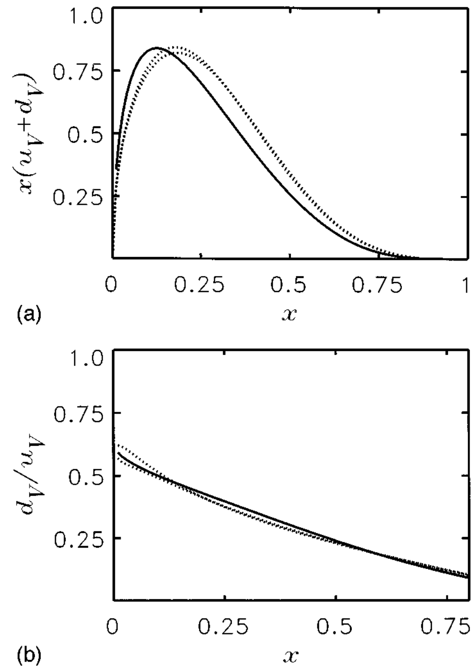

FIG. 2. Unpolarized valence quark distributions $x(u_V+d_V)$ and d_V/u_V. The solid line represents distributions evolved from scale μ^2 to $Q^2=10$ GeV^2. Dotted curves are parametrizations from Ref. [33].

With the same parameters, the polarized valence distributions are then calculated according to the relations[2]

$$\Delta u_V(x)=\tfrac{3}{2}\,\Delta q_0(x)-\tfrac{1}{6}\,\Delta q_1(x), \tag{33}$$

$$\Delta d_V(x)=-\tfrac{1}{3}\,\Delta q_1(x), \tag{34}$$

where Δq_0 and Δq_1 are the polarized quark distributions for scalar and pseudovector spectator states, respectively. The first moments of the polarized valence distributions in the proton then turn out to be $\int_0^1 dx\Delta u_V(x)=0.99$ and $\int_0^1 dx\Delta d_V(x)=-0.27$, which saturates the Bjorken sum rule: $\int_0^1 dx[\Delta u_V(x)-\Delta d_V(x)]=g_A$. The total momentum carried by valence quarks at the scale μ^2 is around 85%, leaving about 15% to be carried by the sea.

[1]While a next-to-leading order analysis is important for a precise determination of the Q^2 dependence of g_1 and the Bjorken sum rule [34], the present treatment is perfectly adequate for the purpose of evaluating the relative sizes of the nuclear corrections.

[2]Note that the formal results do not rely on the use of SU(4) spin-flavor symmetry — these relations merely provide a convenient way to parametrize the polarized quark distributions [30].

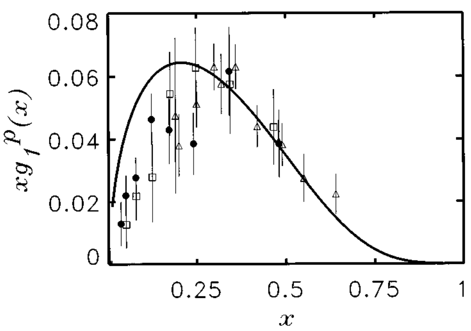

FIG. 3. Valence component of the proton g_1 structure function at $Q^2=10$ GeV^2. The data for the full structure function g_1^p are from Refs. [35–38].

The x dependence of the (valence part of the) polarized proton structure function $xg_1^p(x)=x[4\Delta u_V(x)+\Delta d_V(x)]/18$ is shown in Fig. 3. In the valence quark dominated region ($x>0.3$) the result agrees rather well with the SLAC, EMC, and SMC proton data [35–38]. A negatively polarized sea component at $x<0.3$ would bring the curve even closer to the data points.

Having fixed the nucleon inputs, we next estimate the size of the relativistic corrections to the deuteron structure function.

B. Polarized deuteron structure function

The total valence part of the structure function of the deuteron, calculated from Eq. (25), is shown in Fig. 4. The agreement with the SMC [39] and SLAC E143 [40] data in the valence region is also quite good. Note that care must be taken regarding the normalization of the quark-nucleon vertex functions when the nucleon is no longer on shell. In this simple model, the modifications of the vertex functions are made via the explicit p^2 dependence of the normalization constants $N_S(p^2)$. In practice, because the structure function is not very sensitive to the p^2 dependence in the quark-nucleon vertex functions, $N_S(p^2)$ can be taken to be constant. The numerical values of these normalization constants are fixed through valence quark number conservation in the deuteron's spin-averaged distributions. They turn out to be 0.8% and 1.7% smaller for the scalar and pseudovector vertices, respectively, than for the free nucleon. The insensitivity of the deuteron structure function to the p^2 dependence of the quark-nucleon vertex should limit the uncertainty introduced through the specific choice of vertex functions in Eqs. (31). Any model dependence associated with alternative vertex structures would be compensated to some extent by the necessary adjustments to their momentum dependence — the k^2 dependence is constrained by refitting the nucleon data, and the p^2 dependence by readjusting the normalization constants N_S. Nevertheless, it would be interesting to explore the residual model dependence numerically.

