Comparison of instant form and front form one-particle exchange models

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We study the use of the Okubo method for constructing effective two-particle, instant form and front form, one-particle exchange models, starting from quantum field theory vertices. The truncation of the field theory that is made in applying the Okubo method leads to mass operators that are not exactly Poincaré invariant. It is found that when the spurious terms that destroy the exact Poincaré invariance are eliminated the instant and front form potentials come out essentially the same, however they wind up in slightly different two-particle Lippmann-Schwinger equations. In order to study the practical consequences of this difference, a numerical comparison is made between instant and front form one-meson-exchange models of the two-nucleon system that have been fit to the same partial-wave analysis of the two-nucleon data. These models assume the exchange of π, η, ρ, ω, δ, and σ mesons. No dramatic differences are found between the instant form and front form vertex parameters obtained from the fits. [S0556-2813(96)00508-0]

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I. INTRODUCTION

Recently we have developed techniques for constructing Poincaré-invariant, particle-exchange models for systems such as the nucleon-nucleon and pion-nucleon systems [1–4]. These techniques make it possible to construct generators of the Poincaré group, which act in the Hilbert space of a few-particle system; and moreover lead to quantum-mechanical state vectors which transform from one inertial frame to another according to unitary representations of the Poincaré group. This guarantees that probabilities calculated, for example, from S-matrix elements are invariant under inhomogeneous Lorentz transformations. We have developed these techniques in the context of Dirac's instant and front forms of relativistic quantum mechanics [5,6]. The purpose of the present work is to compare models obtained using these two forms, since it turns out that the final few particle models are somewhat different.

The basic ingredients of a particle exchange model are its vertices. The nature of the coupling at a vertex is specified by a Lorentz invariant, Lagrangian density; as well as a form factor or vertex function. The purpose of the vertex function is to take into account the extension of a strong interaction vertex, which in general involves composite particles. In most cases these vertex functions are phenomenological.

These vertices can be related to the observables of a hadronic system in a manifestly covariant way by using the Bethe-Salpeter equation [7], or one of its three-dimensional reductions. The three-dimensional reductions that are most widely used are due to Blankenbecler-Sugar [8], and to Gross [9]. Tjon and his collaborators [10] have employed both the Bethe-Salpeter equation and the Blankenbecler-Sugar equation. The most recent application of the Gross equation to the two-nucleon system is given in [11], and to the pion-nucleon system in [12].

Extensive use of time-ordered perturbation theory has been used in developing the so-called Bonn meson-exchange model for the nucleon-nucleon interaction, starting from a set of meson-nucleon vertices [13,14]. Johnson's method of folded diagrams [15] has been used to eliminate the energy dependence of the amplitudes obtained from time-ordered perturbation theory. This leads to instantaneous interactions which can be conveniently used in calculating the properties of system with more than two nucleons. This approach is not manifestly covariant; nor does it fall within the framework of one of Dirac's forms of relativistic quantum mechanics [5,6]. It should be noted, however, that the potentials and Lippmann-Schwinger equation obtained in our recent front form, one-boson-exchange model of the two-nucleon system [3] turn out to be almost identical to those employed in the Bonn one-boson-exchange models [13,14].

In constructing our Poincaré invariant models we have used an extension of the Glöckle-Müller implementation [16] of Okubo's formalism [17] to derive two-particle potentials from the field-theory vertices. In the Okubo method an attempt is made to block diagonalize the quantum field theory Hamiltonian, so as to find an effective Hamiltonian that acts in a few particle subspace of Fock space. Of course, some form of perturbation theory must be used in this attempt. At the level of one-particle-exchange models the effective Hamiltonian is given by Eq. (3.7). In our context, this essentially defines instant and front form ladder approximations. If the quantum field theory is quantized on a $t=0$ hypersurface, then the quantum field theory generators define an instant form of relativistic quantum mechanics. The three-momentum operator $\mathbf{P}$ and the angular momentum operator $\mathbf{J}$ are noninteracting, while the Hamiltonian H and the generator of rotationless boosts $\mathbf{K}$ contain interactions. The effective two-particle Hamiltonian obtained with the Okubo method will satisfy the correct commutation relations with respect to the projections of $\mathbf{P}$ and $\mathbf{J}$ onto the two-particle subspace, however it is difficult to find an effective $\mathbf{K}$ to complete the Poincaré algebra. Similar problems arise when the field theory is quantized on the null plane, which leads to a front form of relativistic quantum mechanics.

We have chosen to use the Bakamjian-Thomas method [18,6] for constructing a set of two-particle generators that satisfy the Poincaré algebra. Here the focus is on constructing a Poincaré invariant mass operator. In constructing a mass operator it is important to realize that the relation be-

tween the quantum field theory mass operator and the quantum field theory Hamiltonian is not the same in the instant and front forms. The instant form mass-square operator is quadratic in the instant form Hamiltonian, while the front form mass-square operator is linear in the front form Hamiltonian. As a consequence of this, when the Okubo method is used to obtain a one-particle-exchange interaction to use in a two-particle mass operator, this interaction appears linearly in the instant form *mass* operator, and linearly in the front form *mass-square* operator.

We have found [2–4] that the Okubo method for obtaining one-particle-exchange potentials is equivalent to a slight variation of the standard Feynman diagram rules. The potentials can be obtained by first drawing the relevant second-order Feynman diagrams, and then determining the four-momentum of the virtual, exchanged particle in each diagram by assuming that the total four-momentum is conserved either at the vertex on the right or at the vertex on the left, but not necessarily at both vertices. The potentials are obtained by adding together the two resulting Feynman-like amplitudes and dividing by 2. In order to have a model that is complete in the sense that it leads to a representation of the Poincaré group, it is necessary to specify other operators as well.

In a Bakamjian-Thomas [18,6] construction of an instant form model the other operators are usually taken to be the three-momentum operator **P**, the *canonical spin operator* $\mathcal{J}_c$, and the Newton-Wigner position operator **X**; while in the front form they are usually taken to be three *front form* components of the momentum, denoted collectively here by P, the *front form spin operator* $\mathcal{J}_f$, and the generators of the so-called *front form boosts*, denoted here by K_3 and **B**. In either form only the mass operator M contains an interaction; the other operators are assumed to be the same as those of the corresponding noninteracting system. Instant form generators and front form generators can be constructed from the sets $\{M_c,\mathbf{P},\mathcal{J}_c,\mathbf{X}\}$ and $\{M_f,P,\mathcal{J}_f,\mathbf{B},K_3\}$, respectively. The subscripts on the mass operators indicate that the mass operators associated with the two forms are in general different from each other. The instant form and front form generators will satisfy the Poincaré algebra exactly as long as the sets $\{M_c,\mathbf{P},\mathcal{J}_c,\mathbf{X}\}$ and $\{M_f,P,\mathcal{J}_f,\mathbf{B},K_3\}$ satisfy certain relatively simple commutation relations. When the instant and front form mass operators obtained with the Okubo method are examined it is found that they do not satisfy these simple commutation rules exactly. The approximate instant form interactions contain spurious dependence on the total three-momentum of the two-particle system, while the front form interactions contain spurious dependence on the orientation of the null plane. Thus the violations of Poincaré invariance that result from the truncation of the quantum field theory play out differently in the two forms of relativistic quantum mechanics. The interesting result is that when the spurious terms are eliminated, the instant form potential and the front form potential have essentially the same dependence on the relative momentum variables and spins of the particles. It turns out, however, that the final Lippmann-Schwinger equations that we solve to calculate the scattering amplitudes are somewhat different in the instant and front forms. This is a consequence of the fact that our one-particle-exchange interactions appear linearly in the instant form *mass* operator, but linearly in the front form *mass-square* operator. One of our purposes here is to study the effect of this difference. It should be emphasized that this difference does not imply that there is some fundamental inconsistency between the two forms of relativistic quantum mechanics; it is simply a consequence of the fact that the necessary approximations that are made in deriving the models from the field-theory vertices play out differently in the two forms. Here we compare numerical results obtained with the two forms in the context of a fairly realistic one-boson-exchange model of the two-nucleon system. The exchange mechanisms assumed here are the same as those employed in our earlier work on the two-nucleon system [3], i.e, we assume the exchange of π, η, ρ, ω, δ, and σ mesons. We fit the coupling constants, the masses of some of the mesons, and the cutoff masses in the vertex functions to a recent Nijmegen partial wave analysis of the two-nucleon data [19]; and compare the resulting parameters. We find no dramatic difference between the two parameter sets.

The outline of the paper is as follows. Section II simply gives the relation between the front form and canonical components of the four-momentum operator, as well as the relations between the mass operators and Hamiltonians in the instant and front forms. The relevant features of the Okubo formalism [17] are summarized in Sec. III. Section IV outlines the procedure for deriving two-particle potentials from field-theory vertices, and also introduces the relative three-momentum variables that we employ. Section V summarizes the techniques used to express the potentials obtained with the modified Feynman rules in terms of these relative momentum variables, and also specifies the prescriptions that must be applied to the potentials so as to ensure exact Poincaré invariance. The numerical results for the one-boson-exchange model of the two-nucleon systems are given in Sec. VI. A discussion of the results and suggestions for future work are given in Sec. VII.

