Systematic study of spin assignments and signature inversion of $\pi h_{11/2}\otimes\nu h_{11/2}$ bands in doubly odd nuclei around $A\sim130$

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According to the argument of excitation energy systematics, new spin values are assigned to the lowest observed states of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands for doubly odd nuclei ^{124,126,130,132,134}La, $^{130-134}$Pr, ^{136,138}Pm, and ^{138}Eu, and possible new spin values are discussed for doubly odd nuclei $^{120-130}$Cs. Based on these new spin assignments, two systematic features of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands are revealed. First, the low spin signature of the $\pi h_{11/2}\otimes\nu h_{11/2}$ band is inverted for all doubly odd nuclei discussed in the present work. Second, the observed inversion spin increases with increasing neutron number for Cs and La isotopes. These features are in agreement with the recent systematic calculations based on the particle-triaxial-rotor model, with the inclusion of zero-range residual interaction between unpaired proton and neutron of Semmes and Ragnarsson, performed by Tajima. [S0556-2813(96)01907-3]

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I. INTRODUCTION

In the systematic study on the signature inversion of the $\pi h_{11/2}\otimes\nu i_{13/2}$ bands of deformed doubly odd nuclei in the mass region of $A\sim160$ [1], it was shown that the low-spin signature is inverted for all observed $\pi h_{11/2}\otimes\nu i_{13/2}$ bands. However, the situation is quite different for the mass region of $A\sim130$ where the low-spin signature of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands is not inverted in most cases as reported in the references cited in the later part of this paper. Is this a real phenomenon or was it caused by incorrect spin (I_0) assignment of the lowest observed state of the $\pi h_{11/2}\otimes\nu h_{11/2}$ band? In most cases of this mass region, the I_0's were assigned tentatively and to change the I_0 by an odd number (say, $\Delta I=\pm1$, $\pm3,\ldots$) will make the signature inverted from normal to abnormal or the reverse. The purpose of the present work is intended to clarify this problem through a systematic study on the spin assignments of the lowest observed states of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands in doubly odd nuclei around $A\sim130$. In the cases of $A\sim160$ [1], we were able to pick out the questionable spin assignments and correct them by using three arguments. However, it turned out that the argument of excitation energy systematics was the decisive one, and which can be described as following: The excitation energy of the

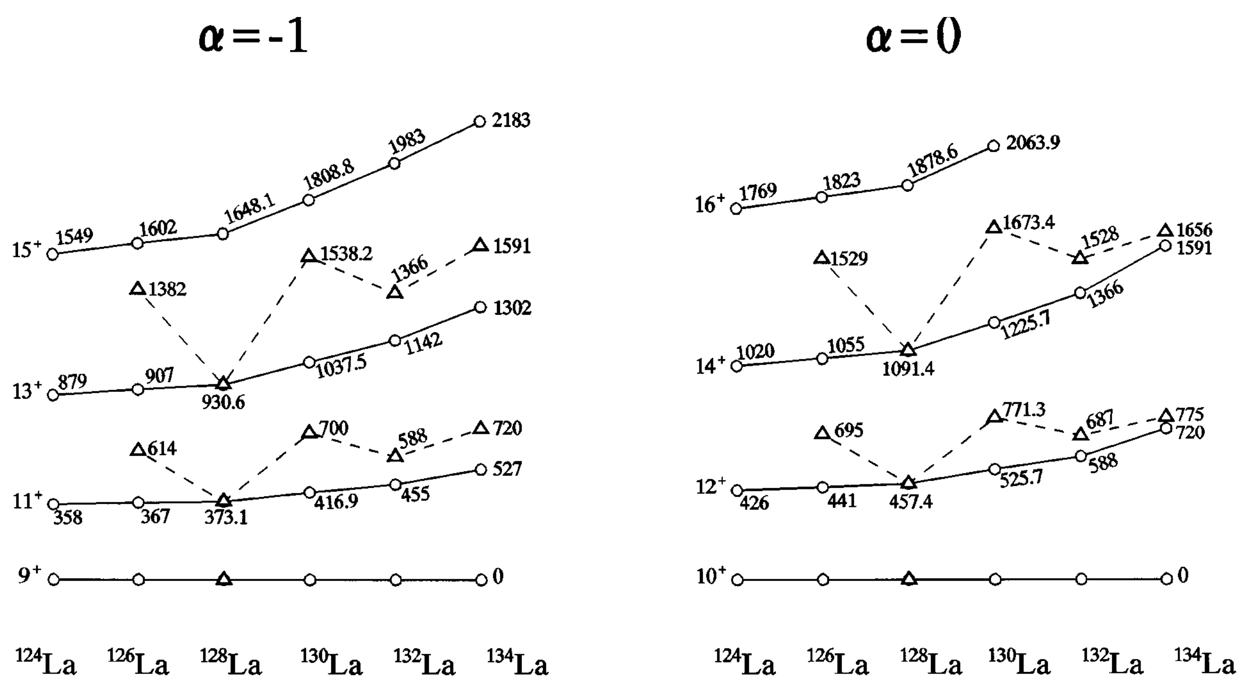

FIG. 1. Excitation energy systematics of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands in $^{124-134}$La. Symbols △ and ○ represent the level positions based on previous and present spin assignments, respectively. Energies indicating level positions are in keV. Data sources are ^{124}La [7], ^{126}La [3], ^{128}La [2,4], ^{130}La [4], ^{132}La [5], and ^{134}La [6].

TABLE I. Lists of previous and present I_0's for the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in La, Pr, Pm, and Eu isotopes, and comparisons of observed and calculated features of signature inversion.

Nuclei and references	I_0 ($\hbar$)		I_{inv} ($\hbar$)			Low-spin signature inverted		
	Previous	Present	Previous	Present	Tajima[a] [21]	Previous	Present	Tajima[b] [21]
^{124}La [7]	—	7	—	18.5	12.0	—	Yes	Yes
^{126}La [3]	4	7	17.5	21.5	13.5	No	Yes	Yes
^{128}La [2]	5	5	⩾23.5	⩾23.5	15.0	Yes	Yes	Yes
^{130}La [4]	6	9	—	—	18.0	No	Yes	Yes
^{132}La [5]	8	9	—	—	>21	No	Yes	Yes
^{134}La [6]	8	9	—	—	—	No	Yes	—
^{130}Pr [8]	8	7	—	—	—	No	Yes	—
^{132}Pr [9]	8	9	17.5	18.5	—	No	Yes	—
^{134}Pr [10]	8	9	16.5	17.5	18.5	No	Yes	Yes
^{136}Pr [6]	8	9	—	—	—	No	Yes	—
^{134}Pm [11]	8	8	17.5	17.5	—	Yes	Yes	—
^{136}Pm [12]	8	9	—	—	15.0	No	Yes	Yes
^{138}Pm [13]	8	9	—	—	—	No	Yes	—
^{138}Eu [14]	8	9	—	—	—	No	Yes	—

[a]Estimated from Figs. 10 and 11 of [21] with uncertainties $\pm 0.5\hbar$.
[b]Seen from Figs. 10 and 11 of [21].

