Hadrodynamic approach to compressible nonuniform nuclear systems

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The energy-momentum tensor for isospin-symmetric semi-infinite nuclear matter (SINM) as well as for finite spherically symmetric nuclei is evaluated in the relativistic Thomas-Fermi (RTF) approximation of the σ-ω model. The T_{33} stress tensor component of self-bound SINM vanishes identically. Our field theoretical RTF approach reflects the elementary relation $p=2\sigma/R$ between the bulk pressure p, the surface tension σ and the radius R of a nucleus. [S0556-2813(96)03407-3]

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I. INTRODUCTION

In conventional nonrelativistic descriptions of nuclei the nucleon density appears as explicit degree of freedom. The meson fields, however, are hidden in an effective nucleon-nucleon interaction that is represented by, e.g., a Skyrme-type energy functional of the nucleon density. Hadrodynamic approaches [1] start from a relativistic treatment of the nucleon motion that is coupled explicitly to meson fields. Thus, these fields enter as new degrees of freedom mediating the interaction between the nucleons.

If uniform infinite symmetric nuclear matter (INM) is treated nonrelativistically, phenomenological thermodynamics defines the (thermodynamic) pressure p_{th} at zero temperature by

$$p_{\mathrm{th}}=\rho^2\frac{d}{d\rho}\left(\frac{\mathcal{E}(\rho)}{\rho}\right), \tag{1}$$

where ρ is the baryon density and $\mathcal{E}$ is the energy density depending only on the baryon density. In the case of a field theoretical description of nuclear matter the energy density $\mathcal{E}$ in addition becomes a function of the meson field potentials. Nevertheless, the expression (1) — now with meson fields depending on the source density ρ — defines a (thermodynamic) pressure for a uniform system with given baryon density and meson fields.

Field theoretically, the energy density $\mathcal{E}$ of a system must be identified with the quantum mechanical expectation value $\langle T_{00}\rangle$ of the T_{00} component of the energy-momentum tensor $T_{\mu\nu}$ defined from the Lagrangian $\mathcal{L}=\mathcal{L}(\phi_a,\partial_\mu\phi_a)$ of the system by

$$T_{\mu\nu}=-g_{\mu\nu}\mathcal{L}+\sum_a \partial_\nu\phi_a\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_a)}, \tag{2}$$

where in the case of a QHD-I [1] nuclear Lagrangian $\mu,\nu=0,1,2,3$ and $a=1,2,3$ with the meson fields ϕ_1 and ϕ_2, and the nucleon field $\phi_3=\psi$. The entity T_{ik} $(i,k=1,2,3)$, composed of the spatial components of the energy-momentum tensor, is called the stress tensor since classically the component F_i of the force on a volume with surface elements dn^k in the matter-field system is given by the surface integral

$$F_i=\oiint T_{ik}dn^k. \tag{3}$$

For a homogeneous and isotropic fluid at rest the diagonal elements of the expectation value of the stress tensor are equal, and the nondiagonal elements vanish. Thus, a hydrostatic scalar pressure p_{hydr} can be defined by

$$p_{\mathrm{hydr}}\equiv\tfrac{1}{3}\sum_{i=1}^{3}\langle T_{ii}\rangle. \tag{4}$$

This simplification of the stress tensor clearly is only possible for such symmetric systems as, e.g., uniform static fluids with rotational invariance.

