

Determination of pion-baryon coupling constants from QCD sum rules

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We evaluate the πNN , $\pi\Sigma\Sigma$, and $\pi\Sigma\Lambda$ coupling constants using QCD sum rules based on pion-to-vacuum matrix elements of correlators of two interpolating baryon fields. The parts of the correlators with Dirac structure $\not{k}\gamma_5$ are used, keeping all terms up to dimension 5 in the OPE and including continuum contributions on the phenomenological side. The ratios of these sum rules to baryon mass sum rules yield stable results with values for the couplings of $g_{\pi NN}=12\pm 5$, $g_{\pi\Sigma\Sigma}=7\pm 4$, and $g_{\pi\Sigma\Lambda}=6\pm 3$. The sources of uncertainty are discussed. [S0556-2813(96)05412-X]

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I. INTRODUCTION

Meson-baryon coupling constants form an important ingredient in many calculations of strong-interaction processes and one would like to determine these quantities from QCD. In the absence of treatments from first principles, the method of QCD sum rules [1] has proved to be a very powerful tool for studying various properties of low-lying hadron states. Here we apply this method to the calculation of the coupling constants of pions to the lowest states of the baryon octet: N , Λ , and Σ .

The pion-nucleon coupling constant $g_{\pi NN}$ has previously been studied within the framework of QCD sum rules by several groups [2–5]. Reinders, Rubinstein, and Yazaki [3] explored two different approaches, one based on the correlator of three interpolating fields sandwiched between vacuum states, and one based on the pion-to-vacuum matrix element of the correlator of two interpolating nucleon fields, η :

$$\langle 0|T\{\eta(x)\bar{\eta}(0)\}|\pi^a(k)\rangle. \quad (1)$$

The particular sum rule they studied was based on the soft-pion limit of the part of the two-point correlator (1) with Dirac structure γ_5 . However those authors took into account only the leading term of the operator product expansion (OPE) and they neglected continuum contributions. Shiomi and Hatsuda [4] extended the analysis of this sum rule to include condensates up to dimension 7 in the OPE as well as a perturbative estimate of continuum contributions.

The sum rules that we use here are also constructed from two-point correlators (1) of the appropriate baryon interpolating fields. The advantage of this method is that it allows one to calculate hadron properties at low values of the momentum transfer to the baryon. In contrast, the straightforward use of the OPE for the three-point correlator is valid only for large spacelike meson momenta and therefore a determination of the coupling constant requires an extrapolation to zero momentum where the OPE is clearly not valid because of large power corrections. Estimates of the coupling constant from the coefficient of $1/k^2$ determined at

large k^2 , as in Refs. [2,3,6], cannot distinguish the pole term of the lowest meson from the contributions of higher-mass states in the same meson channel which give the same $1/k^2$ behavior at large k^2 .

We note that modified versions of the OPE of three-point correlators for processes with small momentum transfer have been developed in Refs. [7,8]. The essence of these methods is the inclusion of ‘‘bilocal power corrections,’’ which effectively sum up the series of power terms in $1/k^2$ by matching them to the contributions of mesonic states in the relevant channel. The contributions of low-lying mesons to the form factors play an increasingly important role as the momentum transfer decreases. Meson-baryon coupling constants can be obtained from the OPE of three-point correlators with bilocal power corrections by going to the meson pole. At the pole this treatment of the three-point correlators yields the same results as the method based on two-point correlators which is used in this paper (cf. [9]).

The particular sum rules that we study here are constructed from the part of the correlator (1) with Dirac structure $\not{k}\gamma_5$. We chose this structure because it provides a determination of the pion-baryon couplings that is not simply related to the sum rules for the baryon masses. In contrast the soft-pion limit of the OPE for the γ_5 piece of the two-point correlator for $g_{\pi NN}$ has exactly the same form as that for the nucleon sum rule [10,8] involving condensates of odd dimension, up a factor of $1/f_\pi$ [3,4]. Shiomi and Hatsuda [4] showed that the ratio of the γ_5 sum rule to one for the nucleon mass takes the form of the Goldberger-Treiman relation with $g_A=1$, provided that the continuum thresholds are taken to be the same in both cases. Those authors took different thresholds in the two sum rules in order to get around this problem with the implied value of g_A .

However, we stress that taking the soft-pion limit of the γ_5 piece of the two-point correlator (1) does not lead to an independent determination of the coupling constant. In the case of $g_{\pi NN}$, the usual soft-pion theorem [11] can be used to express the correlator (1) in the form

$$\begin{aligned} & -\frac{i}{f_\pi}\langle 0|[Q_5^a, T(\eta(x), \eta^\dagger(0))]|0\rangle \\ & = \frac{i}{2f_\pi}\{\gamma_5\tau^a, \langle 0|T(\eta(x), \eta^\dagger(0))|0\rangle\}, \end{aligned} \quad (2)$$

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where Q_5^a is the axial charge and we have made use of the transformation properties of the interpolating field under axial rotations [12,13], $[Q_5^a, \eta] = -1/2 \gamma_5 \tau^a \eta$. The anticommutator with γ_5 picks out the part of the two-point correlator proportional to the unit Dirac matrix. The phenomenological side of the resulting sum rule is thus $i \gamma_5 / f_\pi$ times the corresponding expression for the odd-condensate nucleon sum rule. This matches exactly with the structure found for the OPE side in Refs. [3,4].