Throughout we use units in which $\hbar=c=1$.

II. MOMENTUM AND MASS OPERATORS

In the instant form and the front form it is convenient to use respectively, the canonical (c) and front form (f) components of four-vectors and four-vector operators. For the four-momentum operator we work with the two sets of components,

$$P=(\hat{P}^0,\hat{P}^1,\hat{P}^2,\hat{P}^3)=(H_c,\mathbf{P}), \tag{2.1a}$$

$$P=(P^0,P^1,P^2,P^3)=(\overline{P},H_f), \tag{2.1b}$$

where we have used the caret to distinguish the canonical components from the front form components. Here H_c is the instant form Hamiltonian, **P** is the three-momentum, $\overline{P}=(P^0,P^1,P^2)$, and H_f is the front form Hamiltonian. In the instant form **P** is noninteracting or kinematic and interactions are put into H_c, while in the front form P is kinematic and interactions are put into H_f. The relation between the two sets of components is given by

$$P^0=(\hat{P}^0+\hat{P}^3)/\sqrt{2},\quad P^3=(\hat{P}^0-\hat{P}^3)/\sqrt{2}, \tag{2.2a}$$

$$\mathbf{P}_\perp\equiv(P^1,P^2)=(\hat{P}^1,\hat{P}^2). \tag{2.2b}$$

The mass operator M plays a central role in relativistic quantum mechanics, and is defined in terms of the two sets of components by

$$M^2 \equiv P\cdot P = (\hat{P}^0)^2 - \mathbf{P}^2 = 2P^0P^3 - \mathbf{P}_\perp^2. \tag{2.3}$$

According to Eq. (2.1) the mass-square operator is quadratic in the instant form Hamiltonian H_c, but linear in the front form Hamiltonian H_f. We can easily solve Eq. (2.3) for these Hamiltonians to obtain

$$H_c = (\mathbf{P}^2 + M_c^2)^{1/2}, \quad H_f = (\mathbf{P}_\perp^2 + M_f^2)/(2P^0), \tag{2.4}$$

where we have put subscripts on the instant and front form mass operators to indicate that in general they are different. In Sec. III we outline the Okubo method for constructing effective Hamiltonians that act in a subspace of the vector space of a quantum-mechanical system.

III. THE OKUBO METHOD

In the Okubo method [17] the vector space of a system is divided into two subspaces whose projection operators η and Λ are orthogonal and satisfy

$$\eta + \Lambda = 1. \tag{3.1}$$

The basic idea of the method is to construct a unitary operator U that transforms a Hamiltonian H according to

$$H' = U^{-1}HU, \tag{3.2}$$

such that the transformed Hamiltonian H' is block diagonal, i.e.,

$$\Lambda H' \eta = 0. \tag{3.3}$$

An effective Hamiltonian that acts only in the η subspace can then be defined by

$$H^\eta \equiv \eta H' \eta. \tag{3.4}$$

In general, except for some simple models [17], it is not possible to find U exactly, and it is therefore necessary to resort to perturbation theory [1,2,16,17]. In perturbation theory the Hamiltonian is divided into two parts according to

$$H = H_0 + H_1, \tag{3.5}$$

where H_0 is a solvable Hamiltonian whose eigenstates can be used to construct the projection operators η and Λ. If $\{|\zeta\rangle\}$ is a basis for the η subspace where

$$H_0|\zeta\rangle = \omega(\zeta)|\zeta\rangle, \tag{3.6}$$

then to second order in H_1, the effective Hamiltonian is given by [1,2,16,17]

$$\langle\zeta|H^\eta|\zeta'\rangle = \left\langle \zeta \middle| H + \frac{1}{2}H_1\left[\frac{\Lambda}{\omega(\zeta)-H_0} + \frac{\Lambda}{\omega(\zeta')-H_0}\right]H_1 \middle| \zeta' \right\rangle + \cdots. \tag{3.7}$$

The structure of this result is very similar to *old-fashioned* or time-ordered perturbation theory. We note that the denominators in Eq. (3.7) contain eigenvalues of H_0 which correspond to either the initial state or the final state, and that the two possibilities appear in a symmetric way.

IV. EFFECTIVE TWO-PARTICLE MODELS

In relativistic quantum mechanics it is usually convenient to define particle states by boosting states associated with a rest frame of the particle. We will denote the Lorentz transformations that we use to boost from rest frames by $l_g(\lambda)$ where the subscript g distinguishes the various possible boosts, and λ is a timelike unit vector. If we let x symbolize the components of a four-vector in an arbitrary frame, then the components of this vector in a rest frame are given by

$$x_{g\lambda} = l_g^{-1}(\lambda)x, \quad \lambda^2 = 1, \tag{4.1}$$

where the boost has the property

$$l_g(\lambda)(1,\mathbf{0}) = \lambda. \tag{4.2}$$

As indicated in Eq. (4.1), the rest-frame components depend on both λ and the choice of the boost. In the instant form it is convenient to use the so-called *canonical boost* $(g=c)$ given by

$$\hat{x}^0 = \hat{\lambda}^0\hat{x}^0_{c\lambda} + \boldsymbol{\lambda}\cdot\mathbf{x}_{c\lambda}, \quad \mathbf{x} = \mathbf{x}_{c\lambda} + \left(\hat{x}^0_{c\lambda} + \frac{\boldsymbol{\lambda}\cdot\mathbf{x}_{c\lambda}}{\hat{\lambda}^0+1}\right)\boldsymbol{\lambda}, \tag{4.3}$$

while in the front form it is convenient to use the so-called *front form boost* $(g=f)$ given by

$$x^0 = \sqrt{2}\lambda^0 x^0_{f\lambda}, \quad \mathbf{x}_\perp = \sqrt{2}\boldsymbol{\lambda}_\perp x^0_{f\lambda} + \mathbf{x}_{f\lambda\perp}. \tag{4.4}$$

We note that the first three front form components, $\overline{x} = (x^0, x^1, x^2)$ transform among themselves under front form boosts, so that while these boosts are not as well known as the canonical boosts, they possess a technical advantage.

We let $|i_\nu h_\nu\rangle$ denote a rest-frame state of a particle labeled ν with spin s_ν, where $h_\nu = -s_\nu, -s_\nu+1, \ldots, s_\nu$; and where in general i_ν stands for any other *internal* quantum numbers of interest. For a nucleon, i_ν stands for its isospin. We assume that in the subspace of this particle we have a unitary representation $U_\nu(a)$ of the Lorentz group, where $U_\nu(a)$ is a unitary operator corresponding to a Lorentz transformation a. In particular for a three-rotation r, if we assume that the states $|i_\nu h_\nu\rangle$ are orthonormal, the action of the unitary operator $U_\nu(r)$ corresponding to the rotation r can be defined by

$$U_\nu(r)|i_\nu h_\nu\rangle = \sum_{h'_\nu} |i_\nu h'_\nu\rangle D^{(s_\nu)}_{h'_\nu h_\nu}(r), \tag{4.5}$$

where $D^{(s_\nu)}(r)$ is the standard unitary, matrix representation of SU(2) for the spin s_ν. We define the states of a particle with on-shell momentum p_ν and mass m_ν by

$$|p_\nu i_\nu h_\nu\rangle_g \equiv U_\nu[l_g(p_\nu/m_\nu)]|i_\nu h_\nu\rangle \quad (p_\nu^2 = m_\nu^2), \tag{4.6}$$

where we note that different boosts lead to different single-particle states.

