

Ioffe current constant from a relativistic three quark model

George L. Strobel^{1,*} and K. V. Shitikova^{2,†}

¹Physics Department, University of Georgia, Athens, Georgia 30602

²Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794

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The Ioffe current constants for the proton are evaluated using a Poincaré invariant solution of the three-body Dirac equation, for the $(1/2^+)^3$ positive parity configuration. Results are also compared to the QCD sum rule predictions and with the random instanton liquid model. [S0556-2813(96)06008-6]

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I. INTRODUCTION

The Ioffe currents are an attempt to incorporate what is known about the quark content of a QCD description of strongly interacting baryons. Namely the Δ^{++} has three quarks of the same flavor existing in orbital angular momentum zero states [1]. From SU(3) symmetry, similar considerations apply for the nucleon described as three quarks, two u quarks, and one d quark. If one is restricted to considering currents proportional to the product of three quark field operators without derivatives, then the form of the current is unambiguous for the Δ^{++} . For the nucleon, two forms for the current emerge [1,2]. They are

$$\eta_1 = [u^a(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x)\epsilon^{abc} \quad (1)$$

and

$$\eta_2 = [u^a(x)C\sigma_{\mu\nu}u^b(x)]\gamma_5\sigma_{\mu\nu}d^c(x)\epsilon^{abc}. \quad (2)$$

C is a charge conjugation matrix; a , b , and c are color indices for the quarks; and ϵ is the totally antisymmetric tensor. The sum over repeated indices is understood. $\sigma_{\mu\nu}$ is the combination of Dirac matrices,

$$\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2. \quad (3)$$

These are the only two positive parity, spin one-half currents that can be constructed [3], these are the so-called Ioffe currents for the nucleon. The nucleon coupling to the two Ioffe currents is given by

$$\langle 0 | \eta_{1,2} | N(p, M_z) \rangle = \lambda_{1,2}^N \sqrt{[2E/(2\pi)^3]} U(p, M_z) e^{-ipx}, \quad (4)$$

where $|N(p, M_z)\rangle$ denotes a nucleon state with momentum p , and spin z component M_z . $U(p, M_z)$ is the corresponding free fermion spinor and $\lambda_{1,2}^N$ is the Ioffe current constant. E is the nucleon rest mass energy. The main result of this paper is to calculate the Ioffe current constant using a covariant solution for the composite wave function of three quarks from the relativistic three-body Dirac equation in the center of mass frame. In this frame, the proton rest frame, p is zero.

*Electronic address: gstrobel@hal.physast.uga.edu

†Permanent address: Institute of Nuclear Physics Moscow State University, Moscow, Russia.

This is a continuation of efforts [2] to compare the results of QCD sum rules to the results of quark potential models.

The Ioffe current constant for the proton couples the proton wave function to the vacuum. The Ioffe current constant provides a connection to the electromagnetic form factors [4] at large momentum transfer. It is also a factor in the decay amplitude of the proton in some QCD theories. The Ioffe current constant can be calculated given a theory of the quark structure in the proton. One needs the probability amplitude that all three quarks are at the center of mass of the proton simultaneously.

In a relativistic three-quark model, the charge and spin of the proton are modeled by assigning appropriate charges to the assumed three quark constituents, and assigning them to a $(1/2^+)^3$ configuration coupled to a total angular momentum of one-half. The configuration is labeled by the upper component quantum numbers for each quark. The three-body Dirac equation is used to describe the dynamics of a bound relativistic three-quark system.

The hyperspherical method has been applied to the three-body Dirac equation [5,6], where hyperangular averages of a diagonal central potential and the relativistic kinetic energy operator were evaluated. The basic idea is to use the chain rule of calculus to change the partial derivatives of the kinetic energy operator with respect to r_1 , etc., into partial derivatives with respect to the hyperradius. The hyperspherical formulation expands the three-body bound state wave function into a set of configurations, each of which has a hyperradial and a hyperangular factor.

