First-order eikonal approximation for the elastic scattering of 800 MeV/c pions from ^{12}C and ^{40}Ca nuclei

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We have extended the first-order eikonal model formalism to include charged pions scattering on nuclei. It has been applied satisfactorily to elastic angular distributions of 800 MeV/c pions scattering from ^{12}C and ^{40}Ca nuclei. [S0556-2813(96)02706-9]

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In the past years a number of efforts have been made to study the interaction of pions with nuclei [1–9]. Precise measurements of the cross sections of 800 MeV/c pions scattered from ^{12}C and ^{40}Ca were performed [2] at Brookhaven National Laboratory. The elastic scattering data were analyzed within the framework of the first-order optical model [2] and within the framework of Glauber theory [3]. Reasonable agreements between the predicted results and observed data were obtained in both cases. Recently, elastic scattering data were also calculated [8] by using a strong absorption model (SAM). The overall results of the SAM calculation were reported to be in excellent agreement with the experimental data.

In a previous paper [10], we presented first- and second-order corrections to the zero-order eikonal phase shifts for heavy-ion elastic scatterings based on Coulomb trajectories of colliding nuclei and it has been applied satisfactorily to the ^{16}O$+^{40}$Ca and ^{16}O$+^{90}$Zr systems at $E_{\mathrm{lab}}=1503$ MeV. Hence, it is interesting to extend the formalism of eikonal phase shifts to include charged pions scattering on nuclei and also to obtain potential parameter values applicable to the eikonal phase shift approach for these nuclei. In this paper, we reproduce the elastic scattering angular distributions for the charged pions scatterings on ^{12}C and ^{40}Ca nuclei by using the phase shift analysis based on the Coulomb-modified eikonal phase shift and its first-order correction.

The general expression for the elastic scattering amplitude between a spin-zero charged pion and a spin-zero target nucleus is given by

$$f(\theta)=f_R(\theta)+\frac{1}{ik}\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)e^{2i\sigma_l}(S_l^N-1)P_l(\cos\theta), \quad (1)$$

where $f_R(\theta)$ is the usual Rutherford scattering amplitude and σ_l the Coulomb phase shift. In the above equation, k is the pion wave number in the center of mass coordinate system given by

$$k=\frac{m_N}{\hbar}\left[\frac{(E_\pi^2/c^2)-m_\pi^2c^2}{m_\pi^2+m_N^2+2m_NE_\pi/c^2}\right]^{1/2}, \quad (2)$$

where m_π and m_N are the pion and nuclear rest mass, respectively. E_π is the total pion energy in the laboratory system. In Eq. (1), the nuclear S matrix S_l^N can be obtained from the nuclear phase shifts δ_l with $S_l^N=\exp(2i\delta_l)$. In the present formulation, the Coulomb-modified eikonal phase shift and its first-order correction are given by [10]

$$\delta_l^0=-\frac{\mu}{\hbar^2k}\int_0^\infty V_N(\sqrt{r_c^2+z^2})dz, \quad (3)$$

$$\delta_l^1=-\frac{\mu^2}{2\hbar^4k^3}\left(1+r_c\frac{d}{dr_c}\right)\int_0^\infty V_N^2(\sqrt{r_c^2+z^2})dz. \quad (4)$$

Here μ is the reduced mass and the distance of closest approach r_c is written as

$$r_c=\frac{1}{k}\{\eta+[\eta^2+l(l+1)]^{1/2}\}, \quad (5)$$

with the Sommerfeld parameter η. By taking the nuclear potential $V_N(r)$ as a complex Woods-Saxon form given by

$$V_N(r)=-\frac{V_0}{1+e^{(r-r_vA_N^{1/3})/a_v}}-i\frac{W_0}{1+e^{(r-r_wA_N^{1/3})/a_w}}, \quad (6)$$

TABLE I. Shape parameters of the fitted complex potential from the eikonal phase shift analysis for charged pions elastic scattering on nuclei at an incident momentum of 800 MeV/c.

