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Optical model approach for heavy ion fusion

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The differences between optical model (OM) and the barrier transmission approaches for fusion are studied in detail at low energy. In the heavy ion case at deep subbarrier energies, the absorption mean square spin of an optical model calculation using a short ranged imaginary potential differs significantly from the results of the WKB transmission method. This discrepancy of OM results is shown to be due to absorption occurring beyond the barrier position. The coupled reaction channel calculations for fusion based on OM approach are shown to be sensitive to the choice of imaginary fusion potentials, whereas the coupling effects on fusion are dominant for energies only around the barrier. [S0556-2813(96)03212-8]

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It is well known that the coupled reaction channels (CRC) method successfully accounts for the large enhancement in the fusion cross section and a broad spin distribution at energies around the barrier [1–4]. In this formalism, fusion is obtained by the use of imaginary optical potentials or by the barrier transmission approach. In many of these CRC calculations, fusion was estimated by the condition of the complete traversal of the potential barrier (obtained by the WKB transmission method) [5]. The use of the optical model (OM) approach with short ranged imaginary potentials is usually assumed to be consistent with the transmission approach [6]. In the transmission approach at deep subbarrier energies, the fusion cross sections fall exponentially and the mean square spin [MSS, also denoted by $\langle L^2\rangle$; cf. Eq. (1a)] saturates to a constant. However, Satchler *et al.* have shown [4,7] that large fusion mean square spin values can be obtained by relaxing this transmission condition. In a direct reaction approach, Udagawa *et al.* showed that the effective optical model potential for fusion extends to large distances (radius parameter r_i of about 1.5 fm). This implies that fusion is initiated at larger distances [8], in contrast to the barrier transmission approach. Further, in the heavy ion reaction studies, the optical model is extensively used for fusion reactions. Therefore, in the present work, following Satchler *et al.* [4,7], we study the optical model approach for fusion spin distributions as compared to the barrier transmission approach at different energies.

It is generally believed that the barrier penetration model and the optical model with a short ranged imaginary potential (r_i=1.0 fm) give similar results for fusion spin distributions [6]. However, these two approaches deviate significantly at low energies particularly for heavy ion systems. Figure 1 shows the plot of $\langle L^2\rangle$ versus energy for the flux absorbed in the OM for the imaginary radius parameter values of r_i = 1.0 fm and 0.8 fm. This calculation is for a heavy ion system of $^{16}\mathrm{O}+{}^{208}\mathrm{Pb}$ and the parameters of the OM potential are r_c = 1.23 fm, V_0=60.5 MeV, r_0=1.179 fm, a_0=0.685 fm, W_v =10 MeV, and a_i =0.40 fm. The WKB transmission of flux through the real potential barrier (corresponding to OM potentials) is also calculated and the resulting $\langle L^2\rangle$ values are represented by squares in Fig. 1. As seen in the figure, the MSS obtained from the OM with a short ranged imaginary potential does not agree with the results of the WKB method at low energies. The OM results show an increasing trend, whereas the WKB method predicts saturation of $\langle L^2\rangle$ at low energies. A further decrease of r_i to less than 0.8 fm merely shifts the observed difference between the OM and WKB results to still lower energies, showing the sensitivity to the rms radius of the fusion imaginary potentials.

In order to understand this difference of the OM estimates of the MSS over WKB results, we studied the partial wave distribution of flux absorbed [$A(r)$] as a function of the radial cutoff limit (R_F, also known as the fusion radius), given by

$$\sigma_l=-\frac{8}{\hbar v}\frac{\pi}{k^2}(2l+1)\int_0^{R_F}|u_l(r)|^2W(r)dr=\int_0^{R_F}A(r)dr \tag{1}$$

and the MSS (in units of $\hbar^2$) by

$$\langle L^2\rangle=\sum\ l^2\sigma_l/\sum\ \sigma_l\,. \tag{1a}$$

These calculations show that for small values of R_F, the MSS shows saturation at low energies consistent with predictions of barrier traversal methods. As R_F increases beyond the barrier position, the MSS also increases at low energies, indicating that absorption beyond the barrier accounts for this difference with the WKB method. This effect is better represented by the radial distribution of flux absorbed, i.e., integrand $A(r)$ of Eq. (1). Figure 2 shows $A(r)$ as a function of radial separation for different partial waves at a subbarrier energy of 72 MeV (lab). It can be seen that the $A(r)$ exhibits multiple peaks, the lowest one corresponding to absorption to the left of the barrier position (R_b=11 fm) as used in the standard barrier transmission models. The next significant peak corresponds to absorption around the classi-