In the method of hyperspherical functions, one obtains a self-adjusted quark system in which the parameters of the quark-quark interaction determine the size of the system [7]. This method has been used to make numerous calculations of the structure of heavy baryons and other multi-quark systems using various potentials for the interquark interaction [8,9]. The importance of a relativistic approach for description of the dipole form factor of the proton has been shown [10]. In this paper we are using the wave functions of the proton [5,11] which reproduce the proton size, to calculate the Ioffe current constant.

Thus the eight radial components of the $(1/2^+)^3$ composite three-quark wave function have been given [5,11] a hyperradial dependence of $R(r) = \text{const} r^K \exp(-Lr^2)$. K denotes the orbital angular momentum in a given component, zero for the large cubed first component which survives in the nonrelativistic limit, and up to three for the small cubed eighth component of the $(1/2^+)^3$ composite three-body wave function.

The strength of the hypercentral quadratic potential for a given quark mass was adjusted to reproduce the proton rest energy. The Lorentz character of the potential (scalar, vector, tensor, etc.) is determined by requiring an analytic solution of the three-body Dirac equation in hyperspherical coordinates. This potential vanishes at zero separation, in agreement with the idea of asymptotic freedom.

II. THEORY

The composite field, for a three-body system is given by

$$\Phi(x_1, x_2, x_3) = \Psi(x_1)\Psi(x_2)\Psi(x_3), \quad (5)$$

where the $\Psi(x)$'s are single-particle wave functions, and x is a four-vector describing the particle coordinates. It is important to vary the action with respect to the composite field only, and one will then obtain a linear equation for the composite field. This is in contrast to varying the action with respect to the single-particle wave functions, which results in nonlinear differential integral equations to solve. One obtains for the composite field, the three-body Dirac equation, written in covariant form as

$$\left\{ \begin{array}{l} [(\gamma^\mu i \partial_\mu - m_1) \otimes \gamma^\mu \eta_\mu \otimes \gamma^\mu \eta_\mu + \gamma^\mu \eta_\mu \otimes (\gamma^\mu i \partial_\mu - m_2) \otimes \gamma^\mu \eta_\mu \\ + \gamma^\mu \eta_\mu \otimes \gamma^\mu \eta_\mu \otimes (\gamma^\mu i \partial_\mu - m_3) - \sum_{i < j} V_{ij}(d_{ij})] \end{array} \right\} \Phi(x_1, x_2, x_3) = 0. \quad (6)$$

Here η_μ is a timelike four-velocity vector of the system. In the center of momentum frame, η_μ is (1 0 0 0). d_{ij} is the transverse difference of the two four vectors, x_i and x_j . Here x_{ij} is defined as $(x_i - x_j)^2$, so that the transverse difference is

$$d_{ij} = \{-(x_i - x_j)^2 + (x_{ij} \cdot \eta)^2\}^{1/2}. \quad (7)$$

In the center of momentum frame, the transverse difference simplifies to the magnitude of the usual radial separation of the two particles, r_{ij} . The motion of the center of mass is correctly handled by the three-body Dirac equation in this frame by use of some version of the Jacobi coordinates. This results in a covariant three-body one-time equation with relativistic potentials. The time is the time of the center of mass. In a three-body system, there is no dependence on either relative time. When the three-body composite particle wave function normalization is defined on a three-dimensional spatial surface perpendicular to a unit four-vector, which is (1 0 0 0) in the overall center of momentum frame, then there is no relative time in the composite particle Hamiltonian in this frame.

The three-body Dirac equation is solved in hypercentral approximation. The six space coordinates necessary to specify the location of the particles are taken as a hyperradius, ρ , and five hyperangles, Ω . The hyperradius is defined as

$$\rho^2 = r_1^2 + r_2^2 + r_3^2 = 2r^2/3. \quad (8)$$

The hypercentral approximation utilizes the hyperangular average of the $\sum_{i < j} V_{ij}(d_{ij})$ potential terms. For the solution used here, this is taken as proportional to the hyperradius squared. The hyperangular reduction of these equations has been reported elsewhere [5,6]. The equations and solutions are somewhat simpler expressed in terms of the r above, rather than the hyperradius.

