Axial current conservation in the Bethe-Salpeter approach to the nuclear two-body problem

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We investigate the structure of the one- and two-nucleon axial-current operators necessary for the partial conservation of the nuclear axial-current elastic matrix element in the one-boson-exchange approximation to the two-nucleon Bethe-Salpeter equation. We use three models for this purpose: (a) the linear sigma model, (b) the nonlinear sigma model, (c) a hybrid model, which is, roughly speaking, a linear combination of (a) and (b). We construct a partially conserved nuclear axial-current elastic matrix element in models (a) and (c) *provided* that the associated nuclear wave functions are solutions to the Bethe-Salpeter equation with a potential made of one-boson-exchange diagrams. In the nonlinear sigma model the nuclear one-body axial current is partially conserved by itself, without reference to the nuclear wave function, whereas the two-body axial current partial conservation is violated by terms of order $1/f_{\pi}^2$. The complete axial current in models (a) and (c) and the one-body axial current in model (b) are applicable to the construction of the deuteron electroweak process amplitudes, for example. The divergence of the axial-current matrix element is proportional to the pion absorption nuclear matrix element, which leads to another potential application in the foundation of chiral perturbation theory for pion-two-nucleon processes. Consistency between the nuclear axial-currents and the underlying nuclear dynamics in models (a) and (c) is a new condition imposed by the partial conservation of the nuclear axial-current matrix element. We also examine conditions imposed on the form of the nucleon self-energy by the nucleon and meson axial Ward-Takahashi identities, as well as the approximations that satisfy the said conditions. We show that besides the first Born approximation, the so-called Hartree+random-phase approximation satisfies chiral Ward-Takahashi idenitities in models (a) and (c). [S0556-2813(96)05512-4]

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I. INTRODUCTION AND SUMMARY

The search for non-nucleonic degrees of freedom in nuclei is an old one. The best-known discovery made thus far is the observation of the electromagnetic (EM) meson-exchange currents (MEC) [1]. Close analogy between the vector and axial-vector currents in the standard model seems to imply the existence of axial MEC as well. The present status of axial MEC, however, is not nearly as well established as that of the electromagnetic currents [2]. There may be several reasons for this state of affairs: For one, there is substantially less weak interaction data, the measured cross sections being fewer and smaller, than there are EM ones. That may explain the absence of conclusive experimental evidence for the existence of axial MEC, so far. Another reason, on the theoretical side, may be that the *raison d'etre* for the axial MEC seems somewhat weaker than the one for EM currents: The exact conservation of the nuclear EM current matrix elements has impeccable credentials to be contrasted with ''merely'' *partially conserved* axial current (PCAC). It is well known that EM current conservation, or equivalently gauge invariance, plays a pivotal role in the ''nailing down'' of the EM MEC. We shall show that PCAC can be as good a principle for constraining axial currents, as gauge invariance is for EM ones. Another reason behind the lesser use of PCAC as a guiding principle may be the fact that PCAC is a consequence of both *spontaneously* and *explicitly* broken chiral symmetry of the strong interactions, which even in the case of a single nucleon and in the limit of no explicit breaking, i.e., in the chiral limit, is rather complicated and not fully understood. Specifically, there are (at least) two distinct ways chiral symmetry can be realized in Nature: (i) the linear, and (ii) the nonlinear realization. The jury is still out on the question of which realization is the ''right'' one, or if the question is a meaningful one. The standard approach to the axial current in particle physics is to start from the (unphysical) limit of exact chiral symmetry in which the pion is massless and the axial current exactly conserved, and build on it a ''perturbative'' expansion in the chiral symmetry breaking parameters such as the pion mass for the observables in the theory. This procedure, pioneered by Dashen, Weinberg, and others [3–5], and now referred to as the chiral perturbation theory (χPT) relies on the nonlinear realization of chiral symmetry implemented via the most general allowed effective πN Lagrangian. Efforts at extending this method to nuclear few-body physics began only recently [6,7].

It is the purpose of this paper to examine constraints imposed on the nuclear axial two-body currents and the nuclear dynamics by PCAC, within the one-boson-exchange (OBE) approximation to the two-nucleon Bethe-Salpter (BS) Eq. [8]. We limit ourselves to the study of three specific models, one of which is essentially the two-flavor nonlinear sigma model used in the nuclear applications of χPT [6,7], because we do not see features that are sufficiently common to allow a general discussion, such as the one developed for the electromagnetic current by Gross and Riska [2]. The problem here is the same one that appeared in the single-nucleon case: there are two different realizations of chiral symmetry. Therefore we examine three typical chiral models of the πNN interaction and their associated axial currents: (a) the linear sigma model [4], (b) the nonlinear sigma model [3–5],

(c) the hybrid model [4,9–11], which can roughly be thought of as a linear combination of (a) and (b) that allows a nucleon axial coupling g_A that is different from unity. All three of these models can be straightforwardly extended to include *isoscalar* vector and axial-vector mesons, as well as less than maximal $U_A(1)$ breaking [12]. Isovector spin-one fields, on the other hand, are more difficult to include if we insist on preserving the chiral symmetry. The mixing of isovector axial-vector mesons with the pions greatly complicates the analysis, and will not be pursued here. Furthermore, nonchiral corrections we consider are limited to the finite pion mass—various smaller chiral-symmetry-breaking (χSB) effects, like the pion and nucleon mass splittings and deviations from the Goldberger-Treiman (GT) relation are not considered. Extensions to other, reduced, relativistic two-body equations are not examined here either.

In models (a) and (c) the nuclear axial current matrix element is shown to be *conserved only when the axial one- and two-nucleon operators are consistent with the nuclear wave functions*, i.e., with the underlying nuclear dynamics. This result is, to our knowledge, new.[1] Thus we have progressed to the stage where the nuclear axial-current matrix elements are at the same level of conceptual development as the EM ones.[2] The stated relativistic dynamics is specified by a BS equation. We base our analysis on (a) axial (chiral) Ward-Takahashi identities (WT) for the axial currents, and (b) validity of the appropriate BS equation in the one-boson-exchange approximation. In this way we essentially follow the example set by Gross and Riska in their analysis of EM current conservation in the two-nucleon problem as described by a BS equation [2]. In contrast to Gross and Riska, however, we find that we cannot introduce arbitrary nucleon and/or meson axial form factors without essentially modifying the underlying dynamics. The best example of this is the question of the nucleon axial coupling constant that differs from unity $g_A \neq 1$, which can be thought of as the simplest of all nucleon axial form factors. At least one of the models used here, model (a), does not allow an easy incorporation of such an axial coupling constant. Indeed it is for that reason that model (b) and in particular model (c) were introduced. In the nonlinear sigma model, case (b), there is, however, a curious exception to the rules valid for models (a) and (c), and a potentially important problem in the implementation of our program: In this model the one-body axial current is (partially) conserved *by itself* and without any reference to the nuclear dynamics or wave function. Hence, there is no obligation to include two-body axial currents, as there is in models (a) and (c). If one nevertheless does so, one finds that the (uniquely defined) two-body axial current is *not* exactly conserved *even in the chiral limit*. Rather, the two-body current is only approximately conserved, with a PCAC-violating "remnant" of order $1/f_\pi^2$. Although the least well known of the three, model (c) seems to be the most viable, or at least the most adaptable candidate for the role of a chirally invariant meson-nucleon field theory: We find both a partially conserved nuclear axial current and $g_A \neq 1$. Model (c) is closely related to the nonlinear sigma model at the Lagrangian level, but its chiral transformation properties of hadron fields are *linear*; therefore we classify it as a linear realization of chiral symmetry. All this indicates that in nuclear two-body applications, models with nonlinear realization of chiral symmetry behave substantially differently from the linear ones. Two, (b) and (c), of the model nuclear axial-current matrix elements, as well as several of the hybrid model properties reported here seem to be the first in the literature.

The resulting relativistic nuclear axial-current matrix elements in all three models are ready for applications, such as the calculation of electroweak form factors of the deuteron. Minimal modifications are necessary for the extension to inelastic matrix elements [the final state is a different (excited state) solution to the inhomogeneous BS equation with the same kernel]. Those modifications open the door to applications of this formalism to reactions with astrophysical significance, such as the $pp \to De^+\nu_e$ in stars [14]. Another line of potential applications lies in the direction of pion-nuclear processes [7]. All of the partially conserved axial-current matrix elements contain terms wherein the axial current is first "transformed" into a pion (for two flavors, or some other pseudoscalar meson for higher symmetries) and only then "hits" the nucleus. These ("pion pole") graphs, with the pion propagator "amputated," define exactly the pion-nuclear absorption/production amplitude[3] that is demanded by PCAC. It is well known that PCAC plays an important role in pion-nuclear reactions [1,16], and the χPT is beginning to see its applications to light nuclei [6,7]. The present paper provides a *consistent* relativistic field-theoretic formalism for models based on linear realization of chiral symmetry, on which a systematic χPT expansion for pion-nuclear reactions on few-body nuclei can also be built. In that context one must keep in mind, however, that the nonchiral effects discussed in this paper do not constitute the complete set of χSB terms, but rather represent only the leading term, due to the pion mass.[4] We shall not concern ourselves with nuclear χPT in this paper beyond comments indicating its relationship with the present formalism.

The above results all depend on certain specific assumptions concerning the form of the nucleon propagator and the axial current vertices, which, as pointed out above, prevent an easy incorporation of weak form factors for the hadrons. The simplest approximation that is consistent with these requirements is the first Born, or the "tree" approximation: the nucleon mass is a constant, and the axial Ward identities

[1]Model (a) has been considered before by Bentz [13], but he worked in configuration space and thus did not specify his results in a form that can be easily compared with ours.

[2]One of the important lessons learned in the process of constructing gauge-invariant nuclear EM current matrix elements is that having the correct EM current operators is not enough—the initial- and final-state wave functions must be solutions to the nuclear dynamics that are consistent with the said currents [2].

[3]This rule is by no means new, indeed Blin-Stoyle and Tint made an attempt at relating pion-deuteron absorption with axial MEC in nuclear beta decay as early as 1967 [15]. What is new here is (a) the demand that the pion absorption/emission Hamiltonian and the two-nucleon axial current be *consistent* with the two-nucleon potential binding the nucleus, and (b) the relativistic dynamics.

[4]Pion-nuclear reactions are particularly sensitive to χSB effects, due to their being subject to so-called low-energy theorems.

are satisfied, but the nucleon cannot have any structure that would manifest itself in a nontrivial axial form factor, or otherwise. We look for nonperturbative solutions that satisfy the Ward-Takahashi identities. The said identities concern the nucleon axial current, which ''flows,'' at least some of the time, through a pion. Hence we need a nonperturbative method to describe a nontrivial axial current in the baryon number $B=0$, rather than in the $B=2$ sector. It seems only natural that, in a closed self-consistent approximation to the nuclear dynamics, the crossed channels be described by the same, or at least closely related set of diagrams,[5] as those in the ''direct'' channel. The $B=0$ channel is where NN (bound) states live, besides mesons. Some of the latter, such as the pseudoscalars, are subject to strict constraints imposed by chiral invariance. Hence, we have to show that any NN dynamics that is produced by our model(s) does not spoil the chiral symmetry of the mesons with the same quantum numbers. That we show in an approximation that is one step below the ''true'' one-boson exchange approximation and does not exist in the $B=2$ sector: in the Hartree plus random phase approximation (H+RPA)[6] [17]. H + RPA is the only nonperturbative approximation consistent with our assumption that the nucleon self-energy shows no off-shell variation.[7]

This paper falls into five sections. After the Introduction, in Sec. II we briefly review the elements of our three models and of the methods used to solve them. Section III is devoted to the construction of the partially conserved axial current matrix elements in the three models. The proof of partial conservation of the axial current matrix elements is shown in Sec. IV. In Sec. V we examine chiral symmetry in the NN sector. The proof of chiral Ward identities is shown in Sec. V B. In Sec. VI we summarize and discuss our results.