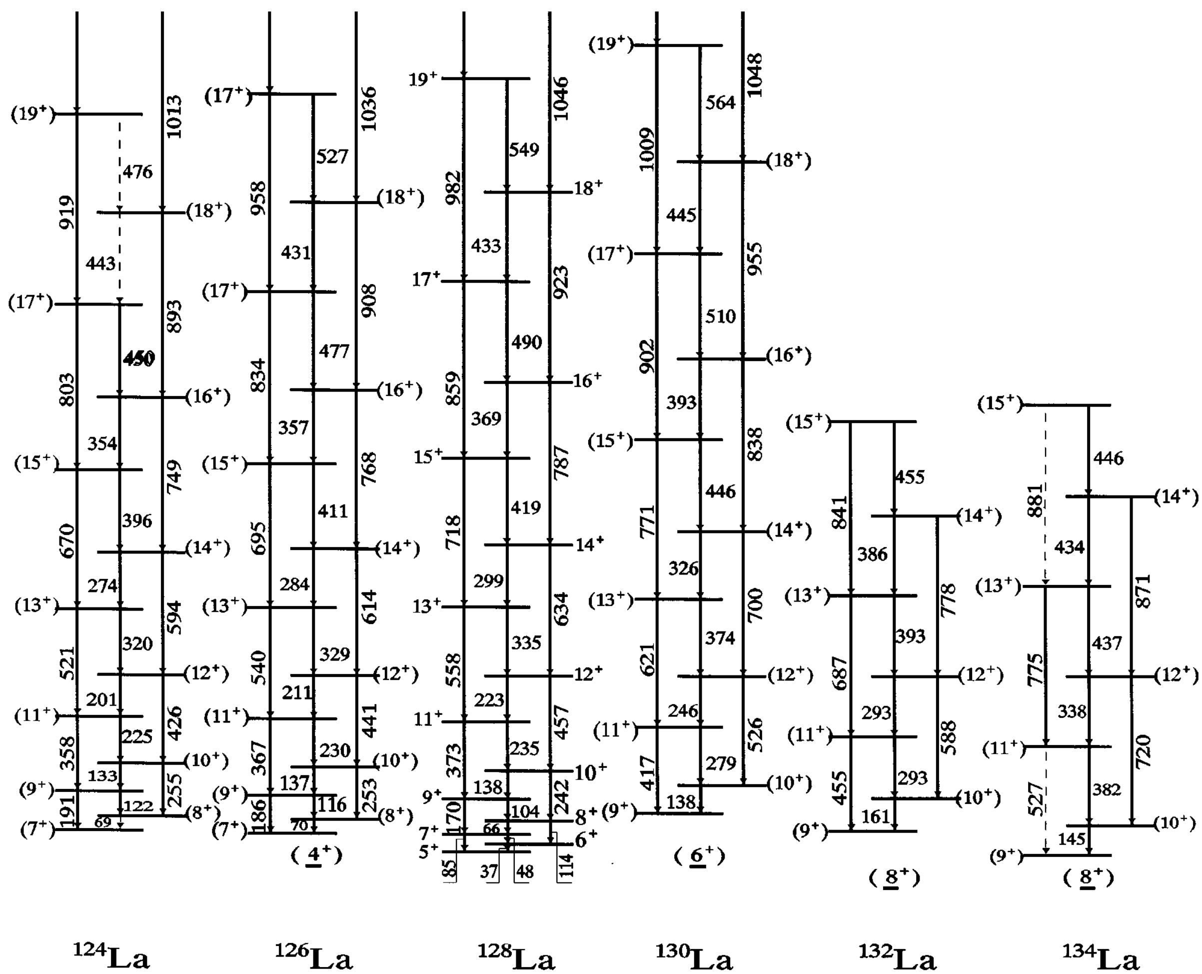

FIG. 2. Level schemes with present spin assignments of $^{124-134}$La. The underlined numbers in the parentheses are the previous spin values of the lowest observed states. Data sources are the same as those of Fig. 1.

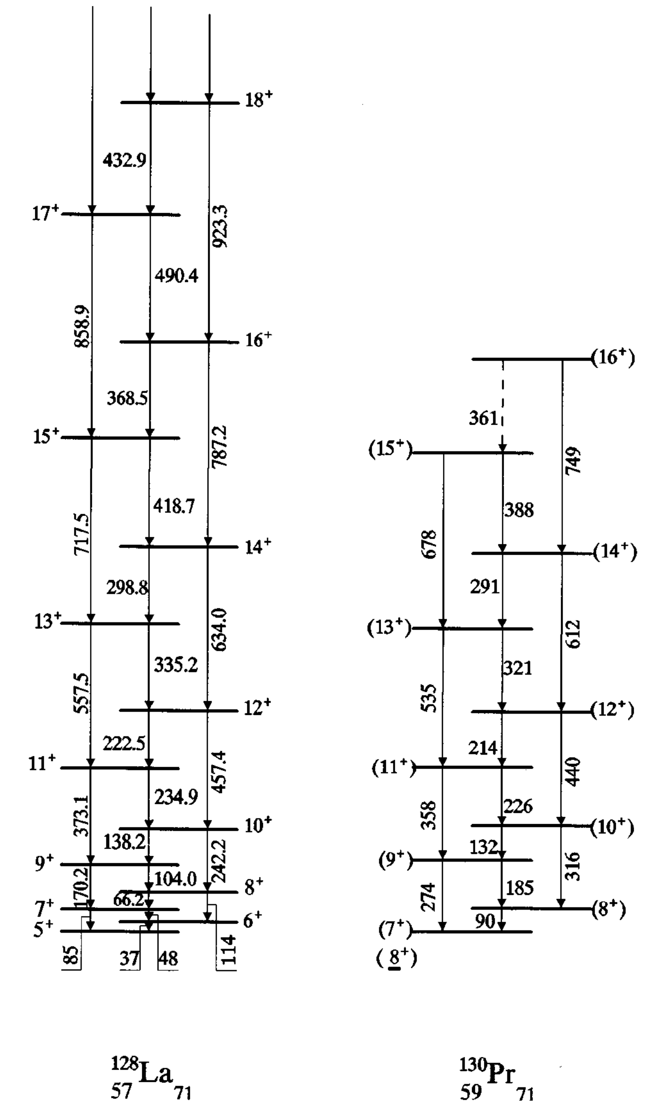

FIG. 3. Level schemes for $N=71$. Data sources: ^{128}La [2,4], ^{130}Pr[8].

levels, with the same spin, of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands in a chain of deformed doubly odd isotopes (isotones) varies with neutron number (proton number) in a smooth way, and a deviation from the smooth variation trend may imply a questionable spin assignment and the spin assignment which can remove the deviation from the smooth curve is considered to be the correct spin assignment. It is natural to assume that this argument can also be applied to the spin assignments of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands of the deformed doubly odd nuclei in the mass region of $A\sim130$.

II. SPIN ASSIGNMENTS OF La ISOTOPES

Among the La isotopes discussed in the present work, ^{128}La is the only case where the spin (I_0) of the lowest observed state of the $\pi h_{11/2}\otimes\nu h_{11/2}$ band was determined through experimental spectroscopy and was given without bracket [2]. The I_0's of ^{126}La [3], ^{130}La [4], ^{132}La [5], and

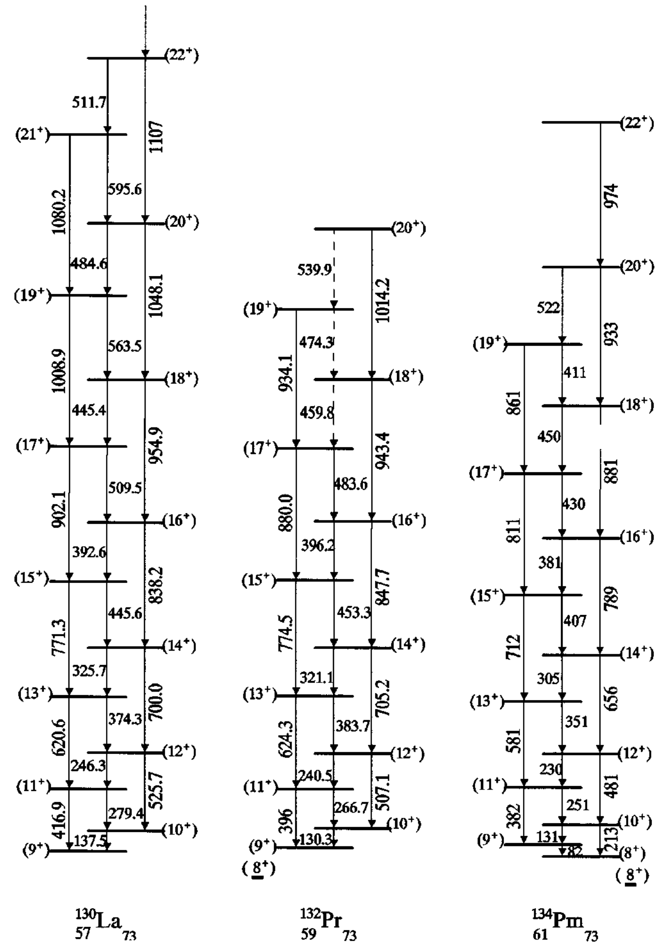

FIG. 4. Level schemes for $N=73$. Data sources: ^{130}La [4], ^{132}Pr [9], ^{134}Pm [11].