The composite wave function is originally a 4^3 equals 64 component wave function. In the Dirac upper and lower two-

component formalism, it reduces to a 2^3 equals eight component wave function. The three-fermion composite wave function, Φ , is written as

$$\Phi = \sum U(\Omega)R(\rho), \quad (9)$$

where the sum is over the various configurations. In general, the sum over configurations is eventually truncated by a multibody angular momentum barrier that favors small orbital angular momenta configurations for short ranged forces. The hypercentral approximation truncates this sum to that of a single configuration. $U(\Omega)$ is a product of the orbital, spin, flavor, and color part of the wave function for each of the particles, and includes the angular momentum coupling. Ω denotes the hyperangles, and the other spin, flavor, and color coordinates of the system. The angular momentum coupling is $[j_1, j_2]J_{12}, j_3 JM_z$. j_1 , j_2 , and j_3 are the total angular momentum of each of the three particles, J_{12} is the intermediate coupling of the first pair. The total angular momentum of the third particle is coupled to J_{12} to produce J , the total angular momentum of the three-body system, and its z component M_z . Sums over the m values are understood. For the nucleon, J is one-half, and J_{12} can be only zero or one for the configurations considered here. Doing the hyperangular integration results in the three-body Dirac equation becoming a set of coupled differential equations involving derivatives with respect to the hyperradius. An eight by eight matrix is obtained for the Hamiltonian which operates on the composite three-body wave function involving products of the upper and lower components for each single-particle wave function. The unknown hyperradial dependence is symmetric upon exchange of any pair of coordinates.

The factors that make up the $U(\Omega)$ part of the composite wave function are now expressed in detail. The color singlet part of the composite wave function of three quarks can be written as a factor:

$$\psi_{\text{color}} = \det(abc)/\sqrt{6}, \quad (10)$$

where a , b , and c denote the three color indices of the quarks. This determinant is totally antisymmetric upon exchange of color indices. The rest of the composite wave function must therefore be totally symmetric upon exchange of coordinates. The composite wave function can be rewritten as

$$\Phi = \psi_{\text{color}} \psi_{fc} \psi_{\text{space}}. \quad (11)$$

The flavor and angular momentum coupling part can be expressed as

$$\psi_{fc} = (\chi_s [j_1, j_2] 1, j_3 JM_z) + \chi_A [j_1, j_2] 0, j_3 JM_z) / \sqrt{2}. \quad (12)$$

Here the flavor part consists of only u or d components, the symmetric upon exchange of the first pair being

$$\chi_s = [duu + udu - 2uud] / \sqrt{6} \quad (13)$$

and the antisymmetric upon exchange of the first pair being

$$\chi_A = [udu - duu] / \sqrt{2}. \quad (14)$$

The combined symmetry of the flavor angular momentum coupling part is maintained by the angular momentum coupling factors having the same symmetry as the corresponding flavor part. J_{12} is 1 for the symmetric flavor part, and J_{12} is zero for the antisymmetric flavor part.

The composite three-body wave function is an eight-component column vector with unknown hyperradial dependence to be determined. The color, flavor, angular momentum coupling, and the orbital factors of the composite wave function are all collected into the factor $U(\Omega)$.

The orbital part of the wave function for each quark is given by

$$\phi_{j\pi}^m(r_1) = \begin{bmatrix} C_l(r_1/\rho)^l Y_l^{ml} \zeta_{1/2}[l, 1/2] jm \\ i \vec{\sigma}_1 \cdot \hat{r}_1 C_{l'}(r_1/\rho)^{l'} Y_{l'}^{m l'} \zeta_{1/2}[l', 1/2] jm \end{bmatrix}. \quad (15)$$