The soft-pion limit for the γ_5 piece of the correlator (1) thus yields a sum rule for $M_N / f_\pi = g_{\pi NN} / g_A$. The value for the coupling determined from such a sum rule follows from the odd-condensate sum rule for the nucleon mass and the Goldberger-Treiman relation (or an approximation to it taking $g_A = 1$). The sum rule can be thought of as just a chiral rotation of the odd-condensate nucleon sum rule and *not* an independent determination of $g_{\pi NN}$. Physically this result is quite natural since in the soft-pion limit πB and B states become degenerate and can be related to each other by chiral transformations. In this paper, by considering terms beyond the soft-pion limit, we obtain values for pion-baryon couplings that are not simply consequences of chiral symmetry.

In addition we note that a potentially important piece of the phenomenological side is missing from previous sum-rule determinations of $g_{\pi NN}$. This term corresponds to transitions where a ground-state baryon created by the interpolating field absorbs the pion and is excited into the continuum. Since they are not suppressed by the Borel transformation such terms should be included in a consistent sum-rule analysis, as pointed out long ago [8,14] and stressed recently by Ioffe [15,16]. In the soft-pion limit of the γ_5 sum rule, such terms generate contact interactions where the pion couples directly to the baryon field, $\langle B(p) | \bar{\eta}_n(0) | \pi(k) \rangle$, and which are essential if the correct soft-pion limit is to be obtained. The omission of these terms in Refs. [3,4] can explain why the correct Goldberger-Treiman relation was not found there. Indeed, as the authors of [4] point out, a quick estimate of these unsuppressed N^* contributions suggests that they could be as large as 25%: enough to remove the discrepancy with the Goldberger-Treiman relation.

As discussed above, the sum rules studied here provide values for the pion-baryon couplings that are not simply related to the baryon masses by chiral symmetry. We include all condensates up to dimension 5 as well as mixed continuum terms. These are essential for assessing the reliability of the sum rules and estimating the uncertainties in the results. The application of these sum rules to $g_{\pi NN}$ has been described briefly in [5]. Similar sum rules have been applied to other pion couplings, especially in the context of D and B mesons, as discussed in [9] and references therein.

The paper is organized as follows: in Sec. II we derive the sum rules for the pion-baryon couplings from the relevant two-point correlators; the numerical analysis of the sum rules is presented in Sec. III; finally our results are summarized in Sec. IV.

II. TWO-POINT CORRELATORS AND SUM RULES

Our sum rules are obtained from the two-point correlator (1) just discussed, but instead of the piece with Dirac structure γ_5 considered in Refs. [3,4] we work with the structure

$\not{k} \gamma_5$, where k is the pion momentum. We work here to leading order in a chiral expansion, neglecting higher-order terms in the pion momentum or current quark mass. To illustrate the derivation of the sum rules for pion-baryon couplings, we consider first the sum rule for $g_{\pi NN}$. The differences that arise for the pion-hyperon couplings will then be discussed and the forms of the resulting sum rules presented.

We consider the two-point correlation function

$$\Pi(p) = i \int d^4x \exp(ip \cdot x) \langle 0 | T \{ \eta_p(x) \bar{\eta}_n(0) \} | \pi^+(k) \rangle, \quad (3)$$

where we use the Ioffe interpolating field [10] for the proton,

$$\eta_p(x) = \epsilon_{abc} [u^a(x)^T C \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu d^c(x), \quad (4)$$

and the corresponding neutron field η_n which is obtained by interchanging u and d quark fields and multiplying by -1 . Here a, b, c are the color indices and C is the charge conjugation matrix. Other choices of interpolating field can be used, as discussed in detail by Leinweber [17]. For the odd-condensate nucleon sum rule, which we make use of in our determination of $g_{\pi NN}$, it turns out that the Ioffe field is close to optimal [17] and so we do not consider more general fields.