We can take for the basis states for the two-particle subspace the direct product states

$$|p_1p_2\beta\rangle_g \equiv |p_1i_1h_1\rangle_g \otimes |p_2i_2h_2\rangle_g , \tag{4.7}$$

where β is the set

$$\beta \equiv \{i_1,h_1;i_2,h_2\}. \tag{4.8}$$

These states satisfy the equations

$$P_0|p_1p_2\beta\rangle_g = p|p_1p_2\beta\rangle_g , \quad M_0|p_1p_2\beta\rangle_g = W|p_1p_2\beta\rangle_g , \tag{4.9}$$

where P_0 and M_0 are the four-momentum and mass operators for the noninteracting system, and the total four-momentum p and invariant mass W are given by

$$p = p_1 + p_2 , \quad W = +(p\cdot p)^{1/2}. \tag{4.10}$$

We define relative momentum variables $\mathbf{q}_g$ for these states as the three-momentum of particle 1 in a rest frame related to an arbitrary frame as in Eq. (4.1), therefore we can write

$$p_{1g\Lambda} = [\epsilon_{m_1}(\mathbf{q}_g), \mathbf{q}_g] = l_g^{-1}(\Lambda)p_1 ,$$

$$p_{2g\Lambda} = [\epsilon_{m_2}(\mathbf{q}_g), -\mathbf{q}_g] = l_g^{-1}(\Lambda)p_2 , \tag{4.11}$$

where the relativistic energies are given by

$$\epsilon_m(\mathbf{q}) = (\mathbf{q}^2 + m^2)^{1/2}, \tag{4.12}$$

and we have defined a timelike unit vector Λ by

$$\Lambda \equiv p/W. \tag{4.13}$$

We note that Eqs. (4.10) and (4.11) imply

$$W = W(\mathbf{q}) = \epsilon_{m_1}(\mathbf{q}) + \epsilon_{m_2}(\mathbf{q}). \tag{4.14}$$

Instead of labeling the states (4.7) with (p_1,p_2) we can label them with $(\mathbf{p},\mathbf{q}_c)$ or $(\overline{p},\mathbf{q}_f)$. We assume that our two-particle states are normalized according to [3,4]

$$\begin{aligned} {}_g\langle p_1p_2\beta|p_1'p_2'\beta'\rangle_g &= (2\pi)^3 2\epsilon_W(\mathbf{p})\delta^3(\mathbf{p}-\mathbf{p}')\Delta(\mathbf{q}_c)\delta^3(\mathbf{q}_c-\mathbf{q}_c')\delta_{\beta\beta'} \\ &= (2\pi)^3 2p^0\delta^3(\overline{p}-\overline{p'})\Delta(\mathbf{q}_f)\delta^3(\mathbf{q}_f-\mathbf{q}_f')\delta_{\beta\beta'} , \end{aligned} \tag{4.15}$$

where

$$\Delta(\mathbf{q}) = (2\pi)^3 2\epsilon_{m_1}(\mathbf{q})\epsilon_{m_1}(\mathbf{q})/W(\mathbf{q}). \tag{4.16}$$

It has been shown previously [2,4] that if the Hamiltonian H in Eq. (3.7) is assumed to be an instant form or front form, quantum field-theory Hamiltonian, appropriate for a particle-exchange model, then the effective instant form and front form Hamiltonians obtained with the Okubo method are given to second order by

$$\begin{aligned} {}_c\langle p_1p_2\beta|H_c^\eta|p_1'p_2'\beta'\rangle_c &= (2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}') \\ &\times[2\epsilon_W^2(\mathbf{p})\Delta(\mathbf{q}_c)\delta^3(\mathbf{q}_c-\mathbf{q}_c')\delta_{\beta\beta'} \\ &+ \widetilde{V}_{c,\beta\beta'}(p_1,p_2;p_1',p_2')], \end{aligned} \tag{4.17a}$$

$$\begin{aligned} {}_f\langle p_1p_2\beta|H_f^\eta|p_1'p_2'\beta'\rangle_f &= (2\pi)^3\delta^3(\overline{p}-\overline{p'})[\Delta(\mathbf{q}_f) \\ &\times\delta^3(\mathbf{q}_f-\mathbf{q}_f')\delta_{\beta\beta'}(\mathbf{p}_\perp^2+W^2) \\ &+ \widetilde{V}_{f,\beta\beta'}(p_1,p_2;p_1',p_2')], \end{aligned} \tag{4.17b}$$

where here particles 1 and 2 are the ones that are interacting through exchange processes; and the interaction terms $\widetilde{V}_{g,\beta\beta'}$ are determined by a slight variation of the standard Feynman diagram rules. As pointed out in Sec. I, we can obtain $\widetilde{V}_{g,\beta\beta'}$ by first drawing the relevant one-particle-exchange Feynman diagrams, and then determining the four-momentum of the virtual exchanged particle in each diagram by assuming that the total four-momentum is conserved *either* at the vertex on the *right or* at the vertex on the *left*, but not necessarily at both vertices. We then add together the two amplitudes, and divide by 2. As an example, the interaction between two nucleons due to the exchange of a scalar meson (δ or σ) is given by [3]

$$\begin{aligned} \widetilde{V}^b_{g,\beta\beta'}(p_1,p_2;p_1',p_2') &= g_b^2\langle i_1i_2|\Gamma_b|i_1'i_2'\rangle f_b[(p_1-p_1')^2]f_b[(p_2-p_2')^2] \\ &\times\frac{1}{2}\left[\frac{\overline{u_g}(p_1,h_1)u_g(p_1',h_1')\overline{u_g}(p_2,h_2)u_g(p_2',h_2')}{(p_1-p_1')^2-m_b^2}+(1\leftrightarrow 2)\right], \quad b=\delta,\sigma, \end{aligned} \tag{4.18a}$$

$$\Gamma_\delta = \tau_1\cdot\tau_2 , \quad \Gamma_\sigma = 1, \tag{4.18b}$$

where $|i_1i_2\rangle$ is an isospin state for the two nucleons. Here the g_b are coupling constants, the $\boldsymbol{\tau}_\nu$ are isospin vectors for the nucleons, the f_b are form factors which take into account the extension of the vertices, and the u_g are Dirac spinors. These spinors are normalized according to

$$\overline{u_g}(p_\nu,h_\nu)u_g(p_\nu,h_\nu') = 2m_N\delta_{h_\nu h_\nu'}, \tag{4.19}$$

and are defined in terms of rest-frame spinors $u(h)$ by

$$u_g(p_\nu,h_\nu) \equiv S[l_g(p_\nu/m_\nu)]u(h_\nu), \tag{4.20}$$

where for an arbitrary Lorentz transformation a the matrix $S(a)$ satisfies [20]

$$S^{-1}(a)\gamma^{\mu}S(a)=a^{\mu}{}_{\nu}\gamma^{\nu}. \tag{4.21}$$

We note that in the instant form and front form the Dirac spinors are defined in terms of canonical and front form boosts, respectively.

V. POINCARÉ INVARIANCE

In order to develop a Poincaré-invariant model it is necessary to construct 10 Hermitian generators that satisfy the Poincaré algebra [5,6]. A practical way of doing this is by means of a so-called Bakamjian-Thomas construction [6,18]. With this procedure the 10 generators are expressed in terms of other operators which satisfy relatively simple commutation rules. In the instant form the generators are usually taken to be H_c, **P**, **J**, and **K**; where the angular momentum operator **J** generates three-rotations, and **K** is the generator of canonical boosts. We can write [4,6]

$$H_c=(\mathbf{P}^2+M_c^2)^{1/2},\quad \mathbf{J}=\mathbf{X}\times\mathbf{P}+\mathcal{J}_c,$$

$$\mathbf{K}=-\frac{1}{2}(H_c\mathbf{X}+\mathbf{X}H_c)-\frac{\mathbf{P}\times\mathcal{J}_c}{M_c+H_c}, \tag{5.1}$$

where M_c is the instant form mass operator, $\mathcal{J}_c$ is called the *canonical spin operator*, and **X** is the Newton-Wigner position operator. The only nonzero commutators of the members of the set $\{M_c,\mathbf{P},\mathcal{J}_c,\mathbf{X}\}$ are

$$[X^j,P^k]=i\delta_{jk},\quad [\mathcal{J}_c^j,\mathcal{J}_c^k]=i\epsilon_{jkl}\mathcal{J}_c^l, \tag{5.2}$$

which makes it simpler to construct models for this set than for the Poincaré generators. If the members of the set $\{M_c,\mathbf{P},\mathcal{J}_c,\mathbf{X}\}$ satisfy the correct commutation relations, then the 10 generators $\{H_c,\mathbf{P},\mathbf{J},\mathbf{K}\}$ defined by Eq. (5.1) satisfy the Poincaré algebra. In an instant form model, the mass operator M_c is interacting while **P**, $\mathcal{J}_c$, and **X** are noninteracting. According to Eq. (5.1) this implies that H_c and **K** are interacting while the other six generators are noninteracting.