^{134}La [6] were assigned tentatively while that of ^{124}La [7] was not assigned previously. Based on the previously I_0, the positions of levels with $I=11$ and 13 relative to those of $I=9$, and the positions of levels with $I=12$ and 14 relative to those of $I=10$ are indicated in Fig. 1 with triangles. These triangles are not situated on slow varying smooth curves. According to the argument of excitation energy systematics, if the I_0's of La isotopes were correctly assigned, the positions of levels with the same spin should fall on a slow varying smooth curve. In order to bring the level positions, designated by triangles, back to smooth curves, and thereby to obtain the correct I_0's, previous I_0's, of some of the La isotopes have to be adjusted. Under the assumption that the spin assignment $I_0=5$ for ^{128}La [2] is correct, the I_0 of ^{128}La is kept unchanged while adjusting I_0's of other isotopes. This means that the smooth curves are required to pass through the level positions of ^{128}La with $I_0=5$. To simplify the later descriptions, we will say that the level scheme of ^{128}La with $I_0=5$ is taken as the reference in the systematic study. After adjustments, the previous I_0's were changed by $+3$, $+3$, $+1$, and $+1$, and the new I_0's of 7, 9, 9, and 9 are assigned to ^{126,130,132,134}La instead of the previous I_0's of 4 [3], 6 [4], 8 [5], and 8 [6], respectively, and the level positions based on the new I_0's are indicated in Fig. 1 with open circles which fall on smooth curves passing through the triangles of ^{128}La

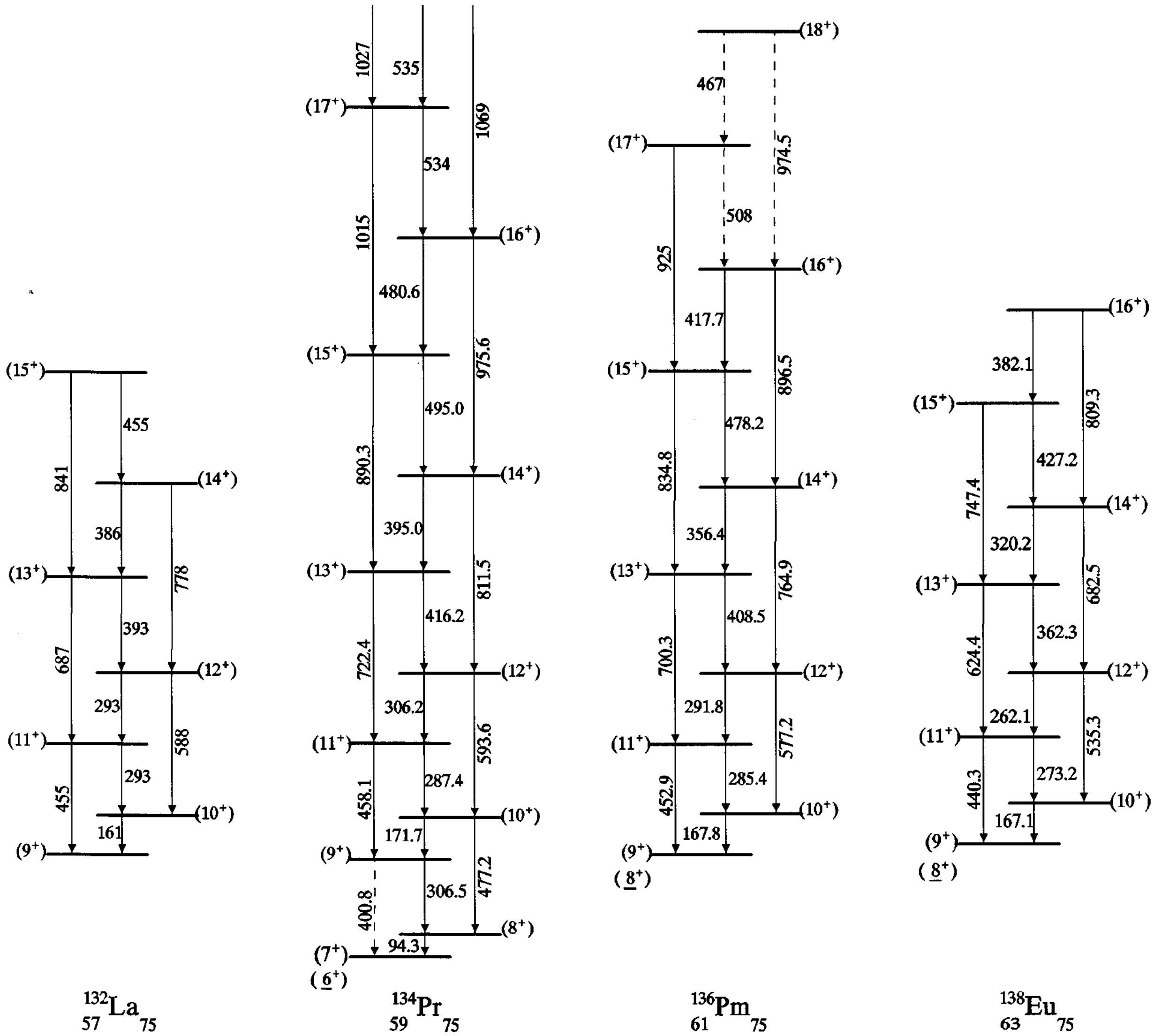

FIG. 5. Level schemes for $N=75$. Data sources: ^{132}La [5], ^{134}Pr [10], ^{136}Pm [12], ^{138}Eu [14].

with $I_0=5$. By assuming $I_0=7$ for the previously not assigned ^{124}La [7], the level positions of ^{124}La fit to the smooth curves very well. Therefore $I_0=7$ is assigned to ^{124}La. The previous and present I_0's are listed in Table I.

The level schemes of the $\pi h_{11/2}\otimes \nu h_{11/2}$ bands of ^{124,126,130,132,134}La, based on the new I_0's obtained from the present systematic study, are presented in Fig. 2 together with that of ^{128}La [2]. The similarity of the level schemes of the La isotopes and the slow varying tendency of the $\Delta I=2$ level spacings revealed in Fig. 2 are consistent with the slow varying smooth curves of Fig. 1, and these systematic features of the level schemes of La isotopes would not be revealed on the basis of the previous I_0's which are indicated at the bottom of the corresponding level scheme by the underlined digit within the bracket. The revelation of the systematic features of the level schemes based on new I_0's may be taken as a support to the I_0's obtained from the excitation energy systematics.

The new I_0's of La isotopes were assigned by assuming that the $I_0=5$ for ^{128}La [2] is correct and its level scheme was taken as reference in the systematic study. If the future experiments show that the spin assignment $I_0=5$ for ^{128}La is incorrect and it has to be changed from $I_0=5$ to $I_0=5+\Delta I$, then in order to obtain the corresponding correct spins for the other La isotopes, what we need to do is to add ΔI to the I_0's of other La isotopes obtained in the present systematic study. This is because that to add the same ΔI to all I_0's of all La isotopes is equivalent to changing the zero-energy reference level in Fig. 1 from 9 to $9+\Delta I$ for $\alpha=1$ (from 10 to $10+\Delta I$ for $\alpha=0$) and this will not effect the shape of the smooth curves.

III. SPIN ASSIGNMENTS OF $^{130-136}$Pr, $^{134-138}$Pm, AND ^{138}Eu

Once the spins of La isotopes were assigned, the I_0's of $^{130-136}$Pr, $^{134-138}$Pm, and ^{138}Eu can be assigned according to the similarity between the level schemes of La and those of