In the deeply Euclidean region, where p^2 is large and negative, the OPE of the product of two interpolating fields takes the following general form

$$i \int d^4x \exp(ip \cdot x) T \{ \eta_p(x) \bar{\eta}_n(0) \} = \sum_n C_n(p) O_n, \quad (5)$$

where $C_n(p)$ are the Wilson coefficients and O_n are local operators constructed out of quark and gluon fields (all renormalized at some scale μ). Using this OPE in correlators of the form (3), we find that only operators of odd dimension contribute. The leading term in this expansion involves operators with dimension 3 and is given by

$$\begin{aligned} \Pi_3(p, k) = & -\frac{1}{2\pi^2} p^2 \ln(-p^2) \\ & \times \langle 0 | \bar{d} \gamma^\alpha \gamma_5 u | \pi^+(k) \rangle \gamma_\alpha \gamma_5 + \dots, \end{aligned} \quad (6)$$

where terms that do not contribute to the Dirac structure of interest, $\not{k} \gamma_5$, have been suppressed. The matrix element here is just the usual one for pion decay:

$$\langle 0 | \bar{d} \gamma^\alpha \gamma_5 u | \pi^+(k) \rangle = i \sqrt{2} f_\pi k^\alpha, \quad (7)$$

where $f_\pi = 93$ MeV is the pion decay constant. Hence we can write the leading term as

$$\Pi_3(p, k) = -i \sqrt{2} \frac{1}{2\pi^2} p^2 \ln(-p^2) f_\pi \not{k} \gamma_5 + \dots \quad (8)$$

At dimension 5 the only relevant contribution arises from the second-order term in the covariant expansion of the non-local operator $\bar{d}(0) \gamma^\alpha \gamma_5 u(x)$. This is a specific feature of the Ioffe nucleon interpolating field [10] which we used to calculate Π^N . This term has the form

$$\Pi_5(p) = \frac{5}{9\pi^2} \ln(-p^2) \langle 0 | \bar{d} \gamma^\alpha \gamma_5 D^2 u | \pi^+(k) \rangle \gamma_\alpha \gamma_5 + \dots \quad (9)$$

Up to corrections of higher order in the current mass, the matrix element here can easily be reexpressed in terms of a mixed quark-gluon condensate

$$\langle 0 | \bar{d} \gamma^\alpha \gamma_5 D^2 u | \pi^+(k) \rangle = \frac{g_s}{2} \langle 0 | \bar{d} \gamma^\alpha \gamma_5 \sigma_{\mu\nu} G^{\mu\nu} u | \pi^+(k) \rangle + \mathcal{O}(m_c^2). \quad (10)$$

With some further manipulation this can be rewritten in the form

$$\begin{aligned} \langle 0 | \bar{d} \gamma^\alpha \gamma_5 D^2 u | \pi^+(k) \rangle &= -g_s \langle 0 | \bar{d} \tilde{G}^{\alpha\mu} \gamma_\mu u | \pi^+(k) \rangle \\ &\quad - i g_s \langle 0 | \bar{d} G^{\mu\alpha} \gamma_\mu \gamma_5 u | \pi^+(k) \rangle, \end{aligned} \quad (11)$$

where $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$. (We use the convention $\epsilon^{0123} = +1$.) The second term in this expression is of higher order in the chiral expansion (see Ref. [18] for details) and so we neglect it here.

The first term in Eq. (11) is of leading order in the chiral expansion. It involves a matrix element that has been extracted by Novikov *et al.* [18] from two QCD sum rules for the pion. They expressed it in the form

$$g_s \langle 0 | \bar{d} \tilde{G}^{\alpha\mu} \gamma_\mu u | \pi^+(k) \rangle = \sqrt{2} i \delta^2 f_\pi k^\alpha, \quad (12)$$

and obtained $\delta^2 = (0.20 \pm 0.02) \text{ GeV}^2$. In both their sum rules the four-quark condensate $\alpha_s \langle 0 | (\bar{q}q)^2 | 0 \rangle$ makes a crucial contribution. Novikov *et al.* [18] used the factorization approximation for this quantity in their analysis. However, direct determinations of it from other sum rules lead to values [19–21] that are at least 2–3 times bigger than those obtained from factorization. These give correspondingly larger values for δ^2 , a point we shall come back to in the analysis of the sum rules in Sec. III. Our final expression for the dimension-5 term in the sum rule is

$$\Pi_5(p) = -i \sqrt{2} \frac{5}{9\pi^2} \ln(-p^2) \delta^2 f_\pi k^\alpha \gamma_5 + \dots \quad (13)$$

To estimate the importance of higher-dimension condensates, we have also calculated the contribution of what we hope is the most important dimension-7 operator in the OPE. This is a mixed quark-gluon condensate, which we evaluate in the factorized approximation. Keeping only this contribution explicitly, the dimension-7 piece of the correlator is

$$\begin{aligned} \Pi_7(p) &= -\frac{1}{12p^2} \langle 0 | \bar{d} \gamma^\alpha \gamma_5 u | \pi^+(k) \rangle \\ &\quad \times \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \gamma_\alpha \gamma_5 + \dots, \end{aligned} \quad (14)$$

where $\langle 0 | (\alpha_s/\pi) G^2 | 0 \rangle$ is the gluon condensate in vacuum. We find that the contribution of this condensate is small, as discussed in the following section.