In the case of the mean-field approximation (MFA) of the quantum hadrodynamic model QHD-I [1] the expectation value of the stress tensor can be specified to

$$\begin{aligned}T_{\mu\nu}=&\langle i\overline{\psi(t,\mathbf{x})}\gamma_\mu\partial_\nu\psi(t,\mathbf{x})\rangle+\partial_\mu\phi(\mathbf{x})\partial_\nu\phi(\mathbf{x})\\&-\partial_\mu V_0(\mathbf{x})\partial_\nu V_0(\mathbf{x})+g_{\mu\nu}(\tfrac{1}{2}\{[\nabla\phi(\mathbf{x})]^2+m_s^2\phi^2(\mathbf{x})\}\\&+\tfrac{1}{3}b\phi^3(\mathbf{x})+\tfrac{1}{4}c\phi^4(\mathbf{x})-\tfrac{1}{2}\{[\nabla V_0(\mathbf{x})]^2+m_v^2V_0^2(\mathbf{x})\}).\end{aligned} \tag{5}$$

For uniform systems the relevant hydrodynamic pressure p_{hydr} turns out to be identical with the thermodynamic pressure p_{th} (see, e.g., [1,2]). One refers to thermodynamic consistency of the two pressure definitions in the case of the MFA for uniform systems. It is a little paradox that if one goes beyond MFA by, e.g., a Dirac-Hartree-Fock (DHF) method, which improves the treatment of the fields by taking into account retardation effects, the thermodynamic consistency between the two pressures is lost (see, e.g., [2]).

Now, saturation of infinite nuclear matter means that it is self-bound without an external pressure. Which pressure, the hydrodynamic or the thermodynamic pressure, should vanish? Also the Hugenholtz–van Hove theorem valid for vanishing pressure has to be reconsidered in view of the fact that there are two pressure definitions [2]. In fact, the trouble calls for a unique adequate definition of a ''real'' pressure. The situation ressembles the case of matter in electromagnetic fields where there is in addition to the material pressure a pressure coming from field effects described by the electromagnetic Maxwell stress tensor. From the standpoint of a unified description of fields and matter by QHD the pressure definition via the stress tensor, which includes field effects consistently, is more satisfying than the thermodynamic definition. The coincidence of the thermodynamic definition in full field theory [3] and also in some approximations with the hydrodynamic definition seems to be remarkable, but not compulsory.

The present investigation aims at working out a preliminary concept for the energy-momentum tensor of *finite* self-bound nuclei. Because of the anisotropy of nonuniform systems with some preferred direction, the stress tensor is (i) not necessarily symmetric and (ii) its diagonal elements need not to be equal. Thus, the concept of a scalar hydrodynamic pressure (4) breaks down, and the correct way to treat the direction dependent mechanical pressure effects is given by the fundamental relation (2). In particular, the question of the consistency of hydrodynamic and thermodynamic (scalar) pressure might become meaningless.

As first steps to finite systems we study semi-infinite isospin-symmetric nuclear matter (SINM) as well as spherically symmetric nuclei treating them in the relativistic Thomas-Fermi (RTF) approximation to QHD-I with scalar σ mesons and vector ω mesons, and disregarding Coulomb effects. Along the surface normal (chosen to be the 3-axis or z axis) the surface region of SINM is nonuniform, and perpendicular to it of homogeneous character, differing, however, from saturated INM. This property of SINM is reflected in its local energy-momentum tensor the form of which is different from that for uniform isotropic infinite nuclear matter. Starting from the minimum principle for the surface tension we carry through pragmatic RTF calculations of SINM. The local pressures—i.e., element T_{33} of the stress tensor (2) and the thermodynamic pressure (1), depending now on the z coordinate—are evaluated. There is a characteristic difference between the two pressures that is identified as a meson field effect depending on the gradients of the field potentials. Thus, thermodynamic consistency is lost for the two local pressures. The hydrostatic pressure T_{33} is found to vanish identically in equilibrated SINM. In the surface region the thermodynamic definition of a local pressure leads to nonzero values. For finite spherical nuclei the RTF approach results in a symmetrical T_{ik} stress tensor with nondiagonal elements in the Cartesian representation. Spherical coordinates (θ,ϕ,r) transform the stress tensor into a diagonal form where the T_{rr} element differs from the others. The T_{rr} element leads to a radial force per surface area which we show to be approximately consistent with the elementary relation for the bulk pressure p,

$$p=\frac{2\,\sigma}{R}, \tag{6}$$

where σ is the surface tension and R the nuclear radius.