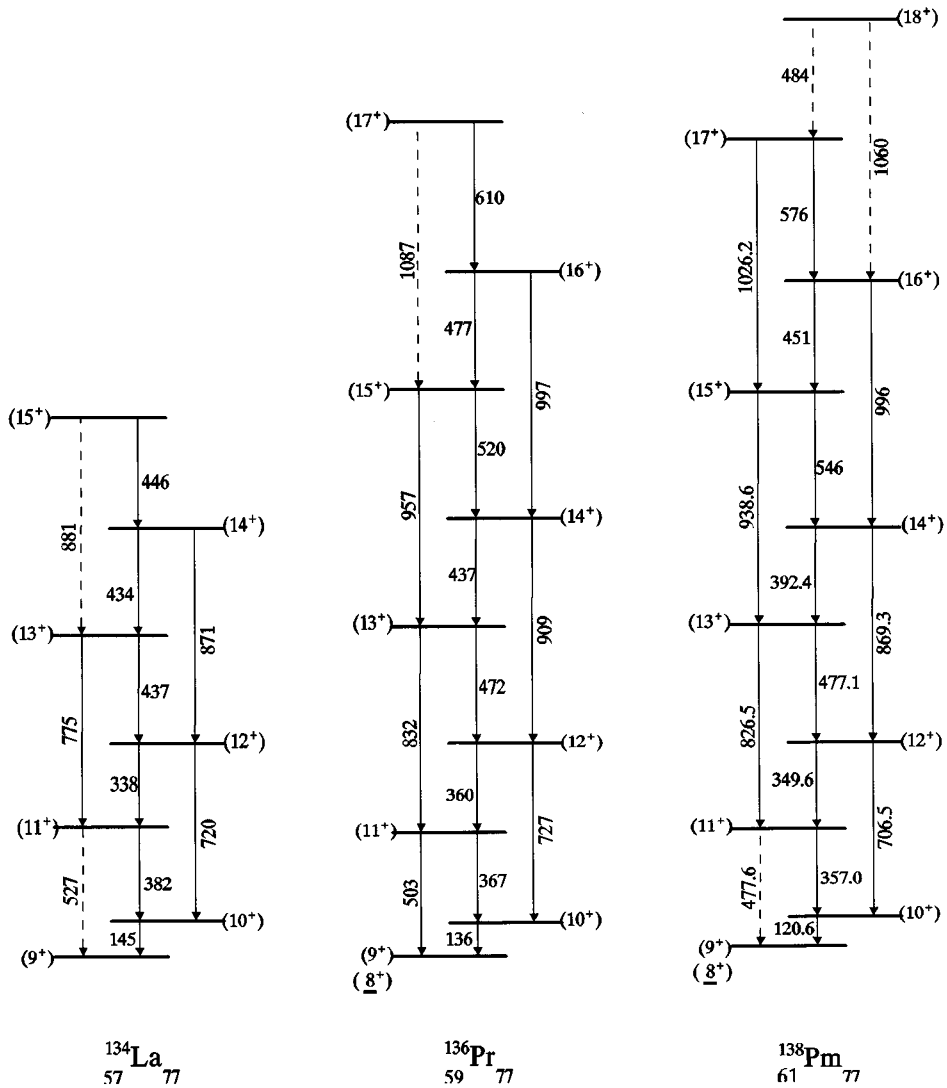

FIG. 6. Level schemes for N=77. Data sources: ^{134}La[6], ^{136}Pr [6], ^{138}Pm [13].

Pr, Pm, and Eu within the isotonic chain. Level schemes of the isotonic chains with N=71, 73, 75, and 77 are shown in Figs. 3–6, respectively where the I_0's of Pr, Pm, and Eu have been assigned on the basis of level scheme similarity. The previous I_0's are indicated at the bottom of the corresponding level scheme by an underlined digit within the bracket. Figures 7 and 8 present the excitation energy systematics of Pr and Pm isotopes based on the new I_0's. The positions of levels with the same spin fit to the smooth curves very well. This can be considered as a support to the new spin assignments obtained from the level scheme similarity.

In summary, the I_0's of the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in ^{130}Pr [8], ^{132}Pr [9], ^{134}Pr [10], ^{136}Pr [6], ^{134}Pm [11], ^{136}Pm [12], ^{138}Pm [13], and ^{138}Eu [14] were previously all assigned as 8, and as the results of the comparison of the level schemes between La and other members of the isotonic chains with N=71, 73, 75, and 77, the previous I_0's were changed by −1, +1, +1, +1, +1, +1, and +1, and new I_0's of 7, 9, 9, 9, 9, 9, and 9 are assigned to ^{130}Pr, ^{132}Pr, ^{134}Pr, ^{136}Pr, ^{136}Pm, ^{138}Pm, and ^{138}Eu, respectively, while the I_0 of ^{134}Pm was not changed. The previous and present I_0's of these nuclei are listed in Table I.

IV. SPIN ASSIGNMENT OF Cs ISOTOPES

The I_0's of the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands of ^{120,122,124,126,128,130}Cs were previously assigned as 8 [15], 6 [16], 7 [7], 5 [7], 9 [18], and 9 [19], respectively. Among them, the I_0's of ^{124}Cs, ^{126}Cs, and ^{130}Cs were assigned through experimental spectroscopy. Similar to the case of La isotopes as discussed in Sec. II, the excitation energy systematics of Cs isotopes is shown in Fig. 9 where the level scheme of ^{124}Cs with I_0=7 [7] was taken as reference, i.e., the slow varying smooth curves were required to pass through the level positions of ^{124}Cs with I_0=7. The level positions based on the previous I_0's, designated by triangles, do not fall on smooth curves. To bring the level positions back to smooth curves, thereby to obtain the correct new I_0's, the previous I_0's are changed by −2, +1, +2, −1, and −2, and new I_0's of 6, 7, 7, 8, and 7 are assigned to ^{120,122,126,128,130}Cs instead of the previous I_0's of 8, 6, 5, 9, and 9 as listed in Table II, respectively. Similarly, if we assume that the previous spin assignment of I_0=9 for ^{130}Cs [19], as shown in Fig. 10(a), is correct, then from the excitation energy systematics with ^{130}Cs (I_0=9) as reference, as shown in Fig. 11, the new I_0's of 8, 9, 9, 9, and 10 are

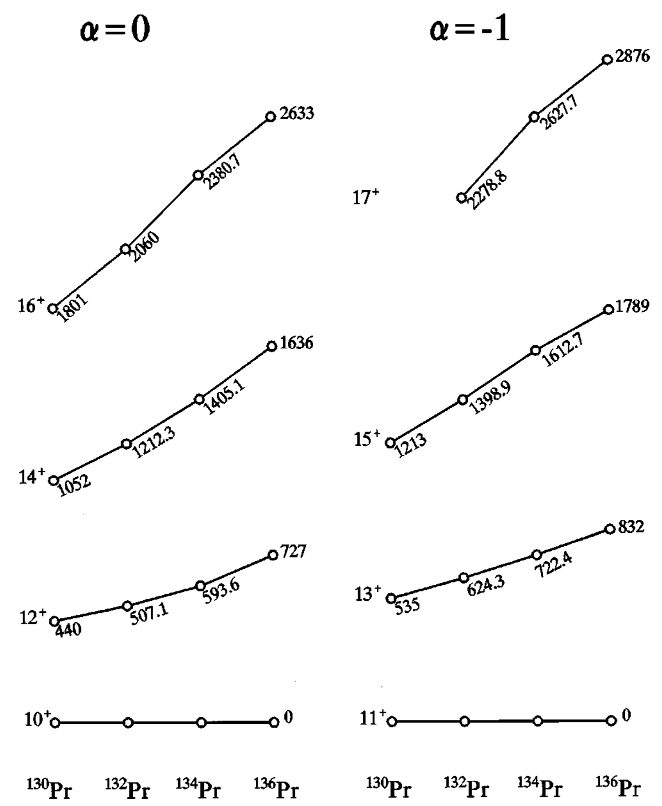

FIG. 7. Excitation energy systematics for $^{130-136}$Pr based on present I_0's. Data sources: ^{130}Pr [8], ^{132}Pr [9], ^{134}Pr [10], ^{136}Pr [6].

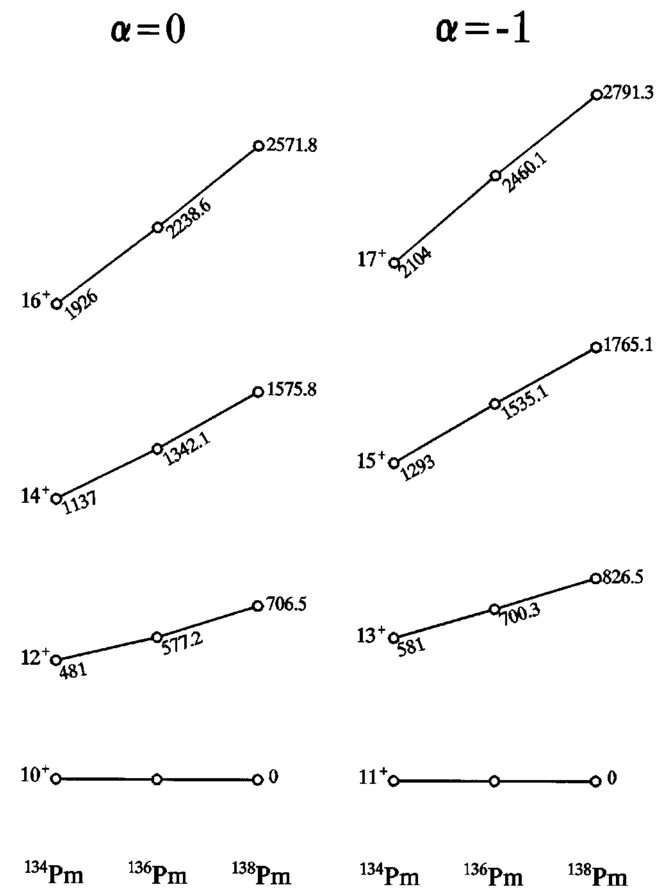

FIG. 8. Excitation energy systematic for $^{134-138}$Pm based on present I_0's. Data sources: ^{134}Pm [11], ^{136}Pm [12], ^{138}Pm [13].

assigned to ^{120,122,124,126,128}Cs, respectively, as listed in Table II.