II. PRELIMINARIES: THE MODELS AND THE METHOD

A. The models

In this paper we confine ourselves to two flavors, i.e., to three $SU(2)_L \otimes SU(2)_R \simeq O(4)$ symmetric models. The minimal set of degrees of freedom includes nucleon and pion fields. As discussed in the Introduction, we shall work with three models, two of which involve an additional scalar isoscalar σ meson field. We can add, one independently from the other, the isoscalar vector (ω) and axial vector (f_1) mesons to all three models without disturbing the chiral symmetry and without adding any new terms to the axial current. The same cannot be said of the isovector vector ($\boldsymbol{\rho}$) and axial vector ($\mathbf{A}_1$) mesons which have to (a) be inserted together into the theory in order to preserve chiral invariance, (b) involve intricate mixing between the pion and the longitudinal component of the axial vector meson, and (c) demand introduction of new terms into the axial current. All of this makes the analysis of axial current conservation substantially more complicated with the $\boldsymbol{\rho},\mathbf{A}_1$ mesons than without them. For this reason we exclude the isovector spin-one mesons from the present paper.

1. The linear sigma model

The Lagrangian density[8] of the linear sigma model is given by

$$\mathcal{L}=\bar{\psi} i \partial\!\!\!/ \psi - g_0 \bar{\psi}[\sigma + i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau}]\psi + \tfrac{1}{2}(\partial_\mu \boldsymbol{\phi})^2 - V(\boldsymbol{\phi}^2), \tag{1}$$

where

$$\boldsymbol{\phi}=(\sigma,\boldsymbol{\pi}),$$

and

$$V(\boldsymbol{\phi}^2)=\varepsilon\sigma - \tfrac{1}{2}\mu_0^2\boldsymbol{\phi}^2 + \frac{\lambda_0}{4}(\boldsymbol{\phi}^2)^2. \tag{2}$$

We assume here λ_0 and μ_0^2 are positive, which ensures spontaneous symmetry breaking at the first Born (tree) approximation level, and $\varepsilon = -f_\pi m_\pi^2$ which ensures explicit breaking of the chiral symmetry.[9]

The vector Nöther (isospin) current in this model reads

$$\mathbf{J}_\mu^a = \left(\bar{\psi}\gamma_\mu \frac{\boldsymbol{\tau}}{2}\psi\right)^a + (\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^a, \tag{3}$$

whereas the axial-vector Nöther current is

$$\mathbf{J}_{\mu 5}^a = \left(\bar{\psi}\gamma_\mu\gamma_5 \frac{\boldsymbol{\tau}}{2}\psi\right)^a - (\boldsymbol{\pi}\partial^\mu\sigma - \sigma\partial^\mu\boldsymbol{\pi})^a. \tag{4}$$

Now we choose a stable, positive-parity ground state of the model, which means shifting the sigma field by its vacuum expectation value f_π, and call the shifted scalar field s. The interaction potential in the new field reads

[5]Conservation laws, such as the baryon number conservation, may prevent certain channels from receiving contributions from some of the diagrams.

[6]Whenever we say RPA, we mean relativistic RPA. Throughout this paper we work *in vacuo*.

[7]Though this approximation has been used widely in many-body physics, nevertheless we do not consider it a realistic one in the NN sector. The true one-boson exchange approximation can be made self-consistent and chirally invariant by addition of the so-called Fock term [17] to the above mentioned Hartree one-body Schwinger-Dyson (SD) equation. This new, Hartree-Fock, approximation provides a self-consistent nonperturbative meson cloud structure to the nucleon, which in turn modifies its electroweak properties (form factors and static moments). Although we do not pursue further that line of research in this paper, we consider it significant to inform the reader that the present Hartree+RPA approximation is the lowest one on a systematically expandable ''tree'' of self-consistent, relativistic, symmetry-preserving nonperturbative approximations.

[8]We use the ''West Coast'' metric defined by the signature $(+---)$; most other conventions coincide with those in Ref. [18].

[9]With explicit chiral symmetry breaking induced in this way one must be careful to identify and separate out terms that come about due to the inadequacy of an approximation from those due to explicit chiral symmetry breaking.

$$V=\tfrac{1}{2}\,(m_s^2 s^2+m_\pi^2\boldsymbol{\pi}^2)-\left(\frac{m_s^2-m_\pi^2}{2f_\pi}\right)s(s^2+\boldsymbol{\pi}^2)+\left(\frac{m_s^2-m_\pi^2}{8f_\pi^2}\right)(s^2+\boldsymbol{\pi}^2)^2. \tag{5}$$

The resulting scalar meson and pion propagators are

$$\Delta_s(p-k)=\frac{1}{(p-k)^2-m_s^2}, \tag{6a}$$

$$\Delta_\pi(p-k)=\frac{1}{(p-k)^2-m_\pi^2}, \tag{6b}$$

where the scalar meson and pion masses squared are

$$m_s^2=-\mu_0^2+3\lambda_0 v^2, \tag{7a}$$

$$m_\pi^2=-\mu_0^2+\lambda_0 v^2. \tag{7b}$$

The axial current becomes

$$\mathbf{J}_{\mu 5}^a=\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a-(\boldsymbol{\pi}\partial^\mu s-s\,\partial^\mu\boldsymbol{\pi})^a+f_\pi\partial^\mu\boldsymbol{\pi}^a. \tag{8}$$

Note that the axial coupling constant of the nucleon is exactly identity in the Born approximation to the linear sigma model.[10] That has been one of the primary reasons for going to the nonlinear sigma model.

2. The nonlinear sigma model

The Lagrangian density of Weinberg's nonlinear sigma model [20] is given by

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-M]\psi+\tfrac{1}{2}\mathcal{R}[\mathcal{R}(\partial_\mu\boldsymbol{\pi})^2-m_\pi^2\boldsymbol{\pi}^2]+\mathcal{R}\left(\frac{f}{m_\pi}\right)(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi)\cdot\partial^\mu\boldsymbol{\pi}-\mathcal{R}\left(\frac{g_{\pi NN}}{2g_A M}\right)^2(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi)\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}), \tag{9}$$

where

$$\mathcal{R}=\left[1+\left(\frac{g_{\pi NN}}{2g_A M}\right)^2\boldsymbol{\pi}^2\right]^{-1}=\left[1+\left(\frac{1}{2f_\pi}\right)^2\boldsymbol{\pi}^2\right]^{-1},$$

and

$$\left(\frac{f}{m_\pi}\right)=\left(\frac{g_A}{2f_\pi}\right)=\left(\frac{g_{\pi NN}}{2M}\right).$$

The nonlinear function of the pion fields is to be understood as a series expansion in powers of $\boldsymbol{\pi}/f_\pi$. Manifestly, such a series has infinitely many terms, which makes it impossible to use in its entirety with our present methods. Rather, the Feynman rules and the associated Nöther currents are also defined by the power series expansion. (That is essentially the method used in chiral perturbation theory.) The above form of the nonlinear Lagrangian (9) differs by the presence of g_A in the denominators of the factors $(g_{\pi NN}/2Mg_A)$ from the standard textbook version [21]. The source of this difference, as emphasized by Weinberg [20], is the need to have both the g_A factor in the axial current and the empirically correct two-pion-nucleon contact interaction.

The exact vector and axial-vector Nöther currents in the nonlinear sigma model are rather complicated, so they will not be shown here—they can be found in Ref. [22]. We expand the Lagrangian (9) to the second nontrivial order, i.e., to $\mathcal{O}(f_\pi^{-2})$, and find

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-M]\psi+\frac{1}{2}\left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}\right)(\partial_\mu\boldsymbol{\pi})^2-\frac{1}{2}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)m_\pi^2\boldsymbol{\pi}^2+\left(\frac{f}{m_\pi}\right)(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi)\cdot\partial^\mu\boldsymbol{\pi}-\left(\frac{1}{2f_\pi}\right)^2\times(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi)\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})+\cdots, \tag{10}$$

which is essentially the Lagrangian that Schwinger first wrote down in Ref. [23]. He introduced a new set of chiral transformation laws that only leave the Lagrangian (10) invariant to first order in $1/f_\pi$. Unfortunately, this means that the associated Nöther current is *not exactly conserved*, but only up to a remnant of finite order in $1/f_\pi$, in this case $\mathcal{O}(f_\pi^{-2})$.

For our purposes it ought to be sufficient to expand everything through $\mathcal{O}(f_\pi^{-1})$. This means keeping terms of the two lowest orders in the polar-vector Nöther (isospin) current

$$\mathbf{J}_\mu^a=\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\right)^a+(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^a\left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}\right)-\left(\frac{g_A}{2f_\pi}\right)\times(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi\times\boldsymbol{\pi})^a+\cdots, \tag{11}$$

since the leading term is of $\mathcal{O}(f_\pi^0)$, and three lowest orders in the axial-vector Nöther current

$$\mathbf{J}_{\mu 5}^a=g_A\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a+f_\pi\partial^\mu\boldsymbol{\pi}^a\left(1-\frac{\boldsymbol{\pi}^2}{2f_\pi^2}\right)-\left(\frac{1}{2f_\pi}\right)\times(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\times\boldsymbol{\pi})^a+\cdots, \tag{12}$$

since the leading term is of $\mathcal{O}(f_\pi)$. Such power expansions can be tedious, so one may ask if there is a model which incorporates the best of models (a) and (b)? Such a model is considered next.

3. The hybrid sigma model

The Lagrangian density of the hybrid sigma model [4,9–11] is given by

$$\mathcal{L}=\bar{\psi}i\partial\!\!\!/\psi-g_0\bar{\psi}[\sigma+i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau}]\psi+\tfrac{1}{2}(\partial_\mu\boldsymbol{\phi})^2-V(\boldsymbol{\phi}^2)+\left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\right)\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})+\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)\cdot(\sigma\partial^\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial^\mu\sigma)\right], \tag{13}$$

[10] The one-loop correction is finite and negative [19], which only exacerbates the problem.

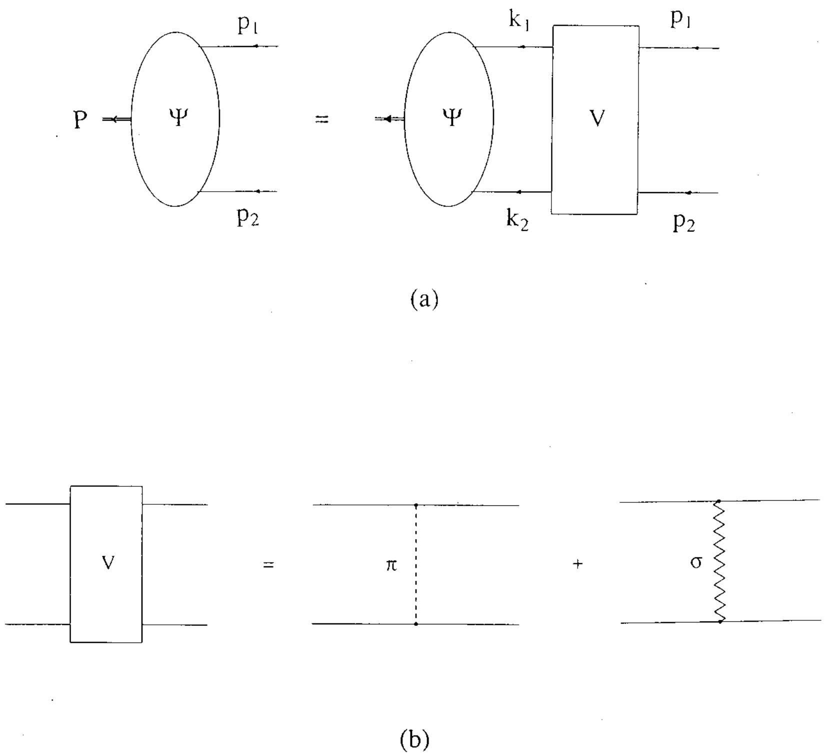

FIG. 1. The two-body Bethe-Salpeter (BS) equation (a) and the kernel (relativistic potential) of the BS equation in the one-boson-exchange (OBE) approximation (b). The dashed line denotes a pion; a zig-zag line denotes a sigma meson, the solid line denotes a nucleon and the double solid line in (a) represents a deuteron. The ''blob'' in (a) represents the deuteron wave function, and the box represents the relativistic potential.

where $\boldsymbol{\phi}=(\sigma,\boldsymbol{\pi})$ is a column vector and V is the same potential as in the linear sigma model, Eq. (2). The hybrid model is the first and, so far, the only example of a chirally symmetric field theoretic model that implements the ''mixed'' pion-nucleon coupling [25]. This model predicts a definite value for the Gross mixing parameter $\lambda=(1/g_A)\simeq 0.8$ in the (first) Born approximation.

