Field-theoretical description of nuclear matter with only the pion-nucleon interaction

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A relativistic Lagrangian for the nuclear matter consisting of nucleons and pions with a pseudoscalar interaction term is considered. It is shown that a nonrelativistic reduction of the problem automatically introduces a Lorentz scalar, isoscalar coupling of nucleons with two correlated pions. The corresponding Hamiltonian is used to study the properties of infinite nuclear matter nonperturbatively, treating both the nucleons and pions as quantized fields. The model is shown to reproduce the characteristic nuclear matter properties very nicely without the necessity of σ and ω fields, as is usually done in the mean field Walecka model. The corresponding equation of state for zero temperature nuclear matter is calculated and is shown to be consistent with the known phenomenological equations of state. The binding energy of nuclear matter is calculated to be 15.3 MeV at a saturation density of 0.153 fm^{-3}. The model reproduces a softer nuclear matter with the incompressibility of 134 MeV. [S0556-2813(96)04906-0]

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I. INTRODUCTION

Since the prediction of pions by Yukawa in the context of the nucleon-nucleon interaction, a considerable amount of effort has been channeled to the sole aim of gaining an understanding of the two nucleon interaction, starting from the basic meson nucleon interaction. The *N-N* potentials generated by including the exchange of several low mass mesons, the so-called one boson exchange potentials (OBEP) [1,2] have achieved a remarkable success in explaining the properties of the two nucleon states, as well as those of the nuclear many body systems. However, a lacuna still persists in all the versions of the OBEP in the fact that a scalar isoscalar meson σ has to be introduced in an *ad hoc* manner to account for the strong intermediate range attraction observed in all *N-N* channels. Such a mesonic state has not yet been conclusively established in experiments.

The structure of the σ meson has been investigated by several authors [2–4]. Since an intermediate state of mass $2m_\pi \approx 280$ MeV clearly has a sufficiently long range to reproduce the observed midrange *N-N* force, these contributions must be included in the interaction kernel, if one already includes the exchange of heavier mesons in the OBEP. Indeed two pions in the *S*-wave, isospin zero state have the same quantum numbers as the scalar isoscalar particle σ, therefore one may naturally suppose that the observed midrange attraction is produced by correlated two pion exchange between the nucleons. Durso *et al.* [4] have shown that about 2/3 of the scalar isoscalar attraction in the *N-N* channel is supplied by the resonant two pion exchange which can be approximated by a scalar particle with a broad mass centered at about 650 MeV. These authors also found significant attraction supplied by the *NN* and $N\Delta$ box diagrams, a fact that has been noted earlier [3,5]. The implied NN attraction in this model is found strong enough to explain the observed nuclear binding in simple nuclear matter calculations [6]. The OBEP models [2,7] which include the resonant two pion exchange in addition to the *NN* and $N\Delta$ box diagrams also achieve accurate quantitative fits to the *NN* scattering data. In all these OBEP models there are still ambiguities about the dynamical models of the $\pi\pi$ interaction and about the consistency of crossed diagrams and the counting of amplitudes when an elementary $\pi N\Delta$ vertex is included in the model [8].

Another general framework which has achieved considerable success in recent years for the description of nuclear matter and finite nuclei is Walecka's σ-ω model [9–11]. The *NN* dynamics arises in this model from the exchange of a Lorentz scalar isoscalar meson ''σ'' which provides the midrange attraction and an isoscalar vector meson ''ω'' which provides the short range repulsion. With a small number of parameters the σ-ω model reproduces the nuclear matter saturation and describes the bulk and single particle properties of nuclei reasonably well [12].

Despite the successes of the Walecka model, several open questions still remain unanswered. Basically, the microscopic nature of the σ field in this model is unclear. The Walecka ''σ'' cannot be interpreted as the representation of a physical particle, since such a particle or resonant state remains still to be confirmed in experiments. The model also produces collapse upon passing to the nonrelativistic limit, which may not correspond to complete reality.