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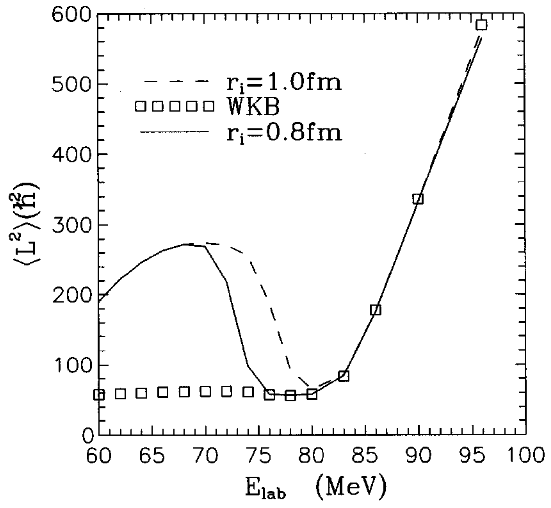

FIG. 1. Absorption mean square spin versus energy from an optical model with short ranged imaginary potentials. The solid and dashed lines represent the results for $r_i = 0.8$ fm and 1.0 fm. The squares are from the WKB transmission method through the corresponding real barrier.

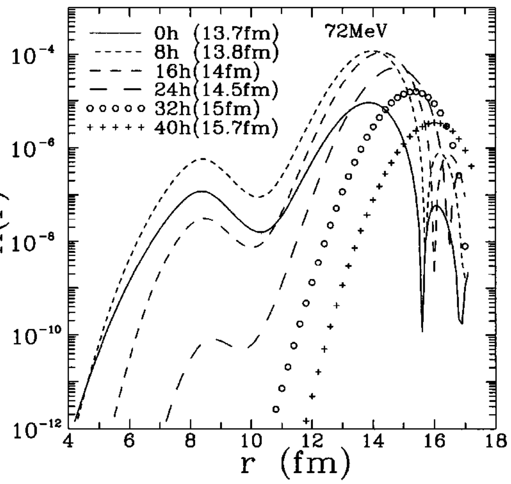

FIG. 2. Absorption of flux in an OM calculation as a function of radial separation. The OM potential parameters are the same as for Fig. 1. The symbols for different partial waves are as shown in the figure. The values in the parentheses are the R_m values obtained from Eq. (2).

cal turning point (CTP). This result is expected at low energies as discussed by Broglia and Winther [9]. Following the procedure of Ref. [9], the distance R_m, where the long range contribution is maximum, can be obtained from

$$R_m = \eta/(kq^2)[1+(1+q^2L^2/\eta^2)^{1/2}]. \tag{2}$$

Here $q^2 = 1+\kappa^2/4k^2$, k is an incident wave number, η is the Sommerfeld parameter, and $\kappa = 1/a_i$ is the inverse of the diffuseness of the fusion imaginary potential $W(r)$ in Eq. (1). These R_m values for different partial waves are indicated in parentheses in Fig. 2. At low energies absorption beyond the barrier position contributes significantly, whereas at high energies only the lowest peak contributes. The other peaks have no significant contribution to the integral. This effect is responsible for the large OM estimates of absorption $\langle L^2 \rangle$ at low energies.

Following Ref. [7], we have studied the long range absorption effects of the OM by including surface terms for the fusion potential and varied the surface radius r_s between 1.2 fm and 1.8 fm. It has been seen that the absorption excitation function exhibits a deviation (plateau) from exponential decrease at low energies and this strongly depends on the r_s parameter. The corresponding MSS exhibits a peak at the low energy region where the cross section exhibits a plateau. This study shows that even a weak imaginary potential at and beyond the barrier position results in a large flux in the absorption channel for higher partial waves. As a result, the MSS for the same cross section differs significantly with the calculations involving only a volume term for the imaginary potential [7].