The vector Nöther (isospin) current in this model reads

$$\mathbf{J}^a_\mu=\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\right)^a+(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^a+\left(\frac{g_A-1}{f_\pi^2}\right)\times\left\{\left[\boldsymbol{\pi}\times\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\times\boldsymbol{\pi}\right)\right]^a+\sigma\left(\boldsymbol{\pi}\times\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a\right\}. \tag{14}$$

This is an exact result—no power expansion was used. Note the additional pieces due to the nonlinear terms in the Lagrangian Eq. (13), which modify the EM current, as well. The partially conserved axial-vector Nöther current in this model reads

$$\mathbf{J}^a_{\mu 5}=\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a-(\boldsymbol{\pi}\partial^\mu\sigma-\sigma\partial^\mu\boldsymbol{\pi})^a+\left(\frac{g_A-1}{f_\pi^2}\right)\times\left[\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\cdot\boldsymbol{\pi}\right)\boldsymbol{\pi}^a+\sigma^2\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a+\sigma\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\times\boldsymbol{\pi}\right)^a\right], \tag{15}$$

[compare with Eq. (4) in Ref. [9] and, Eq. (9) in Ref. [10], which apparently can be traced back to Eq. (5.53) in Ref. [11]]. After shifting the sigma field we find that the axial-vector Nöther current (15) can be written as

$$\mathbf{J}^a_{\mu 5}=g_A\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)+f_\pi\partial^\mu\boldsymbol{\pi}+(s\partial^\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial^\mu s)^a+\left(\frac{g_A-1}{f_\pi^2}\right)\left[\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\cdot\boldsymbol{\pi}\right)\boldsymbol{\pi}^a+s(2f_\pi+s)\times\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\boldsymbol{\tau}}{2}\psi\right)^a+(f_\pi+s)\left(\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2}\psi\times\boldsymbol{\pi}\right)^a\right]. \tag{16}$$

Note that the first two terms on the right-hand side of (16) are identical with those in the nonlinear sigma model axial current (12), i.e., the nucleon has acquired an axial coupling constant $g_A\neq 1$ without any expansions, which was the purpose of this model. The vector and axial currents (14), (15), (16) are, to the best of our knowledge, new results.

B. Nuclear two-body equation

Having defined our models, we next specify the two-body dynamics. This is done by specifying the form of the OBEP kernel $V(p,k;P)$ (''the relativistic potential''), Fig. 1(b), in the BS equation, Fig. 1(a),

$$S_{(1)}^{-1}(p_1)S_{(2)}^{-1}(p_2)\psi(p,P)=i\int\frac{d^4k}{(2\pi)^4}V(p,k;P)\psi(k,P)\ , \tag{17}$$

where

$$p_1=(\tfrac{1}{2}P+p), \tag{18a}$$

$$p_2=(\tfrac{1}{2}P-p) \tag{18b}$$

are the four-momenta of the two nucleons.

1. The linear sigma model

For example, in the linear sigma model and within the OBE approximation, the relativistic potential Fig. 1(b) reads

$$V(p,k;P)=V(p-k)$$
$$=g_0^2[\Delta_s(p-k)-(\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)})\gamma^5_{(1)}\gamma^5_{(2)}\Delta_\pi(p-k)], \tag{19}$$

where $\Delta_s(p-k),\Delta_\pi(p-k)$ are given in Eq. (6a) and (6b).

2. The nonlinear sigma model

In the nonlinear sigma model, in the OBE approximation the relativistic potential reads

$$V(p,k;P)=V(p-k)$$
$$=\left(\frac{f}{m_\pi}\right)^2(\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)})(p-k)_\mu\gamma^\mu_{(1)}\gamma^5_{(1)}$$
$$\times(p-k)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}\Delta_\pi(p-k), \tag{20}$$

where the pion propagator is given in Eq. (6b). We note that the coupling constant is regular in the chiral limit $m_\pi\to 0$, although that is not obvious from the above form.

3. The hybrid sigma model

In the hybrid model the OBE potential is a linear combination of the linear and nonlinear sigma model potentials plus additional ''cross terms'' due to diagrams with one vertex determined by the ''linear sigma model part'' of the model, and the other determined by the ''nonlinear part''

$$V(p,k;P)=V(p-k)$$
$$=g_0^2[\Delta_s(p-k)-(\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)})\Delta_\pi(p-k)\gamma^5_{(1)}\gamma^5_{(2)}]$$
$$-\left(\frac{g_A-1}{2f_\pi}\right)(\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)})\Delta_\pi(p-k)$$
$$\times\Bigg\{g_0[(p-k)_\mu\gamma^\mu_{(1)}\gamma^5_{(1)}\gamma^5_{(2)}-\gamma^5_{(1)}$$
$$\times(p-k)_\mu\gamma^\mu_{(2)}\gamma^5_{(2)}]-\left(\frac{g_A-1}{2f_\pi}\right)$$
$$\times(p-k)_\mu\gamma^\mu_{(1)}\gamma^5_{(1)}(p-k)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}\Bigg\}, \tag{21}$$

where, again, the sigma and the pion meson propagators are shown in Eq. (6a), (b). Despite its complicated structure, the potential (21) can be represented by the same graphs as the potential in the linear sigma model, Fig. 1(b).

III. NUCLEAR AXIAL CURRENT MATRIX ELEMENT

Following Ref. [2], we write the axial current two-body bound state matrix element as

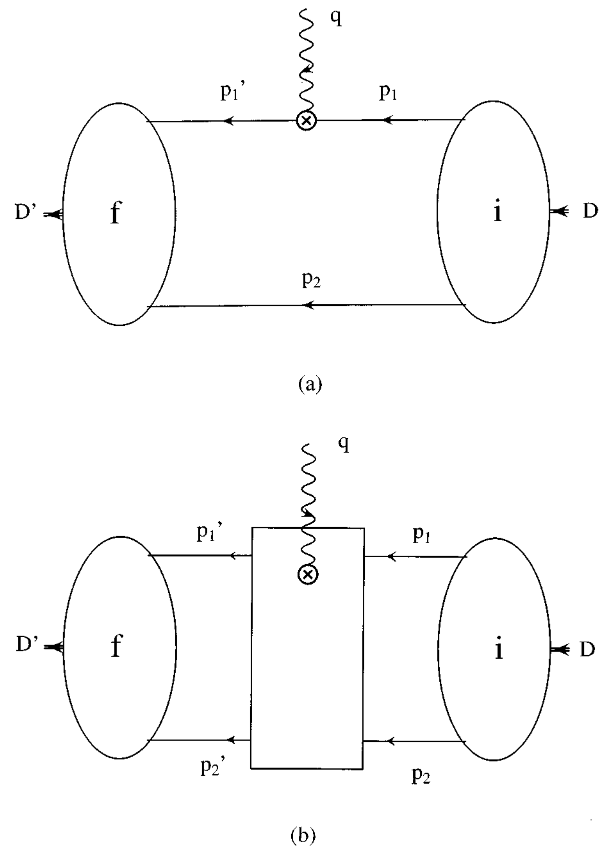

FIG. 2. Feynman diagrams contributing to the one- (a) and the two-body axial current elastic matrix element (b). The circle with a cross denotes a complete axial current single nucleon vertex as depicted in Fig. 3 and a wavy line denotes an external axial current. The box in (b) denotes a two-body, or meson-exchange axial current.

$$\langle J^a_{\mu 5}\rangle=\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')$$
$$\times J^a_{\mu 5}(p',D';p,D)\psi(p,D)\ \ , \tag{22}$$

where

$$J^{\mu 5a}(p',D';p,D)=-i(2\pi)^4S^{-1}_{(2)}(p_2)$$
$$\times\delta^4(p_2-p_2')j^{\mu 5a}_{(1)}(p_1',p_1)$$
$$-i(2\pi)^4S^{-1}_{(1)}(p_1)$$
$$\times\delta^4(p_1-p_1')j^{\mu 5a}_{(2)}(p_2',p_2)$$
$$+J^{\mu 5a}_{2\,\mathrm{body}}(p',D';p,D), \tag{23}$$

is the sum of two parts: (i) the one-body, Fig. 2(a), and (ii) the two-body current, Fig. 2(b). The four-momenta p_1',p_2',p_1,p_2 are related to the relative and center-of-mass (CM) four momenta p',D',p,D via Eqs. (18a), (b). We shall also need the pion absorption nuclear matrix element, which we define as follows:

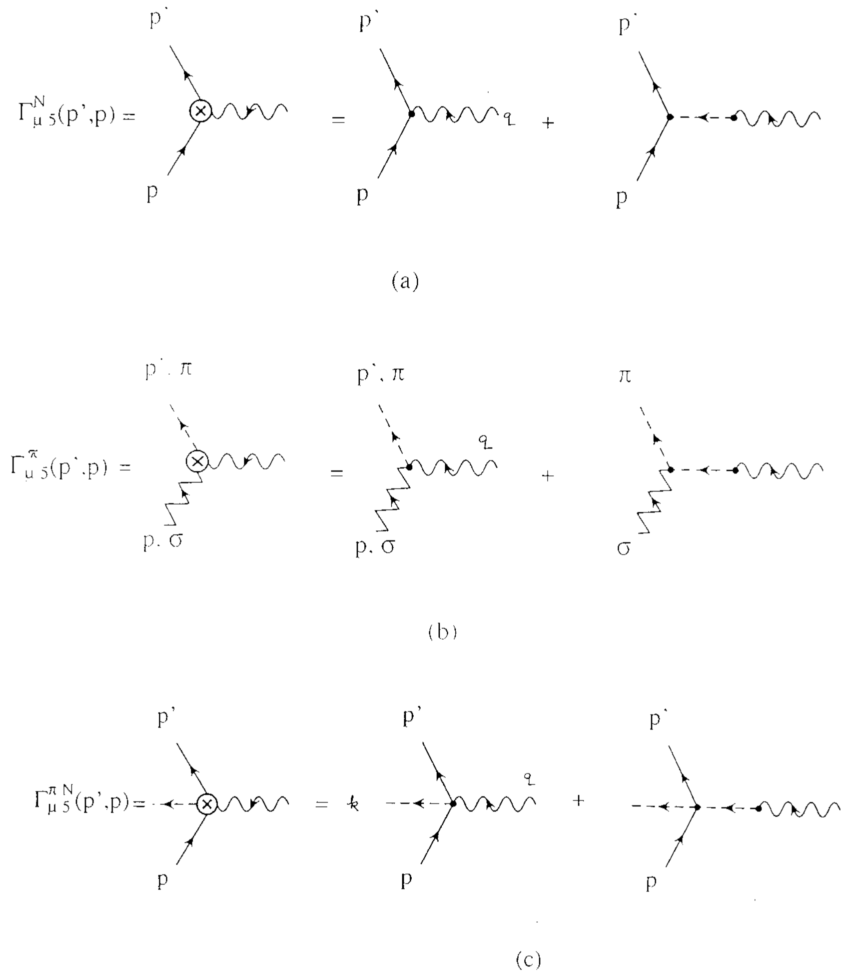

FIG. 3. The complete single-nucleon axial current vertex (a), the single-meson axial current vertex (b), and the nucleon-pion axial current vertex (c). Here a wavy line denotes an external axial current, solid line is a nucleon, the dashed line represents the pion and the ''vertex'' (dot) converting the axial current into the pion is proportional to the pion decay constant. A zig-zag line denotes the sigma meson. Other, ''mixed'' nucleon-meson axial-current vertices such as the ones appearing in Fig. 4(b),(c) are constructed analogously from a ''direct'' and a ''pion pole'' term.

$$\langle \Pi^a \rangle = \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p'}{(2\pi)^4} \overline{\psi}(p',D') \times \Pi^a(p',D';p,D)\psi(p,D), \tag{24}$$

where, as above, the total pion absorption operator breaks down into the sum of the one-body and the two-body pion absorption operators

$$\begin{aligned} \Pi^a(p',D';p,D) &= -i(2\pi)^4 S^{-1}_{(2)}(p_2)\delta^4(p_2-p_2')\Pi^a_{(1)}(p_1',p_1) \\ &\quad - i(2\pi)^4 S^{-1}_{(1)}(p_1)\delta^4(p_1-p_1')\Pi^a_{(2)}(p_2',p_2) \\ &\quad + \Pi^a_{2\,\text{body}}(p',D';p,D). \end{aligned} \tag{25}$$