The original Walecka model [9] does not contain a dynamical description of the pion field. However, the importance of pions in intranuclear dynamics cannot be simply wished away. The dominance of pion degrees of freedom in certain nuclear processes, like the pion-exchange currents, has been established beyond any doubts in the past two decades [13]. The σ-ω model has later been extended to include other mesons: π, ρ, etc. [10,14]; however, for a correct description of pion dynamics in nuclear medium, it has been found necessary to impose an additional chiral symmetry. Unfortunately, even this larger symmetry cannot provide a consistent description of nuclear matter, the nuclear ground state, and the free *N-N* scattering simultaneously [15,16].

Realizing the essential role of pions in the description of nuclear medium, an alternative model for infinite nuclear matter consisting of interacting nucleons and pions has been attempted in recent years [17]. The model is aesthetically appealing in the sense that the scalar isoscalar pion pair-states simulate the effects of σ mesons. The purpose of the present work is to examine some features of the pion-nucleon description of the nuclear matter; especially to examine the bulk properties of nuclear matter in such a simple model, without introducing additional mesonic degrees of freedom or additional symmetry parameters. In the recent extensions of the Walecka model attempts have been made to include the heavier mesons, like ρ and ω, besides the pion and the ''σ'' meson. However, while the ''σ'' is not observed, ρ and ω are the multipion resonant states. If the dynamics of the pions and nucleons is treated correctly, the effects attributed to the multipion resonances should automatically be included. In our model we do not include either σ or any of the pionic resonances; but we treat the strongly interacting pions nonperturbatively. In an earlier work, Schuck *et al.* [18] have estimated the contribution of two pion correlation to nuclear matter binding by introducing the π-π interaction through a phenomenological Hamiltonian. We have, however, shown explicitly below that a nonrelativistic reduction of the relativistic Hamiltonian for the nuclear matter automatically gives rise to an N-N interaction term involving the square of the pion field φ, which is amenable to the introduction of the Bogoliubov transformations.

It may be argued that the nonrelativistic model presented here is somewhat incomplete, because several important questions concerning the ω meson repulsion and relativistic corrections have been left unanswered. However, an exact treatment of multipion correlations should reproduce the σ,ρ, and ω contributions automatically. Our attempt is only a first step in this direction. Much more work is needed to arrive at satisfactory answers to these questions.

In Sec. II we introduce an effective Lagrangian for a system consisting of interacting nucleons and pions and a nonrelativistic Hamiltonian for the system is deduced. Section III describes a Bogoliubov transform to handle the correlated S-wave pion pairs. Section IV calculates the nuclear matter properties from the above Hamiltonian. A discussion of the physical content of the model and comparison of the calculated bulk properties of nuclear matter with their phenomenological values are presented in Sec. V.

II. NONRELATIVISTIC HAMILTONIAN

Recognizing the important role of the pion field as a constituent of nuclear matter we can write down an effective Lagrangian for the nuclear matter as

$$\mathcal{L}=\overline{\Psi}(i\gamma^{\mu}\partial_{\mu}-M+G\gamma_5\phi)\psi+\tfrac{1}{2}\{(\partial_{\mu}\varphi_i)(\partial^{\mu}\varphi_i)-\mu^2\varphi_i\varphi_i\}, \tag{1}$$

where $\psi=\binom{\psi_I}{\psi_{II}}$ is the doublet nucleon field and $\phi=\tau_i\varphi_i;\varphi_i$'s are the pion fields.

The representation for the γ matrices are

$$\gamma=\begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix},\quad \gamma^0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \gamma^5=\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}.$$

From the above Lagrangian, the equations of motion are

$$(E-M)\psi_I+(\vec{\sigma}\cdot\vec{p}-iG\phi)\psi_{II}=0, \tag{2}$$

$$(E+M)\psi_{II}+(\vec{\sigma}\cdot\vec{p}+iG\phi)\psi_I=0. \tag{3}$$

In the above equations $E\equiv H=i(\partial/\partial t)$ and $\vec{p}=-i(\partial/\partial\vec{x})$.