These two sets of I_0's are different and we are not able, at present, to judge which of them is really correct. We have failed to find reasonable slow varying smooth curves which can pass through the level positions of ^{124}Cs ($I_0=7$) and those of ^{130}Cs ($I_0=9$) at the same time. This implies that the spin assignment of $I_0=7$ for ^{124}Cs and $I_0=9$ for ^{130}Cs cannot both be correct.

To meet the need of later discussions, the I_0's of Cs isotopes were also deduced by taking the level scheme of ^{130}Cs ($I_0=11$), as shown in Fig. 10(b), as reference and the corresponding new I_0's are listed in Table II.

On the other hand, we can also discuss the spin assignments of Cs isotopes by means of the excitation energy systematics of isotonic chains. Figure 12 shows the excitation

TABLE II. Lists of previous I_0's and possible choices of present I_0's for the $\pi h_{11/2} \otimes \nu h_{11/2}$ bands in $^{120-130}$Cs, and comparisons of observed and calculated features of signature inversion.

Nuclei and references	I_0 ($\hbar$)				I_{inv} ($\hbar$) inversion spin					Low-spin signature inverted		
	Previous	Present			Previous	Present			Tajima[a] [21]	Previous	Present	Tajima[b] [21]
		^{124}Cs ($I_0=7$)	^{130}Cs ($I_0=9$)	^{130}Cs ($I_0=11$)		^{124}Cs ($I_0=7$)	^{130}Cs ($I_0=9$)	^{130}Cs ($I_0=11$)				
^{120}Cs [15]	8	6	8	10	16.5	14.5	16.5	18.5	14.5	Yes	Yes	Yes
^{122}Cs [16]	6	7	9	11	14.5	15.5	17.5	19.5	—	No	Yes	—
^{124}Cs [7]	7	7	9	11	16.5	16.5	18.5	20.5	16.0	Yes	Yes	Yes
^{126}Cs [7]	5	7	9	11	17.5	19.5	21.5	23.5	19.0	Yes	Yes	Yes
^{128}Cs [18]	9	8	10	12					20.5	No	Yes	Yes
^{130}Cs [19]	9	7	9	11					—	Yes	Yes	—

[a]Estimated from Fig. 9 of [21] with uncertainties $\pm 0.5\hbar$.

[b]Seen from Fig. 9 of [21].

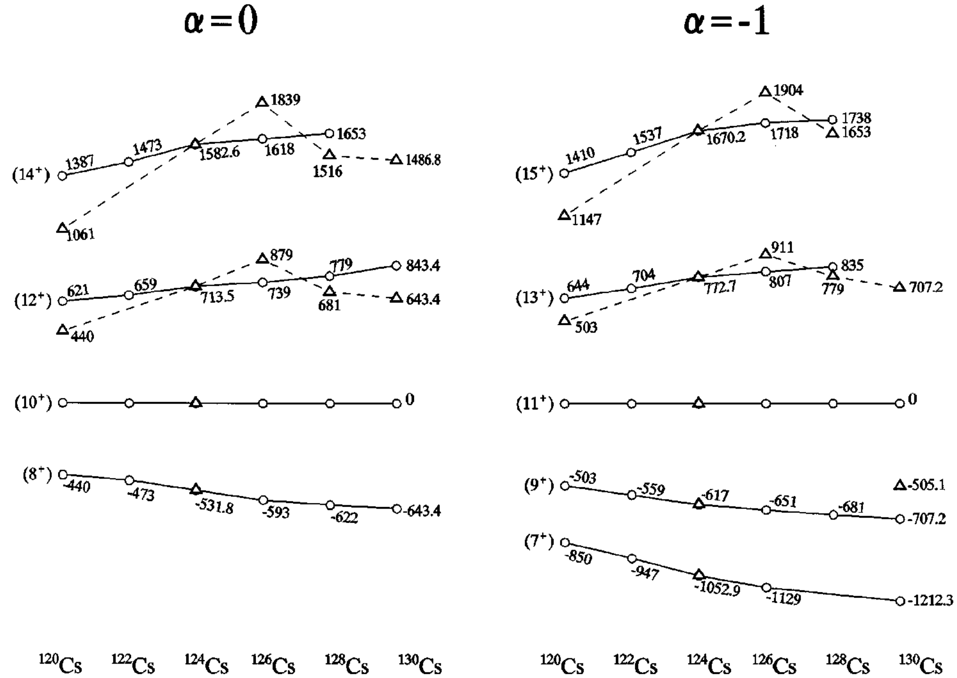

FIG. 9. Excitation energy systematics for $^{120-130}$Cs with ^{124}Cs ($I_0=7$) as reference. Data sources: ^{120}Cs [15], ^{122}Cs [16], ^{124}Cs [7,17], ^{126}Cs [7], ^{128}Cs [18], ^{130}Cs [19].

energy systematics of the isotonic chain with $N=73$ where the level positions of ^{130}La, ^{132}Pr, and ^{134}Pm are based on I_0's assigned in Secs. II and III as listed in Table I, and, for ^{128}Cs, the level positions designated by a, b, and c are based on the level schemes of ^{128}Cs with $I_0=8$, 10, and 12 deduced from the systematics by taking ^{124}Cs ($I_0=7$), ^{130}Cs ($I_0=9$), and ^{130}Cs ($I_0=11$) as reference respectively as seen in Table II. The level positions of ^{128}Cs with $I_0=6$, which would be deduced by taking ^{126}Cs ($I_0=5$) as reference in the systematic study of Cs isotopes, will be even higher than the level positions designated by a in Fig. 12. Judged by the variation trend of the curves of Fig. 12, this possibility is unlikely. This is why the I_0's of Cs isotopes deduced by taking ^{126}Cs ($I_0=5$) as reference, are not listed in Table II. Curves similar to that of Fig. 12 exist for isotonic chains with $N=75$ and 77. Figure 13 shows the systematics of isotonic chain with $N=75$ where the level positions of ^{132}La, ^{134}Pr, and ^{136}Pm are based on I_0's assigned in Secs. II and III, and, for ^{130}Cs, the upper circles are based on level scheme of ^{130}Cs with $I_0=11$ as shown in Fig. 10(b), and because only a small number of levels were observed in ^{130}Cs, the level positions of ^{130}Cs with $I_0=7$ [deduced by taking ^{124}Cs ($I_0=7$) as reference] do not show up in Fig. 13.

Because there are not enough points on the smooth curves of Figs. 12 and 13, we are not able to judge definitely which variation trend (from La to Cs) of the smooth curves is the correct one, that is to say the systematics of isotonic chains cannot help us to decide which reference [^{124}Cs ($I_0=7$), ^{130}Cs ($I_0=9$), or ^{130}Cs ($I_0=11$)] should be taken in the systematic study of Cs isotopes, or which spin assignment is really correct. However, to facilitate the following discussions, we tentatively assume that $I_0=11$ is the correct spin assignment for ^{130}Cs and based on this tentative assumption the level schemes of $^{120-128}$Cs are shown in Fig. 14 and that of ^{130}Cs ($I_0=11$) is shown in Fig. 10(b). If the future experiments confirm that the spin assignment of $I_0=9$ for ^{130}Cs is correct, what we need to do is to change all the spin values in Fig. 14 from I to $I-2$. If $I_0=7$ for ^{124}Cs is confirmed to be correct, then all spin values in Fig. 14 have to be changed from I to $I-4$.