II. ENERGY-MOMENTUM TENSOR FOR SELF-BOUND NONUNIFORM SYSTEMS

A. Semi-infinite nuclear matter

The energy density $\mathcal{E}$ of isospin-symmetric SINM in the RTF approximation to QHD-I with nonlinear scalar meson terms is given by

$$\begin{aligned}\mathcal{E}(\phi,\phi',V_0,V_0',\rho)=&+\tfrac{1}{2}[(\phi')^2+m_s^2\phi^2]+\tfrac{1}{3}b\phi^3+\tfrac{1}{4}c\phi^4\\&-\tfrac{1}{2}[(V_0')^2+m_v^2V_0^2]+g_vV_0\rho\\&+\frac{4}{(2\pi)^3}\int_{\Theta_F(z)}\varepsilon^\star(k,z)d^3k,\end{aligned} \tag{7}$$

where $\varepsilon^\star(k,z)=\sqrt{k^2+M^{\star 2}(z)}$ and $M^\star(z)=M-g_s\phi(z)$. A prime denotes derivation with respect to the z coordinate. $\Theta_F(z)$ is the local Fermi sphere defined by $\Theta_F(z)\equiv\{\boldsymbol{k}|k\leq k_F(z)\}$, where the local Fermi momentum $k_F(z)$ follows from the baryon density $\rho(z)$ via

$$\rho(z)=\frac{2}{3\pi^2}k_F^3(z). \tag{8}$$

The integration over $\Theta_F(z)$ in the last term of Eq. (7) can be carried through analytically and yields

$$\frac{4}{(2\pi)^3}\int_{\Theta_F(z)}\varepsilon^\star(k,z)d^3k=3\,k_F(z)\rho(z)g(a(z)),$$

$$a(z)\equiv\frac{M^\star(z)}{k_F(z)}, \tag{9}$$

where the function g is given by

$$\begin{aligned}g(a)&=\int_0^1x^2\sqrt{x^2+a^2}dx\\&=\frac{1}{8}\left[(1+a^2)^{3/2}+\sqrt{1+a^2}-\frac{a^4}{2}\ln\frac{\sqrt{1+a^2}+1}{\sqrt{1+a^2}-1}\right].\end{aligned} \tag{10}$$

There are pure meson field contributions together with terms arising from the interaction of fields and matter. All these relations follow in a straightforward way by specializing the general RTF expressions, derived in Ref. [1], to the SINM system. $\mathcal{E}(z)$, Eq. (7), can be decomposed into two terms,

$$\mathcal{E}(\phi,\phi',V_0,V_0',\rho)=\mathcal{E}_\infty(\phi,V_0,\rho)+\mathcal{E}_{\rm gr}(\phi',V_0'),$$

$$\mathcal{E}_{\rm gr}(\phi',V_0')\equiv\tfrac{1}{2}[(\phi')^2-(V_0')^2], \tag{11}$$

where the "volume part" $\mathcal{E}_\infty$ at a position z follows from the energy density of INM when in a local density approximation the INM density is replaced by the density $\rho(z)$, and meson fields are treated correspondingly. The additional term in Eq. (11) depends on gradients of the meson fields, only. Thus, the INM limit of Eq. (11) is correctly obtained.