Eliminating the small component ψ_{II} from Eq. (2) and (3) we have

$$[(E^2-M^2)-(E+M)(\vec{\sigma}\cdot\vec{p}-iG\phi)(E+M)^{-1}\times(\vec{\sigma}\cdot\vec{p}+iG\phi)]\psi_I=0. \tag{4}$$

$(E+M)$ in the nonrelativistic limit may be assumed to commute with ϕ so that Eq. (4) can be rewritten as

$$[E^2-M^2-p^2+iG\{(\vec{\sigma}\cdot\vec{p})\phi\}-G^2\phi\cdot\phi]\psi_I=0. \tag{5}$$

In getting to Eq. (5) from Eq. (4), a term of the magnitude of (E_π/M) has been neglected, so that what follows will be valid in the limit of low excitation energies of the pions in the nuclear medium. From Eq. (5) we can immediately identify an effective Hamiltonian for the nucleons:

$$H_N=\psi_I^{\dagger}(x)[p^2+M^2-iG\{(\vec{\sigma}\cdot\vec{p})\phi\}+G^2\phi^2]^{1/2}\psi_I(x)\approx\psi_I^{\dagger}(x)\left[(p^2+M^2)^{1/2}-\left\{\frac{iG}{2M}(\vec{\sigma}\cdot\vec{p})\phi\right\}+\frac{G^2}{2M}\phi^2\right]\psi_I(x). \tag{6}$$

The effective Hamiltonian consists of two distinct parts, the free nucleon part

$$H_N^{(0)}(x)=\psi_I^{\dagger}(x)[(-\nabla^2+M^2)^{1/2}]\psi_I(\vec{x}), \tag{7}$$

and the effective pion-nucleon interaction term

$$H_{\rm int}(x)=\psi_I^{\dagger}(\vec{x})\left[-\frac{iG}{2M}\{(\vec{\sigma}\cdot\vec{p})\phi\}+\frac{G^2}{2M}\phi^2\right]\psi_I(\vec{x}). \tag{8}$$

In Eqs. (6)–(8) both ψ_I and ϕ are taken to be quantized fields. In the succeeding section we shall use Bogoliubov transformation to diagonalize the Hamiltonian.

III. BOGOLIUBOV TRANSFORMATION

The ϕ^2 term in Eq. (8) constitutes a scalar-isoscalar interaction of the nucleons. To treat the ϕ^2 term we introduce a Bogoliubov transformation generating the creation and annihilation of pseudopions. With $a_{\vec{k}},a_{\vec{k}}^{\dagger}$ as pion annihilation and creation operators, the pseudopion annihilation operator is

$$\alpha_{\vec{k}}=u_{\vec{k}}a_{\vec{k}}+v_{\vec{k}}a_{-\vec{k}}^{\dagger}. \tag{9}$$

It is easy to check that

$$[\alpha_{\vec{k}},\alpha_{\vec{q}}^{\dagger}]=\delta_{\vec{k}\vec{q}},\qquad [\alpha_{\vec{k}},\alpha_{\vec{q}}]=[\alpha_{\vec{k}}^{\dagger},\alpha_{\vec{q}}^{\dagger}]=0, \tag{10}$$

provided that

$$u_{\vec{k}}^2-v_{\vec{k}}^2=0 \tag{11}$$

and

$$u_{-\vec{k}}=u_{\vec{k}}, \quad v_{-\vec{k}}=v_{\vec{k}}.$$

Such transformations are canonical. The bare nucleon states however get dressed with pions. These states are necessary for the correct description of nuclear matter. To find them we construct the dressed nucleon states as

$$|\psi_{\text{dressed}}\rangle=U|\psi_{\text{bare}}\rangle=U|0\rangle, \tag{12}$$

where U is a unitary and Hermitian operator. The pseudopions are the result of a unitary transformation, namely

$$\alpha_k=U^{\dagger}a_kU, \quad \alpha_k^{\dagger}=U^{\dagger}a_k^{\dagger}U. \tag{13}$$