In front form dynamics the 10 Poincaré generators are usually taken to be $H_f,P,J_3\equiv\mathbf{e}_3\cdot\mathbf{J}$, $K_3\equiv\mathbf{e}_3\cdot\mathbf{K}$, $\mathbf{B}\equiv(\mathbf{K}_\perp-\mathbf{e}_3\times\mathbf{J})/\sqrt{2}$, and $\mathbf{S}\equiv(\mathbf{K}_\perp+\mathbf{e}_3\times\mathbf{J})/\sqrt{2}$. Here $\mathbf{e}_3$ is a unit vector along the three-axis and $\perp$ indicates spatial components transverse to this axis. The three generators K_3 and **B** generate the front form boosts defined by Eq. (4.4). In the front form the three generators H_f and **S** are taken to be interacting while the other seven generators are taken to be noninteracting. As a consequence of the fact that the front form boost generators, K_3 and **B**, are noninteracting, the unitary operator $U(l_f)$ that maps a quantum-mechanical state vector from a rest frame to an arbitrary frame is noninteracting; thus it is simpler to boost state vectors in the front form than in the instant form. On the other hand, the transverse components of the angular momentum, $\mathbf{J}_\perp=[\mathbf{e}_3\times(\mathbf{B}-\mathbf{S})]/\sqrt{2}$, are interacting, which makes the treatment of angular momentum in the front form more complicated than in the instant form. In the front form a Bakamjian-Thomas construction focuses on the set of operators $\{M_f^2,P,\mathcal{J}_f,\mathbf{B},K_3\}$ where M_f^2 and $\mathcal{J}_f$ are the front form mass-square and spin operator, respectively. The instant form and front form spin operators differ from each other due to the fact that these operators are related to rest-frame angular momenta, and the two forms involve different rest frames [3,4,6]. The six operators P, **B**, and K_3 are Poincaré generators, so their commutation relations are part of the Poincaré algebra. The commutation relations among these six operators are satisfied by taking them to be noninteracting. In the usual Bakamjian-Thomas construction, $\mathcal{J}_f$ is taken to be noninteracting, which implies [3,6] that in order to ensure that the members of the set $\{M_f^2,P,\mathcal{J}_f,\mathbf{B},K_3\}$ satisfy the correct commutation relations, it is only necessary to ensure that M_f^2 commute with the other members of the set. If the Poincaré generators that do not belong to this set are defined by

$$H_f\equiv(\mathbf{P}_\perp^2+M_f^2)/(2P^0),$$

$$J_3\equiv(B_2/P^0)P^1-(B_1/P^0)P^2+\mathcal{J}_f^3,$$

$$\mathbf{S}\equiv(1/P^0)[\mathbf{P}_\perp K_3+H_f\mathbf{B}+\mathbf{e}_3\times(M_f\mathcal{J}_f^\perp+\mathbf{P}_\perp\mathcal{J}_f^3)], \tag{5.3}$$

then as long as the members of the set $\{M_f^2,\overline{P},\mathcal{J}_f,\mathbf{B},K_3\}$ satisfy the correct commutation relations, the above generators in combination with P, **B**, and K_3 will satisfy the Poincaré algebra.

In ensuring that the various operators satisfy the correct commutation relations, it is very helpful to work with basis states that lead to simple representations for these operators. Unfortunately the simple direct product states (4.7) do not have this property. For the instant form it is convenient to take as basis states [4]

$$|\mathbf{p}\mathbf{q}_c\beta\rangle\equiv U[l_c(\Lambda)]|p_{1c\Lambda}p_{2c\Lambda}\beta\rangle_c\quad\text{(instant form)} \tag{5.4}$$

while for the front form we use [3]

$$|\overline{p}\mathbf{q}_f\beta\rangle\equiv U[l_f(\Lambda)]|p_{1f\Lambda}p_{2f\Lambda}\beta\rangle_c\quad\text{(front form)}, \tag{5.5}$$

where the kets on the right-hand sides of these equations are defined by (4.5)–(4.8), and the single-particle momenta $p_{\nu g\Lambda}$ are defined by (4.11)–(4.13). We note that the kets on the right-hand sides of these equations are both defined as the direct product of single-particle states which are obtained by *canonical* boosts from the single-particle rest frames. The states defined by Eqs. (5.4) and (5.5) can be expressed as linear combinations of the simple direct product states (4.7), with coefficients that are the elements of a unitary transformation (see Eq. (3.25) of [3] and Eq. (3.6) of [4]). This implies that Eqs. (5.4) and (5.5) also satisfy the orthogonality relations (4.15).

The spin operators have particularly simple, as well as familiar, representations in these new bases [3,4], i.e.,

$$\langle\mathbf{p}\mathbf{q}_c i_1h_1i_2h_2|\mathcal{J}_c=\sum_{h_1'h_2'}[\mathcal{J}(\mathbf{q}_c)]_{h_1h_2,h_1'h_2'}\langle\mathbf{p}\mathbf{q}_c i_1h_1'i_2h_2'|, \tag{5.6a}$$

$$\langle \overline{p\mathbf{q}}_f i_1 h_1 i_2 h_2 | \mathcal{J}_f = \sum_{h_1' h_2'} [\mathcal{J}(\mathbf{q}_f)]_{h_1 h_2, h_1' h_2'} \langle \overline{p\mathbf{q}}_f i_1 h_1' i_2 h_2' |, \tag{5.6b}$$

where

$$\mathcal{J}(\mathbf{q}) \equiv I_1 \otimes I_2 (i\nabla_{\mathbf{q}} \times \mathbf{q}) + \mathbf{S}_1 \otimes I_2 + I_1 \otimes \mathbf{S}_2, \tag{5.7}$$

with I_ν and $\mathbf{S}_\nu$ the unit matrix and spin-matrix vector for particle ν, respectively. For nucleons, $\mathbf{S}_\nu = \boldsymbol{\sigma}_\nu/2$. The Newton-Wigner position operator, which only plays a role in the instant form, has the representation [4]

$$\langle \mathbf{p}\mathbf{q}_c \beta | \mathbf{X} = \left[i\nabla_p - \frac{i\mathbf{p}}{2\epsilon_W^2(\mathbf{p})} \right] \langle \mathbf{p}\mathbf{q}_c \beta |. \tag{5.8}$$

It can be shown [3,4] that replacing the basis states $|p_1 p_2 \beta\rangle_c$ and $|p_1 p_2 \beta\rangle_f$ in Eq. (4.17) with $|\mathbf{p}\mathbf{q}_c\beta\rangle$ and $|\overline{p\mathbf{q}}_f\beta\rangle$, respectively; is equivalent to replacing the Dirac spinors [in (4.18a), for example] according to the rules

$$u_g(p_\nu, h_\nu) \rightarrow S[l_g(\Lambda)] u_c(p_{\nu g\Lambda}, h_\nu),$$

$$u_g(p_\nu', h_\nu') \rightarrow S[l_g(\Lambda')] u_c(p'_{\nu g\Lambda'}, h_\nu'), \tag{5.9}$$

where

$$u_c(p_{\nu g\Lambda}, h_\nu) = [\epsilon_{m_\nu}(\mathbf{q}_g) + m_\nu]^{1/2} \begin{bmatrix} \chi_{h_\nu} \\ (-1)^{\nu+1} \boldsymbol{\sigma}_\nu \cdot \mathbf{x}_g \chi_{h_\nu} \end{bmatrix},$$

$$\mathbf{x}_g = \frac{\mathbf{q}_g}{\epsilon_{m_\nu}(\mathbf{q}_g) + m_\nu}, \tag{5.10}$$

with

$$\chi_{1/2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \chi_{-1/2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{5.11}$$

According to Eq. (4.17), in both the instant and front forms only three of the components of the four-momentum are conserved, which implies that the final-state, two-particle rest frame is not the same as the initial one. We see that it follows from Eq. (4.1) that these two frames are related by

$$x_{g\Lambda} = l_g^{-1}(\Lambda) l_g(\Lambda') x_{g\Lambda'}, \quad \Lambda = p/W, \quad \Lambda' = p'/W', \tag{5.12}$$

where $W = W(\mathbf{q}_g)$ and $W' = W(\mathbf{q}_g')$. The front form boosts form a subgroup of the Lorentz group [6], therefore the Lorentz transformation in Eq. (5.12) is a front form boost when $g = f$. In general, the canonical boosts do not form a subgroup, however the delta function in Eq. (4.17a) implies that the boosts $l_c(\Lambda')$ and $l_c(\Lambda')$ are along the common $\mathbf{p} = \mathbf{p}'$ direction; which in turn implies that the Lorentz transformation in Eq. (5.12) is a canonical boost when $g = c$. In fact it can be shown that [3,4]

$$l_g^{-1}(\Lambda) l_g(\Lambda') = l_c(\Omega_g), \quad \Omega_g = [\cosh(\zeta_g), \mathbf{u}_g \sinh(\zeta_g)], \tag{5.13a}$$

$$\zeta_c = \tanh^{-1}\left[\frac{\epsilon_W(\mathbf{p}) - \epsilon_{W'}(\mathbf{p})}{\epsilon_W(\mathbf{p})\epsilon_{W'}(\mathbf{p}) - \mathbf{p}^2} |\mathbf{p}| \right], \quad \mathbf{u}_c = \frac{\mathbf{p}}{|\mathbf{p}|}, \tag{5.13b}$$

$$\zeta_f = \ln(W/W'), \quad \mathbf{u}_f = \mathbf{e}_3 = (0,0,1). \tag{5.13c}$$

We note that the initial and final rest frames are related by a canonical boost in both forms; with this canonical boost along the common $\mathbf{p} = \mathbf{p}'$ direction in the instant form, and along the three-axis in the front form.