The RTF equations for the *self-bound* SINM matter system follow by minimizing the surface tension

$$\sigma=\int_{-\infty}^{\infty}[\mathcal{E}(\phi,\phi',V_0,V_0',\rho)-\mu\rho]dz, \tag{12}$$

with respect to the three degrees of freedom, i.e., the two meson field potentials ϕ and V_0, and the baryon density ρ,

$$\frac{\delta\mathcal{E}}{\delta\phi}=0, \quad \frac{\delta\mathcal{E}}{\delta V_0}=0, \quad \frac{\delta\mathcal{E}}{\delta\rho}=\frac{\partial\mathcal{E}}{\partial\rho}=\mu,$$

$$\frac{\delta}{\delta\chi_a}=\frac{\partial}{\partial\chi_a}-\frac{d}{dz}\frac{\partial}{\partial\chi_a'} \quad (\chi_a=\phi,V_0,\rho). \tag{13}$$

The Lagrangian parameter μ in this SINM case coincides with the Fermi energy in INM, and due to the Hugenholtz–van Hove theorem with the average energy per nucleon in INM. The variational derivatives occuring in Eqs. (13) lead to the field equations

$$\left(\frac{d^2}{dz^2}-m_s^2\right)\phi(z)=-g_s\rho_S(z)+b\phi^2(z)+c\phi^3(z), \tag{14}$$

$$\left(\frac{d^2}{dz^2}-m_v^2\right)V_0(z)=-g_v\rho(z), \tag{15}$$

and the proper RTF equation

$$g_vV_0(z)+\sqrt{k_F^2(z)+M^{\star 2}(z)}=\mu. \tag{16}$$

As in the case of any Thomas-Fermi approximation one is faced with the technical problem of boundary conditions.

Now, a *local* RTF energy-momentum tensor $T_{\mu\nu}$ can be defined following the general prescription (5) with a view to the fact that SINM is nonuniform in the z direction. For the ground state expectation value one obtains the expression

$$T_{\mu\nu}=\begin{bmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mu\rho-\mathcal{E} & 0 & 0 \\ 0 & 0 & \mu\rho-\mathcal{E} & 0 \\ 0 & 0 & 0 & \mu\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr} \end{bmatrix}. \tag{17}$$

This local SINM tensor $T_{\mu\nu}$ turns out to have vanishing nondiagonal elements as in the INM case. However, the structure of its diagonal elements differs from the INM case. In the surface region of SINM they are no longer equal, reflecting the nonuniformity or anisotropy of SINM.

The T_{33} element of the SINM stress tensor T_{ik} ($i=1,2,3$) is of special interest. Following Eq. (3) it is the force per unit area in the z direction in SINM, produced by matter and field. Explicitly written,

$$T_{33}(z)=\mu\rho(z)-\mathcal{E}(z)+\{[\phi'(z)]^2-[V_0'(z)]^2\}$$
$$=\mu\rho(z)-\mathcal{E}(z)+2\,\mathcal{E}_{gr}(z), \tag{18}$$

with

$$\mathcal{E}(z)\equiv\mathcal{E}(\phi(z),\phi'(z),V_0(z),V_0'(z),\rho(z)),$$

$$\mathcal{E}_{\rm gr}(z)\equiv\mathcal{E}_{\rm gr}(\phi'(z),V_0'(z))$$

[see Eq. (11)]. The first two terms in the expression for T_{33} are identical with the negative Swiatecki integrand entering into the definition (12) of the surface tension. In addition, there is the characteristic gradient term $2\,\mathcal{E}_{\rm gr}$.

Differentiating $T_{33}(z)$ with respect to z, one gets

$$T_{33}'=\mu\rho'-\frac{\partial\mathcal{E}}{\partial\chi_a}\chi_a'-\frac{\partial\mathcal{E}}{\partial\chi_a'}\chi_a''+\frac{d}{dz}\left(\chi_a'\frac{\partial\mathcal{E}}{\partial\chi_a'}\right)$$
$$=\mu\rho'-\chi_a'\frac{\delta\mathcal{E}}{\delta\chi_a'}, \tag{19}$$

with summation over the index $a=1,2,3$ and $\mathcal{E}$ independent of $\chi_3'=\rho'$. Taking into account the variational Eqs. (13) that define the SINM saturation state one can see that

$$T_{33}'(z)=0\Rightarrow T_{33}=\text{const} \quad \forall z, \tag{20}$$

and since $T_{33}(z)$ in the bulk interior of the saturated SINM system is zero, $T_{33}(z)$ vanishes identically. Because of this property, the stress tensor T_{ik} can be rewritten as

$$T_{ik}=\begin{bmatrix} -2\,\mathcal{E}_{\rm gr} & 0 & 0 \\ 0 & -2\,\mathcal{E}_{\rm gr} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{21}$$