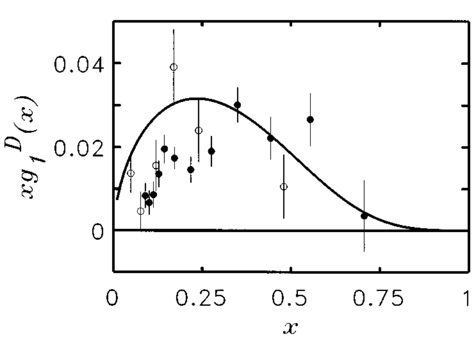

FIG. 4. Valence part of the deuteron structure function g_1^D, compared with the data for the full g_1^D structure function from the SMC and SLAC E143 [39,40].

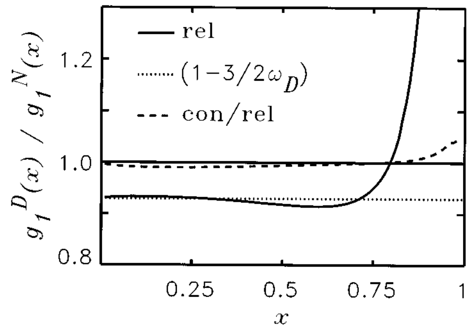

FIG. 5. Ratio of deuteron and nucleon structure functions in the full model (solid), and with a constant depolarization factor $1-3/2\omega_D$ (dotted), with $\omega_D=4.7\%$ [26]. The dashed curve is the ratio of g_1^D calculated via convolution to g_1^D calculated in the relativistic model.

The resulting ratio, g_1^D/g_1^N, is displayed in Fig. 5. For large x it exhibits the same characteristic shape as for the (unpolarized) nuclear EMC effect, namely a dip of $\sim$ 7–8 % at $x\approx 0.6$ and a steep rise due to Fermi motion for $x>0.6$. For small x it stays below unity, where it can be reasonably well approximated by a constant depolarization factor, $1-3/2\omega_D$, as is typically done in data analyses [39,40]. Also shown in Fig. 5 is the ratio of the convolution ansatz [Eqs. (25)–(27)] to the full calculation (dashed curve). As one can see, this ansatz works remarkably well for most x, the only sizable deviations occurring for $x>0.8$, which is outside the range covered by previous experiments. Nevertheless, future experiments, both inclusive and semi-inclusive, will be able to access the very large-x region, in which case the issue of nuclear — and in particular relativistic — effects will need to be seriously addressed.

VI. CONCLUSION

We have discussed polarized nuclear deep-inelastic scattering within a covariant framework. In this context we analyzed the structure of the forward scattering amplitude of a virtual photon from a bound, off-mass-shell nucleon, focusing especially on its symmetry properties. Within the impulse approximation, we derived the most general, relativistic expression for the nuclear structure function g_1^A. Our results clearly demonstrate that, in general, nuclear and nucleon pieces do not factorize in a relativistic treatment of nuclear structure functions. The conventional convolution model can only be recovered by taking the nonrelativistic limit for the nucleon-nucleus amplitude and assuming the on-shell limit for the off-shell nucleon structure function.

We showed numerical results for the deuteron structure function g_1^D, where the off-shell nucleon tensor was calculated within a relativistic spectator model. At moderate x (<0.7) we found that nuclear effects in the deuteron are dominated by the D-state probability of the deuteron. Binding and Fermi motion become significant only at large x. Also off-shell corrections turned out to be small at moderate x, but increase considerably in the region $x>0.8$.

With respect to the present experimental situation the main uncertainty in the extraction of the neutron structure function g_1^n still comes from the deuteron D-state probability. For practical purposes, the off-shell modification of the bound nucleon structure function has to be taken into account when high precision data at large x become available.

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APPENDIX A: TRUNCATED STRUCTURE FUNCTIONS

Here we present the complete expressions for the truncated structure functions $G_{(i)}$ from Eq. (4) in terms of the functions $f_{1\dots 10}$ and $g_{1\dots 9}$ in Eqs. (13):

$$G_{(p)}\left(\frac{x}{y},p^2\right)=\int d\widetilde{k}\left\{\left(m^2+k^2-2p\cdot k\frac{x}{y}\right)\left(g_1+\frac{x}{y}g_2\right)-\frac{x}{y}(k^2-m^2)\left(g_3+\frac{x}{y}g_4\right)+2\left(\frac{x}{y}\right)^2 g_7 +2m\left(k^2-p\cdot k\frac{x}{y}\right)\left(f_7+\frac{x}{y}f_8\right)+2m\frac{x}{y}\left(f_9+\frac{x}{y}f_{10}\right)\right\}, \tag{A1}$$