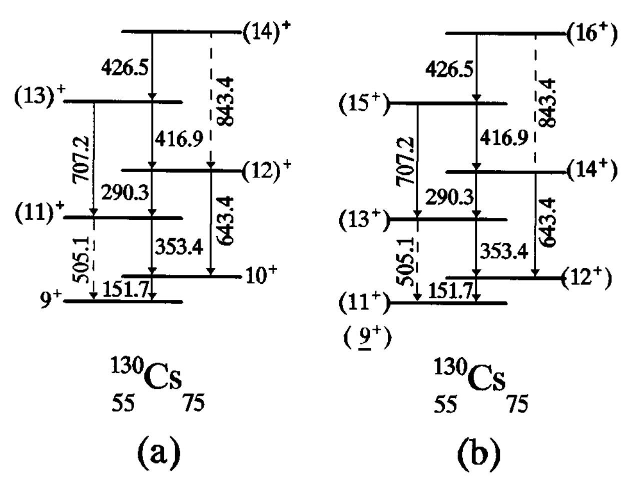

FIG. 10. Level scheme of ^{130}Cs with $I_0=9$ (a). Level scheme of ^{130}Cs with $I_0=11$ (b). Data source: [19].

V. COMMENTS AND REMARKS ON SPIN ASSIGNMENTS

A. Comment on the spin assignments of $^{124-134}$La, $^{130-136}$Pr, $^{134-138}$Pm, and ^{138}Eu

As can be seen from Table I, the I_0's of 13 nuclei have previously been assigned. For 11 of them, their previous I_0's are changed and the values of changes [$\Delta I=I_0$ (previous)$-I_0$ (present)] all are odd numbers. These changes lead to the changes of signature dependence from normal to inverted. One would not be surprised by these results if the following facts are noticed: First, the signature inversion point has been observed in several cases which implies that

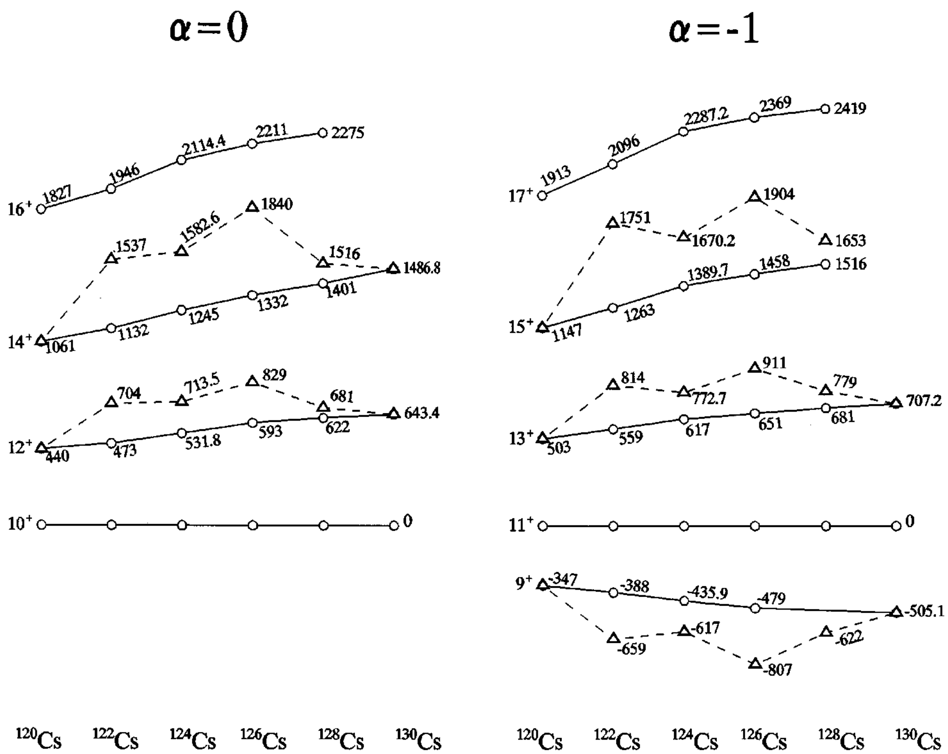

FIG. 11. Excitation energy systematics for $^{120-130}$Cs with ^{130}Cs (I_0=9) as reference. Data sources: Same as Fig. 9.

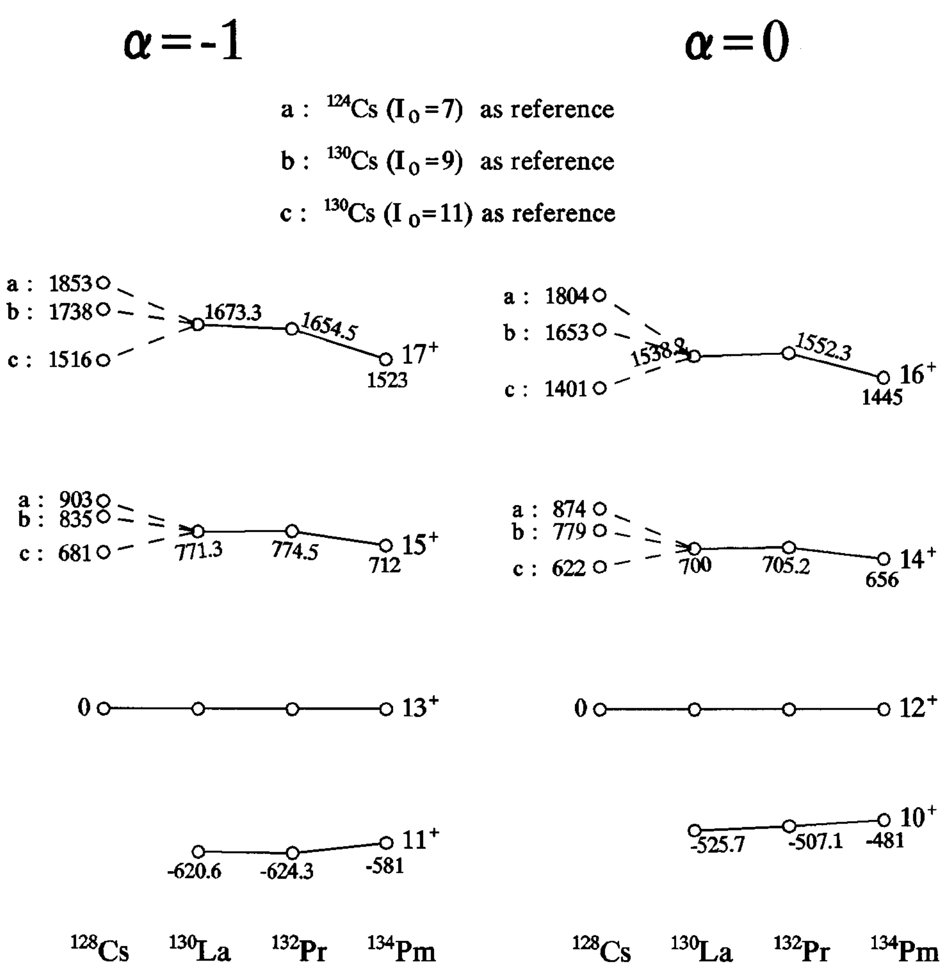

FIG. 12. Excitation energy systematics for $N=73$ isotonic chain. Data sources: ^{128}Cs [18], others see Fig. 4.

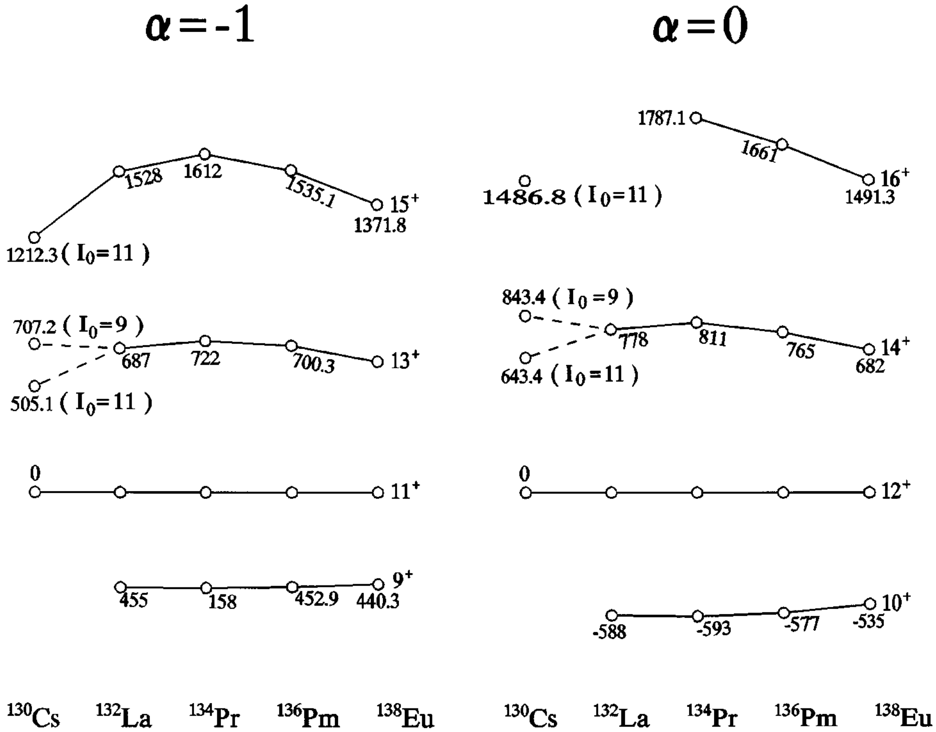

FIG. 13. Excitation energy systematics for $N=75$ isotonic chain. Data sources: same as Figs. 5 and 10.