In the CRC method for fusion, as discussed above, one generally uses either transmission approach or the OM approach with short ranged imaginary potentials for estimating fusion. As these approaches were shown to differ at low energies, the fusion predictions of the CRC method with the OM basis will be sensitive to the choice of imaginary potentials and this study is discussed next.

In the present study, we performed the CRC calculations for $^{16}O+^{208}Pb$ system, using the code FRESCO [10]. This system is well studied within the CRC approach, with a comprehensive coupling scheme [2]. The channels considered include the 3^-, 2^+, and 5^- states of ^{208}Pb, the 3^- state of ^{16}O, the neutron and proton transfers to ground state as well as a few excited states, and the alpha transfer channel. Details of the method are described in Ref. [2]. The results of these calculations indicate that the CRC method can adequately describe the elastic scattering, total reaction cross section, and the fusion cross section in the energy range of 80–102 MeV. The fusion was obtained as the difference of the total reaction and the sum of the cross sections in all the channels included in the CRC method. It was observed that at low energies the fusion obtained by this difference of cross sections method is very sensitive to input parameters such as radial step size, maximum radius, and convergence limits for S-matrix elements (R_{max} and δS parameters in FRESCO). In order to get convergent results for fusion by the difference method, it was necessary to use a large R_{max} value, a small step size, and very low δS limits, with large computation time and memory requirements. We therefore used the overlap integral method [cf. Eq. (1) above] to estimate fusion, as given previously [11, Eq. (1)], as this measure is not so sensitive to the attainment of exact convergence. At very low energies, it was found that fusion predominantly takes place from coupled elastic channels alone and, in comparison, the contribution of other diagonal and nondiagonal terms is negligible. At higher energies ($E_{lab}>80$ MeV) the contribution of other diagonal and nondiagonal terms is significant. We estimated fusion as the sum of all diagonal terms in the absorption matrix at low energy. These results were further verified using a different coupled channels (cc) code ECIS for the same input as FRESCO and with couplings to only inelastic channels.

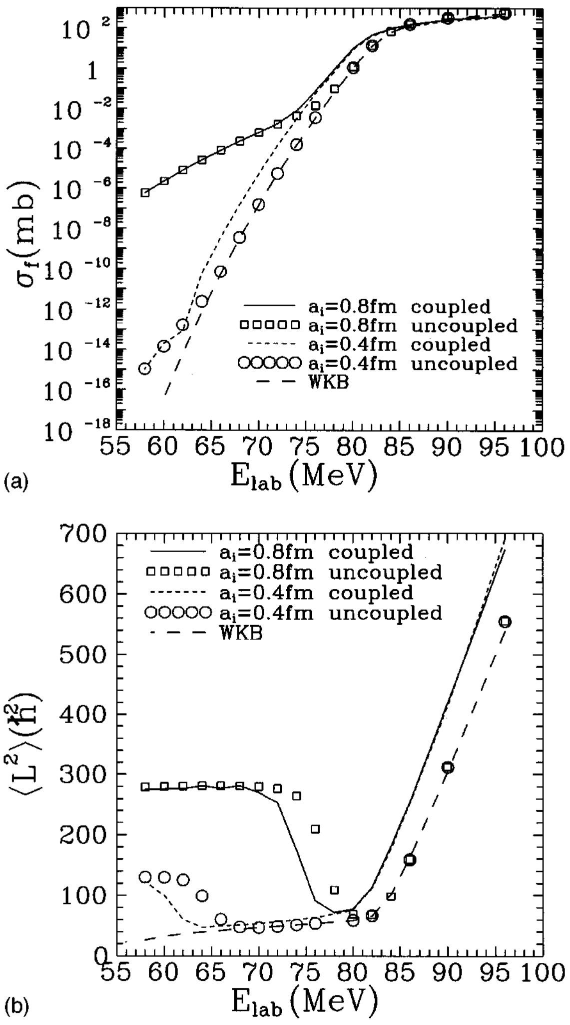

FIG. 3. Fusion excitation function by the CRC method (diagonal absorption) with only five channels included. The solid and dashed curves represent the CRC fusion results for a_i values of 0.8 fm and 0.4 fm, respectively. The corresponding results for uncoupled elastic channel cases are represented by squares and circles. The results of the WKB transmission method using the real part of elastic potential are shown by long dashes. (b) The fusion MSS plots corresponding to the different cases of (a).