A. The one-body current

(a) The one-body current of the ith nucleon in the linear sigma model, as defined in Fig. 3(a), i.e., with a unit (''normalized'') nucleon axial coupling reads

$$j^{(i)a}_{\mu 5}(p_i',p_i) = \left[\gamma^{(i)}_\mu - 2 f_\pi g_0 \left(\frac{q_\mu}{q^2 - m_\pi^2} \right) \right] \gamma^{(i)}_5 \frac{\boldsymbol{\tau}^a_{(i)}}{2}, \tag{26}$$

and satisfies the elementary fermion axial Ward-Takahashi identity

$$\begin{aligned} q_\mu j^{\mu 5 a}_{(i)}(p_i',p_i) &= [S^{-1}_{(i)}(p_i')\gamma^{(i)}_5 + \gamma^{(i)}_5 S^{-1}_{(i)}(p_i)] \frac{\boldsymbol{\tau}^a_{(i)}}{2} \\ &\quad - f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2} \right) g_0 \boldsymbol{\tau}^a_{(i)} \\ &= [S^{-1}_{(i)}(p_i')\gamma^{(i)}_5 + \gamma^{(i)}_5 S^{-1}_{(i)}(p_i)] \frac{\boldsymbol{\tau}^a_{(i)}}{2} \\ &\quad - f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2} \right) \Pi^a_{(i)}(p_i',p_i), \end{aligned} \tag{27}$$

which follows from the algebraic identity

$$\not{q}\gamma_5 = 2M\gamma_5 + S^{-1}(p+q)\gamma_5 + \gamma_5 S^{-1}(p), \tag{28}$$

and the Goldberger-Treiman (GT) relation $M = g_0 f_\pi$. The second line on the right-hand side (r.h.s.) of Eq. (27) is the single-nucleon pion absorption operator multiplied by the divergence of the axial current factor $f_\pi m_\pi^2$ and the pion propagator $\Delta_\pi(q)$. We see that the pion absorption operator arose naturally from the divergence of the axial current. Here we have tacitly assumed that neither g_0 nor f_π have any momentum dependence. This is a crucial assumption, as we will see later, for it implies a momentum-independent nucleon self-energy, i.e., a constant nucleon mass M. This, in turn, limits the type of one-body nuclear dynamics to which the present discussion pertains (see Sec. V), and implicitly limits the extensions to models with form factors.

(b) In the nonlinear sigma model the GT relation is modified to $g_A M = g_{\pi NN} f_\pi$, where $g_A = 1.26$ and the one-body current reads

$$j_{\mu 5}^{(i)a}(p_i', p_i) = g_A \gamma_\mu^{(i)} \gamma_5^{(i)} \frac{\boldsymbol{\tau}_{(i)}^a}{2} - f_\pi \frac{f}{m_\pi}\left(\frac{q_\mu}{q^2 - m_\pi^2}\right) \not{q}^{(i)} \gamma_5^{(i)} \boldsymbol{\tau}_{(i)}^a$$
$$= g_A \left[\gamma_\mu^{(i)} - \left(\frac{q_\mu}{q^2 - m_\pi^2}\right) \not{q}^{(i)}\right] \gamma_5^{(i)} \frac{\boldsymbol{\tau}_{(i)}^a}{2}. \quad (29)$$

Consequently the appropriate WT identity becomes

$$q_\mu j_{(i)}^{\mu 5a}(p_i', p_i) = -f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right)\left(\frac{f}{m_\pi}\right) \not{q}^{(i)} \gamma_5^{(i)} \boldsymbol{\tau}_{(i)}^a$$
$$= -f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right) \Pi_{(i)}^a(p_i', p_i), \quad (30)$$

i.e., the single-nucleon pion absorption operator in this model multiplied by the usual factor $f_\pi m_\pi^2$ and the pion propagator, thus rendering *all* one-nucleon (on- or off-shell) axial-current matrix elements partially conserved in this model. That, of course, also means that (i) whatever two-nucleon currents there are, they will also have to be partially conserved separately, i.e., by themselves, (ii) there is no compelling *need* for such two-body currents.

(c) In the hybrid model, on the other hand, the simpler version of the GT relation $M = g_0 f_\pi$ still holds, but there are two kinds of πNN couplings: (i) the (''ordinary'' linear sigma model) pseudoscalar coupling g_0, and (ii) the new pseudovector coupling $(g_A - 1)/2f_\pi$. When added to (i), term (ii) increases the value of the ''effective'' on-shell πNN coupling to $g_A g_0 = g_{\pi NN}$. The complete axial current vertex, for off-shell nucleons, is now

$$j_{\mu 5}^{(i)a}(p_i', p_i) = g_A \gamma_\mu^{(i)} \gamma_5^{(i)} \frac{\boldsymbol{\tau}_{(i)}^a}{2} - f_\pi \left(\frac{q_\mu}{q^2 - m_\pi^2}\right)$$
$$\times \left(g_0 + \frac{g_A - 1}{2f_\pi} \not{q}\right) \gamma_5^{(i)} \boldsymbol{\tau}_{(i)}^a. \quad (31)$$

Contract this with q^μ to find

$$q_\mu j_{(i)}^{\mu 5a}(p_i', p_i) = \{g_A \not{q} - [2 g_0 f_\pi + (g_A - 1)\not{q}]\} \gamma_5 \frac{\boldsymbol{\tau}_{(i)}^a}{2}$$
$$- f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right)\left[g_0 + \left(\frac{g_A - 1}{2f_\pi}\right)\not{q}\right] \boldsymbol{\tau}_{(i)}^a$$
$$= [\not{q} - 2M]\gamma_5 \frac{\boldsymbol{\tau}_{(i)}^a}{2} - f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right)$$
$$\times \left[g_0 + \left(\frac{g_A - 1}{2f_\pi}\right)\not{q}\right] \boldsymbol{\tau}_{(i)}^a$$
$$= [S_{(i)}^{-1}(p_i')\gamma_5^{(i)} + \gamma_5^{(i)} S_{(i)}^{-1}(p_i)] \frac{\boldsymbol{\tau}_{(i)}^a}{2}$$
$$- f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right) \Pi_{(i)}^a(p_i', p_i), \quad (32)$$

which is closely related to the WT identity (27) of the linear sigma model. Thus we see that a renormalized (''nontrivial'') axial coupling constant *need not* cause the ''inverse propagator'' part of the whole one-body axial current divergence to vanish, as it does in the nonlinear sigma model. Rather, those parts of the one-body current that are induced by the ''nonlinear part'' of the interaction are partially transverse by construction, whereas the ''linear part'' satisfies a nontrivial WT identity (32).

The one-body axial currents in the three models we considered are essentially identical for on-shell nucleons, modulo the overall factor g_A. Yet, they satisfy two profoundly different kinds of WT identities. This is a consequence of two different realizations of chiral symmetry being implemented in these models.

B. The two-body current

1. The linear sigma model

The two-body axial current in the linear sigma model, Fig. 4(a), reads

$$J_{2\,\mathrm{body}}^{\mu 5a}(p_1', p_2'; p_1, p_2) = g_0^2 [\gamma_5^{(1)} \boldsymbol{\tau}_{(1)}^a \Delta_s(p_2 - p_2') \Delta_\pi(p_1' - p_1)$$
$$\times j_{s\pi}^{\mu 5a}(p_2' - p_2; p_1 - p_1') + (1 \leftrightarrow 2)], \quad (33)$$

where the sigma-pion axial current depicted in Fig. 3(b) reads

$$j_{s\pi}^{\mu 5a}(k', k) = -\left[(k' + k)^\mu + \left(\frac{q^\mu}{q^2 - m_\pi^2}\right)(m_s^2 - m_\pi^2)\right], \quad (34)$$

and satisfies the elementary Ward identity

$$q_\mu j_{s\pi}^{\mu 5a}(k', k) = -[\Delta_\pi^{-1}(k') - \Delta_s^{-1}(k)] - f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right) g_{s\pi\pi}. \quad (35)$$

This, in turn, leads to the following WT identity for the complete two-body current

$$q_\mu J_{2\,\mathrm{body}}^{\mu 5a}(p_1', p_2'; p_1, p_2)$$
$$= -g_0^2 \{\gamma_5^{(1)} \boldsymbol{\tau}_{(1)}^a \Delta_s(p_2 - p_2') \Delta_\pi(p_1' - p_1)$$
$$\times [\Delta_s^{-1}(p_2' - p_2) - \Delta_\pi^{-1}(p_1' - p_1)] + (1 \leftrightarrow 2)\}$$
$$- f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right) g_{s\pi\pi} g_0^2 [\gamma_5^{(1)} \boldsymbol{\tau}_{(1)}^a \Delta_s(p_2 - p_2')$$
$$\times \Delta_\pi(p_1' - p_1) + (1 \leftrightarrow 2)]$$
$$= -g_0^2 \{\gamma_5^{(1)} \boldsymbol{\tau}_{(1)}^a [\Delta_\pi(p_1' - p_1)$$
$$- \Delta_s(p_2 - p_2')] + (1 \leftrightarrow 2)\} - f_\pi \left(\frac{m_\pi^2}{q^2 - m_\pi^2}\right)$$
$$\times \Pi_{2\,\mathrm{body}}^a(p_1', p_2'; p_1, p_2). \quad (36)$$

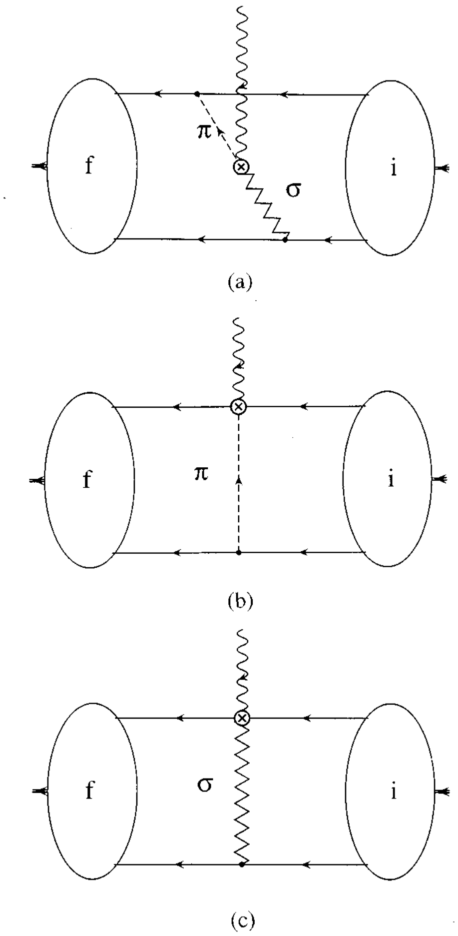

FIG. 4. Diagrams contributing to the two-body axial current deuteron elastic matrix element considered in this paper: The two-nucleon axial, or meson-exchange current (MEC) in the linear sigma model (a); the axial MEC in the nonlinear sigma model (b); the axial MEC in the hybrid sigma model (c), where the graphical symbols have the same meaning as in Fig. 3. For each diagram explicitly shown there is another diagram that can be obtained from the first one by the exchange of nucleon No. 1 by the nucleon No. 2.

2. The nonlinear sigma model

The elementary building block of the two-body axial current in the nonlinear sigma model is the new axial-current–nucleon-pion vertex

$$\Gamma^{(i)a}_{\mu 5(A)}(p',k;p,q)=\left(\frac{1}{2f_\pi}\right)\gamma^{(i)}_\mu(\boldsymbol{\tau}_{(i)}\times\boldsymbol{\pi})^a, \tag{37}$$

stemming from the third term in Eq. (12), where $\boldsymbol{\pi}$ is the isospin wave function of the pion. Another, ''compensating,'' piece comes from the second interaction term in Eqs. (9), (10):

$$\Gamma^{(i)a}_{\mu 5(B)}(p',k;p,q)=\left(\frac{-1}{4f_\pi}\right)\left(\frac{q^\mu}{q^2-m_\pi^2}\right)(k+q)_\nu\gamma^\nu_{(i)} \times(\boldsymbol{\tau}_{(i)}\times\boldsymbol{\pi})^a, \tag{38}$$

leading to the complete vertex

$$\begin{aligned}\Gamma^{(i)a}_{\mu 5}(p',k;p,q)&=\Gamma^{(i)a}_{\mu 5(A)}(p',k;p,q)+\Gamma^{(i)a}_{\mu 5(B)}(p',k;p,q)\\&=\left(\frac{1}{2f_\pi}\right)\left[\gamma^{(i)}_\mu-\frac{1}{2}\left(\frac{q_\mu}{q^2-m_\pi^2}\right)(k+q)_\nu\gamma^\nu_{(i)}\right]\\&\quad\times(\boldsymbol{\pi}\times\boldsymbol{\tau}_{(i)})^a.\end{aligned} \tag{39}$$

Hence, the two-body axial current in the nonlinear sigma model, shown in Fig. 4(b), reads

$$\begin{aligned}&J^{\mu 5a}_{2\,\text{body}}(p_1',p_2';p_1,p_2)\\&\quad=ig_A\left(\frac{1}{2f_\pi}\right)^2(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a\\&\quad\quad\times\left\{\left[\gamma^\mu_{(1)}+\frac{1}{2}\left(\frac{q^\mu}{q^2-m_\pi^2}\right)(p_1'-p_1-q)_\nu\gamma^\nu_{(1)}\right]\right.\\&\quad\quad\left.\times(p_2'-p_2)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}\Delta_\pi(p_2'-p_2)-(1\leftrightarrow 2)\right\}.\end{aligned} \tag{40}$$

Note the factor 1/2 in front of the second term in Eq. (40), which can be traced back to the third line of Eq. (10). That factor is the source of axial-current nonconservation in this model, within the present approximation, even in the chiral limit. If the said factor were unity, the axial current would be partially conserved. As it stands, however, this MEC does not conform with the tenets of PCAC. To be sure, this was to be expected: the nonlinear sigma model as we have used it in this paper, is defined only through $\mathcal{O}(f_\pi^{-1})$, and the nonconserving term is $\mathcal{O}(f_\pi^{-2})$.