The correlated pion states can be seen if we calculate U explicitly. Since U is Hermitian and unitary, in general it will be

$$U=\exp\left[\tfrac{1}{2}\int d\vec{k}\ f(\vec{k})a_{\vec{k}}^{\dagger}a_{-\vec{k}}^{\dagger}-\tfrac{1}{2}\int d\vec{k}\ f(\vec{k})a_{\vec{k}}a_{-\vec{k}}\right]=e^{B}, \tag{14}$$

so that

$$[B,a_{\vec{k}}]=-f(\vec{k})a_{\vec{k}}^{\dagger} \tag{15}$$

and

$$[B_{+},a_{\vec{k}}^{\dagger}]=-f(\vec{k})a_{\vec{k}}.$$

Let us define the operators

$$U(\lambda)=e^{\lambda B},$$

and

$$F(\lambda)=U^{\dagger}(\lambda)a_{\vec{k}}U(\lambda), \tag{16}$$

so that

$$U=U(1),$$

and

$$\alpha_{\vec{k}}=F(1).$$

Differentiating the expression for $F(\lambda)$ twice in Eq. (16) one gets

$$\frac{d^2F(\lambda)}{d\lambda^2}=f^2(\vec{k})F(\lambda). \tag{17}$$

Equation (17) yields

$$F(\lambda)=\cosh\{\lambda f(\vec{k})\}a_{\vec{k}}+\sinh\{\lambda f(\vec{k})\}a_{-\vec{k}}^{\dagger}, \tag{18}$$

since

$$F(0)=a_{\vec{k}}$$

and

$$F'(0)=a_{-\vec{k}}^{\dagger}f(\vec{k}). \tag{19}$$

From Eq. (18) one gets

$$\alpha_{\vec{k}}=\cosh f(\vec{k})a_{\vec{k}}+\sinh f(\vec{k})a_{-\vec{k}}^{\dagger}$$
$$=u_{\vec{k}}a_{\vec{k}}+v_{\vec{k}}a_{-\vec{k}}^{\dagger}, \tag{20}$$

which is the Bogoliubov transformation to be used later. $f(\vec{k})$ in Eq. (20) will be determined when the meson pairing energy will be maximized.

IV. NUCLEAR MATTER

Besides the effective nucleonic Hamiltonian given in Eqs. (7) and (8), the free mesonic Hamiltonian in nuclear matter is given as

$$H_M=\int d^3x\ \tfrac{1}{2}[\dot{\varphi}_i^2+\vec{\nabla}\varphi_i\cdot\vec{\nabla}\varphi_i+m_\pi^2\varphi_i^2]. \tag{21}$$

When the quantized pion field is represented as

$$\varphi_i(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2w_k}}[e^{i\vec{k}\cdot\vec{x}}a_{i\vec{k}}+e^{-i\vec{k}\cdot\vec{x}}a_{i\vec{k}}^{\dagger}], \tag{22}$$

the meson Hamiltonian can be written as

$$H_M=\int\frac{d^3k}{(2\pi)^3}w_k a_{i\vec{k}}^{\dagger}a_{i\vec{k}}, \tag{23}$$

where $w_k^2=k^2+m_\pi^2$. So the kinetic energy density due to mesons in nuclear matter is given by

$$h_k=\langle\psi_{\text{dressed}}|H_M|\psi_{\text{dressed}}\rangle$$
$$=\sum_{i,k} w_k\langle 0|\alpha_{i\vec{k}}^{\dagger}\alpha_{ik}|0\rangle$$
$$=3/(2\pi)^3\int d^3k\ w_k\sinh^2 f_k. \tag{24}$$