The calculated local energy-momentum tensor therefore fulfils the differential conservation laws

$$\partial_\mu T^{\mu\nu}=0, \tag{22}$$

which in general come from the invariance of $\mathcal{L}$ under an infinitesimal space-time translation.

From the local SINM energy density (11) (identical with the T_{00} element) a *local* (thermodynamic) pressure can be obtained following the definition (1),

$$p_{\rm th}=\rho^2\frac{\delta}{\delta\rho}\left(\frac{\mathcal{E}}{\rho}\right), \tag{23}$$

with the variational derivative denoted by δ. Since $\mathcal{E}$, Eq. (11), does not depend on the gradient of the baryon density ρ and because of Eqs. (13), the expression for $p_{\rm th}$ is obtained explicitly by

$$p_{\rm th}=\rho^2\frac{\partial}{\partial\rho}\left(\frac{\mathcal{E}}{\rho}\right)=\mu\rho-\mathcal{E}. \tag{24}$$

The contributions from the $\mathcal{E}_{\rm gr}$ term are different in the expression (24) for $p_{\rm th}$ and in T_{33}, Eq. (18). Thus, the thermodynamic pressure in the surface region at a given point z is different from T_{33}. The value (24) will be nonzero in the surface region even in saturated SINM, whereas the bulk value for saturated SINM comes out to be zero.

There is no thermodynamic consistency between the two local RTF pressures in the surface region. In the field theoretical calculation leading to $T_{33}(z)$ the field contribution to the pressure is treated correctly. Also intuitively one expects the pressure T_{33} in saturated SINM to vanish everywhere. In any case, because of the anisotropy of T_{ik}, Eq. (17) the consistency of the scalar thermodynamic pressure and the hydrostatic pressure following from T_{ik} is no more a reasonable question. Therefore, we suppose that the hydrodynamic definition of a local pressure via the stress tensor is the adequate one.

B. Spherically symmetric nuclei

The RTF energy density of a spherical nucleus with mass number A in QHD-I follows from the T^A_{00} component of the energy-momentum tensor,

$$T^A_{00}=\mathcal{E}=\mathcal{E}_\infty(\phi,V_0,\rho)+\mathcal{E}_{\rm gr}(\phi',V_0'), \tag{25}$$

where $\mathcal{E}_{\rm gr}(\phi',V_0')=\frac{1}{2}[(\phi')^2-(V_0')^2]$, with the prime denoting derivation with respect to the r coordinate. The local Fermi momentum $k_F(r)$ and the function $g(a(r))$ are defined in correspondence to the SINM case Eqs. (8) – (10). The self-bound ground state of a spherical nucleus is defined by the minimum of its total energy under the constraint of a fixed nucleon number A, which is coupled to the energy density $\mathcal{E}$ with a Lagrange parameter μ_A, i.e.,

$$\delta\left\{4\pi\int_0^\infty[\mathcal{E}(r)-\mu_A\rho(r)]r^2dr\right\}=0. \tag{26}$$

The requirement of the vanishing of the functional derivatives in Eq. (26) with respect to the meson fields χ_i (i denoting σ or ω, respectively) and with respect to the baryon density ρ leads to the RTF equations for a nucleus,

$$\frac{\partial\mathcal{E}}{\partial\chi_i}-\frac{d}{dr}\frac{\partial\mathcal{E}}{\partial\chi_i'}-\frac{2}{r}\frac{\partial\mathcal{E}}{\partial\chi_i'}=0 \quad (i=1,2),$$

$$\frac{\partial\mathcal{E}}{\partial\rho}=\mu_A. \tag{27}$$

In contrast to the SINM case (14) and (15), there are now characteristic curvature terms in the equations for the fields, coming from the Laplacian written in spherical coordinates. The proper RTF equation (16) remains formally unchanged, with the chemical potential μ_A depending, however, on the nucleon number A.