3. The hybrid sigma model

There are three kinds of axial MEC's in this model.

(a) First, the sum of the familiar linear sigma model MEC, Eq. (33) and a variation obtained from it by replacing the pseudoscalar πNN coupling with a pseudovector one in Fig. 4(a),

$$\begin{aligned}&J^{\mu 5a}_{2\,\text{body},(A)}(p_1',p_2';p_1,p_2)\\&\quad=-g_0\left[\left(g_0+\left(\frac{g_A-1}{2f_\pi}\right)(p_1'-p_1)_\nu\gamma^\nu_{(1)}\right)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}\right.\\&\quad\quad\times\Delta_s(p_2-p_2')j^{\mu 5a}_{s\pi}(p_2'-p_2;p_1-p_1')\\&\quad\quad\left.\times\Delta_\pi(p_1'-p_1)+(1\leftrightarrow 2)\right],\end{aligned} \tag{41}$$

where $j^{\mu 5a}_{s\pi}(k',k)$ is defined in Eq. (34). This MEC has the following divergence:

$$q_\mu J^{\mu 5a}_{2\,\mathrm{body},(A)}(p_1',p_2';p_1,p_2)$$

$$=g_0\left[\left(g_0+\left(\frac{g_A-1}{2f_\pi}\right)(p_1'-p_1)_\nu\gamma^\nu_{(1)}\right)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}\right.$$
$$\left.\times(\Delta_s(p_2-p_2')-\Delta_\pi(p_1'-p_1))+(1\leftrightarrow 2)\right]$$
$$-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\Pi^a_{2\,\mathrm{body},(A)}(p_1',p_2';p_1,p_2). \quad (42)$$

(b) There is a purely pionic axial MEC, very much as in the nonlinear sigma model of the previous subsection. [One of two main differences between the hybrid and the nonlinear sigma model is that in the former we have two kinds of πNN couplings (pseudoscalar and pseudovector) at the second vertex versus pure pseudovector in the latter.] Its elementary building block is new axial-current–pion-nucleon vertex, shown in Fig. 3(c), which consists of two parts:

$$\Gamma^{\mu 5a}_{(i)c'}(p',k;p,q)=i\left(\frac{g_A-1}{2f_\pi}\right)\gamma^\mu_{(i)}(\boldsymbol{\pi}\times\boldsymbol{\tau}_{(i)})^a, \quad (43)$$

stemming from the second line in Eq. (12), and

$$\Gamma^{\mu 5a}_{(i)c''}(p',k;p,q)=i\left(\frac{g_A-1}{2f_\pi}\right)\left(\frac{q^\mu}{q^2-m_\pi^2}\right)(k-q)_\nu\gamma^\nu_{(i)}$$
$$\times(\boldsymbol{\pi}\times\boldsymbol{\tau}_{(i)})^a, \quad (44)$$

coming from the second interaction term in Eq. (13). It is in this second term that we find the crucial difference from the nonlinear sigma model: there is no factor 1/2 multiplying the second term. When put together the two yield a new axial-current–nucleon-pion vertex [Fig. 3(c)],

$$\Gamma^{\mu 5a}_{(i)c}(p',k;p,q)=i\left(\frac{g_A-1}{2f_\pi}\right)\left[\gamma^\mu_{(i)}-\left(\frac{q^\mu}{q^2-m_\pi^2}\right)(k+q)_\nu\gamma^\nu_{(i)}\right]$$
$$\times(\boldsymbol{\pi}\times\boldsymbol{\tau}_{(i)})^a, \quad (45)$$

which satisfies the WT identity

$$q_\mu\Gamma^{\mu 5a}_{(i)c}(p',k;p,q)=i\left(\frac{g_A-1}{2f_\pi}\right)\left[k_\nu\gamma^\nu_{(i)}+\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\right.$$
$$\left.\times(k+q)_\nu\gamma^\nu_{(i)}\right](\boldsymbol{\pi}\times\boldsymbol{\tau}_{(i)})^a. \quad (46)$$

[The complete nonlinear sigma model vertex (39) does *not* satisfy this WT identity due to the aforementioned factor of 1/2. This is the second major difference between the hybrid and the nonlinear sigma models.] This leads to the following two-body current, depicted in Fig. 4(b):

$$J^{\mu 5a}_{2\,\mathrm{body},(B)}(p_1',p_2';p_1,p_2)$$
$$=i\left(\frac{g_A-1}{2f_\pi}\right)(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a\left\{\left(g_0+\left(\frac{g_A-1}{2f_\pi}\right)\right.\right.$$
$$\left.\times(p_2'-p_2)_\nu\gamma^\nu_{(2)}\right)\gamma^5_{(2)}\left[\gamma^\mu_{(1)}+\left(\frac{q^\mu}{q^2-m_\pi^2}\right)\right.$$
$$\left.\left.\times(p_2'-p_2-q)_\nu\gamma^\nu_{(1)}\right]\Delta_\pi(p_2'-p_2)-(1\leftrightarrow 2)\right\}, \quad (47)$$

with the divergence

$$q_\mu J^{\mu 5a}_{2\,\mathrm{body},(B)}(p_1',p_2';p_1,p_2)$$
$$=i\left(\frac{g_A-1}{2f_\pi}\right)(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a\left\{\left(g_0+\left(\frac{g_A-1}{2f_\pi}\right)(p_2'\right.\right.$$
$$\left.\left.-p_2)_\nu\gamma^\nu_{(2)}\right)\gamma^5_{(2)}(p_2'-p_2)_\nu\gamma^\nu_{(1)}\Delta_\pi(p_2'-p_2)-(1\leftrightarrow 2)\right\}$$
$$-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\Pi^a_{2\,\mathrm{body},(A)}(p_1',p_2';p_1,p_2), \quad (48)$$

(c) The third two-body axial current in the hybrid sigma model is the σ-exchange MEC. Its elementary building block is the new axial-current–nucleon-sigma meson vertex, graphically identical to the vertex shown in Fig. 3(c) when the outgoing pion is replaced by a sigma, which together with its compensating piece, coming from the second interaction term in Eq. (13), reads

$$\Gamma^{\mu 5a}_{(i)C}(p',k;p,q)=\left(\frac{g_A-1}{2f_\pi}\right)\left[2\gamma^{(i)}_\mu\gamma^{(i)}_5-\left(\frac{q^\mu}{q^2-m_\pi^2}\right)\right.$$
$$\left.\times(k+q)_\nu\gamma^\nu_{(i)}\gamma^5_{(i)}\right]\boldsymbol{\tau}^a_{(i)}, \quad (49)$$

and satisfies the WT identity

$$q_\mu\Gamma^{\mu 5a}_{(i)C}(p',k;p,q)=\left(\frac{g_A-1}{2f_\pi}\right)\left[(q_\nu-k_\nu)\gamma^\nu_{(i)}-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\right.$$
$$\left.\times(k+q)_\nu\gamma^\nu_{(i)}\right]\gamma_5^{(i)}\boldsymbol{\tau}^a_{(i)}. \quad (50)$$

This leads to the following σ-exchange two-body current, Fig. 4(c):

$$J^{\mu 5a}_{2\,\mathrm{body},(C)}(p_1',p_2';p_1,p_2)=g_0\left(\frac{g_A-1}{2f_\pi}\right)\left[\boldsymbol{\tau}^a_{(1)}\Delta_\sigma(p_2'-p_2)\right.$$
$$\times\left(2\gamma^\mu_{(1)}\gamma^5_{(1)}-\left(\frac{q^\mu}{q^2-m_\pi^2}\right)\right.$$
$$\left.\times(q+p_2'-p_2)_\nu\gamma^\nu_{(1)}\gamma^5_{(1)}\right)$$
$$\left.+(1\leftrightarrow 2)\right], \quad (51)$$

with the divergence

$$q_\mu J^{\mu 5a}_{2\,\mathrm{body},(C)}(p_1',p_2';p_1,p_2) = g_0\left(\frac{g_A-1}{2f_\pi}\right)[\boldsymbol{\tau}^a_{(1)}\Delta_\sigma(p_2'-p_2)(q-p_2'+p_2)_\nu\gamma^\nu_{(1)}\gamma^5_{(1)} + (1\leftrightarrow 2)] - f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\Pi^a_{2\,\mathrm{body},(C)}(p_1',p_2';p_1,p_2). \tag{52}$$

The total two-body axial current is the sum of the two-body axial currents (A-C) (41), (47), (51). This is a new result, as is the pion absorption two-nucleon operator defined by Π_{A+B+C}. The linear sigma model result, Eq. (33), might be retrievable from the configuration space work by Bentz [13]. The axial meson exchange current (40) in the nonlinear sigma model is not acceptable as a result within the BS equation approach due to its violation of PCAC. The complete axial-current matrix elements, defined in Eq. (22) as the sum of the relevant one- and two-body axial currents, obey PCAC in the linear and hybrid sigma models as we shall show in the next section. In the nonlinear sigma model only the one-body axial current ought to be used in the matrix element, where the nuclear wave function is *not* constrained by PCAC. The total axial currents are ready for their applications, provided the respective BS relativistic wave functions are used for the linear and the hybrid models. For use in weak interactions the necessary polar-vector weak current is to be calculated using the same Lagrangian as for the axial current and applying the Gross-Riska method [2].

IV. CONSERVATION OF NUCLEAR AXIAL CURRENT MATRIX ELEMENTS

We can effectively separate this task into an evaluation of the divergences of the one- and two-body currents using the above Ward identities.

A. One-body axial current divergence

The generic form of the divergence of the axial one-body current in the linear and hybrid sigma models is

$$q_\mu\langle J^{\mu 5a}_{1\,\mathrm{body}}\rangle = \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\Bigg\{-i(2\pi)^4\delta^4(p_2-p_2')S^{-1}_{(2)}(p_2)\left[S^{-1}_{(1)}(p_1')\gamma_5^{(1)}\frac{\boldsymbol{\tau}^a_{(1)}}{2} + \gamma_5^{(1)}S^{-1}_{(1)}(p_1)\frac{\boldsymbol{\tau}^a_{(1)}}{2}\right] - i(2\pi)^4\delta^4(p_1-p_1')\times S^{-1}_{(1)}(p_1)\left[S^{-1}_{(2)}(p_2')\gamma_5^{(2)}\frac{\boldsymbol{\tau}^a_{(2)}}{2} + \gamma_5^{(2)}S^{-1}_{(2)}(p_2)\frac{\boldsymbol{\tau}^a_{(2)}}{2}\right]\Bigg\}\psi(p,D) - f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\times\langle\Pi^a_{1\,\mathrm{body}}\rangle. \tag{53}$$

Now we use the BS equation (17) to write this as

$$q_\mu\langle J^{\mu 5a}_{1\,\mathrm{body}}\rangle = \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\times\Bigg\{V(p'-p-\tfrac{1}{2}q)\gamma_5^{(1)}\frac{\boldsymbol{\tau}^a_{(1)}}{2} + \gamma_5^{(1)}\frac{\boldsymbol{\tau}^a_{(1)}}{2}\times V(p'-p-\tfrac{1}{2}q) + V(p'-p+\tfrac{1}{2}q)\times\gamma_5^{(2)}\frac{\boldsymbol{\tau}^a_{(2)}}{2} + \gamma_5^{(2)}\frac{\boldsymbol{\tau}^a_{(2)}}{2}V(p'-p+\tfrac{1}{2}q)\Bigg\}\times\psi(p,D) - f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\mathrm{body}}\rangle, \tag{54}$$

which is as far as simplification of this term can go without specifying the relativistic potential V. We emphasize again that the above formula holds for the linear and hybrid models only; the nonlinear model contains only the pion absorption term and is independent of the potential. We will now examine the one-body current divergence, model by model.