From the effective interaction Hamiltonian of Eq. (8), the first term containing a single pion field φ does not contribute to the interaction energy density. So the interaction energy density is given by [10]

$$
\begin{aligned}
h_{\text{int}}&=\langle\psi_{\text{dressed}}|\text{tr}[\hat{\rho}_N H_{\text{int}}(x)]|\psi_{\text{dressed}}\rangle\\
&=\frac{G^2}{2M}\rho\int d^3x\langle\psi^\dagger_{\text{dressed}}(x)|:\phi_i(x)\phi_i(x):|\psi_{\text{dressed}}\rangle\\
&=\frac{G^2}{2M}\rho\int\frac{d^3k}{(2\pi)^3}\frac{1}{2w_k}\\
&\quad\times\langle 0|\alpha^\dagger_{i\vec{k}}\alpha^\dagger_i(-\vec{k})+\alpha_{i\vec{k}}\alpha_{i(-\vec{k})}+2\alpha^\dagger_{i\vec{k}}\alpha_{i\vec{k}}|0\rangle\\
&=\frac{3G^2}{2M}\rho\int\frac{d^3k}{(2\pi)^3}\frac{1}{w_k}\left\{\frac{\sinh 2f(\vec{k})}{2}+\sinh^2 f(\vec{k})\right\}.
\end{aligned}
\tag{25}
$$

So the meson energy density from the kinetic and interaction terms is

$$
h_m=h_k+h_{\text{int}}=3\int\frac{d^3k}{(2\pi)^3}\left[\left(w_k+\frac{G^2\rho}{2M}\frac{1}{w_k}\right)\sinh^2 f_k+\frac{G^2\rho}{2M}\frac{1}{2w_k}\sinh 2f_k\right].
\tag{26}
$$

Now extremizing Eq. (26) with respect to f_k gives

$$
\tanh 2f_k=-\frac{G^2\rho}{2M}\frac{1}{w_k^2+\frac{G^2\rho}{2M}}.
\tag{27}
$$

Making use of Eq. (27), Eq. (26) can be rewritten as

$$
h_m=\frac{3}{2}\left(\frac{G^2\rho}{2M}\right)^2\frac{1}{(2\pi)^3}\int_0^\infty\frac{d^3k}{w_k}\times\frac{1}{\left[w_k\left(w_k^2+\frac{G^2\rho}{M}\right)^{1/2}+\left(w_k^2+\frac{G^2\rho}{M}\right)\right]}.
\tag{28}
$$

The energy density given by Eq. (28) is not finite since the integral diverges. To get rid of the divergence we introduce a cutoff, an upper limit Λ for the integration variable k. This is equivalent to introducing a form factor for the pions, instead of treating them as point particles, as has been done in Ref. [17]. However, the present procedure has the advantage of introducing only one parameter instead of the two of Ref. [17]. Besides, with a cutoff upper limit, as will be seen later, the integral can be evaluated analytically and hence a better understanding of the contributions from different terms can be obtained. Λ is treated purely as a parameter; however, an order of magnitude estimate for this quantity can be obtained from simple physical reasoning. Schuck *et al.* [18]. have reported through detailed calculations that the pion self-energy $\overline{w_q}$ with pion pair correlation, as introduced above, jumps back to its free value $w_q(=\sqrt{q^2+m_\pi^2})$ beyond a certain q value ≈ 400 MeV/c. Once $\overline{w_q}$ approaches w_q, the coherent state disappears. They have also shown that, if one uses the pion dispersion relation with nuclear medium corrections, the two pions form a bound state which is located 10 MeV below the $2m_\pi$ threshold. So for correlated pion pair excitation, we expect the maximum pion energy $w(\Lambda)\leqslant 2m_\pi$. That such a pion momentum cutoff is necessary for the pairing to occur is also evident from the nonrelativistic limit used in getting to Eq. (5).

