Local approximations to the exchange nonlocality for neutron-^{16}O scattering

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We investigate the accuracy of two methods used to approximate the exchange nonlocality for the case of neutrons scattering off ^{16}O: the Taylor series expansion approach of Sinha and the zero range exchange approximation. These two approximations are evaluated analytically by methods previously employed in our microscopic folding calculations. The resulting local potential is compared to a phase-equivalent local potential that is derived from rigorously obtained S-matrix elements through the inversion method of Mackintosh, which, being very accurate, serves as our benchmark. The two approximations compare favorably to this inversion potential. The deviations in the potentials decrease with increasing energy, for both the real and imaginary parts. At 20 MeV the differences in the imaginary part are as large as 40% in the nuclear interior. At 50 MeV and 100 MeV the differences in the imaginary parts are comparable to those of the real parts: on the order of 10% far inside the nucleus and close to zero at the nuclear surface. The effects of these differences on the cross sections are far from negligible, as is also shown. [S0556-2813(96)03608-4]

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I. INTRODUCTION

Phenomenological local optical models are still in wide use for describing the final state interaction in reactions such as (p,p'), (p,n), and $(e,e'p)$, even though nonlocal microscopic potentials are now in an advanced stage of development and should be used instead [1]. The transition to nonlocal microscopic potentials has been slow because they are cumbersome to use and require a much higher degree of expertise than local phenomenological optical potentials (LPOP's). Because LPOP's are still being used, it is of interest to examine the validity of two local approximations which are commonly used to simulate the exchange nonlocality. Especially noteworthy is the local optical potential of Kelly and collaborators which utilizes the zero range exchange approximation (ZREA) [2]. This is currently being applied with increasing frequency by the electromagnetic-nuclear physics community to evaluate the $(e,e'p)$ reaction. The other approximation to note is the Taylor series expansion of Sinha [3].

We are currently in a good position to examine these two approximations for the case of n-^{16}O elastic scattering at low energies $E\leqslant 100$ MeV, because we have, for comparison, a reliable local equivalent potential obtained by an inversion method due to Mackintosh and collaborators [4]. This potential *inverts* the S-matrix elements obtained from the nonlocal optical model [5]. It is reliable because, not only are the local equivalent potentials (LEP's) obtained from the inversion quite close to the ones based on Sinha's approximation of the exchange nonlocality, but also because we found that the scattering wave functions for the l-independent inversion potential were very close (within a reasonable Perey factor) to the wave functions of the nonlocal potential [5,6].

In Sec. II we briefly review the definitions of Sinha's approximation and the ZREA. In Sec. III we display the errors. We find that the errors are small but not negligible. The Perey damping factors for the inversion LEP have been examined previously [5,6] and will not be discussed here.

II. FORMALISM

The standard form of the Hartree-Fock potential is given as

$$V_{\mathrm{HF}}(r_1)=\int t^D(\vec{r_1},\vec{r_2})\rho(\vec{r_2})d^3r_2 + \int t^X(\vec{r_1},\vec{r_2})\rho(\vec{r_1},\vec{r_2})\Psi(\vec{r_2})d^3r_2, \qquad (1)$$

where $\vec{r_1}$ is the position of the incident or scattered nucleon and $\vec{r_2}$ is the position of any target nucleon. t^D and t^X represent the direct and exchange parts of the effective N-N interaction, respectively. In this and in our previous works [5,6] the t^D and t^X are obtained from the effective N-N interactions of Yamaguchi *et al.* [7]. We call these CEG, for complex effective Gaussian; other authors also call these YNM (Yamaguchi, Nagata, and Michiyama). In Sinha's approach, $\Psi(\vec{r_2})$ is approximated as described in [3,8,9]. The result for this method's local replacement to the Fock portion [the term in the second line of Eq. (1)] is

$$V_{\mathrm{Sinha}}(r_1)=\int t^X(\vec{r_1},\vec{r_1}+\vec{s})\rho(\vec{r_1},\vec{r_1}+\vec{s})j_0(ks)d^3s. \qquad (2)$$

At this point, Sinha utilizes approximations to the mixed density such as Slater's, for infinite nuclear matter [3], or the form developed by Negle and Vautherin, based on an averaging method [10]. We, on the other hand, treat the mixed density exactly, within the independent particle model, without further approximation. In addition, there exist other approaches in dealing with the two-nucleon density [11].