On the phenomenological side, the πN coupling constant is contained in the term of the correlator (3) with a double pole at the nucleon mass. However there are also continuum contributions which cannot be ignored. These include continuum-to-continuum pieces which can be modeled in the usual manner, in terms of the spectral density associated with the imaginary part of the OPE expression for the correlator. This continuum is assumed to start at some threshold $S_{\pi N}$. After Borel transformation, it can be taken over to the OPE side of the sum rule where it modifies the coefficients of the terms involving $\ln(-p^2)$. In addition one must include nucleon-to-continuum terms since Borel transformation does not suppress these with respect to the double-pole term [8,14–16]. To first order in k , the correlator has the form

$$\begin{aligned} \Pi(p) &= i \sqrt{2} k^\alpha \gamma_5 \left[\frac{\lambda_N^2 M_{\pi NN} g_{\pi NN}}{(p^2 - M_N^2)^2} \right. \\ &\quad \left. + \int_{W^2}^{\infty} ds b(s) \frac{1}{s - M_N^2} \left(\frac{1}{p^2 - M_N^2} + \frac{a(s)}{s - p^2} \right) \right] + \dots, \end{aligned} \quad (15)$$

where the continuum-continuum terms (and terms with other Dirac structures) have not been written out. Here λ_N is the strength with which the interpolating field couples to the nucleon:

$$\langle 0 | \eta_N(0) | N(p) \rangle = \lambda_N u(p). \quad (16)$$

The sum rule is obtained by equating the OPE and phenomenological expressions for the correlator (3) and Borel transforming [1]. Keeping only condensates up to dimension 5, this has the form

$$\begin{aligned} \frac{1}{2\pi^2} M^4 E_2(x_{\pi N}) + \frac{5}{9\pi^2} M^2 E_1(x_{\pi N}) \delta^2 \\ = \left(\frac{\lambda_N^2 M_{\pi NN} g_{\pi NN}}{f_\pi M^2} + A \right) \exp(-M_N^2/M^2), \end{aligned} \quad (17)$$

where M is the Borel mass and $E_n(x) = 1 - (1 + x + \dots + x^n/n!) e^{-x}$ with $x_{\pi N} = S_{\pi N}/M^2$. The second term on the right-hand side of this sum rule is the Borel transform of the nucleon pole term of the nucleon-to-continuum piece in Eq. (15). It involves an undetermined constant A but, since it contains the same exponential as the nucleon double-pole term, it cannot be ignored. The second nucleon-to-continuum term in Eq. (15) leads to a term that is suppressed by an exponential involving the masses of states in the continuum. It is thus typically a factor of 3–4 smaller than the term included in Eq. (17). Provided that the first of these mixed terms is a reasonably small correction to the sum rule, it should be safe to neglect the second, as discussed by Ioffe [15,16].

The construction of sum rules for the pion-hyperon couplings follows similar lines. For the $\Sigma^{+,0}$ and Λ we use the following fields, obtained by SU(3) rotations of Eq. (4) [10]:

$$\eta_{\Sigma^+}(x) = \epsilon_{abc} [u^a(x)^T C \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu s^c(x), \quad (18)$$

$$\eta_{\Sigma^0}(x) = \sqrt{2} [\eta_{Y_2}(x) + \eta_{Y_1}(x)], \quad (19)$$

$$\eta_{\Lambda}(x) = \sqrt{\frac{2}{3}} [\eta_{Y_2}(x) - \eta_{Y_1}(x)], \quad (20)$$

where we have introduced

$$\eta_{Y_1}(x) = \epsilon_{abc} [d^a(x)^T C \gamma_{\mu} s^b(x)] \gamma_5 \gamma^{\mu} u^c(x), \quad (21)$$

$$\eta_{Y_2}(x) = \epsilon_{abc} [u^a(x)^T C \gamma_{\mu} s^b(x)] \gamma_5 \gamma^{\mu} d^c(x). \quad (22)$$

It is convenient to evaluate the correlators of η_{Y_1} and η_{Y_2} with the Σ^+ field separately. Considering η_{Y_1} first, we find that its correlator has the same basic form as the proton-neutron one just discussed. The only difference is that it is smaller by a factor of 2 since it contains only one strange-quark field. For the $k \gamma_5$ piece of this correlator we therefore have

$$\begin{aligned} \Pi^{Y_1}(p) &= -i \sqrt{2} \frac{1}{4\pi^2} p^2 \ln(-p^2) f_{\pi} k \gamma_5 \\ &\quad - i \sqrt{2} \frac{5}{18\pi^2} \ln(-p^2) \delta^2 f_{\pi} k \gamma_5 + \dots \end{aligned} \quad (23)$$

The OPE for the correlator of η_{Y_2} starts with a dimension-3 term of the form

$$\Pi_3^{Y_2}(p) = i \sqrt{2} \frac{1}{24\pi^2} p^2 \ln(-p^2) f_{\pi} k \gamma_5 + \dots \quad (24)$$