Besides the pion kinetic and interaction energies discussed above, we must also include the nucleonic energy density

$$
h_N=\int d^3x\,\psi_I(x)\varepsilon_X\psi_I(x)=\frac{\gamma}{6\pi^2}k_F^3\left(M+\frac{3}{10}\frac{k_f^2}{10M}\right).
\tag{29}
$$

$\gamma=4$ for nuclear matter and the Fermi momentum k_F is related to the nucleon density ρ by

$$
\rho=\frac{\gamma k_f^3}{6\pi^2}.
$$

Collecting all terms, finally we have the energy per nucleon

$$
E/A=\frac{h_N+h_m}{\rho}.
\tag{30}
$$

With the integrations done, the pion energy of Eq. (28) can be simply expressed as

$$
\frac{h_m}{\rho}=\frac{h_m^{(1)}}{\rho}+\frac{h_m^{(2)}}{\rho},
\tag{31}
$$

where

$$
\frac{h_m^{(1)}}{\rho}=\frac{3}{32\pi^2}(G^2/M)\frac{(\Lambda^2-m_\pi^2)^{1/2}}{\Lambda+\left(\Lambda^2+\frac{G^2\rho}{M}\right)^{1/2}}\left[\Lambda^2+\frac{G^2\rho}{M}-\Lambda\left(\Lambda^2+\frac{G^2\rho}{M}\right)^{1/2}-m_\pi^2\right]
\tag{32}
$$

and

$$
\begin{aligned}
\frac{h_m^{(2)}}{\rho}&=\frac{3}{32\pi^2}(m_\pi^4/\rho)\ln\frac{\Lambda+\sqrt{\Lambda^2-m_\pi^2}}{m_\pi}\\
&\quad+\frac{3}{16\pi^2}\left(\frac{G^2m_\pi^2}{M}\right)\ln\frac{\Lambda+\sqrt{\Lambda^2-m_\pi^2}}{m_\pi}-\frac{3}{64\pi^2}\frac{1}{\rho}\left(\frac{G^2\rho}{M}\right.\\
&\quad\left.+m_\pi^2\right)^2\ln\frac{\left(\Lambda^2+\frac{G^2\rho}{M}\right)^{1/2}+(\Lambda^2-m_\pi^2)^{1/2}}{\left(\Lambda^2+\frac{G^2\rho}{M}\right)^{1/2}-(\Lambda^2-m_\pi^2)^{1/2}}.
\end{aligned}
\tag{33}
$$

The incompressibility of nuclear matter is given as

$$
K=9\rho^2\frac{\partial^2E}{\partial\rho^2}.
\tag{34}
$$

The second derivative of energy can easily be evaluated from Eqs. (30)–(33).

TABLE I. *E/A* and incompressibility K as obtained from Eqs. (29)–(34) for different saturation densities ρ_c.

ρ_c (fm^{-3})	Λ (fm^{-1})	*E/A* (MeV)	K (MeV)
0.153	1.206	−15.34	133
0.16	1.216	−16.36	138
0.17	1.23	−17.77	145

V. RESULTS AND DISCUSSION

Using Eq. (30) in conjunction with Eqs. (31)–(33) the single particle binding energy in nuclear matter *E/A* is calculated as a function of ρ, the nuclear matter density. For the pseudoscalar pion-nucleon coupling constant we have used the recent value $(G^2/4\pi)=13.5$ obtained by the Nijmegen group [19,20] from a comprehensive and sophisticated phase shift analysis of the *NN* (*pp* and *np*) scattering data. This is somewhat smaller than the earlier quoted value of 14.6 for the same quantity. The only parameter Λ, the pion momentum cutoff, is fixed by demanding that $(dE/d\rho)_{\rho=\rho_c}=0$, where ρ_c is the equilibrium nuclear matter density. The other physical quantities *E/A* and K then follow directly from the closed form expression given in Sec. IV.