The ZREA [2] method modifies Eq. (2) by taking it in the limit of $s\rightarrow 0$. Thus, $j_0(ks)$ is replaced by unity, $\rho(\vec{r_1},\vec{r_2})$ is replaced by $\rho(\vec{r_2})$, and t^X is replaced by $V^X(r_1)\delta(\vec{s})$. These approximations are compensated for by obtaining the strength factor V^X according to

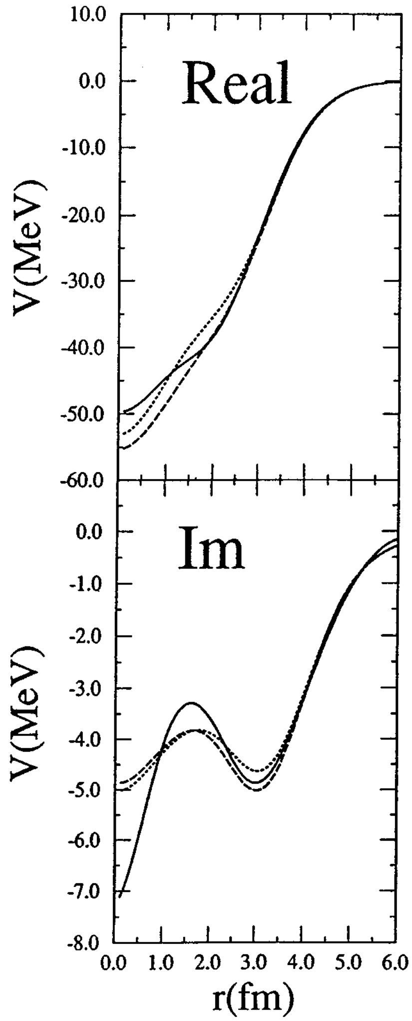

FIG. 1. (a) The real parts of the local potentials for 20 MeV neutrons scattering on ^{16}O vs radial distance. The solid curve represents the phase-equivalent local inversion potential (IP), the dotted curve represents the local approximation due to the Sinha approach, Eq. (2), and the dashed curve represents the local approximation due to the zero range exchange approximation (ZREA), Eq. (4). (The direct or Hartree part has been included.)(b) Same as (a), for the imaginary parts at 20 MeV.

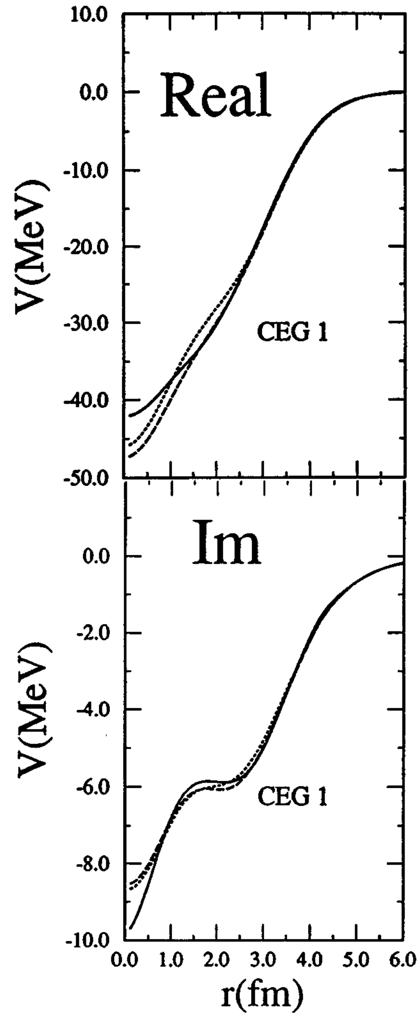

FIG. 2. (a) Same as Fig. 1(a), for the real parts at 50 MeV. (b) Same as Fig. 1(a), for the imaginary parts at 50 MeV.

$$V^X(r_1) = \int t^X(\vec{r_1}, \vec{r_1} + \vec{s}) j_0(ks) d^3s \qquad (3)$$

and, finally,

$$V_{\mathrm{ZREA}}(r_1) = \int V^X(r_1)\, \delta(\vec{s})\, \rho(\vec{r_2}) d^3r_2$$
$$= V^X(r_1)\rho(r_1). \qquad (4)$$

The most difficult portion of this computation is the evaluation of V^X. The Hartree or direct portion remains unchanged. Its analytic evaluation is described in detail in [12].

The analytic expression for V_{Sinha} of Eq. (2) is given in detail in [12]. However, the analytic expression we obtain for V_{ZREA}, given by the evaluation of Eq. (4), is new and is available upon request. These analytic expressions were programmed and then processed on the University of Connecticut's ES9000 mainframe computing system.

III. RESULTS

As stated previously, the above local potentials were calculated for laboratory energies of 20, 50, and 100 MeV. In addition, the imaginary part of $k(r_1)$ was included as suggested in Georgiev and Mackintosh [13]. The local momenta $k(r_1)$ were computed utilizing Mackintosh's local complex inversion potential (IP) where $k^2(r_1) = (2m/\hbar^2) - E - U(r_1)$, and $U(r_1)$ is replaced by the IP. This was done over a radial mesh of 0.1 fm for all three energies. The IP was calculated by Mackintosh and Cooper at the Open University in England and it was derived from the S-matrix elements of a previously published full nonlocal model [5].