The upper and lower components of this equation are now also named F and G , respectively, for a shorthand notation for the upper and lower components of the single-particle wave function. $\zeta_{1/2}$ is the intrinsic spin one-half (up or down spinor component) wave function for the quark intrinsic spin. The coefficients are

$$C_{\ell} = [2\Gamma(\ell + 3/2)]^{1/2}. \quad (16)$$

$\Gamma(n)$ is the gamma function of order n . ℓ and ℓ' differ by one and sum to twice j . The one-body parity, π , determines ℓ . The orbital part is based on solid harmonics, $(r_i/\rho)^\ell$ times a spherical harmonic, $Y_{\ell}^{m\ell}(\hat{r}_i)$. The orbital part can be expressed as solid harmonics involving only the Jacobi relative coordinates. We use the individual coordinates of the quarks however, constrained as

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0 \quad (17)$$

in writing the composite wave function.

In terms of F and G , the single-particle upper and lower component to the one-body Dirac equation, the composite wave function radial components from particles 1, 2, and 3 are:

Component	123
R_1	FFF
R_2	GFF
R_3	FGF
R_4	GGF
R_5	FFG
R_6	GFG
R_7	FGG
R_8	GGG

(18)

We define a $\Lambda(x, x') = K(x) + K(x')$. Here x denotes the composite wave function component index listed above, ranging from 1 to 8. This angular momentum coefficient varies from element to element in the Hamiltonian matrix. The orbital angular momentum potential barrier is in part of the kinetic energy operator defined as

$$D(n) = (d/dr + n/r). \quad (19)$$

The normalization of the wave function is now considered. Each configuration is here separately considered normalized to unity. After the hyperangular integration, the normalization for a configuration is

$$1 = (2/\pi^{3/2}) N^2 \sum_x \int \rho^5 d\rho (R_x)^2 / \Gamma\{\{\Lambda(x, x) + 6\}/2\}. \quad (20)$$

The $(1/2^+)^3$ configuration has been solved for harmonic-oscillator-like potentials [5]. This assumed hyperradial dependence correctly handles the angular momentum dominated shape of the wave function at small distances, and also the charge radius of the proton, if the quark mass is 0.110 GeV or less. The hypercentral potential required for the solution is obtained analytically by substitution into the three-body Dirac equation above, and equating coefficients.

III. THE $(1/2^+)^3$ CONFIGURATION SOLUTION

For the $(1/2^+)^3$ configuration the K values are

x	$L1$	$L2$	$L3$	K
1	0	0	0	0
2	1	0	0	1
3	0	1	0	1
4	1	1	0	2
5	0	0	1	1
6	1	0	1	2
7	0	1	1	2
8	1	1	1	3

The Hamiltonian matrix, after hyperangular integration for the $(1/2^+)^3$ configuration, is

$$H = \begin{bmatrix} (3M-E) & -D(5)/3 & -D(5)/3 & 0 & -D(5)/3 & 0 & 0 & 0 \\ D(0) & (M-E) & 0 & -D(6)/4 & 0 & -D(6)/4 & 0 & 0 \\ D(0) & 0 & (M-E) & -D(6)/4 & 0 & 0 & -D(6)/4 & 0 \\ 0 & D(-1) & D(-1) & (-M-E) & 0 & 0 & 0 & -D(7)/5 \\ D(0) & 0 & 0 & 0 & (M-E) & -D(6)/4 & -D(6)/4 & 0 \\ 0 & D(-1) & 0 & 0 & D(-1) & (-M-E) & 0 & -D(7)/5 \\ 0 & 0 & D(-1) & 0 & D(-1) & 0 & (-M-E) & -D(7)/5 \\ 0 & 0 & 0 & D(-2) & 0 & D(-2) & D(-2) & (-3M-E) \end{bmatrix}. \quad (21)$$