Unlike the corresponding terms in Eqs. (8) and (23), which have the form $k \gamma_5 / x^6$ in coordinate space, this term arises from one of the form $kx \cdot k / x^8$. This difference in the coordinate-space structure means that the corresponding dimension-5 term coming from the expansion of $\bar{d}(0) \gamma^{\alpha} \gamma_5 u(x)$ has a different relative coefficient compared to that in Eqs. (13) and (23). It involves the same matrix element (12) discussed above and has the form

$$\Pi_5^{Y_2}(p) = i \sqrt{2} \frac{5}{72\pi^2} \ln(-p^2) \delta^2 f_{\pi} k \gamma_5 + \dots \quad (25)$$

One might have expected an additional contribution of this form from the background gluon field in the quark propagator. However it turns out that such a term vanishes for the $k \gamma_5$ piece of the correlator of η_{Y_2} and $\bar{\theta}_{\Sigma^+}$ because of a cancellation of contributions from the coordinate-space forms $k \gamma_5 / x^4$ and $kx \cdot k / x^6$.

At dimension 7 there are mixed quark-gluon condensate terms, which are similar to the term in the nucleon correlator (14). The first SU(3)-breaking term also appears at this order. This involves a condensate of the form $m_s \langle 0 | \bar{q} q \bar{d} \gamma^{\alpha} \gamma_5 u | \pi^+(k) \rangle$, stemming from the mass term in the strange-quark propagator. The term can be estimated in the factorization approximation and we find that it gives a very small (less than 5%) contribution to the OPE side of the sum rules. We therefore neglect it in our analyses.

The phenomenological expressions for the hyperon correlators are

$$\Pi^{\Sigma}(p) = i k \gamma_5 \frac{\lambda_{\Sigma}^2 M_{\Sigma} g_{\pi\Sigma\Sigma}}{(p^2 - M_{\Sigma}^2)^2} + \dots, \quad (26)$$

$$\Pi^{\Lambda}(p) = -i k \gamma_5 \frac{\lambda_{\Sigma} \lambda_{\Lambda} M_Y g_{\pi\Sigma\Lambda}}{(p^2 - M_Y^2)^2} + \dots, \quad (27)$$

where only the pole terms have been written out. In the $\Lambda\Sigma$ correlator M_Y denotes the average hyperon mass since we neglect the mass difference between the Σ and Λ . (The numerical coefficients in the definitions of the coupling constants can be found in [22].)

Taking the combinations of the η_{Y_1} and η_{Y_2} correlators that correspond to the Σ^0 and Λ and equating them to the phenomenological expressions, we obtain the sum rules

$$\begin{aligned} &\frac{5}{12\pi^2} M^4 E_2(x_{\pi\Sigma}) + \frac{5}{12\pi^2} M^2 E_1(x_{\pi\Sigma}) \delta^2 \\ &= \left(\frac{\lambda_{\Sigma}^2 M_{\Sigma} g_{\pi\Sigma\Sigma}}{f_{\pi} M^2} + A_{\Sigma} \right) \exp(-M_{\Sigma}^2/M^2), \end{aligned} \quad (28)$$

$$\begin{aligned} &\frac{7}{12\pi^2} M^4 E_2(x_{\pi\Lambda}) + \frac{25}{36\pi^2} M^2 E_1(x_{\pi\Lambda}) \delta^2 \\ &= \sqrt{3} \left(\frac{\lambda_{\Sigma} \lambda_{\Lambda} M_Y g_{\pi\Sigma\Lambda}}{f_{\pi} M^2} + A_{\Lambda} \right) \exp(-M_Y^2/M^2). \end{aligned} \quad (29)$$

In the limit of exact SU(3) symmetry there two independent couplings of pseudoscalar mesons to the baryon octet, usually denoted F and D corresponding to antisymmetric and symmetric combinations of the octet fields. The πN coupling is proportional to $F + D$ and the hyperon couplings can be written as

$$g_{\pi\Sigma\Sigma} = 2\alpha g_{\pi NN}, \quad (30)$$

$$g_{\pi\Sigma\Lambda} = \frac{2}{\sqrt{3}} (1 - \alpha) g_{\pi NN}, \quad (31)$$

where

$$\alpha = \frac{F}{F + D} \quad (32)$$

(see, for example, [23,24]). Comparing our sum rules (28) and (29) with these forms we see that, if the strengths λ_B are SU(3) symmetric, the correlator of η_{Y_1} contributes to the coupling $F + D$, while η_{Y_2} contributes to $F - D$. In this limit the dimension-3 terms in these sum rules would lead to an F/D ratio of 5/7 [3], although the dimension-5 terms would tend to reduce this value. For comparison, SU(6) quark models give $F/D = 2/3$, and SU(3)-symmetric analyses of pion-baryon couplings [23,24] or baryon axial couplings [25] tend to give values around 0.58. One should remember that SU(3) is significantly broken by the strange quark mass and so it may not be possible to represent the couplings in terms of F and D .