By evaluating the invariant momentum transfers that occur [in Eq. (4.18a), for example] in the $x_{g\Lambda}$ frame, and using Eqs. (5.12), (5.13), and (4.11); we can express them in terms of the relative momentum variables $\mathbf{q}_g$ and $\mathbf{q}_g'$. The result is

$$\begin{aligned}(p_\nu - p_\nu')^2 = {} & \epsilon_{m_\nu}^2(\mathbf{q}_g) - 2\epsilon_{m_\nu}(\mathbf{q}_g)\epsilon_{m_\nu}(\mathbf{q}_g')\cosh(\zeta_g) + \epsilon_{m_\nu}^2(\mathbf{q}_g') \\ & - (\mathbf{q}_g - \mathbf{q}_g')^2 + 2(-1)^\nu \sinh(\zeta_g) \\ & \times [\epsilon_{m_\nu}(\mathbf{q}_g)\mathbf{u}_g \cdot \mathbf{q}_g' - \epsilon_{m_\nu}(\mathbf{q}_g')\mathbf{u}_g \cdot \mathbf{q}_g] \\ & + 2[\cosh(\zeta_g) - 1](\mathbf{u}_g \cdot \mathbf{q}_g)(\mathbf{u}_g \cdot \mathbf{q}_g'), \quad \nu = 1,2.\end{aligned} \tag{5.14}$$

The spinor products that occur in Eq. (4.18a) can be expressed in terms of $\mathbf{q}_g$ and $\mathbf{q}_g'$ by using Eqs. (5.9), (5.10), and the relation [3,4]

$$S^{-1}[l_g(\Lambda)] S[l_g(\Lambda')] = S[l_c(\Omega_g)] = \exp\left(\frac{1}{2} \mathbf{u}_g \cdot \boldsymbol{\alpha} \zeta_g \right), \tag{5.15}$$

where the components of $\boldsymbol{\alpha}$ are the usual Dirac matrices. After making the replacements (5.9) in Eq. (4.18), and using Eq. (5.15) we find the following scalar meson-exchange potential in the spin-isospin space of the two-nucleons:

$$\begin{aligned}V_g^b(p_1, p_2; p_1', p_2') = {} & \frac{g_b^2 \Gamma_b}{2} f_b[(p_1 - p_1')^2] f_b[(p_2 - p_2')^2] \\ & \times \left[\frac{1}{(p_1 - p_1')^2 - m_b^2} + \frac{1}{(p_2 - p_2')^2 - m_b^2} \right] [\epsilon_{m_N}(\mathbf{q}_g) \\ & + m_N][\epsilon_{m_N}(\mathbf{q}_g') + m_N][\cosh(\zeta_g/2)(1 - \boldsymbol{\sigma}_1 \cdot \mathbf{x}_g \boldsymbol{\sigma}_1 \cdot \mathbf{x}_g') \\ & + \sinh(\zeta_g/2)(\boldsymbol{\sigma}_1 \cdot \mathbf{u}_g \boldsymbol{\sigma}_1 \cdot \mathbf{x}_g' - \boldsymbol{\sigma}_1 \cdot \mathbf{x}_g \boldsymbol{\sigma}_1 \cdot \mathbf{u}_g)] \\ & \times [\cosh(\zeta_g/2)(1 - \boldsymbol{\sigma}_2 \cdot \mathbf{x}_g \boldsymbol{\sigma}_2 \cdot \mathbf{x}_g') - \sinh(\zeta_g/2) \\ & \times (\boldsymbol{\sigma}_2 \cdot \mathbf{u}_g \boldsymbol{\sigma}_2 \cdot \mathbf{x}_g' - \boldsymbol{\sigma}_2 \cdot \mathbf{x}_g \boldsymbol{\sigma}_2 \cdot \mathbf{u}_g)], \quad b = \delta, \sigma.\end{aligned} \tag{5.16}$$

In order to establish Poincaré-invariant instant and front form models, we now turn our attention to the mass operator. As pointed out above, in the instant form we focus on the set of operators $\{M_c, \mathbf{P}, \mathcal{J}_c, \mathbf{X}\}$, and take $\mathbf{P}$, $\mathcal{J}_c$, and $\mathbf{X}$ to be the same as the operators for the noninteracting system. In order to ensure Poincaré invariance it is only necessary to choose the mass operator so that it commutes with $\mathbf{P}$, $\mathcal{J}_c$, and $\mathbf{X}$; the other commutation relations of the set $\{M_c, \mathbf{P}, \mathcal{J}_c, \mathbf{X}\}$ are automatically satisfied by our choice of $\mathbf{P}$, $\mathcal{J}_c$, and $\mathbf{X}$. With the

help of Eqs. (5.6a) and (5.7a) we can show that our model will be Poincaré invariant if the matrix elements of the mass operator are of the form [4]

$$\langle \mathbf{p}\mathbf{q}_c\beta|M_c|\mathbf{p}'\mathbf{q}_c'\beta'\rangle=(2\pi)^3 2[\epsilon_W(\mathbf{p})\epsilon_{W'}(\mathbf{p})]^{1/2}\delta^3(\mathbf{p}-\mathbf{p}') \times\langle\beta|M_c(\mathbf{q}_c,\mathbf{q}_c')|\beta'\rangle, \tag{5.17}$$

where $M_c(\mathbf{q}_c,\mathbf{q}_c')$ is a rotationally invariant function of $\mathbf{q}_c$, $\mathbf{q}_c'$, and $\boldsymbol{\sigma}$'s, and is independent of $\mathbf{p}$. Here $|\beta\rangle=|i_1,h_1;i_2,h_2\rangle$ is a spin-isospin state vector for the two nucleons. Setting $\mathbf{p}=\mathbf{0}$ in Eq. (4.17a), and using Eqs. (5.1) and (5.17), suggests that an $M_c(\mathbf{q}_c,\mathbf{q}_c')$ with the desired properties can be defined by

$$M_c(\mathbf{q}_c,\mathbf{q}_c')=W(\mathbf{q}_c)\Delta(\mathbf{q}_c)\delta^3(\mathbf{q}_c-\mathbf{q}_c')+\frac{V_c(\mathbf{q}_c,\mathbf{q}_c')}{2[W(\mathbf{q}_c)W(\mathbf{q}_c')]^{1/2}}, \tag{5.18}$$

with

$$V_c(\mathbf{q}_c,\mathbf{q}_c')=\sum_b V_c^b(p_1,p_2;p_1',p_2')|_{\mathbf{p}=\mathbf{p}'=0}. \tag{5.19}$$

The $b=\delta,\sigma$ contributions to the sum in Eq. (5.19) are determined by Eq. (5.16), while the contributions from pseudoscalar (π,η) and vector (ρ,ω) meson exchange are determined by Eqs. (4.11) and (4.12) of [3], respectively.

As pointed out above, in constructing a Poincaré invariant front form model we focus on the set of operators $\{M_f^2,P,\mathcal{J}_f,\mathbf{B},K_3\}$, and choose $P,\mathcal{J}_f,\mathbf{B}$, and K_3 to be noninteracting. In order to ensure Poincaré invariance we then only need to require that the mass-square operator commute with $P,\mathcal{J}_f,\mathbf{B}$, and K_3. We can show [3] that this requirement will be satisfied if the matrix elements of the mass-square operator are of the form

$$\langle\overline{p}\mathbf{q}_f\beta|M_f^2|\overline{p'}\mathbf{q}_f'\beta'\rangle=(2\pi)^3 2p^0\delta^3(\overline{p}-\overline{p'}) \times\langle\beta|M_f^2(\mathbf{q}_f,\mathbf{q}_f')|\beta'\rangle, \tag{5.20}$$

where $M_f^2(\mathbf{q}_f,\mathbf{q}_f')$ is a rotationally invariant function of $\mathbf{q}_f$, $\mathbf{q}_f'$, and the $\boldsymbol{\sigma}$'s; and is independent of $\overline{p}$. Using Eqs. (2.1b), (2.3), (4.15), and (4.17b), we find that a $M_f^2(\mathbf{q}_f,\mathbf{q}_f')$ with the desired properties can be defined by

$$M_f^2(\mathbf{q}_f,\mathbf{q}_f')=W^2(\mathbf{q}_f)\Delta(\mathbf{q}_f)\delta^3(\mathbf{q}_f-\mathbf{q}_f')+V_f(\mathbf{q}_f,\mathbf{q}_f'), \tag{5.21}$$

with

$$V_f(\mathbf{q}_f,\mathbf{q}_f')=\sum_b V_f^b(p_1,p_2;p_1',p_2') \quad(\mathbf{u}_f\text{-dependent terms are dropped}). \tag{5.22}$$

The fact that it is necessary to impose the requirements indicated in Eqs. (5.19) and (5.22) on potentials such as Eq. (5.16) in order to ensure Poincaré invariance is related to our use of Bakamjian-Thomas constructions in which the Newton-Wigner position operator $\mathbf{X}$ in the instant form and the spin operator $\mathcal{J}_f$ in the front form are assumed to be noninteracting. There is no fundamental reason for $\mathbf{X}$ and $\mathcal{J}_f$ to be noninteracting, and in fact in general they can contain interactions [6,21,22]. The prescriptions adopted here seem to be the simplest possibilities. Another possibility [2] for the front form is to average over the direction of $\mathbf{u}_f$ in Eq. (5.16). This cannot be done analytically, so this prescription complicates calculations. According to Eqs. (5.13b), and (5.13c), on shell, i.e., when $W=W'$, we have $\zeta_c=\zeta_f=0$; which implies that the prescriptions (5.19) and (5.22) do not effect the on-shell limit of Eq. (5.16). These prescriptions also have no effect in the nonrelativistic limit. This is true of the other one-boson-exchange potentials, as well; therefore we can think of the prescriptions (5.19) and (5.22) as defining the high-energy, off-shell extensions of the potentials. In this regime the vertex functions $f_b[(p_\nu-p_\nu')^2]$ come into play, and since in general these are phenomenological and contain adjustable parameters, our prescriptions are both sensible and practical.