The elements $T^A_{\mu\nu}$ of the energy-momentum tensor can be calculated in the standard way starting from Eqs. (2) and (5). In the Cartesian representation the energy-momentum tensor has the symmetrical form

$$T^A_{\mu\nu}=\begin{bmatrix}\mathcal{E} & 0 & 0 & 0\\ 0 & \mu_A\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr}\frac{x^2}{r^2} & 2\,\mathcal{E}_{\rm gr}\frac{xy}{r^2} & 2\,\mathcal{E}_{\rm gr}\frac{xz}{r^2}\\ 0 & 2\,\mathcal{E}_{\rm gr}\frac{xy}{r^2} & \mu_A\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr}\frac{y^2}{r^2} & 2\,\mathcal{E}_{\rm gr}\frac{yz}{r^2}\\ 0 & 2\,\mathcal{E}_{\rm gr}\frac{xz}{r^2} & 2\,\mathcal{E}_{\rm gr}\frac{yz}{r^2} & \mu_A\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr}\frac{z^2}{r^2}\end{bmatrix}. \tag{28}$$

The spatial part T^A_{ik} $(i,k=1,2,3)$, the relevant stress tensor, can be shortly written as

$$T^A_{ik}=-(\mu_A\rho-\mathcal{E})g_{ik}+2\,\mathcal{E}_{\rm gr}\frac{1}{r^2}x_ix_k. \tag{29}$$

Note that symmetries of the (electromagnetic) stress tensor and angular momentum conservation are interrelated (see e.g., [4]). Because of nondiagonal elements, T^A_{ik} is more complicated than the SINM tensor, which is obtained in the limit $A\to\infty$, setting $x=y=0$ and $r=|z|$. By inserting the field equations (27) into the relation

$$r^2\partial^iT^A_{ik}=\left[\mu_A\rho'(r)-\frac{d}{dr}\,\mathcal{E}(r)+\frac{d}{dr}\{2\mathcal{E}_{\rm gr}(r)\}\right]rx_k +4\mathcal{E}_{\rm gr}(r)x_k, \tag{30}$$

which we verified by a Maple program, it can be proved that the conservation laws (22) are valid for the energy-momentum tensor (28), too.

Starting from spherical coordinates (θ,ϕ,r), which are more appropriate for a spherical nucleus, one gets after some lengthy algebra the tensor T^A_{ab} in spherical representation as (the indices representing the spherical coordinates θ, ϕ, and r, respectively)

$$T^A_{ab}=\begin{bmatrix}\mu_A\rho-\mathcal{E} & 0 & 0\\ 0 & \mu_A\rho-\mathcal{E} & 0\\ 0 & 0 & \mu_A\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr}\end{bmatrix}. \tag{31}$$

It can be shown explicitly that $T^A_{rr}(r)$ cannot vanish identically since its derivative with respect to r is nonzero in general. Starting from

$$T^A_{rr}=\mu_A\rho-\mathcal{E}+2\,\mathcal{E}_{\rm gr}=\mu_A\rho-\mathcal{E}+\frac{\partial\mathcal{E}}{\partial\chi_a'}\chi_a', \tag{32}$$

differentation with respect to r leads to

$$\frac{d}{dr}T^A_{rr}=-\frac{4}{r}\mathcal{E}_{\rm gr} \tag{33}$$

[where the field equations (27) had to be used]. T^A_{rr} therefore cannot vanish identically as the corresponding SINM T_{zz} element, Eq. (21).