III. ANALYSIS

We now turn to the numerical analysis of these sum rules. First, one should get rid of the unknown constants A_B . Multiplying the sum rules by $M^2 \exp M_{\Sigma}^2/M^2$, we see that the right-hand sides become linear functions of M^2 . By acting on

these forms of the sum rules with $(1 - M^2 \partial / \partial M^2)$ [8] (or equivalently by fitting a straight line to the left-hand sides and extrapolating to $M^2 = 0$ [14]) we can in principle determine value for the couplings. However we are unable to find a region of Borel mass in which the left-hand sides are approximately linear functions of M^2 , and hence there is no region of stability for the extracted $g_{\pi BB}$.

This lack of stability is similar to the situation for the

nucleon sum rules, where two sum rules can be derived [10] (involving either odd or even dimension operators) but neither shows good stability. Nonetheless the ratio of these leads to a more stable expression for the nucleon mass. We have therefore taken the ratio of our sum rules to those for the corresponding baryons. We obtain the most stable results from the ratios to the following baryon sum rules [10,8,3] (see also [26–28]),

$$-\frac{1}{4\pi^2} M^4 E_1(x_N) \langle 0 | \bar{q}q | 0 \rangle + \frac{1}{24} \langle 0 | \bar{q}q | 0 \rangle \langle 0 | (\alpha_s / \pi) G^2 | 0 \rangle = \lambda_N^2 M_N \exp(-M_N^2 / M^2), \quad (33)$$

$$\frac{m_s}{16\pi^4} M^6 E_2(x_\Sigma) - \frac{1}{4\pi^2} M^4 E_1(x_\Sigma) \langle 0 | \bar{s}s | 0 \rangle + \frac{4}{3} m_s \langle 0 | (\bar{q}q)^2 | 0 \rangle = \lambda_\Sigma^2 M_\Sigma \exp(-M_\Sigma^2 / M^2), \quad (34)$$

$$-\frac{m_s}{48\pi^4} M^6 E_2(x_\Lambda) - \frac{M^4}{12\pi^2} (4 \langle 0 | \bar{q}q | 0 \rangle - \langle 0 | \bar{s}s | 0 \rangle) E_1(x_\Lambda) + \frac{4}{9} m_s [3 \langle 0 | (\bar{q}q)^2 | 0 \rangle - \langle 0 | (\bar{q}q)(\bar{s}s) | 0 \rangle] = M_\Lambda \lambda_\Lambda^2 \exp(-M_\Lambda^2 / M^2), \quad (35)$$

and so we present here only the results for these cases. Taking such ratios also has the advantage of eliminating the experimentally undetermined strengths λ_B from the sum rules. Note that we have allowed for a different continuum threshold S_B in each of the sum rules and have defined $x_B = S_B / M^2$.

Again we describe first the sum rule for $g_{\pi NN}$ and then discuss the additional features that arise for the hyperons. We take the ratio of the sum rules (17) and (33)

$$f_\pi \frac{\frac{1}{2\pi^2} M^6 E_2(x) + \frac{5}{9\pi^2} M^4 E_1(x) \delta^2 + \frac{1}{12} M^2 E_0(x) \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle}{-\frac{1}{4\pi^2} M^4 E_1(x_N) \langle 0 | \bar{q}q | 0 \rangle + \frac{1}{24} \langle 0 | \bar{q}q | 0 \rangle \langle 0 | (\alpha_s / \pi) G^2 | 0 \rangle} = g_{\pi NN} + A_N' M^2, \quad (36)$$

and use the method discussed above to eliminate the unknown mixed nucleon-to-continuum term, $A_N' M^2$ (where $A_N' = A_N f_\pi / \lambda_N^2 M_N$). The results for $g_{\pi NN}$ are shown in Fig. 1 as a function of the Borel mass M^2 . These have been obtained using the following typical values of the condensates and thresholds: $\langle 0 | \bar{q}q | 0 \rangle = -(0.245 \text{ GeV})^3$, $\langle 0 | (\alpha_s / \pi) G^2 | 0 \rangle \approx 0.012 \text{ GeV}^4$, $\delta^2 = 0.35 \text{ GeV}^2$, $S_N = 2.5 \text{ GeV}^2$, and $S_{\pi N} = 2.15 \text{ GeV}^2$. Stable values of $g_{\pi NN} \approx 11.7$ are found over a region $M^2 \approx 0.8 - 1.8 \text{ GeV}^2$. Corrections due to the $A_N' M^2$ term are small, at most 5%. The second such term in Eq. (15) is expected to be smaller by a factor of 3–4, and so we are justified in neglecting it.

The threshold $S_{\pi N}$ has been adjusted so that stable results are obtained for Borel masses around 1 GeV^2 , since one may hope that in this region the Borel transformed sum rule is not too sensitive to the approximations that have been made on both the OPE and phenomenological sides of the sum rule. The existence of a window of stability provides a check on the consistency of this assumption. We also demand that the thresholds S_N and $S_{\pi N}$ should lie significantly above this window so that the continuum is not too heavily weighted in the Borel transform. We find that the window of stability moves rapidly upwards as $S_{\pi N}$ is increased for fixed S_N . For the typical parameter values above, only the region $2.05 \text{ GeV}^2 \leq S_{\pi N} \leq 2.22 \text{ GeV}^2$ satisfies these requirements. The value of $g_{\pi NN}$ varies by at most ± 0.2 over this region.