In the following CRC calculations, couplings to only four inelastic channels were taken into account with different imaginary diffuseness parameters. Similar calculations were reported earlier [5,11–13] at high energies and the fusion calculations of Refs. [5,12] were based on the barrier traversal method. The present calculations show that the effects of long range absorption can be seen only at very low energies. The fusion optical potential parameters for all channels consist of short ranged imaginary potential [Woods-Saxon (WS) square form] of depth 10 MeV and 1.0 fm range. Figure 3(a) shows the absorption excitation function by the CRC method (diagonal absorption) for a_i values of 0.4 fm and 0.8 fm. The corresponding results for uncoupled elastic channel excitation function along with the results of WKB transmission through its real barrier are also shown. It can be seen that the CC excitation function for the flux absorbed deviates from the expected exponential fall at low energy (depending on the a_i value) and merges with the corresponding curve for uncoupled case. This means that the enhancement over the WKB estimates at very low energies is completely due to absorption of flux at the large distances generated by the optical potentials. However, for the energies around the barrier (72–82 MeV) the fusion enhancement is due to coupling effects and not due to long range absorption. This is evident from the fact that curves for the coupled and uncoupled cases deviate in this region. It can also be noticed that in this region and at higher energies the CRC fusion estimates are not sensitive to the imaginary diffuseness parameter.

The corresponding absorption MSS plots are shown in Fig. 3(b). It can be seen that the WKB transmission estimates saturate at low energy whereas the CRC results show an increasing trend depending on the a_i value. The absorption MSS for coupled cases for a given a_i value can be seen to merge with the results for the corresponding uncoupled case at low energies. As shown in Fig. 3(a), the absorption excitation function exhibits a plateau in the subbarrier region and falls exponentially at deep subbarrier energies. In the corresponding energy regions, the MSS increases and saturates to a high value. The magnitude of the MSS enhancement and the corresponding absorption cross section are both related and depend strongly on the coupling parameters.

The CRC results as derived from difference of cross sections depend strongly on the maximum radius $R_{\max}$. It was observed at subbarrier energies that the total reaction and the fusion l distributions exhibit a sharp fall in the l distributions for the case $R_{\max}=17$ fm. Semiclassically, as l is the product $kR_{\max}$ of $R_{\max}$ and the local wave number, a smaller value of $R_{\max}$ results in the artificial cutoff in the partial wave distribution. These l distributions converge for large values of $R_{\max}$ (depending on energy and type of couplings). The fusion cross section for large l values decreases in magnitude in a manner especially dependent on $R_{\max}$, showing the importance of this parameter for fusion calculations. It was shown by one of the authors (I.J.T.) for the full coupling scheme and parameters as used in Ref. [2] that $R_{\max}$ should be as large as 50 fm for 76 MeV incident energy in order to get converging results for fusion by the difference method. For this case, the fusion MSS predicted is around $700\hbar^2$ for an $R_{\max}$ value of 17 fm and converges to around $60\hbar^2$ for an $R_{\max}$ value of 50 fm.

The mean square spin values have been studied for both OM absorption with short ranged imaginary potentials as well as the WKB transmission method. It is observed that while both these methods agree at above barrier energies, they differ significantly at low energies for heavy ion systems. It is further shown that in the OM at low energies, a significant amount of absorption occurs beyond the barrier position. In the CRC calculations one generally uses either the OM approach or a barrier transmission approach for estimating the flux for the fusion channel. Therefore, the CRC results for fusion based on these two approaches will also differ significantly at low energies due to their inherent differences. In the CRC calculations using the OM approach, the couplings strongly enhance the fusion MSS for energies only around the barrier. The long range absorption effects are dominant at deep subbarrier energies whereas these effects are not significant for energies close to and above the barrier. Therefore, while using the optical model approach for fusion, one has to keep in mind the artifacts of the optical model at deep subbarrier energies in order to identify the true mechanism for fusion.

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