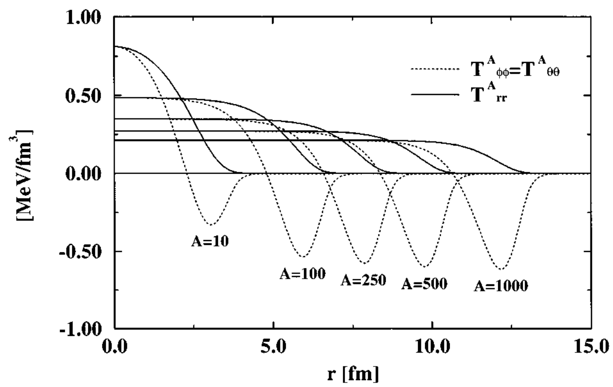

FIG. 1. The matrix elements T^A_{ab} of the stress tensor for some fictitious isospin-symmetric nuclei with mass numbers A as a function of the radial coordinate r. The NL1 parametrization [5] was used.

We have evaluated numerically in the RTF approach the elements T^A_{ab} of the spherical coordinate representation of the stress tensor for some spherical nuclei including large fictitious ones, using the linear parameter set introduced in Ref. [1] as well as the standard realistic parametrization NL1 Ref. [5] of the nuclear QHD-I Lagrangian. The results and conclusions obtained for the two parameter sets are qualitatively identical.

For the NL1 parametrization the stress tensor elements (31) are displayed in Fig. 1 as a function of the radial coordinate r for some nuclei with mass numbers A. The asymptotic behavior of the stress tensor for large mass number can be seen. All elements $T^A_{ab}(r=0)$ become equal. Thermodynamic consistency is reached in the bulk with the thermodynamic pressure equal to the hydrodynamic one. The surface values of $T^A_{rr}(\text{surf})$ approach zero in the large A limit as the bulk values do. Therefore the surface values of T_{zz} of SINM are obtained. Also $T_{\phi\phi}(\text{surf})=T_{\theta\theta}(\text{surf})$ approach nonzero values as SINM T_{xx} and T_{yy} do.

Obviously, there is a nonvanishing bulk pressure $T^A_{rr}(0)$ in finite nuclei. This reflects the intuitive picture of a pressure from the surface tension squeezing the bulk density. For a liquid droplet with a surface layer much smaller than the bulk part the bulk pressure p is given by the surface tension σ and the radius R of the droplet by the well known elementary relation (6). In Fig. 2 we compare the bulk values $T^A_{rr}(0)$ with this pressure approximating the nuclear radius R by

$$R(A)=r_0(A)A^{1/3}, \quad r_0(A)=\left(\frac{3}{4\pi\rho_c(A)}\right)^{1/3}, \qquad (34)$$

where r_0 is given in terms of the central density ρ_c. The RTF value of σ was found in Ref. [9] to be $\sigma=1.16$ MeV/fm^2. Nuclei with mass numbers A larger than about 50 fulfill Eq. (6) quite well. For smaller nuclei there is a deviation that follows from the fact that these nuclei are no more of saturated structure necessary for the derivation of Eq. (6).

In Fig. 3 the *density change* in the center of a finite nucleus with respect to the INM saturation value is plotted for the relativistic RTF calculations. In a strict droplet model (DM) picture this change can be easily expressed by the bulk