		Zero order				First order			
Pions	Nucleus	r_v (fm)	a_v (fm)	r_w (fm)	a_w (fm)	r_v (fm)	a_v (fm)	r_w (fm)	a_w (fm)
π^+, π^-	^{12}C	0.89	0.50	1.00	0.47	0.91	0.50	0.97	0.49
π^+, π^-	^{40}Ca	1.03	0.60	1.17	0.48	1.03	0.63	1.16	0.47

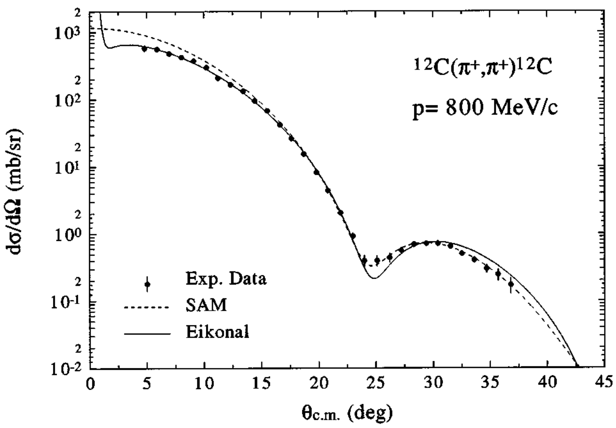

FIG. 1. Elastic scattering angular distribution for positive pions of an incident momentum 800 MeV/c from ^{12}C. The observed data are taken from Ref. [2]. The result of the strong absorption model (SAM) is taken from Ref. [8].

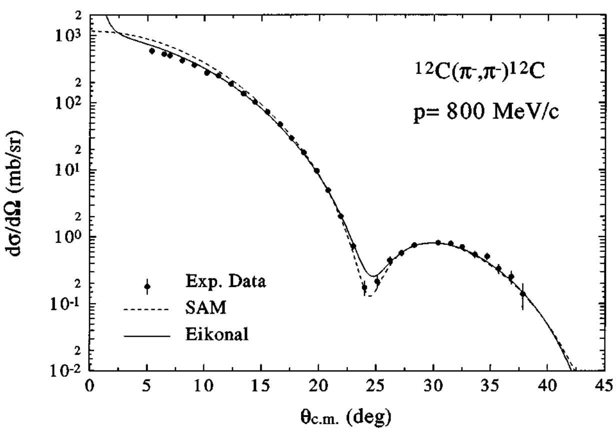

FIG. 2. Elastic scattering angular distribution for negative pions of an incident momentum 800 MeV/c from ^{12}C. The observed data are taken from Ref. [2]. The result of the strong absorption model is taken from Ref. [8].

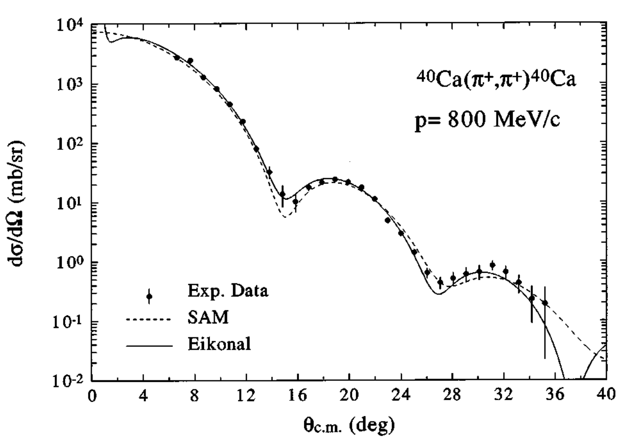

FIG. 3. Elastic scattering angular distribution for positive pions of an incident momentum 800 MeV/c from ^{40}Ca. The observed data are taken from Ref. [2]. The result of the strong absorption model is taken from Ref. [8].

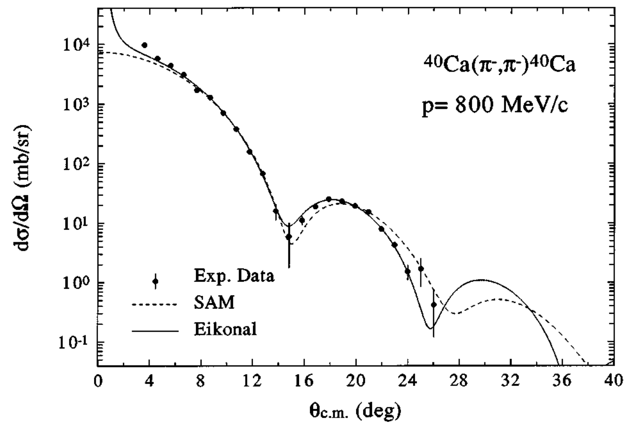

FIG. 4. Elastic scattering angular distribution for negative pions of an incident momentum 800 MeV/c from ^{40}Ca. The observed data are taken from Ref. [2]. The result of the strong absorption model is taken from Ref. [8].

with nuclear mass number A_N, we can use the two Coulomb-modified eikonal phase shifts (3) and (4) instead of the commonly used eikonal phase shift (3).