1. Linear sigma model

We insert Eq. (19) into Eq. (54) above and simplify to find

$$q_\mu\langle J^{\mu 5a}_{1\,\mathrm{body}}\rangle = g_0^2\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\times\{\Delta_s(p'-p-\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)} - \Delta_\pi(p'-p-\tfrac{1}{2}q)\times\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)} + \Delta_s(p'-p+\tfrac{1}{2}q)\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)} - \Delta_\pi(p'-p+\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}\}\psi(p,D) - f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\mathrm{body}}\rangle. \tag{55}$$

2. Nonlinear sigma model

As a consequence of the Ward identity Eq. (30), the nuclear axial one-body current divergence matrix element is equal to the nuclear one-body pion absorption matrix element, *independently of the nuclear wave functions*

$$q_\mu\langle J^{\mu 5a}_{1\,\mathrm{body}}\rangle = -f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\mathrm{body}}\rangle. \tag{56}$$

This fact alleviates the need for a two-body axial current in this model.

3. The hybrid model

In the hybrid model the NN potential Eq. (21) consists of the linear sigma model potential Eq. (19) plus three other terms, which we shall call the ''mixed'' (two) terms and the ''nonlinear'' (one) term. The latter is essentially identical, up to an overall multiplicative factor, to the NN potential in the nonlinear sigma model. Upon inserting the hybrid potential (21) into Eq. (54) above and simplifying we find

$$
\begin{aligned}
q_\mu\langle J^{\mu 5a}_{1\,\text{body}}\rangle = &\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\Bigg(g_0^2[\Delta_s(p'-p-\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}-\Delta_\pi(p'-p+\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}+(1\leftrightarrow 2;q\leftrightarrow -q)] \\
&+i\left(\frac{g_A-1}{2f_\pi}\right)^2(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a[(p'-p-\tfrac{1}{2}q)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}(p'-p-\tfrac{1}{2}q)_\mu\gamma^\mu_{(1)}\Delta_\pi(p'-p-\tfrac{1}{2}q) \\
&-(1\leftrightarrow 2;q\leftrightarrow -q)]+g_0\left(\frac{g_A-1}{2f_\pi}\right)\{[\boldsymbol{\tau}^a_{(1)}(p'-p-\tfrac{1}{2}q)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}\Delta_\pi(p'-p-\tfrac{1}{2}q)+(1\leftrightarrow 2;q\leftrightarrow -q)] \\
&+i(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a[(p'-p+\tfrac{1}{2}q)_\nu\gamma^\nu_{(1)}\gamma^5_{(2)}\Delta_\pi(p'-p+\tfrac{1}{2}q)-(1\leftrightarrow 2;q\leftrightarrow -q)]\}\Bigg)\psi(p,D) \\
&-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\text{body}}\rangle \quad . \qquad (57)
\end{aligned}
$$

B. Two-body axial current divergence

1. Linear sigma model

Using the two-body WT identity (36), we see that the divergence of the axial two-body current in the linear sigma model reads

$$
\begin{aligned}
q_\mu\langle J^{\mu 5a}_{2\,\text{body}}\rangle = &\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\Bigg\{ g_0^2[\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}[\Delta_\pi(p_1'-p_1)-\Delta_s(p_2-p_2')]+(1\leftrightarrow 2)] \\
&-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\Pi^a_{2\,\text{body}}(p',D';p,D)\Bigg\}\psi(p,D) \\
=-&\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\{g_0^2[\Delta_s(p'-p-\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)} \\
&-\Delta_\pi(p'-p-\tfrac{1}{2}q)\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)}+\Delta_s(p'-p+\tfrac{1}{2}q)\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)}-\Delta_\pi(p'-p+\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}]\}\psi(p,D) \\
&-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{2\,\text{body}}\rangle. \qquad (58)
\end{aligned}
$$

But, comparison of this result with Eq. (55) shows that the first term on the left-hand side (l.h.s.) of Eq. (58) is exactly the negative of the first term in the one-body current divergence (55), leading to

$$
q_\mu\langle J^{\mu 5a}_{1\,\text{body}}+J^{\mu 5a}_{2\,\text{body}}\rangle=-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\text{body}}+\Pi^a_{2\,\text{body}}\rangle, \qquad (59)
$$

which completes the proof of partial conservation of the axial-current matrix element in the linear sigma model.

2. Nonlinear sigma model

The axial two-body current divergence is

$$
\begin{aligned}
q_\mu\langle J^{\mu 5a}_{2\,\text{body}}\rangle = &-ig_A\left(\frac{g_A}{2f_\pi}\right)^2(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\{\tfrac{1}{2}(p_1'-p_1+q)_\nu\gamma^\nu_{(1)}(p_2'-p_2)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}\Delta_\pi(p_2'-p_2) \\
&-(1\leftrightarrow 2)\}\psi(p,D)-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{2\,\text{body}}\rangle \\
=&\,\mathcal{O}(f_\pi^{-2})-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{2\,\text{body}}\rangle. \qquad (60)
\end{aligned}
$$

This is manifestly not conserved even in the chiral limit: a term of $\mathcal{O}(f_\pi^{-2})$ remains, as mentioned earlier. It is interesting to note that the same MEC diagram is conserved when evaluated between on-shell nucleon states. This is a rather stark illustration of differences between on- and off-shell nucleons. Future studies ought to take the present analysis one step further in powers of $1/f_\pi$, which procedure ought to produce the correct leading-order axial MEC in the nonlinear sigma model.

3. The hybrid model

The divergence of the axial two-body current in the hybrid sigma model reads

$$
\begin{aligned}
q_\mu\langle J^{\mu 5a}_{2\,\text{body}}\rangle = \int \frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\overline{\psi}(p',D')\Bigg(& -g_0^2[\Delta_s(p'-p-\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}-\Delta_\pi(p'-p-\tfrac{1}{2}q)\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)} \\
& +\Delta_s(p'-p+\tfrac{1}{2}q)\gamma_5^{(2)}\boldsymbol{\tau}^a_{(2)}-\Delta_\pi(p'-p+\tfrac{1}{2}q)\gamma_5^{(1)}\boldsymbol{\tau}^a_{(1)}]-g_0\left(\frac{g_A-1}{2f_\pi}\right)\{[\boldsymbol{\tau}^a_{(1)}(p'-p-\tfrac{1}{2}q)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)} \\
& \times\Delta_\pi(p'-p-\tfrac{1}{2}q)+(1\leftrightarrow 2;q\leftrightarrow -q)]-i(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a[(p'-p+\tfrac{1}{2}q)_\nu\gamma^\nu_{(1)}\gamma^5_{(2)}\Delta_\pi(p'-p+\tfrac{1}{2}q) \\
& -(1\leftrightarrow 2;q\leftrightarrow -q)]\}-i\left(\frac{g_A-1}{2f_\pi}\right)^2(\boldsymbol{\tau}_{(1)}\times\boldsymbol{\tau}_{(2)})^a[(p'-p-\tfrac{1}{2}q)_\nu\gamma^\nu_{(2)}\gamma^5_{(2)}(p'-p-\tfrac{1}{2}q)_\mu \\
& \times\gamma^\mu_{(1)}\Delta_\pi(p'-p-\tfrac{1}{2}q)-(1\leftrightarrow 2;q\leftrightarrow -q)]\Bigg)\psi(p,D)-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{2\,\text{body}}\rangle,
\end{aligned}
\tag{61}
$$

where we used the elementary Ward identity Eq. (35). But, this is the negative of the one-body current divergence (57), modulo terms of $\mathcal{O}(f_\pi m_\pi^2)$,

$$
q_\mu\langle J^{\mu 5a}_{1\,\text{body}}+J^{\mu 5a}_{2\,\text{body}}\rangle=-f_\pi\left(\frac{m_\pi^2}{q^2-m_\pi^2}\right)\langle\Pi^a_{1\,\text{body}}+\Pi^a_{2\,\text{body}}\rangle,
\tag{62}
$$

which completes the proof of partial conservation of the axial current in the hybrid sigma model.

C. Summary and discussion

The axial one- and two-body current operators derived in Sec. III and graphically depicted in Figs. 2, 3, and 4, and their nuclear matrix elements are the main results of this paper. We have shown that these nuclear axial-current matrix elements are partially conserved in the OBE potential approximation to the two-nucleon dynamics. In two, the linear and the hybrid, of three chiral models considered the two-body currents and consistent nuclear wave functions play a crucial role in the proof of PCAC. In the third (the nonlinear sigma) model, however, the one-body current obeys PCAC by itself, irrespective of the nuclear wave functions used. The corresponding two-body axial current is not conserved, leaving a divergence of $\mathcal{O}(f_\pi^{-2})$, even in the chiral limit. The remedy for that is, presumably, to keep going to higher orders in the expansions of the axial-current operator and of the relativistic potential in powers of $1/f_\pi$, which will further increase the order in $1/f_\pi$ of the chiral divergence remnant. Such axial-current matrix elements can be used in the evaluation of electroweak processes involving the deuteron, for example, which are *not* very sensitive to the divergence of the axial current. Processes such as the low-energy pion production/absorption/scattering are not to be evaluated without a careful inclusion of all possible chiral-symmetry-breaking (χSB) effects, since the results are highly sensitive to the latter.

Thus we have learned how to ensure (partial) conservation of the nuclear axial-current elastic matrix element for two-nucleon nuclei based on the Bethe-Salpeter equation and (almost) chirally invariant nuclear dynamics. To our knowledge, the nonlinear and hybrid sigma model results are new. The linear sigma model has been treated by Bentz [13] but with an emphasis on formal renormalization questions and in configuration space.[11] We suspect, but have not proven, that a similar analysis can be carried through for at least some three-dimensional reductions of the BS equation, such as the Gross [2,25] and the Blankenbecler-Sugar [26] equations.

Another lesson stemming from our deliberations is of some importance for the pion-nuclear physics: the divergence of the axial current matrix element yields, due to PCAC and the pion-pole dominance, the correct soft-pion nuclear absorption/emission amplitude with otherwise unchanged initial and final states (which need not coincide). In other words, our procedure determines automatically the pion-nuclear amplitudes that are consistent with chiral symmetry of the underlying nuclear dynamics. The rule is as follows: Take the diagrams entering Fig. 2, as defined in their respective models, and remove the external axial current and the pion propagator from the pion-pole diagrams, see, e.g., Figs. 3, 4, to find the consistent nuclear pion-absorption/emission graphs. (The initial and final states need not be the same as long as they both satisfy the same BS equation.) This prescription is by no means new—Blin-Stoyle and Tint [15] knew about it almost thirty years ago. Rather, it is the emphasis on the consistency between the reaction mechanism and the underlying nuclear dynamics, that removes the *ad hoc* nature of many such calculations [15,27].