A summary of the results is shown in Table I, for three nuclear saturation densities $\rho_c=0.153$, 0.16, and 0.17. The corresponding equations of state are shown in Fig. 1. It is interesting to note that, with the only parameter of the theory having been fixed by the energy minimization condition stated above, we have a theory with no free parameter which is able to reproduce the single particle binding energy and the nuclear compressibility agreeing with the known properties of nuclear matter [21]. Such a cutoff is not unphysical and, in fact, occurs in most of the field-theoretic models in attempts to avoid the logarithmic divergences. The cutoff essentially parametrizes our ignorance about the extended structure of the particles involved in a physical process. It is to be noted that the same effect could be achieved by demanding that the Bogoliubov transformation introducing the correlated pion pairs is valid only in the low energy region where the pions appear to condense around the nucleons. In such a picture an energy cutoff $w<2m_\pi$ is quite natural.

As suggested from analyses of experimental data [22], the incompressibility parameter K of nuclear matter points to a value of about 200 MeV. In the standard σ model the value of K turns out to be quite large, several times the above mentioned value, for plausible values of the coupling constants involved, and can be reduced only by introducing in the theory terms due to scalar field self-interactions and/or vacuum fluctuations with extra adjustable parameters. It is therefore interesting to note that our model gives a value for $K\approx 135$ MeV which is well within the permissible limits for this parameter from experiments and at the same time it reproduces the saturation density and binding energy very nicely.

In conclusion it may be worthwhile to mention that, starting from a Hamiltonian for interacting pions with pseudoscalar coupling of the pions to the nucleons, we find an interesting analytic expression for the ground state energy of nuclear matter. The model consists of only one parameter, i.e., the cutoff pion momentum Λ in the integrals. From the constraint defined above $\Lambda\simeq 1.2\ \mathrm{fm}^{-1}$, which is well within the range of admissible values, as expected in our model from other physical considerations. The model then dynamically generates the binding energy per particle, the saturation density, the incompressibility parameter and the equation of state for nuclear matter. The prescription generates a softer nuclear matter, as is generally expected experimentally. It is also seen that no *ad hoc* inclusion of the σ meson becomes necessary, the scalar isoscalar interaction of the nucleons being generated by the correlated pion pairs. Another important feature seen from our model is that it is not necessary to include the multipion resonances, like the ρ and ω. As has already been pointed out, the effects attributed to them

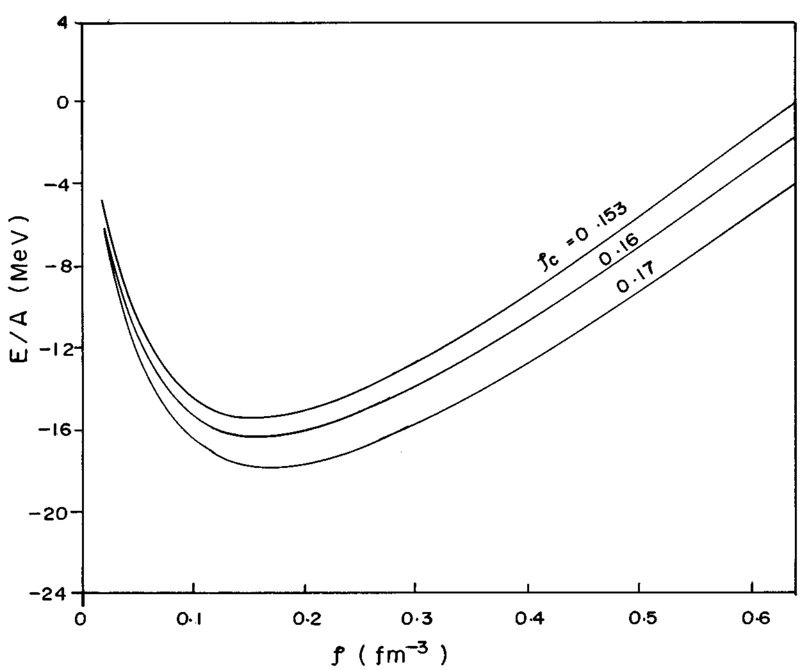

FIG. 1. Binding energy per nucleon *E/A* as a function of nuclear matter density at zero temperature, for three saturation densities 0.153, 0.16, and 0.17 fm^{-3}.