The results for V_{Sinha} (dotted curve) and V_{ZREA} (dashed curve) are illustrated in Figs. 1–3 along with the IP (solid curve). (The direct or Hartree part has been included in both the Sinha and ZREA results.) These figures show both the real and imaginary parts of the total optical potential for each energy. As stated previously, the IP serves as our benchmark for local equivalent potentials since it reproduces the full

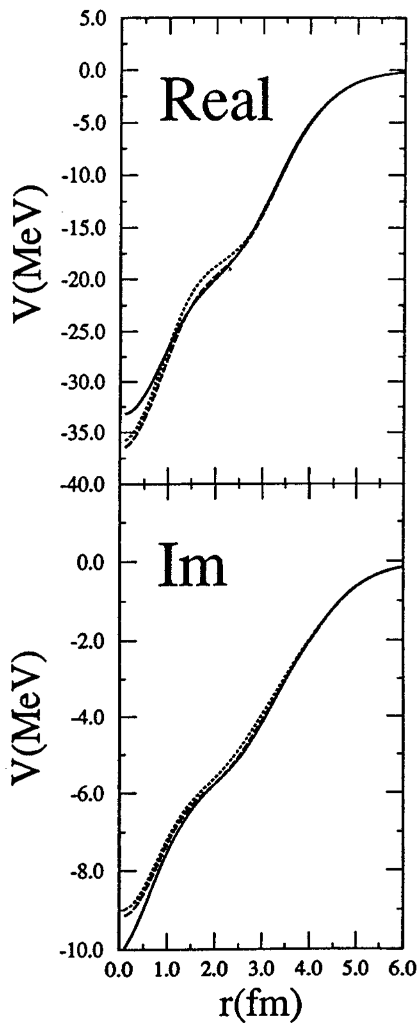

FIG. 3. (a) Same as Fig. 1(a), for the real parts at 100 MeV. (b) Same as Fig. 1(a), for the imaginary parts at 100 MeV.

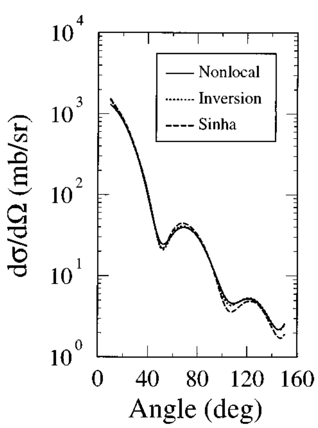

FIG. 4. The differential scattering cross sections for 20 MeV neutrons scattering on ^{16}O. The solid curve represents those obtained from the full nonlocal model, the dotted curve represents those obtained from the phase-equivalent local inversion potential (IP), and the dashed curve represents those obtained from the Sinha potential, Eq. (5). (The Sinha potential includes the direct or Hartree part.)

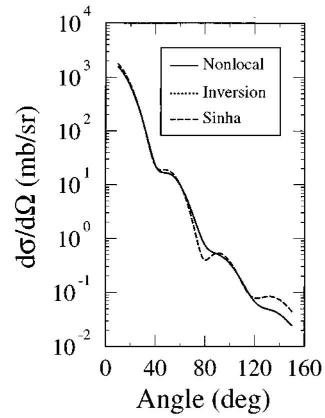

FIG. 5. Same as Fig. 4, for 50 MeV.

nonlocal S-matrix elements [5]. As one can see the agreement between the two approximations and the IP is quite good, and the error decreases with increasing energy. But how do the differences between the potentials manifest themselves in the differential cross sections? In Figs. 4–6 we show the differential cross sections from the full nonlocal model [5] (solid curve), the IP (dotted curve), and the Sinha approach (dashed curve). Here, as one can see, the differences between the Sinha cross sections and the exact result increase with increasing energy, even though the differences between the potentials are decreasing. This result is an indication that the cross sections become more sensitive to changes in the potentials as the energy increases.

IV. SUMMARY AND CONCLUSIONS

We have analytically evaluated two local potentials that approximate the exchange nonlocality: the Sinha approach and the zero range exchange approximation (ZREA), since these are used quite frequently in the literature. We make no approximations to either the single- or two-particle densities other than using harmonic oscillator functions to represent the single-particle target nucleon states. We checked the validity of these local approximations by comparing their results with the phase-equivalent local complex inversion potential (IP) [5] which serves as our benchmark for local

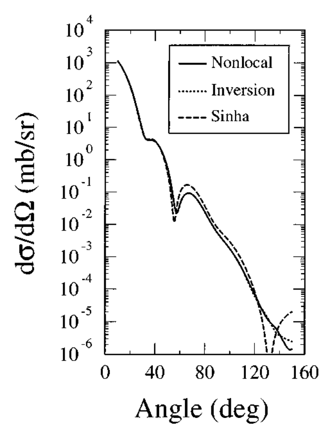

FIG. 6. Same as Fig. 4, for 100 MeV.

potentials. We find that the two approximations compare favorably to the IP. As far as the potentials go, the agreement gets better with increasing energy; however, the differences between these potential models becomes apparent when the differential cross sections are computed and compared. Here, the differences increase with increasing energy; i.e., at higher energies the cross sections become more sensitive to differences between the potentials even though the potential differences are smaller. In conclusion, we feel that the IP is the best of the LEP's we have considered. However, both the Sinha approach and the ZREA are very good approximations for a large range of energies.

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