$$G_{(q)}\left(\frac{x}{y},p^2\right)=\int \frac{d\widetilde{k}}{p\cdot q}\left\{\left((m^2+2k^2)p\cdot k-((m^2+k^2)p^2+4(p\cdot k)^2)\frac{x}{y}+3p^2p\cdot k\left(\frac{x}{y}\right)^2\right)g_2 +\left(k^2-4p\cdot k\frac{x}{y}+3p^2\left(\frac{x}{y}\right)^2\right)\left(\frac{1}{2}(k^2-m^2)g_4-mf_{10}\right)+m\left(3k^2p\cdot k-2(p^2k^2+2(p\cdot k)^2)\frac{x}{y}+3p^2p\cdot k\left(\frac{x}{y}\right)^2\right)f_8 +2m\left(p\cdot k-p^2\frac{x}{y}\right)f_9-\left(k^2-4p\cdot k\frac{x}{y}+3p^2\left(\frac{x}{y}\right)^2\right)g_7\right\} \tag{A2}$$

$$G_{(\gamma)}\left(\frac{x}{y},p^2\right)=\int d\widetilde{k}\left\{\left(k^2-2p\cdot k\frac{x}{y}+p^2\left(\frac{x}{y}\right)^2\right)\left(-p\cdot kg_2-\frac{1}{2}(k^2-m^2)g_4+g_7\right)-(m^2+k^2)g_7 +m\left(k^2-2p\cdot k\frac{x}{y}+p^2\left(\frac{x}{y}\right)^2\right)(-p\cdot kf_8+f_{10})-2m(p\cdot kf_9+k^2f_{10})\right\}, \tag{A3}$$

$$G_{(\sigma p)}\left(\frac{x}{y},p^2\right)=\int d\widetilde{k}\left\{\left(k^2-2p\cdot k\frac{x}{y}+p^2\left(\frac{x}{y}\right)^2\right)\left(-p\cdot kg_5-\frac{1}{2}(k^2-m^2)g_6-g_8\right)+(k^2+m^2)\left(g_8+\frac{x}{y}g_9\right)-2m\left(p\cdot k-p^2\frac{x}{y}\right)f_2 -m\left(k^2-p^2\left(\frac{x}{y}\right)^2\right)f_3+2m\frac{x}{y}f_6\right\}, \tag{A4}$$

$$G_{(\sigma q)}\left(\frac{x}{y},p^2\right)=\int \frac{d\widetilde{k}}{p\cdot q}\left\{\left(p\cdot k-p^2\frac{x}{y}\right)[(k^2+m^2)g_9+2m(p^2f_2+p\cdot kf_3+f_6)]\right\} \tag{A5}$$

$$G_{(\sigma pq)}\left(\frac{x}{y},p^2\right)=\int \frac{d\widetilde{k}}{p\cdot q}\left\{-\left(p\cdot k(m^2+2k^2)-((k^2+m^2)p^2+4(p\cdot k)^2)\frac{x}{y}+3p^2p\cdot k\left(\frac{x}{y}\right)^2\right)g_5 +\left(k^2-4p\cdot k\frac{x}{y}+3p^2\left(\frac{x}{y}\right)^2\right)\left(-\frac{1}{2}(k^2-m^2)g_6-g_8+mf_3\right)-2m\left(p\cdot k-p^2\frac{x}{y}\right)f_2\right\}, \tag{A6}$$

where $p\cdot k=(p^2+k^2-m_S^2)/2$ and $x/y=k\cdot q/p\cdot q=Q^2/2p\cdot q$.

APPENDIX B: RELATIVISTIC CORRECTIONS TO NUCLEAR STRUCTURE FUNCTIONS

As outlined in Sec. IV the pseudovector and tensor components of the nucleon-deuteron scattering amplitude, $\mathcal{A}^\alpha$ and $\mathcal{B}^{\alpha\beta}$ in Eq. (18), can be separated into an on-shell part, which is proportional to the nonrelativistic deuteron wave function, and an off-shell component: $q\cdot\mathcal{A}\equiv q\cdot\mathcal{A}_{\rm on}+q\cdot\mathcal{A}_{\rm off}$, and $\mathcal{T}\cdot\mathcal{B}\equiv\mathcal{T}\cdot\mathcal{B}_{\rm on}+\mathcal{T}\cdot\mathcal{B}_{\rm off}$. The on-shell component $q\cdot\mathcal{A}_{\rm on}=\mathcal{T}\cdot\mathcal{B}_{\rm on}$ is given in Eq. (24). The off-shell contributions, which are of higher order in v/c or involve relativistic P states, are