We have examined the dependence of our results to the threshold in the nucleon sum rule S_N . Varying this from 2.2 to 2.8 GeV^2 , readjusting $S_{\pi N}$ to maintain stability, changes $g_{\pi NN}$ by ± 0.2 . To estimate the sensitivity of our sum rules to the contributions of dimension-7 condensates and to uncertainties in the gluon condensate, we have varied the dimension-7 term in Eq. (17) between zero and twice its standard value. Our results for $g_{\pi NN}$ change by ± 0.5 over this range.

As a further check on our results, we have examined whether the individual sum rules (17) and (33) satisfy the criteria suggested by Leinweber [17]. We find that the highest dimension condensates contribute less than 10% of the OPE to both sum rules for $M^2 > 0.8 \text{ GeV}^2$. The procedure of differentiation with respect to M^2 does tend to increase the size of the continuum contribution. Nonetheless it does remain within Leinweber's limit, forming about 40% of the phenomenological side of the differentiated version of the sum rule (17) for M^2 up to 1.4 GeV^2 , the point at which the continuum reaches 50% of the odd-condensate sum rule (33). We therefore use the region $M^2 \approx 0.8 - 1.4 \text{ GeV}^2$ since this provides a window within which our results are both stable with respect to the Borel mass and not too sensitive to our approximations.

We have also examined the dependence of our results on the other input parameters. One of the most important of

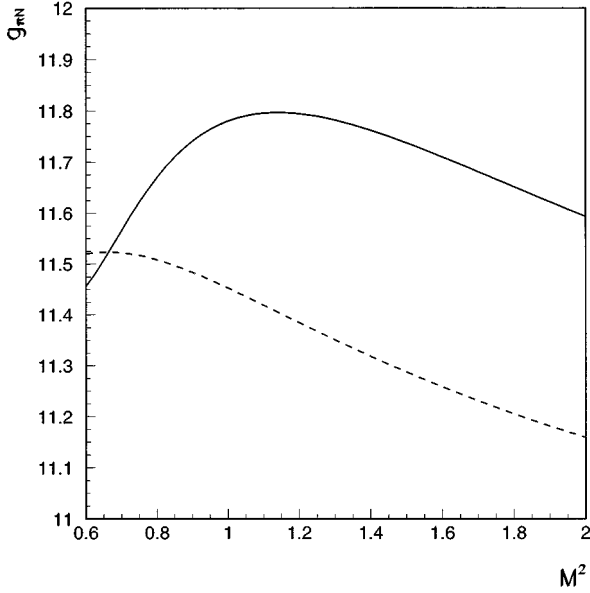


FIG. 1. Dependence on the square of the Borel mass of the πNN coupling constant determined from the ratio of sum rules for M_N and $g_{\pi NN}$. The values of the parameters used are given in the text. The solid line shows the value of $g_{\pi NN}$ corrected for the mixed continuum term $A'_N M^2$, the dashed line corresponds to the uncorrected value of $g_{\pi NN}$.

these is the matrix element δ^2 , defined by Eq. (12). As already mentioned, this parameter was extracted by Novikov *et al.* [18] from an analysis of two sum rules for the pion. Their results depend crucially on the four-quark condensate, $\alpha_s \langle 0 | (\bar{q}q)^2 | 0 \rangle$, for which they made the factorization approximation and took a value of about $2 \times 10^{-4} \text{ GeV}^6$. With this input, both of their sum rules yield consistent results for δ^2 in the region $0.20 \pm 0.02 \text{ GeV}^2$. However, sum-rule analyses of τ decay and e^+e^- annihilation into hadrons lead to significantly larger values of the four-quark condensate (see [19–21] and references therein), in the range $(4–6) \times 10^{-4} \text{ GeV}^6$. Using these in the sum rules of Ref. [18] leads to values for δ^2 ranging from 0.28 to 0.45, although the two sum rules do not then give consistent results. As a conservative estimate of the uncertainty in δ^2 we have considered the range 0.20 to 0.45 GeV^2 . The corresponding variation in $g_{\pi NN}$ is ± 2 when the other parameters are held at their values above and $S_{\pi N}$ is changed to keep the window of stability around 1 GeV^2 .