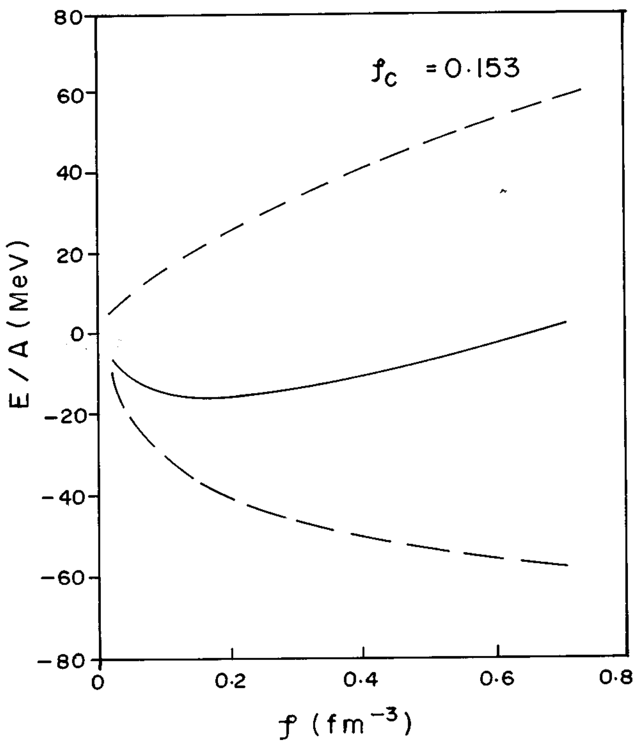

FIG. 2. Repulsive nucleonic contribution and the attractive pion contribution to *E/A* as a function of nuclear matter density (dashed lines). The resultant *E/A* is shown by the continuous line.

should ideally be generated from the correct handling of pion-nucleon interaction. In Fig. 2 we have plotted the repulsive part and the attractive part of the contribution to the binding energy for $\Lambda = 1.2$ fm^{-1} and $\rho_c = 0.153$ fm^{-3}; the two at $\rho = \rho_c$ being 21.6 and -36.7 MeV, respectively. In an earlier work [17] where the pairing of pions was considered, although in a slightly different manner, which also included the ω meson, the ''ω'' coupling treated as a free parameter was found to be $(g_\omega^2/4\pi) = 0.79$. The value is too small compared to the experimentally established value ≈ 20 [23], and it was suggestive enough that ''ω'' has little role to play. How this suppression of ω coupling in nuclear matter comes into effect is not clear from the present stage of our analysis. Janssen *et al.* [24] have shown that a suppression of the ωNN coupling in the case of *N-N* interaction is achieved by introducing a correlated $\pi\rho$ exchange. This should arise in the present model as a $\pi-(2\pi)$ correlation. For quantitative results, however, this point needs further investigation. In conclusion, the present work establishes beyond doubt that it is possible to have a nonperturbative description of nuclear matter as an interacting pion nucleon system. Even though it may be argued that the model is unrealistic because of the nonrelativistic nature and the absence of explicit ''ω'' repulsion, we believe that the simple model presented here has features which may hint to something interesting in the pion-pion correlation. An exact treatment of multipion correlation should reproduce the σ, ρ, and ω contributions automatically. The present work is only a first step in this direction. The model can be looked upon as an alternative to the Walecka model. Applications of the present model to describe nuclear matte at finite temperatures and finite nuclei will be reported later.

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