The above shown proofs of axial current matrix element (partial) conservation crucially depend on the validity of elementary axial Ward-Takahashi identities, such as Eqs. (27), (30), which typically involve the divergence of an axial current vertex and various particle propagators. These propagators differ from one approximation to another. In all of the above considerations we used propagators with constant, i.e.,

[11] The controversy about the presence or absence of cancellations between the two-body and wave-function renormalization terms arising in the papers of Bentz [13] and Zhu *et al.* [24] cannot be resolved on the basis of the present work without further detailed calculations.

four-momentum squared independent self-energies.[12] That fact greatly restricts the shape and form of the axial current vertices used. For example, neither electroweak nor strong nucleon and/or pion form factors are allowed by this choice of propagator.[13] We will show in the next section that such a nucleon self-energy is closely tied to the so-called Hartree approximation to the one-body Schwinger-Dyson (JD) equation which in turn plays a crucial role in maintaining chiral symmetry in the NN channel.[14]

In conclusion we emphasize that the chiral-symmetry-breaking (χSB) effects are not *complete* in our three models. There are many sources of nonchiral corrections, such as the aforementioned pion and nucleon mass differences, deviations from the Goldberger-Treiman (GT) relation, isospin-breaking effects in the πN coupling constants, etc. Indeed each source of isospin breaking corrections in nuclear physics is also a source of nonchiral corrections. The latter admittedly constitute effects smaller than those of the pion mass itself, but are not entirely without consequences.[15] Such nonchiral terms are not always important, but when they are, they may carry the day. These χSB effects are *not* included in our simple versions of these models (which does not mean that they cannot be incorporated). Moreover, the way of including the χSB pion mass term into, say, the nonlinear sigma model is not unique [22]. Fortunately, that ambiguity does not afflict the vertices we use in this calculation, but it is sufficiently important to cause a discrepancy of up to a factor of 2 in the calculation of the $\pi-\pi$ scattering lengths [22,23,20]. Careful extensions of these models beyond the chiral limit are the subject of chiral perturbation theory (χPT) and thus beyond the purview of the present article.

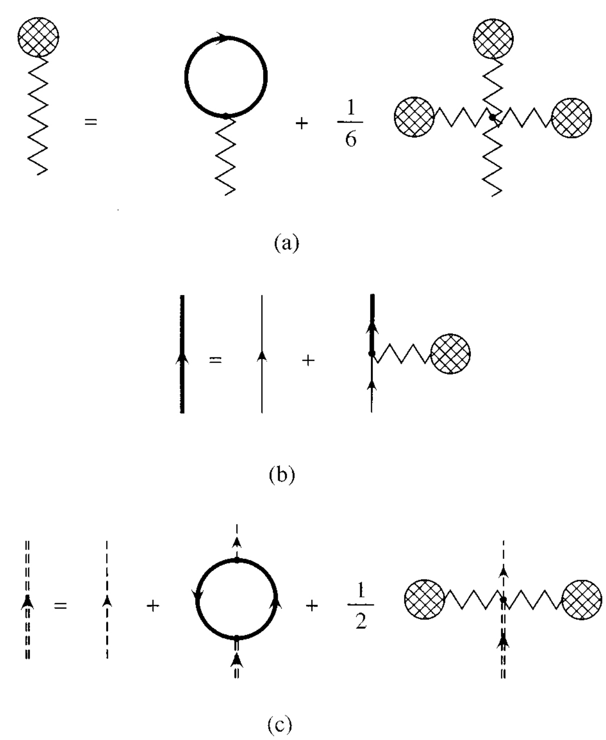

FIG. 5. Schwinger-Dyson equations defining the Hartree + RPA approximation: the vacuum alignment equation (a), the one-nucleon, or the gap equation (b), and the one-meson, or the $N\bar{N}$ Bethe-Salpeter equation (c); (a + b) constitute the Hartree, and (c) the RPA approximation. The dashed double line in (c) represents a dressed pion, and the hatched bubbles together with the zig-zag line leading to it represent the sigma meson vacuum expectation values.

V. CHIRAL SYMMETRY IN THE $N\bar{N}$ SYSTEM

So far we have only been concerned with axial current conservation in the baryon number $B=2$ sector of the two nucleon problem. In the following we shall briefly consider the $B=0$ sector, mostly to show the interdependence of apparently unrelated parts of nuclear dynamics and in particular the importance of self-consistency, i.e., of the one-body SD equation for the nucleon, in preserving chiral symmetry in this sector of the linear sigma model and its relative with isoscalar scalar mesons, the hybrid model. It turns out that in such models one must make the theory self-consistent in order to preserve the underlying chiral symmetry. This will be shown next in an approximation that is, however, one step short of the ''true'' one-boson exchange approximation[16] in the NN channel: in the H+RPA, Fig. 5. The latter is consistent with our assumption that the nucleon self-energy has no four-momentum squared dependence. The nonlinear sigma model receives only a vanishing contribution from the Hartree term to the one-body SD equation, thus rendering the H+RPA trivial therein. The H+RPA in hybrid models is nontrivial only when at least one isoscalar scalar meson is

[12]One can only have a theoretician's prejudice about the viability of that assumption: the self-energy is not an observable except at the pole of the propagator, where it equals the observed mass.

[13]One can, of course, introduce purely transverse structures into the vertices by *fiat*, à la Gross and Riska [2], but that is not a very satisfactory solution since it introduces further uncertainties in the form of new free parameters and is essentially *ad hoc*. We will seek a solution that provides the *nucleon* with a ''meson cloud'' structure that is consistent with the *nuclear* dynamics implemented, yet is sufficiently flexible to allow for additional substructure due to, e.g., the quarks.

[14]Hartree plus random phase approximation (H+RPA) is not the only approximation that satisfies these conditions on the self-energies: the first Born approximation satisfies them as well. (That was the tacit assumption all along, since we used the bare model parameters in all of the derivations). Rather, H + RPA is the only *nonperturbative* approximation that does so.

[15]As an illustration of this point, note that even in the limit of current quark masses going to zero, the charged pions retain a mass of $\simeq 35$ MeV, only the neutral pion becoming massless. The source of this mass, non-negligible on nuclear physics scale, is the EM interaction within the pion. In the nonchiral case, the latter produces only a small (4.6 MeV) pion mass difference.

[16]The true one-boson exchange approximation can be made self-consistent, along with being chirally invariant, by addition of the so-called Fock term to the one-body SD equation. Such an expanded approximation provides a nonperturbative meson cloud structure to the nucleon, which modifies its electroweak properties such as the form factors and the EM and axial static moments. These statements will not be proven here.

present, as it is in our hybrid model. In that model we can reduce the H+RPA analysis to that in the linear sigma model.

We start by reviewing the H+RPA scheme and its properties. The most important property of H+RPA is that it preserves the underlying chiral symmetry of the theory. This symmetry is made manifest in three properties of the observables, or chiral Ward identities: (i) the Goldstone theorem, or the masslessness of the pion mode in the chiral limit, (ii) the axial Ward-Takahashi identity for the nucleon (27), or conservation of the on-shell nucleon axial current elastic matrix element, and (iii) the Goldberger-Treiman relation $f_\pi g_{\pi NN}=M$, where $g_0\neq g_{\pi NN}$ between the pion decay constant f_π, the pion-nucleon coupling constant $g_{\pi NN}$ and the constituent nucleon M. We shall prove them one by one.[17]

A. Hartree plus random phase approximation

The Hartree + RPA approximation can be defined by three Schwinger-Dyson integral equations: (i) the zero-body, or vacuum equation, Fig. 5(a), (ii) the one-fermion, or gap equation, Fig. 5(b), and (ii) the one-meson, or two-fermion, i.e., Bethe-Salpeter equation shown in Fig. 5(c). Traditionally [17,29], the lowest order approximation goes under the name of Hartree for the gap equation, that corresponds to keeping only the diagrams in Fig. 5(a) and (b); and the ''chain,'' or random phase approximation (RPA) for the two-body equation, where only the diagrams in Fig. 5(c) are kept.[18]

The Bethe-Salpeter equation for the $N\overline{N}$ scattering amplitudes is separable and has as the exact solution in the H+RPA the following expression:

$$-iD_{\pi,\sigma}(k)=\frac{-i}{k^2-\Sigma^{(\mathrm{RPA})}_{\pi,\sigma}(k)}, \tag{63}$$

where $\Sigma^{(\mathrm{RPA})}(k)$ consists of a single one-nucleon-loop polarization diagram and one ''tree'' diagram shown in Fig. 5(c). Note that the above solution is just a geometric series in $\Sigma^{(\mathrm{RPA})}(k)$. The Hartree nucleon self-energy $\Sigma^{(\mathrm{H})}$ is momentum independent, but the RPA scalar $\Pi^{(\mathrm{RPA})}_\sigma(k)$ and pseudoscalar $\Pi^{(\mathrm{RPA})}_\pi(k)$ polarization functions [Fig. 5(c)] depend on the four-momentum squared k^2. The Schwinger-Dyson equations now read

$$v=\lambda_0\frac{v^3}{\mu_0^2}+\frac{\mathrm{i}}{\mu_0^2}4g_0MN_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}, \tag{64a}$$

$$\Sigma^{(\mathrm{H})}=M=M_0+g_0v, \tag{64b}$$

$$\Sigma^{(\mathrm{RPA})}_\pi(k)=-\mu_0^2+\lambda_0v^2+g_0^2\Pi^{(\mathrm{RPA})}_\pi(k), \tag{64c}$$

$$\Sigma^{(\mathrm{RPA})}_\sigma(k)=-\mu_0^2+3\lambda_0v^2+g_0^2\Pi^{(\mathrm{RPA})}_\sigma(k). \tag{64d}$$

Note that Eq. (64b) is nothing but the Goldberger-Treiman (GT) relation for Dirac fermions ($g_A=1$), which becomes exact in the chiral limit (here that means vanishing bare nucleon mass $M_0=0$). Several comments are in order here.

(i) Note that the nucleon self-energy $\Sigma^{(\mathrm{H})}$ is a k-independent constant M, in contrast to the pion and sigma self-energies $\Sigma^{(\mathrm{RPA})}_{\pi,\sigma}(k)$ that have received k-dependent contributions from the nucleon loop diagram in Fig. 5(c). We have thus introduced a distinction between nucleons and mesons in this approximation.[19] The first Born approximation can be obtained from the H+RPA by setting all nucleon-loop contributions in Eqs. (64a), (64c), (64d) equal to zero. Equation (64a) remains unchanged, however.

(ii) These self-energies satisfy the following algebraic identities that will be useful in the proof of chiral Ward identities:

$$-i\Pi^{(\mathrm{RPA})}_\pi(k)=4N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}-2N_fk^2I(k), \tag{65a}$$

$$-i\Pi^{(\mathrm{RPA})}_\sigma(k)=4N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}-2N_f(k^2-4M^2)I(k), \tag{65b}$$

where we have introduced the logarithmically divergent integral

$$I(k)=\int\frac{d^4p}{(2\pi)^4}\frac{1}{[p^2-M^2][(p+k)^2-M^2]}, \tag{66}$$

which plays an important role in subsequent calculations.

(iii) It is important to appreciate that Eqs. (65a) and (65b) are a result of a formal manipulation of divergent integrals. In order to make this procedure legitimate one must regulate (''cut off'') this and all other divergent integrals appearing in these calculations in a chirally invariant manner, i.e., so as to retain the above formal relations. We will therefore handle them as if they were Pauli-Villars (PV) regularized [18] even though we will not indicate that explicitly, in order to keep the notation as simple as possible. This step is particularly important if one decides to treat this calculation as an ''effective field theory,'' i.e., if one does not renormalize, but rather keeps the momentum cutoff, as one often does in nuclear few-body physics. It is then crucial to ensure that this cutoff does not spoil the chiral WT identities.

[17]The axial WT identities in the linear sigma model, especially for nucleons immersed in nuclear matter, have been investigated by the University of Tokyo group and W. Bentz in particular, see [28]. The overlap of the present paper with those cited in Ref. [28] seems minimal, however.

[18]As shown by Barnes and Ghandour [30], the Hartree + RPA can equally well be viewed as a variational calculation in the Schrödinger representation of quantum field theory (QFT) [31,32]. The associated ansatz for the ground-state wave functional is a Gaussian one, thereby earning the title of Gaussian functional approximation for the method [31,32].

[19]It is perhaps worth noting that the present H + RPA can be viewed as incomplete: one does not have any one-boson-loop diagrams. Introduction of the said graphs does not make a change in principle, since all of the chiral Ward identities can be preserved, as shown in Ref. [33].