$$q\cdot\mathcal{A}_{\rm off}=2\pi^2P\cdot qM\left[\cos^2\theta\left(\frac{E_p}{M}-1\right)\left(u-\frac{w}{\sqrt{2}}\right)^2-\frac{3}{2}\left(\frac{p_z}{M}-\frac{E_p}{M}\cos^2\theta\right)v_t^2+\frac{3}{\sqrt{2}}(1-\cos^2\theta)v_sv_t\right.$$
$$\left.+\sqrt{6}\left(\cos\theta-\frac{|\mathbf{p}|}{2M}(1-\cos^2\theta)\right)uv_t-\sqrt{3}\left(\cos\theta-\frac{|\mathbf{p}|}{M}(1-\cos^2\theta)\right)wv_t+\frac{\sqrt{3}|\mathbf{p}|}{M}(1-\cos^2\theta)\left(u-\frac{w}{\sqrt{2}}\right)v_s\right],\quad\text{(B1)}$$

and

$$T\cdot\mathcal{B}_{\rm off}=2\pi^2P\cdot qM^2\left[\left(\frac{\varepsilon}{M}+\frac{\mathbf{p}^2}{M(M+E_p)}\left(-\cos^2\theta+\frac{\varepsilon-2E_p}{M}(1-\cos^2\theta)\right)\right)u^2+\left(\frac{E_p-M}{M}-\frac{\mathbf{p}^2}{M^2}(1-\cos^2\theta)\right.\right.$$
$$\left.-\frac{\varepsilon}{2M}\frac{E_p-\cos^2\theta(E_p+2M)}{M}\right)\sqrt{2}uw+\left(\frac{M-E_p}{M}\left(2-\frac{3}{2}\cos^2\theta\right)+2\frac{\mathbf{p}^2}{M^2}(1-\cos^2\theta)-\frac{\varepsilon}{M}\frac{E_p-\cos^2\theta(E_p+M)}{M}\right)w^2$$
$$+\frac{3}{2}\left(-\frac{p_z}{M}+\frac{E_p-\varepsilon-2M}{M}\cos^2\theta\right)v_t^2+\frac{3}{\sqrt{2}}\left(1-\frac{E_p}{M}\left(2+\frac{\varepsilon}{M}\right)\right)(1-\cos^2\theta)v_sv_t$$
$$+\sqrt{\frac{3}{2}}\frac{|\mathbf{p}|}{M}\left(-2\frac{p_z}{M}+2\frac{E_p-\varepsilon}{M}\cos^2\theta-1-3\cos^2\theta\right)uv_t+\sqrt{3}\frac{|\mathbf{p}|}{M}\left(\frac{p_z}{M}-\frac{E_p-\varepsilon}{M}\cos^2\theta-1+3\cos^2\theta\right)wv_t$$
$$\left.+\sqrt{3}\frac{|\mathbf{p}|}{M}(1-\cos^2\theta)\left(u-\frac{w}{\sqrt{2}}\right)v_s\right],\quad\text{(B2)}$$

where $p_z=|\mathbf{p}|\cos\theta$, $E_p=\sqrt{M^2+\mathbf{p}^2}$ and $\cos\theta=(yM_D-p_0)/|\mathbf{p}|$. Also the convolution breaking contribution $p\cdot\mathcal{A}$ in Eq. (19) represents a relativistic correction in comparison with the ''on-shell'' amplitude:

$$\mathcal{A}\cdot p=2\pi^2M_DM\left[(M_D-2E_p)\frac{p_z}{M}\left(u-\frac{w}{\sqrt{2}}\right)^2-\frac{3M_Dp_z}{2M}v_t^2+\sqrt{6}(M_D-E_p)\cos\theta\left(u-\frac{w}{\sqrt{2}}\right)v_t\right].\quad\text{(B3)}$$

Taking into account these relativistic contributions to the nucleon-deuteron amplitude, one obtains the relativistic correction to the convolution component of g_1^D in Eq. (25):

$$\delta^{\rm (off)}g_1^D(x)=\frac{P\cdot q}{4\pi^2M_DS\cdot q}\int dy\,dp^2\left[\frac{q\cdot\mathcal{A}_{\rm on}}{2p\cdot q}\left(\tilde{g}_1^N(x/y,p^2)-g_1^N(x/y)\frac{E_p}{\mathcal{N}M}\right)+q\cdot\mathcal{A}_{\rm off}(p\cdot qG_{(q)}+G_{(\gamma)})\right.$$
$$\left.+\mathcal{T}\cdot\mathcal{B}_{\rm off}M(G_{(\sigma p)}-p\cdot qG_{(\sigma pq)})+p\cdot\mathcal{A}p\cdot qG_{(p)}\right].\quad\text{(B4)}$$

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