When the prescriptions (5.19) and (5.22) are applied to the momentum transfers (5.14) we find that in the instant form $(p_\nu-p_\nu')^2\to[\epsilon_{m_v}(\mathbf{q}_c)-\epsilon_{m_v}(\mathbf{q}_c')]^2-(\mathbf{q}_c-\mathbf{q}_c')^2$, while in the front form $(p_\nu-p_\nu')^2\to-(\mathbf{q}_f-\mathbf{q}_f')^2$. Since the front form result gives better convergence at high momenta, we also adopt it for the instant form. We also improve convergence in the front form by approximating the factor [see Eq. (5.16)] $\cosh^2(\zeta_f/2)=(W+W')^2/4WW'$ by 1. Both of these modifications have no effect on the on-shell potential, or in the nonrelativistic limit.

Our final result for the potentials due to scalar meson exchange is

$$V^b(\mathbf{q}_g,\mathbf{q}_g')=-g_b^2\Gamma_b\frac{f_b^2[-(\mathbf{q}_g-\mathbf{q}_g')^2]}{m_b^2+(\mathbf{q}_g-\mathbf{q}_g')^2}[\epsilon_{m_N}(\mathbf{q}_g)+m_N] \times[\epsilon_{m_N}(\mathbf{q}_g')+m_N](1-\boldsymbol{\sigma}_1\cdot\mathbf{x}_g\boldsymbol{\sigma}_1\cdot\mathbf{x}_g') \times(1-\boldsymbol{\sigma}_2\cdot\mathbf{x}_g\boldsymbol{\sigma}_2\cdot\mathbf{x}_g'), \quad b=\delta,\sigma. \tag{5.23}$$

We see that our instant form and front form potentials have the same functional form; the only difference is in the definitions of the relative momentum variables $\mathbf{q}_g$ and $\mathbf{q}_g'$ [see Eqs. (4.11), (4.3), and (4.4)]. This identity of forms also occurs with the potentials due to pseudoscalar and vector meson exchange. With the same prescriptions that led to Eq. (5.23), the potentials due to pseudoscalar and vector meson exchange are given by Eqs. (4.31) and (4.32) of [3], respectively. As pointed out previously [3,23], the potentials (5.23), as well as the pseudoscalar meson-exchange potentials obtained in the same approximation, agree exactly with the Bonn, one-boson-exchange potentials given by Eqs. (E.32)–(E.34c) of [13]. The vector-meson-exchange potentials obtained in the same approximation differ slightly in the so-called tensor-tensor terms.

VI. THE SCATTERING AMPLITUDES

The fact that the instant and front form potentials have the same dependence on the relative momentum variables and the nucleon spins does not imply that they lead to the same

TABLE I. One-boson-exchange model parameters obtained with two forms of relativistic quantum mechanics. All masses are in MeV, and $n_b=1$ except for $n_\rho=n_\omega=2$.

Meson	Parameters	Front form	Instant form
π	$g_\pi^2/4\pi$	13.740	13.897
	$g_\pi^2(0)/4\pi$	13.541	13.684
	m_π	138.03	138.03
	Λ_π	1619.7	1572.7
η	$g_\eta^2/4\pi$	2.7110	3.4066
	$g_\pi^2(0)/4\pi$	1.7831	1.6636
	m_η	548.8	548.8
	Λ_η	1262.4	1000.0
ρ	$g_\rho^2/4\pi$	1.0062	1.0309
	$g_\rho^2(0)/4\pi$	0.56899	0.58296
	κ_ρ	5.01	5.01
	m_ρ	769.0	769.0
	Λ_ρ	2110.0	2110.0
ω	$g_\omega^2/4\pi$	26.561	29.593
	$g_\omega^2(0)/4\pi$	11.778	11.451
	κ_ω	0.0	0.0
	m_ω	782.6	782.6
	Λ_ω	1824.6	1702.5
δ	$g_\delta^2/4\pi$	2.9746	3.6529
	$g_\delta^2(0)/4\pi$	1.8031	2.0015
	m_δ	983.0	983.0
	Λ_δ	2089.0	1928.6
σ, $t=0$, 1	$g_\sigma^2/4\pi$	17.957, 8.6603	16.143, 7.9127
	$g_\sigma^2(0)/4\pi$	13.716, 7.4169	13.010, 6.9497
	m_σ	704.66, 534.28	674.72, 516.77
	Λ_0	1984.8, 1956.6	2110.0, 2061.7

scattering amplitudes. According to Eqs. (5.17), (5.18), (5.20), and (5.21) the mass operators in the two forms are given by

$$M_c=M_0+V_c \quad M_f^2=M_0^2+V_f, \tag{6.1}$$

where the matrix elements of the potentials are given by

$$\langle \mathbf{p}\mathbf{q}_c\beta|V_c|\mathbf{p}'\mathbf{q}_c'\beta'\rangle=(2\pi)^3 2[\epsilon_W(\mathbf{p})\epsilon_{W'}(\mathbf{p})]^{1/2}\delta^3(\mathbf{p}-\mathbf{p}')\times\left\langle \beta\left|\frac{V(\mathbf{q}_c,\mathbf{q}_c')}{2[W(\mathbf{q}_c)W(\mathbf{q}_c')]^{1/2}}\right|\beta'\right\rangle, \tag{6.2a}$$

$$\langle \overline{p}\mathbf{q}_f\beta|V_f|\overline{p'}\mathbf{q}_f'\beta'\rangle=(2\pi)^3 2p^0\delta^3(\overline{p}-\overline{p'})\langle\beta|V(\mathbf{q}_f,\mathbf{q}_f')|\beta'\rangle, \tag{6.2b}$$

with

$$V(\mathbf{q},\mathbf{q}')=\sum_b V^b(\mathbf{q},\mathbf{q}'). \tag{6.3}$$

The $b=\delta,\sigma$ contributions to Eq. (6.3) are given by Eq. (5.23).

The transition operators for the two forms satisfy the equations

TABLE II. Deuteron and low-energy parameters. Scattering lengths and effective ranges are denoted by a and r, respectively, with the subscript referring to spin singlet (s) and spin triplet (t). A_S and A_D are asymptotic normalization parameters for the deuteron's S and D components, respectively. The experimental values are from Table 4.2 of Ref. [10].

Parameter	Front form	Instant form	Experiment
$-\epsilon_d$ (MeV)	2.225	2.224	2.224575
P_D (%)	4.64	4.41	
A_S (GeV$^{1/2}$)	0.3918	0.3938	0.3930±0.0004
A_D/A_S	0.0256	0.0257	0.0256±0.0004
a_s (fm)	−23.72	−23.75	−23.748±0.010
r_s (fm)	2.71	2.73	2.75±0.05
a_t (fm)	5.407	5.411	5.419±0.007
r_t (fm)	1.737	1.741	1.754±0.008

$$T_c(z)=V_c+V_c\,\frac{1}{z-M_0}\,T_c(z),$$

$$T_f(z)=V_f+V_f\,\frac{1}{z^2-M_0^2}\,T_f(z). \tag{6.4}$$

Since the orthogonality relations for the states that appear in Eq. (6.2) are the same as Eq. (4.15), these operator equations lead to the momentum space, T-matrix equations

$$T_g(\mathbf{q},\mathbf{q}';z)=V(\mathbf{q},\mathbf{q}')+\int\frac{d^3q''}{\Delta(\mathbf{q})}\,\frac{V(\mathbf{q},\mathbf{q}'')}{D_g(\mathbf{q}'';z)}\,T_g(\mathbf{q}'',\mathbf{q}';z), \tag{6.5a}$$

$$D_c(\mathbf{q};z)=2W(\mathbf{q})[z-W(\mathbf{q})], \quad D_f(\mathbf{q};z)=[z^2-W^2(\mathbf{q})]. \tag{6.5b}$$

The transition operators, $T_g(z)$, are related to the T-matrix elements, $T_g(\mathbf{q}_g,\mathbf{q}_g';z)$, as in Eq. (6.2).