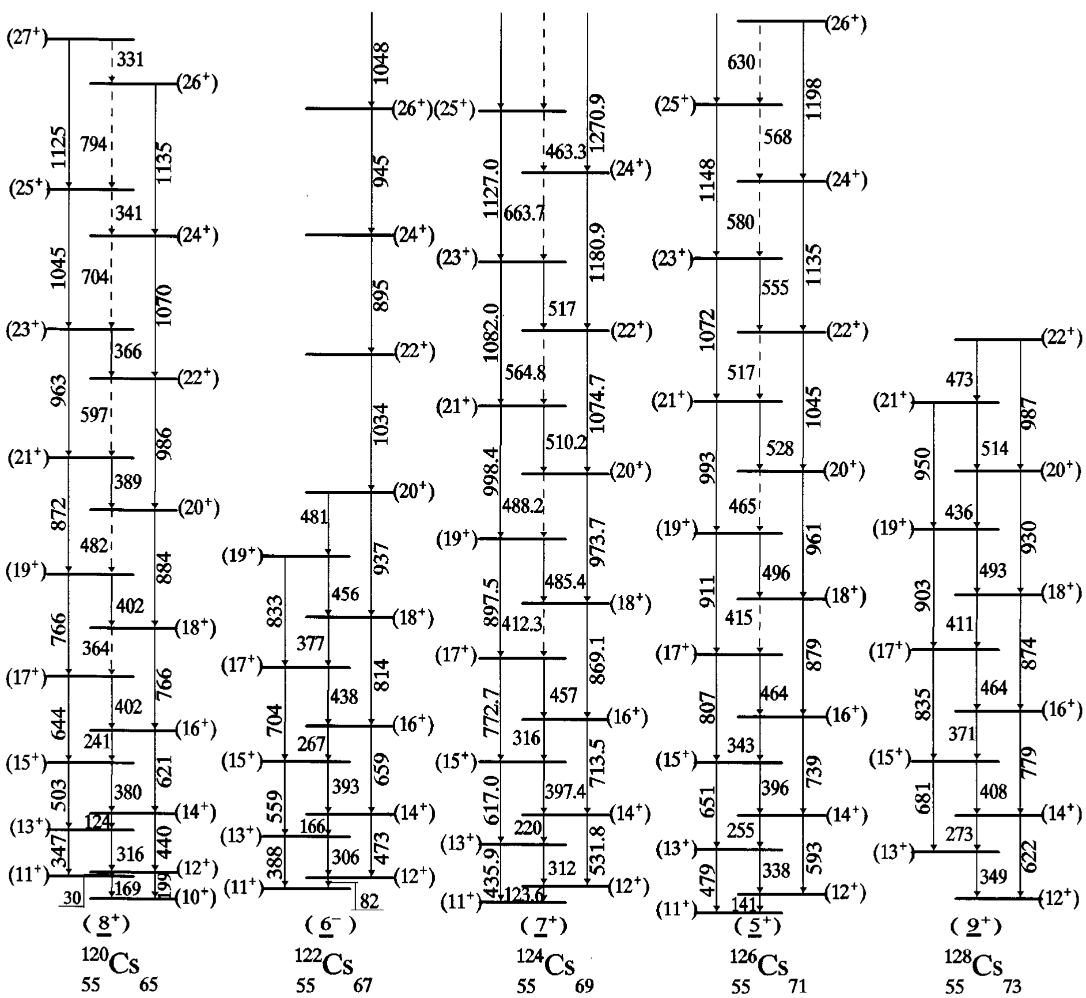

FIG. 14. Level schemes of $^{120-128}$Cs with I_0's assigned by taking ^{130}Cs ($I_0=11$) as reference. Data sources: same as Fig. 9.

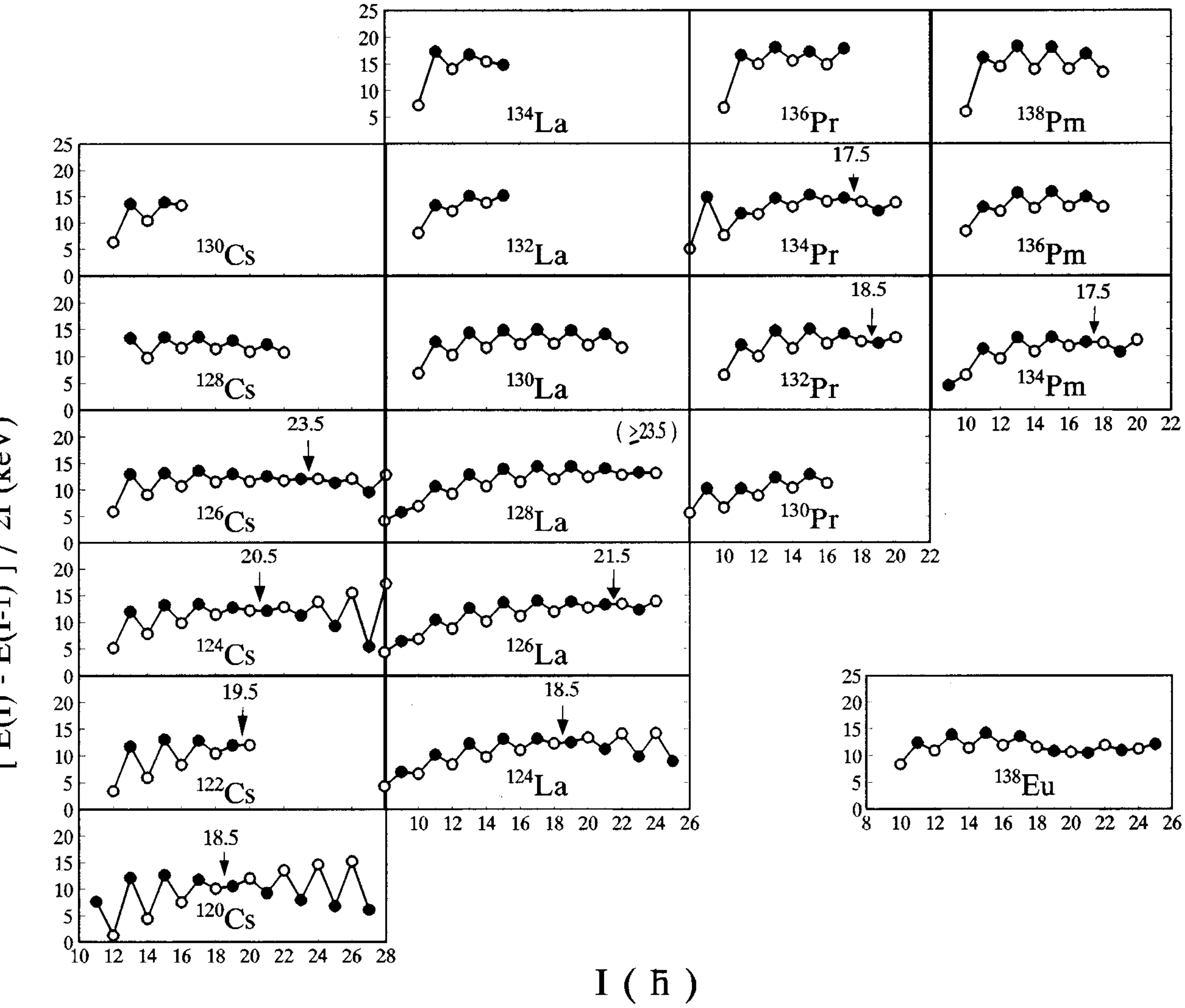

FIG. 15. $[E(I)-E(I-1)]/2I$ vs I plots of $\pi h_{11/2}\otimes\nu h_{11/2}$ bands of doubly odd nuclei around $A\sim130$. Solid and open circles correspond to favored ($\alpha=1$) and unfavored ($\alpha=0$) signatures, respectively. Inversion points are indicated by arrows and inversion spin values with uncertainties $\pm0.5\hbar$. Data of ^{138}Eu are taken from [14,23].

signature inversion occurred in these nuclei. Second, for almost all of the 11 cases (except ^{126}La [3]) the previous I_0's were assigned under the assumption that the signature dependence is normal, i.e., the signature $\alpha=1$ (corresponds to odd spins) lies lower in energy.