The above eikonal phase shifts δ_l^0 and δ_l^1 have been used to calculate the elastic scattering cross secctions for 800 MeV/c charged pions incident upon the target nuclei ^{12}C and ^{40}Ca. The shape parameters r and a for π^+ and π^- ions on a given nucleus are set to be equal during the fit as given in Table I. In Figs. 1–4, the calculated differential cross sections of our model are compared with the results of the strong absorption model and with the observed data for $\pi^\pm + {}^{12}$C and $\pi^\pm + {}^{40}$Ca systems. In these figures, the solid curves represent the cross sections of the first-order eikonal model and the short-dashed curves denote the results of the strong absorption model. The calculated results for ^{12}C$(\pi^\pm,\pi^\pm)^{12}$C elastic scattering are shown in Figs. 1 and 2. It is seen that the agreements of the first-order eikonal model with the observed data are satisfactorily good in both cases compared to the results of the SAM. The results for ^{40}Ca$(\pi^\pm,\pi^\pm)^{40}$Ca elastic scatterings are shown in Figs. 3

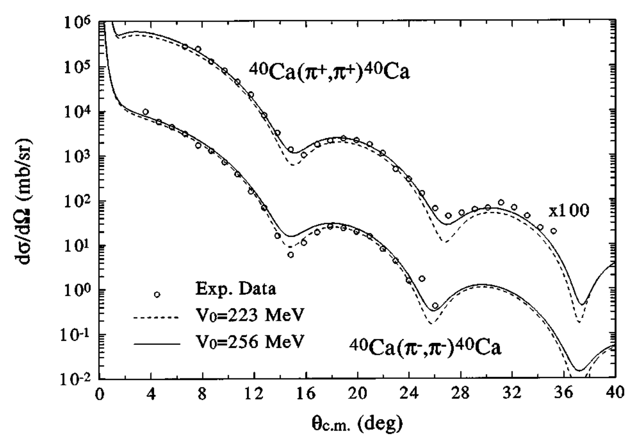

FIG. 5. Elastic scattering angular distributions for the positive and negative pions on ^{40}Ca nuclei with $V_0 = 223$ and 256 MeV. Shape parameters and imaginary potential depths of these systems are given in Tables I and II.

TABLE II. The complex potential depths, reaction cross sections, and χ^2/N values from the zero-order and first-order eikonal phase shift analysis for pion+nucleus elastic scattering at $p=800$ MeV/c.

		Zero order				First order			
Pion	Nucleus	V_0 (MeV)	W_0 (MeV)	σ_R (mb)	χ^2/N	V_0 (MeV)	W_0 (MeV)	σ_R (mb)	χ^2/N
π^+	^{12}C	229	240	196	2.39	236	225	188	2.79
π^-	^{12}C	232	255	204	2.88	238	245	199	2.69
π^+	^{40}Ca	258	140	446	1.94	256	123	416	1.90
π^-	^{40}Ca	224	170	496	2.96	223	162	484	2.73

and 4. It is seen that the agreements of our model with the observed data are also satisfactorily good in both cases. In Table II, we can see that the first-order eikonal model decreases somewhat the values of reaction cross section compared with the values of the zero-order one. The χ^2/N values of the first-order eikonal approach are also decreased in some degree compared with ones of the zero-order one except for π^+ scattering on ^{12}C. One of the overall characteristics appearing in Table II is that the values of an imaginary potential parameter W_0 for positive pion scatterings are smaller than the case for negative pions. The value of W_0 for the $\pi^+ + {}^{12}$C system is less about 8% than the value for $\pi^- + {}^{12}$C, while the difference of W_0 between $\pi^+ + {}^{40}$Ca and $\pi^- + {}^{40}$Ca systems amounts to 24% due to an increase of the target charge. Such differences in the values of the parameter W_0 are mainly due to repulsive Coulomb fields experienced by the positively charged pions compared with the negatively charged pions. Hence from these decreases in the values of W_0 for the positive pions one would expect that repulsive Coulomb fields lead to less absorption of the incident wave than the case for negative pions as pointed out in Ref. [8]. Such absorption effects are reflected in the values of the reaction cross section listed in Table II. We can also see in this table that the potential strength V_0 is almost the same for π^+ and π^- on ^{12}C, but is quite different on ^{40}Ca. Such differences of the real potential depth affect the differential cross section as shown in Fig. 5. In this figure, we can find that the angular distributions for $V_0=256$ MeV are shifted to the upper part as a whole compared with ones for $V_0=223$ MeV.

In this Brief Report we have presented a first-order correction to the zero-order eikonal phase shifts for the charged pions scattering on nuclei. We have found that the overall results of the present first-order eikonal model are in excellent agreement with the observed data for the elastic scattering of 800 MeV/c pions from ^{12}C and ^{40}Ca nuclei.

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