We see that with the reasonably natural procedures that we have followed in determining the instant and front form potentials, the only difference in the two forms occurs in the denominator defined by Eq. (6.5b). In order to assess the significance of this difference we have fit our potentials to the Nijmegen partial wave analysis [19] using both possibilities for $D_g(\mathbf{q};z)$. For the vertex functions we have used a form that was used earlier by us [3,23], as well as by other workers [13,14], i.e.,

$$f_b[-(\mathbf{q}-\mathbf{q}')^2]=\left[\frac{\Lambda_b^2-m_b^2}{\Lambda_b^2+(\mathbf{q}-\mathbf{q}')^2}\right]^{n_b}, \tag{6.6}$$

where Λ_b is a cutoff mass. The two sets of parameters that result from the fits are given in Table I, while the deuteron properties and the low-energy parameters are given in Table II. The two sets of phases that result from the fits are compared with each other, and with the Nijmegen phases, in Figs. 1 and 2. The couplings for the πNN and ηNN vertices have been assumed to be pure γ^5 couplings, also known as *pseudoscalar coupling*; which is consistent with the Bonn model [13,14]. The parameters κ_ρ and κ_ω determine the strength of the tensor-tensor terms in the vector-meson-

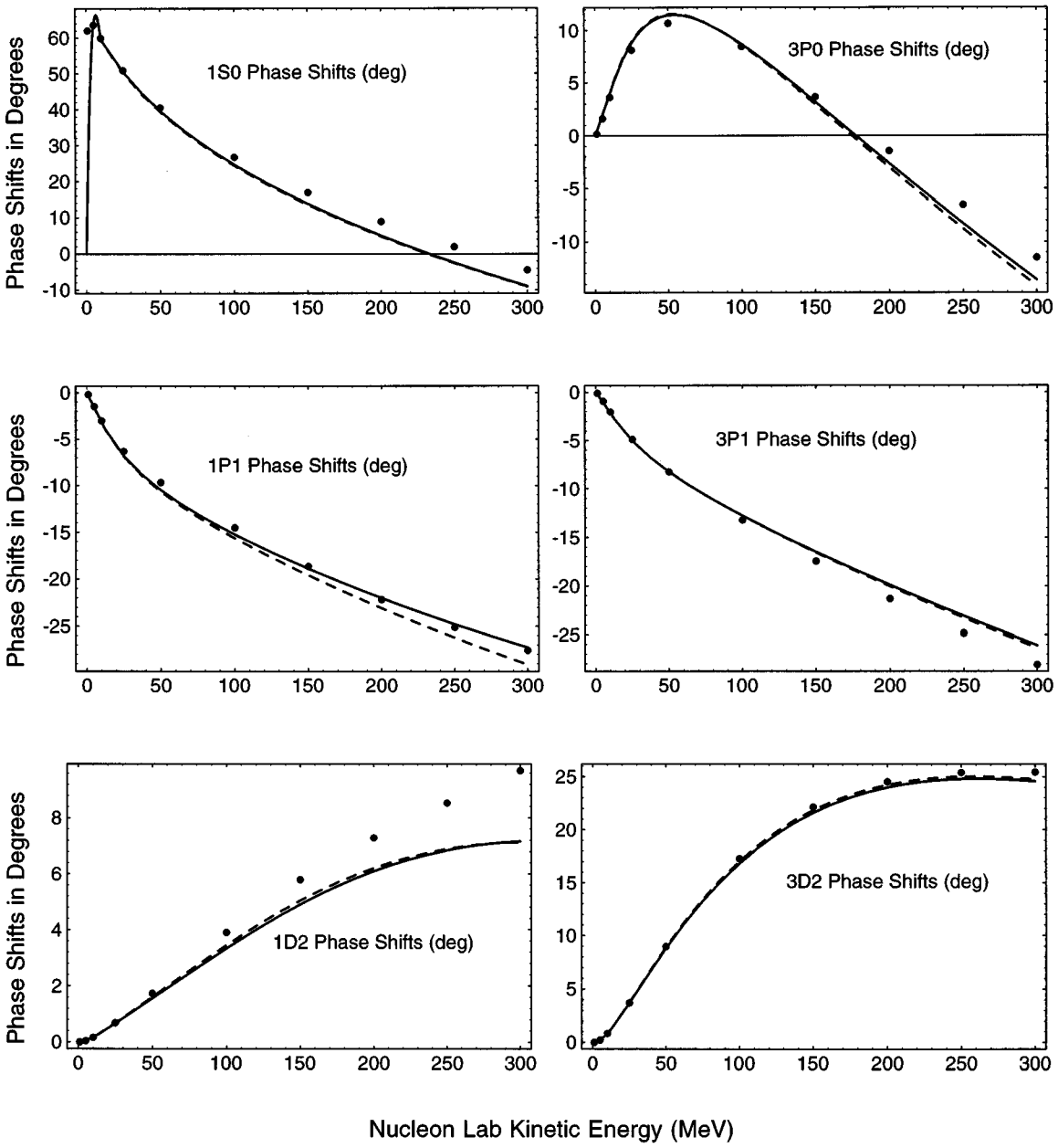

FIG. 1. Nucleon-nucleon np bar phase shifts for uncoupled states as a function of the nucleon laboratory kinetic energy. The solid lines and dashed lines are the front form and instant form results, respectively. The solid circles are from the Nijmegen partial wave analysis [19].

exchange potentials. In Table I we have also included parameters which measure the coupling strength at zero-momentum transfer, i.e.,

$$g_b^2(0)/4\pi \equiv g_b^2 f_b^2(0)/4\pi. \tag{6.7}$$

The parameters that show the most dramatic change in going from the instant form to the front form are $g_\eta^2/4\pi$ and $g_\delta^2/4\pi$, however these change are compensated to some extent by the change in the cutoff masses Λ_η and Λ_σ; as can be seen by comparing the parameters $g_\eta^2(0)/4\pi$ and $g_\delta^2(0)/4\pi$. It is worth noting that there is quite a bit of variation in the values for $g_\eta^2/4\pi$ and $g_\delta^2/4\pi$ among the various relativistic Bonn, one-boson-exchange potentials [14]; so it appears that in general these parameters are not well determined by fitting to the two-nucleon data. It seems reasonable to conclude that dramatic differences between the instant form and the front form do not show up in an analysis of the elastic, nucleon-nucleon scattering information; at least for lab kinetic energies below 300 MeV.

VII. DISCUSSION

We have found that our final results for the instant and front form, one-boson-exchange models for the two-nucleon system turn out to be quite similar. The Lippmann-Schwinger equations (6.5), that we solve to obtain the nucleon-nucleon scattering amplitudes, differ only in the nucleon-nucleon propagators. For the instant form this propagator is