The angular momentum barrier is in the n/ρ term of the kinetic energy operator, $D(n)$. This equation is the hyperangular integrated form of the equation $[H-E]\Psi=0$ for the $(1/2^+)^3$ configuration. The ordering of the eight components of the hyperradial part of the composite three-body wave function is as listed in Eq. (18). An analytic solution has been found for these equations when a specific harmonic oscillator potential is utilized for the case of all three particles identical. For three identical particles, and with each particle with the same set of quantum numbers, one expects the components R_2 , R_3 , and R_5 to be equal, and also for the components R_4 , R_6 , and R_7 to equal each other. Then the wave function has only four unknown components, R_1 , R_2 , R_4 , and R_8 . For the $(1/2^+)^3$ configuration, including central diagonal quadratic interactions along the diagonal, results in the 4×4 Hamiltonian matrix that operates on the four unknown components:

$$\begin{bmatrix} (3M-E+n_1r^2) & -D(5) & 0 & 0 & R_1 \\ D(0) & (M-E+n_2r^2) & -D(6)/2 & 0 & R_2 \\ 0 & 2D(-1) & (-M-E+n_4r^2) & -D(7)/5 & R_4 \\ 0 & 0 & 3D(-2) & (-3M-E+n_8r^2) & R_8 \end{bmatrix} = 0. \quad (22)$$

This matrix operates on the hyperradial components R_1, R_2, R_4 , and R_8 . Solutions of this equation were found [5] of the form

$$\begin{aligned} R_1 &= A \exp(-Lr^2), \\ R_2 &= R_3 = R_5 = Br \exp(-Lr^2), \\ R_4 &= R_6 = R_7 = Cr^2 \exp(-Lr^2), \\ R_8 &= Dr^3 \exp(-Lr^2), \end{aligned} \quad (23)$$

where

$$\begin{aligned} A &= 1, \\ B &= -(E-3M)/6, \\ C &= B^2, \\ D &= B^3, \\ L &= (E-3M)(E+3M)/36, \end{aligned} \quad (24)$$

when the oscillator potential parameters are chosen so that

$$\begin{aligned} n_1 &= (E-3M)^2(E+3M)/108, \\ n_2 &= n_1/2, \\ n_4 &= n_1/5, \\ n_8 &= 0. \end{aligned} \quad (25)$$

To get the Gaussian-type solution, the interaction in the small component must vanish. The ansatz solution is substituted into the four by four Hamiltonian matrix, the differentiation is carried out, and coefficients of like powers of r are equated. Setting the coefficient A to unity, the above solution is obtained. Normalizing this configuration to unity multiplies all the components by a constant N , determined from Eq. (20), compared to the values quoted above. The energy E must be greater than the sum of the rest masses, $3M$, in order for the Gaussian size parameter, L , to be positive. This is necessary for bound, normalizable wave functions.

IV. RESULTS

This solution has two parameters, the system energy, E , and the quark mass, M . E is set to the proton rest energy. The quark mass is varied. This is almost an unconstrained parameter in the potential model solution. The proton charge

radius is nearly reproduced for any value of the quark mass up to about 0.110 GeV.

Using the three-quark composite wave function detailed above as the proton wave function for spin up, i.e., $M_z = +1/2$, we consider

$$I_1 = \langle 0 | \eta_1 | \Phi(0, +1/2) \rangle. \quad (26)$$

The color factor to I_1 is

$$CF = \epsilon^{abc}(abc)\psi_{\text{color}} = \sqrt{6}. \quad (27)$$

The flavor part to I_1 is

$$FP = \langle uud | \psi_{\text{fc}} \rangle = -|(j_1 j_2) 1 j_3 J M_z\rangle / \sqrt{3}, \quad (28)$$

using the flavor spin coupling parts of the wave function, Eqs. (12)–(14). Calling m_1 , m_2 , and m_3 the z components of spin of the three quarks, we have merely the product to two Clebsch Gordan coefficients,

$$\begin{aligned} \langle m_1 m_2 m_3 | (j_1 j_2) 1 j_3 J M_z \rangle \\ = (1/2 m_1, 1/2 m_2 | 1 m_1 + m_2) (1 m_1 + m_2, 1/2 m_3 | J M_z). \end{aligned} \quad (29)$$