B. Proof of chiral Ward identities

In order to prove the Goldstone theorem in the chiral limit ($M_0=0$), we divide Eq. (64a) by v, and then use the GT relation (64b) to rewrite it as

$$1=\lambda_0\frac{v^2}{\mu_0^2}+\frac{i}{\mu_0^2}4g_0\frac{M}{v}N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2},$$

$$-\mu_0^2+\lambda_0v^2=-4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}. \tag{67}$$

Now we insert this into Eqs. (64c), (64d) to find

$$\Sigma_\pi^{(\mathrm{RPA})}(0)=-4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}+g_0^2\Pi_\pi^{(\mathrm{RPA})}(0), \tag{68a}$$

$$\Sigma_\sigma^{(\mathrm{RPA})}(0)=2\lambda_0v^2-4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}+g_0^2\Pi_\sigma^{(\mathrm{RPA})}(0). \tag{68b}$$

Next we apply Eqs. (65a), (65b)

$$\Sigma_\pi^{(\mathrm{RPA})}(0)=-4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}+4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}=0, \tag{69a}$$

$$\begin{aligned}\Sigma_\sigma^{(\mathrm{RPA})}(0)&=2\lambda_0v^2-4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}+4ig_0^2N_f\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}+8ig_0^2N_fM^2I(0)\\&=2\lambda_0v^2+8ig_0^2N_fM^2I(0)\\&=2\lambda_0v^2-4M^2\frac{g_0^2}{g^2}.\end{aligned} \tag{69b}$$

The function $g(k)$ that appears in these equations is defined by

$$g^{-2}(k)=-2N_fiI(k)=g^{-2}F_\pi(k), \tag{70a}$$

$$g^{-2}=(\partial_{k^2}\Pi_\pi^{(\mathrm{RPA})})_{k^2=0}, \tag{70b}$$

$$F_\pi(k)=I(k)/I(0). \tag{70c}$$

Equation (69a) guarantees the existence of a massless Goldstone mode in the pseudoscalar isovector channel, whereas Eq. (69b) determines the mass of the scalar isoscalar meson.

Since the dynamically generated quark mass M that enters all these expressions is determined by the gap equation $M=\Sigma^{(\mathrm{H})}$, one finds in the H+RPA approximation that

$$-iD_\pi(k)=\frac{-i}{Z(k)k^2}, \tag{71a}$$

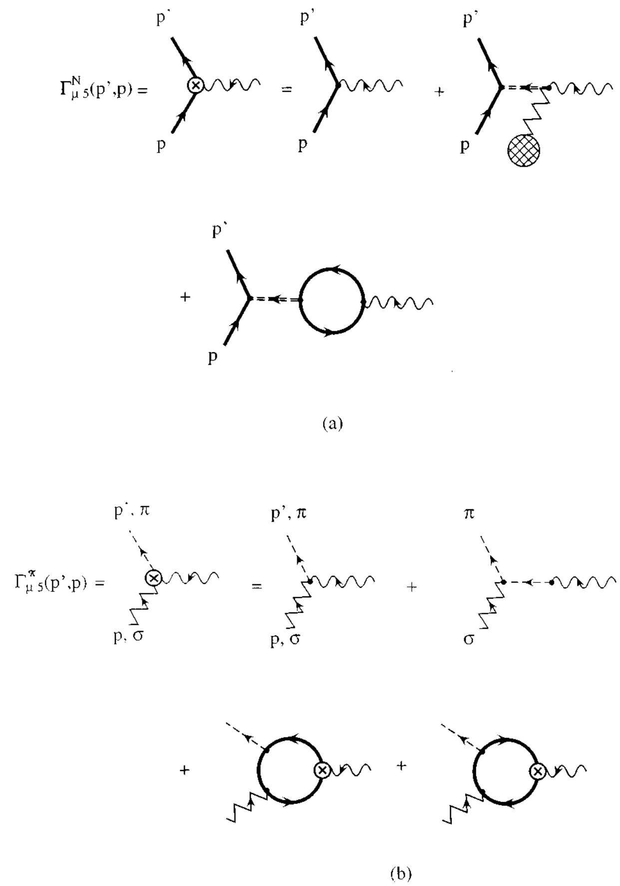

FIG. 6. Diagrams contributing to the conserved nucleon (a) and meson (b) axial current matrix elements in Hartree + RPA. The nucleon effective axial current (encircled cross) in (b) is defined in (a). Compare with Fig. 3(a),(b).

$$-iD_\sigma(k)=\frac{-i}{Z(k)[k^2-\Sigma'_\sigma(k)]}, \tag{71b}$$

where

$$Z(k)=1-\frac{g_0^2}{g^2(k)}, \tag{72a}$$

$$Z(k)\Sigma'_\sigma(k)=2\lambda_0v^2-4M^2\left(\frac{g_0}{g}\right)^2. \tag{72b}$$

We deliberately left Eqs. (70a)–(c), (72a)–(b) in a form in which they can be rendered finite either by the process of renormalization, or by keeping a momentum cutoff in the integrals.

Equations (71a), (b) summarize the linear sigma model in H+RPA: the validity of the gap equation and the GT relation ensures the existence of a massless pseudoscalar excitation that is associated with the pion as the Goldstone mode of the linear sigma model model. Furthermore they also ensure the conservation of the nucleon's axial current (CAC) in H+RPA. To prove this last claim, we start from the one-nucleon axial current as given in Fig. 6(a):

$$j^a_{\mu 5} = \gamma_\mu \gamma_5 \frac{\boldsymbol{\tau}^a}{2} + i[v - 4g_0 M i I(q)] q_\mu i D_\pi(q) g_0 \gamma_5 \boldsymbol{\tau}^a \quad . \tag{73}$$

We contract Eq. (73) with q^μ to find

$$\begin{aligned} q^\mu j^a_{\mu 5} &= q^\mu \gamma_\mu \gamma_5 \frac{\boldsymbol{\tau}^a}{2} - q^2 [v - 4g_0 M i I(q)] D_\pi(q) g_0 \gamma_5 \boldsymbol{\tau}^a \\ &= q^\mu \gamma_\mu \gamma_5 \frac{\boldsymbol{\tau}^a}{2} - q^2 [v - 4g_0 M i I(q)] \\ &\quad \times \frac{1}{q^2(1 - 4g_0^2 i I(q))} g_0 \gamma_5 \boldsymbol{\tau}^a \\ &= q^\mu \gamma_\mu \gamma_5 \frac{\boldsymbol{\tau}^a}{2} - g_0 v \gamma_5 \boldsymbol{\tau}^a \\ &= (p' - p)^\mu \gamma_\mu \gamma_5 \frac{\boldsymbol{\tau}^a}{2} - M \gamma_5 \boldsymbol{\tau}^a \\ &= [S^{-1}(p') \gamma_5 + \gamma_5 S^{-1}(p)] \frac{\boldsymbol{\tau}^a}{2} , \end{aligned} \tag{74}$$

which proves that the H + RPA nucleon axial current $j^a_{\mu 5}$ and propagator $S(p)$ satisfy the WT identity (27), (32), as noted. Note that we have used the vacuum and one-body SD equations (64a), (64b) on several occasions in these proofs. Without them, i.e., without self-consistency, the proof would have been impossible in the linear and the hybrid sigma models.

Similarly, the reader can convince himself that the one-meson axial current defined in Fig. 6(b) and the π, σ meson propagators (63), (71a), (b) satisfy the WT identity (35).[20] Thus we have demonstrated the necessity of a self-consistent gap equation for the chiral invariance of the solutions to the few-body nuclear dynamics in the present approximation. This completes our discussion of Hartree + RPA in the linear sigma model.

Similar considerations hold in the hybrid sigma model within H+RPA. Finally, as far as the nonlinear sigma model is concerned, self-consistency does not seem to make a difference since there are no scalar mesons in the model. Isoscalar scalars are the only mesons that make the Hartree gap equation nontrivial *in vacuo*, i.e., at zero density.

VI. SUMMARY AND CONCLUSIONS

In summary, in this paper we have done the following.

We constructed the Nöther currents in three typical chirally symmetric hadronic models. These currents in the hybrid model of Refs. [11,10,9] are new.

We constructed one- and two-nucleon axial current operators that are necessary for the construction of the partially conserved nuclear axial current matrix elements in three models. The resulting nuclear axial current matrix elements are new and ready for applications.

We have proven partial conservation of the nuclear axial-current matrix elements in the baryon number $B=2$ sector under the assumption of validity of axial Ward-Takahashi identities for nucleon and meson vertices. We showed that these WT identities are true for vertices in the first Born approximation for the three models considered. We discussed the need for consistency between the axial current operators and the nuclear two-body dynamics, as defined by the Bethe-Salpeter equation within the one-boson-exchange approximation.

We examined the relation between the WT identities and the one-body Schwinger-Dyson (SD) equation. We established and discussed the relation between the Hartree + random phase approximation in the linear sigma model and the two-body equation in the $B=0$ sector.

We proved the Goldstone theorem, the Goldberger-Treiman relation and axial-current conservation in the Hartree + random phase approximation to the $B=0$ sector, and showed the interdependence between this sector and the Ward identities in the $B=2$ sector.

We have *not* attempted to do the following.

Include the vector and axial vector mesons.

Examine the chiral symmetry breaking effects beyond the terms induced by the finite pion mass.

Include form factors for nucleons or mesons, either electroweak or strong. We believe that this problem is inextricably related to the question of the meson cloud around hadrons, that, as pointed out above, is closely tied to the Fock terms and the vector and axial vector mesons.

Include Fock terms in the self-consistency equation (see comments above).

Discuss the chiral symmetry in relativistic reductions of the Bethe-Salpeter equation, such as the spectator (Gross) and the Blankenbecler-Sugar equations.

Treat mesons in a self-consistent way when working within the Hartree + RPA, as was done in Ref. [33].

Axial current (partial) conservation ought to be an important criterion in the construction of the elastic parity-violating electron or neutrino nuclear scattering matrix elements. Despite their great importance for nuclear astrophysics [35,14], the axial two-body, or meson-exchange, currents had not, to our knowledge, been examined from the viewpoint of the principle of partial conservation of the axial current (PCAC) in a relativistic BS equation formalism, with the exception of work by Bentz [13]. The present paper presents a solution to that problem. We have not, as yet, applied our results to specific physical processes.

Furthermore, we have not tackled the problem of chiral symmetry in the NN system beyond the Hartree + RPA approach in this paper. This is somewhat unsatisfactory since one of our conclusions was that we have to go beyond Hartree + RPA in order to achieve the true chirally invariant OBE approximation that dresses the nucleon with a ''meson cloud'' to the same degree as it binds the nucleons in the nucleus. We will display a solution to this problem in the sequel to this paper. There we will show that the true one-boson exchange approximation can be made self-consistent, along with being chirally invariant, by addition of the so-called Fock terms to the one-body SD equation. Such an extended approximation provides the *nucleon* with a nonperturbative meson cloud structure that is consistent with the

[20] If the reader has trouble proving this, let him consult Sec. III C, and in particular Eq. (3.14) of Ref. [34].

nuclear dynamics. Nevertheless, we believe that the present paper had to be written in order to (a) show the necessity of this type of investigation, which can lead to relations among seemingly unrelated structures within a theory, such as the axial Ward-Takahashi identities (27),(30) on one side and the one-body SD equation (64a)–(d); and (b) prepare the ground for the technically more demanding true one-boson exchange approximation.

Nonperturbative approximation schemes of this type for relativistic meson-nucleon models have been formulated and elaborated under the name of quantum hadrodynamics (QHD) (for a review see Ref. [29]). The main aim of that effort has been the nuclear many-body problem and its applications to astrophysics, the approximate methods of solution were directly inspired by similar methods in the nonrelativistic many-body theory [17]. Some of these, or similar methods were independently developed by particle theorists interested in implementing chiral symmetry in theories with bound states [36] and in ''finite QED'' [37]. Unfortunately the influence of these methods on the nuclear few-body problem practitioners has been weak in the past. We hope to remedy that situation with this paper. We also hope to have made it manifest that only a few nonperturbative methods make sense in the relativistic nuclear many-body problem with chiral symmetry in the sense that they alone form a chirally invariant self-consistent approximation scheme—this does not seem to have been realized before [29].

In the process of construction of axial currents obeying PCAC we observed that the results have important consequences for chiral perturbation theory (χPT) of pion-nuclear processes. Inclusion of chiral symmetry into nuclear physics is a subject that was begun only recently: some work on *nonrelativistic* many-body systems has even entered a textbook [16], but there is no attempt at a *systematic relativistic* approach in the literature even for the simplest of few-body problems. Various *nonrelativistic* models of two-body axial currents constrained by PCAC are reviewed in Ref. [1]. The main drawback of such calculations, as compared with similar calculations of EM MEC's is that, although the axial MEC's do obey PCAC, constraints on the axial currents imposed by the nuclear dynamics are often not considered at all. In other words, the two-nucleon potential used to calculate the nuclear wave function in such models is *not* necessarily related to the two-nucleon axial current. In this paper we have shown that such independence of axial two-body current from the nuclear two-body potential is highly model-dependent, at least in relativistic formalisms based on the BS equation. A similar analysis of nonrelativistic theories still remains to be done.

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