$$\frac{1}{2W(\mathbf{q})[z-W(\mathbf{q})]}, \tag{7.1}$$

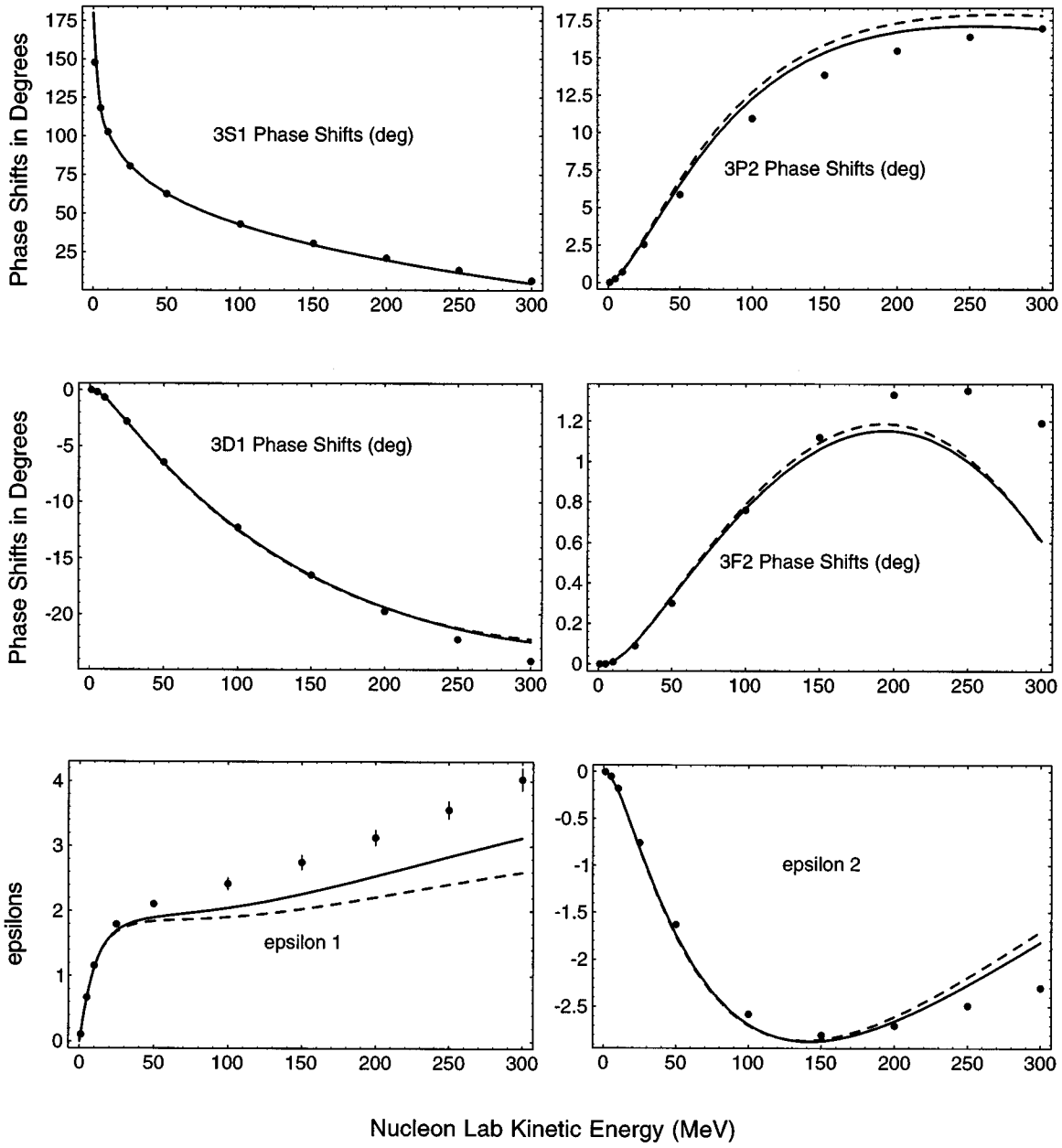

FIG. 2. Nucleon-nucleon np bar phase shifts and mixture parameters for coupled states as a function of the nucleon laboratory kinetic energy. The solid lines and dashed lines are the front form and instant form results, respectively. The solid circles are from the Nijmegen partial-wave analysis [19].

whereas for the front form we have

$$\frac{1}{z^2 - W^2(\mathbf{q})}, \tag{7.2}$$

the potentials $V(\mathbf{q},\mathbf{q}')$ are the same. The front form propagator has a practical advantage, for if we write $z^2 = W^2(\mathbf{k}) + i\epsilon$, then Eq. (7.2) becomes

$$\frac{1}{4(\mathbf{k}^2 + i\eta - \mathbf{q}^2)}, \tag{7.3}$$

which is the same as the nonrelativistic form. Thus the front form potential can be used in a nonrelativistic formalism without refitting to the nucleon-nucleon scattering information. It is interesting to note that the front form propagator (7.3) is identical to the one obtained when a Blancenbecler-Sugar reduction [8] is applied to the Bethe-Salpeter equation [7]. This propagator has also been used in developing some of the relativistic, Bonn, one-boson-exchange potentials [14]; in particular it has been used for the well-known Bonn-B potential. In fact, as pointed out previously [23]; our front form, one-boson-exchange model turns out to be identical to the Bonn-B model except for a few terms in the tensor-tensor contribution to the ρ meson-exchange potential. Our instant form model is similar to the Bonn, one-boson-exchange models based on the Thompson equation [24]; except for the just-mentioned difference in the ρ-meson-exchange poten-

tials, and the fact that the Bonn models assume $\gamma^\mu\gamma^5$ coupling for the π and η mesons, whereas we have assumed γ^5 coupling.

In a way it is not too surprising that our instant form and front form potentials come out the same, since they both derive from Feynman-like amplitudes such as Eq. (4.18). However differences arise, essentially for two reasons: in the instant form and front form we use canonical and front form spinors, respectively; and in the instant form the total three-momentum $\mathbf{p}$ is conserved, while in the front form the first three front form components, $\overline{p}=(p^0,p^1,p^2)$, of the total four-momentum are conserved. What is surprising is that for the most part these differences disappear when we apply the prescriptions that are necessary to achieve Poincaré invariance. To some extent we forced the identity of the final potentials by imposing a couple of prescriptions beyond those necessary to guarantee Poincaré invariance. Applying the prescriptions (5.19) and (5.22) led to meson propagators given by

$$\frac{1}{[\epsilon_{m_N}(\mathbf{q})-\epsilon_{m_N}(\mathbf{q}')]^2-(\mathbf{q}-\mathbf{q}')^2-m_b^2} \tag{7.4}$$

in the instant form, and by

$$\frac{-1}{(\mathbf{q}-\mathbf{q}')^2+m_b^2} \tag{7.5}$$

in the front form. We chose to use the front form propagator in both cases so as to improve convergence in the instant form at high momenta. As an alternative to this prescription we could have stayed with Eq. (7.4) in the instant form, and modified the instant form vertex functions to improve convergence. We improved convergence in the front form by approximating the factor [see Eq. (5.16)] $\cosh^2(\zeta_f/2)=(W+W')^2/4WW'$ by 1. We could have kept this factor and modified the front form vertex functions to improve convergence. We intend to explore these alternatives numerically, in the future. In this connection, it is intesting to note that Haidenbauer and Holinde [25] have pointed out that the Blancenbecler-Sugar reduction [8] of the Bethe-Salpeter equation [7] leads to the meson-propagator (7.5), while Gross's reduction [9,11] leads to Eq. (7.4).

It appears that once the decision is made to use the Bakamjian-Thomas method [6,18] for constructing Poincaré-invariant models, there is no simple alternative to the instant form prescription (5.19). As pointed out in Sec. V, there is an alternative prescription for the front form. In the *new picture* formalism for front form dynamics [26] the unit vector $\mathbf{u}_f$, defined in Eq. (5.13c), becomes a variable; and it has been shown [2] that another plausible prescription for producing Poincaré-invariant front form models within the Bakamjian-Thomas framework is to average over the direction of $\mathbf{u}_f$. Although this leads to numerical complications, we intend to explore this possibility in the future.

The basic idea of the Bakamjian-Thomas method is to express the Poincaré generators in terms of a set of operators that satisfy simpler commutation rules than the generators. It is of course conceivable that Poincaré invariant few particle models can be constructed in terms of the generators directly. The possibility of doing this by applying the Okubo method [17] simultaneously to the Poincaré generators of a quantum field theory has been studied by Glöckle and Müller [16], and by one of us [2]. The difficulty here lies with the use of perturbation theory to determine the unitary transformation that simultaneously block diagonalizes all 10 generators. If, for example, the quantum field theory is formulated in the instant form, i.e., quantized on a spacelike surface, the quantum field-theory three-momentum operator $\mathbf{P}$ and angular momentum operator $\mathbf{J}$ are noninteracting. The effective generators that act in the few particle subspace of interest can simply be taken to be

$$\mathbf{P}^\eta=\eta\mathbf{P}\eta, \quad \mathbf{J}^\eta=\eta\mathbf{J}\eta, \tag{7.6}$$

where η is the projection operator onto this subspace. The interacting quantum field theory generators are the Hamiltonian H and the generator of rotationless boosts $\mathbf{K}$, and the effective few particle generators, H^η and $\mathbf{K}^\eta$, must be obtained from these by perturbation theory. The commutators which involve two noninteracting generators, such as $[P_j^\eta,J_k^\eta]$, or one noninteracting generator and one interacting generator, such as $[\mathbf{P}^\eta,H^\eta]$, are exactly correct in any order of perturbation theory; however commutators that involve two interacting generators, such as $[H,\mathbf{K}]$, are only approximately correct. This is the reason we have chosen to use the Bakamjian-Thomas method.

It is clear that the methods described here for constructing the one-boson-exchange model for the two-nucleon system can be extended so as to allow a treatment of more complicated exchange mechanisms, such as two-pion exchange. Also the method developed for constructing an exactly Poincaré-invariant model for the pion-nucleon system [27] can be applied to the two-nucleon system in order to allow for coupling to $N\Delta$ and $\Delta\Delta$ channels. This will make it possible to extend the analysis to higher energies, and to take inelasticity into account.

It will be of interest to further compare the instant and front forms. It is known that the various forms of relativistic quantum mechanics are related by unitary transformations [28,29], so in principle any form can be used; however as we saw here approximations can play out differently in the various forms. As far as the two-nucleon system is concerned there does not seem to be any practical advantage of one form over the other. Most likely this indifference does not extend to the calculation of electromagnetic form factors and structure functions, or to the three-nucleon system. It is already known that the front form has certain advantages over the instant form with regard to constructing current operators for composite system. In particular it is possible to develop a sensibly consistent impulse approximation for electromagnetic processes in the front form, but not in the instant form [6,30]. In going to the three-nucleon system it is necessary to take into account *cluster separability* [6,29,31,22]. Roughly speaking, this is the requirement that a model of the three-nucleon system decomposes into a model of the two-nucleon system and an independent nucleon when this nucleon has a large spacelike separation from the other two. Because problems with cluster separability arise mainly in connection with the interacting Poincaré generators, the implementation of this requirement plays out differently in the various forms of relativistic quantum mechanics.

We conclude by noting that the methods discussed here can also be applied to few-particle systems in which the interactions take place through the exchange of gluons or photons.

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