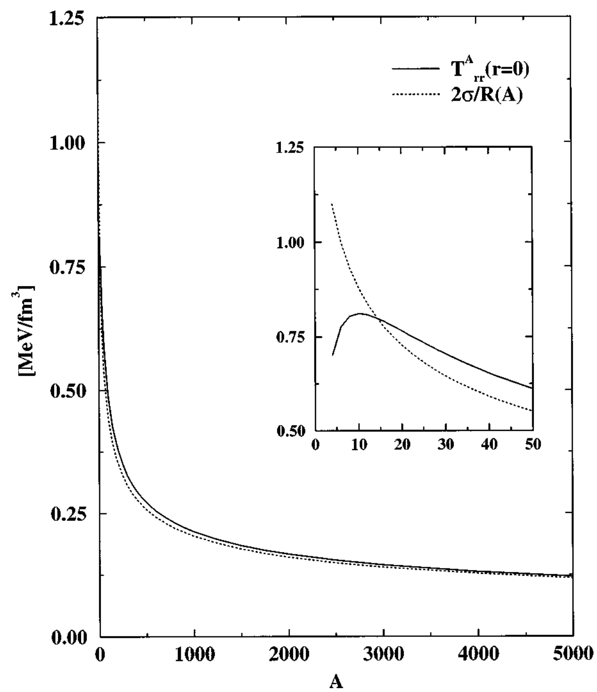

FIG. 2. The calculated RTF stress tensor element $T^A_{rr}(r=0)$ for the NL1 parametrization as a function of the mass numbers A of fictitious isospin-symmetric nuclei compared to the liquid drop expression for the central pressure $p=2\sigma/R(A)$.

compressibility modulus and the surface tension that is stationary around saturation density ρ_0 with respect to changes of the central density (see, e.g., Ref. [6]). Using a nonrelativistic Skyrme approach, the effect was studied extensively in Ref. [7], also for small nuclei. For a pure DM picture the straight curve in Fig. 3 is obtained in our case.

Thus, the large A part of the curve in Fig. 3 for the reduced change of the central density is explained as coming from this compression effect. Smaller nuclei stretch themselves in order to get the finite-range nucleon-nucleon potential in an optimal way. As a consequence their central density falls below the INM saturation value. This desaturation effect has been studied in conventional nuclear structure theory in Refs. [7,8].

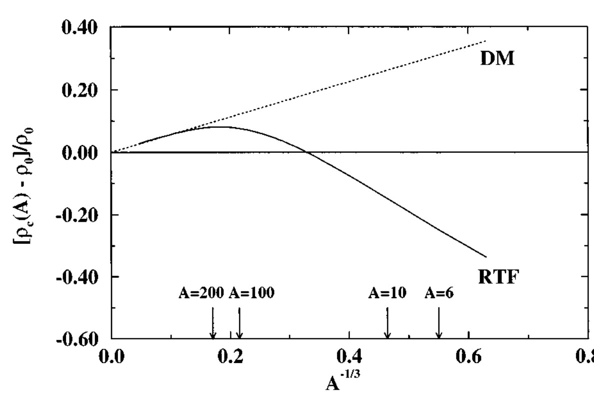

FIG. 3. The RTF relative density change at the center of nuclei with mass numbers A with respect to saturated INM as a function of $A^{-1/3}$. The dotted straight line is the droplet model value. The NL1 parametrization was used.

III. OUTLOOK

We have studied the local energy-momentum tensor for self-bound SINM as well as for finite spherical nuclei starting from the RTF approximation to the QHD-I model. In order to treat the effect of external fields compressing SINM or finite nuclei, respectively, in nonrelativistic nuclear structure theory constraints depending on the nuclear density were introduced (see, e.g., Refs. [10]). In relativistic approaches such as RTF one could analogously add to the formalism external constraints constructed in such a way that they reflect the specific external influence that leads to nuclear compression. Compression effects could be produced by external baryonic as well as mesonic effects. Thus, the external constraint could depend on the baryon as well as on the meson fields. In particular, the dependence of the nuclear surface tension on the compression of the baryonic density could be studied. It is well known in nonrelativistic nuclear structure theory (see, e.g., Refs. [6,10]) that the surface tension of isospin-symmetric SINM is stationary with respect to density changes around the saturation value. This so-called $\dot{\sigma}=0$ theorem had an impact on the theoretical foundation of semiempirical mass formulas since it facilitates the calculation of the expansion coefficients [6].

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