B. Comment on spin assignments of Cs isotopes

I_0's of ^{124}Cs [7], ^{126}Cs [7], and ^{130}Cs [19] were determined through experimental spectroscopy (I_0s of ^{124}Cs were given in brackets). If the excitation energy systematics for Cs isotopes described in the present work is acceptable, then the spin assignments of ^{124}Cs ($I_0=7$) [7], ^{126}Cs ($I_0=5$ [7], and ^{130}Cs ($I_0=9$) [19] cannot all be correct, and, at the utmost, only one of them is correct.

C. Configuration of the rotational band of ^{122}Cs

The configuration of the rotational band of ^{122}Cs shown in Fig. 14 was previously assigned as $\pi h_{11/2}\otimes\nu g_{7/2}$ with $I_0^\pi=6^-$ [16]. This band was treated as the $\pi h_{11/2}\otimes\nu h_{11/2}$ band in [7] but no argument was given. The systematic features of the level schemes of Cs isotopes, as shown in Fig. 14, support the configuration assignment of $\pi h_{11/2}\otimes\nu h_{11/2}$. This configuration assignment is also consistent with the facts that in the experiment of [16] this band was populated with larger population and the $\pi h_{11/2}$ and $\nu h_{11/2}$ bands were populated as yrast band in neighboring odd-Z and odd-N nuclei, respectively.

D. Lowest observed state of the $\pi h_{11/2}\otimes\nu h_{11/2}$ band in ^{130}Cs

The level $I^\pi=9^+$ in Fig. 10(a) was excluded from the $\pi h_{11/2}\otimes\nu h_{11/2}$ yrast band of ^{130}Cs and the level with $I^\pi=10^+$ was adopted as band head of the yrast band in [19] for the reason that the level with 9^+ is higher in energy than the level with $I^\pi=9^-$ belonging to other band in the level scheme of ^{130}Cs [19], and the inclusion of the 9^+ level in the yrast band would not be consistent with the characteristic feature of the yrast band. However, the systematic features of the $\pi h_{11/2}\otimes\nu h_{11/2}$ bands of Cs isotopes as revealed in the smooth curves of Figs. 9 and 11, and in the level schemes of Figs. 14 and 10 suggest that the level with 9^+ in Fig. 10(a) is a member of yrast band. Therefore the level with 9^+ of Fig. 10(a) was adopted as the lowest observed state of the $\pi h_{11/2}\otimes \nu h_{11/2}$ band of ^{130}Cs in the present work.

VI. DISCUSSIONS AND CONCLUSIONS

A. Systematic features of signature inversion of $\pi h_{11/2}\otimes \nu h_{11/2}$ bands around $A\sim130$

The plots $[E(I-1)-E(I)]/2I$ versus I for the $\pi h_{11/2}\otimes \nu h_{11/2}$ bands of all nuclei discussed in the present study are shown in Fig. 15. For Cs isotopes, the plots are based on the I_0's which were obtained by taking the level scheme of ^{130}Cs ($I_0=11$) as reference in the systematic study. Solid dots correspond to favored signature ($\alpha=-1$, odd spin) while the open circles correspond to unfavored signature ($\alpha=0$, even spin). Two systematic features can be seen from Fig. 15. First, the low-spin signature of the $\pi h_{11/2}\otimes \nu h_{11/2}$ band is inverted for all nuclei. This is what happened in the $\pi h_{11/2}\otimes \nu i_{13/2}$ bands of doubly odd nuclei around $A\sim160$ [1]. Second, inversion spin ($I_{\rm inv}$) increases with increasing neutron number for Cs and La isotopes in the cases where the inversion point has been observed. This variation trend is opposite to that of the $\pi h_{11/2}\otimes \nu i_{13/2}$ bands around $A\sim160$, where $I_{\rm inv}$ decreases with increasing neutron number and increases with increasing proton number. The opposite variation trend of $I_{\rm inv}$ for the two cases of $A\sim160$ and 130 is expected if one considers that the roles played by neutron and proton in the shell-filling process are exchanged in the cases of $A\sim160$ and 130. In the case of $A\sim130$, the proton starts to fill the $h_{11/2}$ subshell from its bottom and the neutron starts to fill the $h_{11/2}$ subshell from its middle. In the case of $A\sim160$, the proton starts to fill the $h_{11/2}$ subshell from its middle and the neutron starts to fill the $i_{11/2}$ subshell from its bottom. However, the variation trend of $I_{\rm inv}$ with increasing proton number is not clear for the case of $A\sim130$ as seen in Fig. 15.

B. Comment on the previous discussions of γ deformation of equilibrium shape

One of the motivations for studying the rotational bands built on the high-j configuration, for example the $\pi h_{11/2}\otimes \nu h_{11/2}$ configuration, has been to discuss the γ deformation of equilibrium shape resulted from the competition of the driving force induced by proton quasiparticle situated in the low $-\Omega$ $h_{11/2}$ subshell and the driving force induced by neutron quasiparticle situated in the high $-\Omega$ $h_{11/2}$ subshell. The discussions of γ deformation [9,13] were conducted by comparing the magnitude of signature splitting deduced from the observed rotational band and the magnitude of signature splitting at equilibrium shape calculated by using the formalism of Frauendorf and May [20] based on the cranked shell model. The basis for such a comparison is that the observed and calculated total Routhians have the same sign of signature splitting, i.e., the observed and calculated signature dependence both are normal or both are inverted. The signature dependence of the calculated total Routhian (E') for all previous studied cases (for example, [9,13]) has always been normal, i.e., the total Routhian E' ($\alpha=-1$) lies lower in energy than E' ($\alpha=0$) for the $\pi h_{11/2}\otimes \nu h_{11/2}$ band, and the signature dependence of observed E' deduced from the $\pi h_{11/2}\otimes \nu h_{11/2}$ bands based on the previous I_0's are normal too. The previous discussions of γ deformation were conducted on this basis. If the new I_0's proposed in the present work are correct, then the observed sign of signature splitting will be opposite to that of calculated [9,13]. Therefore, the conjectures on the γ deformation of equilibrium shape made previously have to be reconsidered.

C. Comparison with particle-triaxial-rotor model

A systematic calculation of signature splittings of the $\pi h_{11/2}\otimes \nu h_{11/2}$ bands was performed by Tajima [21] for Cs and La isotopes, and $N=75$ isotones by using the particle-triaxial-rotor model with the inclusion of p-n interaction of Semmes and Ragnarsson [22]. The calculation of Tajima [21] predicts that the low-spin signature of the $\pi h_{11/2}\otimes \nu h_{11/2}$ bands are inverted for all calculated cases and $I_{\rm inv}$ of La and Cs isotopes increase with increasing neutron number as indicated in Tables I and II. These predictions are in agreement with the results deduced from rotational bands based on the present I_0's, as compared in Tables I and II.

The major discrepancy between experiment based on previous I_0's and theory [21] is the opposite sign of signature splittings for the calculated cases ^{128}Cs, ^{126}La, ^{130}La, ^{132}La, and ^{136}Pm as indicated in Tables I and II. (The inverted and not inverted signatures in Tables I and II correspond to negative and positive signs of signature splittings, respectively.) This major discrepancy is completely removed by adopting the new I_0's proposed in the present work. Taking this fact into account, the overall agreement between experiment (based on present I_0's) and theory [21] is very encouraging.

The puzzling opposite sign of signature splittings between experiment and theory has led Tajima [21] to conclude that this systematic discrepancy in the sign might signify that the experimental spin assignment were incorrect by odd values of ΔI, or it would indicate the necessity of a more elaborate model. This question is clearly answered by the results of the present work although further experimental check are desirable.

Additionally, for Cs isotopes, the calculated values of $I_{\rm inv}$ agree very well with those obtained by taking ^{124}Cs ($I_0=7$) as a reference as seen in Table II. This agreement cannot doubtlessly be taken as a support to the spin assignment of $I_0=7$ for ^{124}Cs because the main parameters used in Tajima's calculation were obtained by fitting the experimental data of ^{124}Cs rotational band based on the spin assignment of $I_0=7$.

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