J is one-half, and $M_z = +1/2$. The charge conjugation matrix in the current is symmetric,

$$C = i \gamma^2 \gamma^0 = -i \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix} \quad (30)$$

and we also need

$$\gamma^5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \quad (31)$$

So, collecting these factors, we have

$$\begin{aligned} I_1 = -\sqrt{2} \sum_{ms_2, ms_3} \langle m_2 | C \gamma_\mu | ms_2 \rangle \langle m_3 | \gamma^5 \gamma_\mu | ms_3 \rangle \\ \times \langle m_1 ms_2 ms_3 | (j_1 j_2) 1 j_3 J M_z \rangle \psi_{\text{space}}. \end{aligned} \quad (32)$$

This must also be summed over the three-quark z components of spin, m_1 , m_2 , and m_3 . At the origin, only one of the eight components of the space part of the relativistic composite particle wave function is nonzero. The first, upper component cubed survives. Each quark has zero orbital angular momentum in this component of the composite wave function. The normalized wave function there is given by

$$\psi(0) = AN(C_0 / \sqrt{4\pi})^3, \quad (33)$$

where N is the normalizing coefficient determined by Eq. (20), and the coefficients C_0 are given in Eq. (16).

For this Gaussian-type wave function for the quarks in a proton, the normalization can be obtained analytically as

$$N^2 = \pi^{3/2} (3L)^3 / (1 + S/2)^3, \quad (34)$$

where

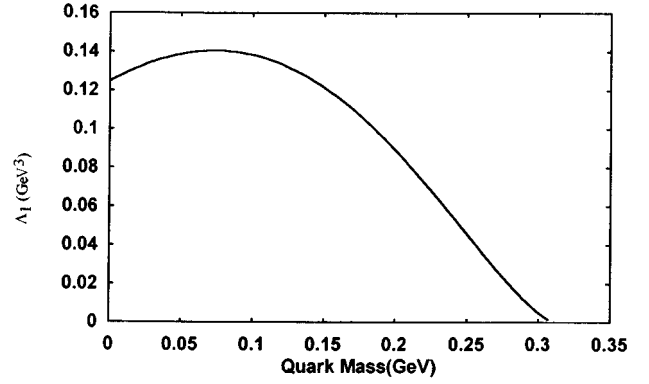


FIG. 1. Ioffe coupling constant λ_1 for the proton.

$$S = (E - 3M) / (E + 3M). \quad (35)$$

This leads to

$$AN = \pi^{3/4} (3L)^{3/2} / (1 + S/2)^{3/2}. \quad (36)$$

We note L has the units of $(\text{GeV})^2$ in the above equation.

Summing over the quark spin z components m_1 , m_2 , and m_3 , we obtain

$$I_1 = 10AN / \pi^{9/4} \sqrt{3}. \quad (37)$$

This evaluation of the first Ioffe current allow the determination of the Ioffe coupling constant, λ_1^N . This Ioffe coupling constant is shown versus assumed quark mass in Fig. 1. The calculation of the second Ioffe constant is similiar. With

$$I_2 = \langle 0 | \eta_2 | \Phi(0, +1/2) \rangle \quad (38)$$

this eventually simplifies to

$$\begin{aligned} I_2 = -\sqrt{2} \sum_{ms_2, ms_3} \langle m_2 | C \sigma_{\mu\nu} | ms_2 \rangle \langle m_3 | \gamma^5 \sigma_{\mu\nu} | ms_3 \rangle \\ \times \langle m_1 ms_2 ms_3 | (j_1 j_2) 1 j_3 J M_z \rangle \psi_{\text{space}}. \end{aligned} \quad (39)$$

This must also be summed over the three-quark z components of spin, m_1 , m_2 , and m_3 . The final result is

$$I_2 = 20AN / \pi^{9/4} \sqrt{3}. \quad (40)$$

The second Ioffe coupling constant is shown in Fig. 2. It is

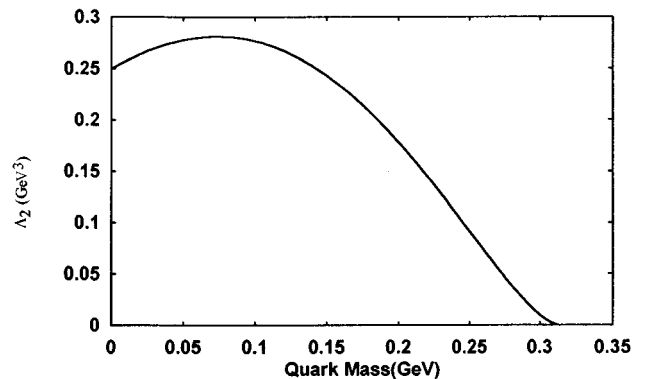


FIG. 2. Ioffe coupling constant λ_2 for the proton.

twice the first Ioffe coupling constant for the proton wave function utilized here.

Equation (41) shows the results of our calculations for the Ioffe coupling constant with quark masses of zero and 0.25 GeV which can be compared with the QCD sum rule predictions and also with the random instanton liquid model (RILM):

Quantity	Dirac model $M=0$	Dirac model $M=0.25$ GeV	RILM [3]	QCD [1]
λ_1	0.122	0.032	0.032 ± 0.001	0.035 +0.008
λ_2	0.244	0.064	0.080 ± 0.004	

(41)

Our calculation results are in good agreement with the RILM for λ_1 and not so good agreement with λ_2 . The explanation for that is as follows. The parameters for the nucleon and the delta were obtained by a global fit to the six nucleon and the four delta correlation functions. The simple ‘‘nucleon pole plus continuum’’ model gives a very good simultaneous description for the complete set of correlation functions. This agreement is particularly good for correlators involving the first Ioffe current, while it is somewhat worse for the other ones. Note [3] that the correlation functions $P3$ and $P6$ (which show deviations from the simple nucleon

pole plus continuum model) involve at least two quarks which have to flip their chirality. It might be that the RILM has problems reproducing those amplitudes.

V. CONCLUSIONS

Zero quark masses are possible in this relativistic potential model of the proton. For a zero mass quark, the first Ioffe current constant is 0.122 (GeV)³. The maximum coupling constant occurs for a quark mass of about 0.075 GeV. A quark mass of about 0.25 GeV is required for the potential model to reproduce the Ioffe current constant value of 0.032 (GeV)³ as determined by QCD sum rules [3]. The second Ioffe current constant, for an assumed quark mass of 0.25 GeV, agrees with QCD sum rule estimates.

The relativistic potential quark model is able to be parametrized to successfully predict Ioffe current constants in agreement with QCD sum rule predictions. With the harmonic oscillatorlike proton wave function used here, the second Ioffe current constant is twice the first constant.

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- [1] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981).
 [2] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
 [3] T. Schafer, E. V. Shuryak, and J. J. M. Verbaarschot, Nucl. Phys. **B412**, 143 (1994).
 [4] G. B. West, Phys. Rep. **18**, 263 (1975).
 [5] G. L. Strobel and C. A. Hughes, Few Body Syst. **2**, 155 (1987).
 [6] G. L. Strobel and T. Pfenninger, Phys. Lett. B **195**, 7 (1987).
 [7] R. I. Djhibuti, K. V. Shitikova, and M. Energoatomizdat, In-

- stitution Report No. ISBN5-283-03989-7, 1993, p. 277.
 [8] A. M. Badalyan, B. L. Jaffe, and A. V. Smilga, Nucl. Phys **B281**, 85 (1987).
 [9] Yu. S. Kalashnikova, I. M. Narodetskii, and Yu A. Simonov, Sov. J. Nucl. Phys. **46**, 698 (1987).
 [10] V. V. Burov and K. V. Shitikova, Sov. J. Nucl. Phys. **55**, 528 (1992).
 [11] G. L. Strobel, Few Body Syst. (to be published).