The second significant source of uncertainty is the quark condensate $\langle 0 | \bar{q}q | 0 \rangle$ which appears in the odd-dimension sum rule for the nucleon. “Standard” values for this lie in the range $-(0.21 \text{ GeV})^3$ and $-(0.26 \text{ GeV})^3$. The values of the baryon masses determined from sum rules [10] are strongly correlated with this condensate. There is also a weaker correlation with the chosen value of the threshold S_B . Since we are dividing our sum rules by baryon sum rules, our results are rather sensitive to the value of this condensate. One would like to use values of $\langle 0 | \bar{q}q | 0 \rangle$ and S_N that, for example, give the nucleon mass correctly, but the ratio of the odd and even dimension nucleon sum rules does not yield completely stable results for M_N . The best we can do is to rule out values of $-\langle 0 | \bar{q}q | 0 \rangle$ below $(0.23 \text{ GeV})^3$

since they cannot reproduce the nucleon mass within the region of Borel mass and threshold that we consider. Varying the quark condensate between $-(0.23 \text{ GeV})^3$ and $-(0.26 \text{ GeV})^3$, we find that $g_{\pi NN}$ changes by ± 2 .

Including all of these sources of uncertainty, our final result for the pion-nucleon coupling constant is thus $g_{\pi NN} = 12 \pm 5$, where the error is dominated by δ^2 and $\langle 0 | \bar{q}q | 0 \rangle$. This value is to be compared with those deduced from NN and πN scattering. For many years the accepted value was $g_{\pi NN} = 13.4$ [29] but this coupling has been the subject of some debate in recent years. More recent analyses lead to values in the range 12.7–13.6 [30]. Our result is obviously consistent with any of these.

The analysis of the pion-hyperon sum rules follows similar lines. In these cases additional input parameters are needed to describe the effects of SU(3) breaking in the hyperon mass sum rules (34) and (35). For the strange quark mass, we consider values in the range $m_s = 130–230 \text{ MeV}$ [31]. We write the strange quark condensate in the form $\langle 0 | \bar{s}s | 0 \rangle = \gamma \langle 0 | \bar{q}q | 0 \rangle$ and consider γ in the range 0.7–0.9. To allow for deviations from the factorization approximation, we write the four-quark condensates in the form $\langle 0 | (\bar{q}q)^2 | 0 \rangle = K (\langle 0 | \bar{q}q | 0 \rangle)^2$ and vary K between 1 and 2.

For the $g_{\pi\Sigma\Sigma}$ sum rule we find a similar window of Borel stability for values of $S_{\pi\Sigma}$ in the region 1.8 to 2 GeV^2 , provided we take S_Σ in the range 2.8 to 3.0 GeV^2 . With the typical values for the parameters above and $m_s = 180 \text{ MeV}$, $\gamma = 0.7$, and $K = 1$, we get $g_{\pi\Sigma\Sigma} \approx 6.8$. The relative uncertainties in this arising from δ^2 and the quark condensate are similar to those for $g_{\pi NN}$. There are also significant further uncertainties from m_s , γ , and K , which add another ± 1 . Our final result for this coupling is $g_{\pi\Sigma\Sigma} = 7 \pm 4$. A similar analysis for the $g_{\pi\Lambda\Sigma}$ sum rule leads to $g_{\pi\Lambda\Sigma} = 6 \pm 3$. We should also point out that there is an additional uncertainty in our determination of the latter coupling since we have ignored the Σ - Λ mass splitting in obtaining the sum rule (29).

Within our large error bars, these results for the pion-hyperon couplings are compatible with the empirical values quoted in Ref. [22], $g_{\pi\Sigma\Sigma} = 13 \pm 2$ and $g_{\pi\Lambda\Sigma} = 12 \pm 2$, as well as more recent determinations [23,24], which yield values in the range 10–12 for both couplings. However one should note that Refs. [23,24] assume SU(3) symmetry of the couplings whereas our results show significant SU(3) breaking and cannot be expressed in terms of F and D couplings.

The rather large uncertainties in these results could be reduced if the quark condensate could be determined more precisely. In addition, the sum rules of Novikov *et al.* [18] should be reexamined using larger values of the four-quark condensate to try to pin down the value of δ^2 more exactly. We also note that there are correlations among the parameters used, for example between δ^2 and the four-quark condensate, and so we may have overestimated the total uncertainties to some extent. It might therefore be worth applying the techniques of Leinweber [17] to these sum rules. However we note that recent applications of that approach to sum rules for the axial coupling also lead to results with $\sim 50\%$ uncertainties [32].

IV. SUMMARY

We have calculated the pion-nucleon and pion-hyperon coupling constants using QCD sum rules based on the pion-

to-vacuum matrix element of a two-point correlator of interpolating baryon fields. We have included baryon-to-continuum terms omitted from previous analyses. Our sum rules are based on the part of the correlator with Dirac structure $\not{k}\gamma_5$ and include all terms up to dimension 5 in the OPE. Stable results are obtained from the ratio of these sum rules to ones for the baryon masses and the unsuppressed baryon-to-continuum contributions are found to be small. Contributions from higher-dimension operators and omitted continuum terms are estimated to be small. Within admittedly rather large errors, our results for the coupling constants are consistent with the empirical values.

One should note that the uncertainties in our results are large. While we have indicated ways in which one might

hope to reduce some of these uncertainties, our results and those of [32] for g_A indicate that sum rules for baryon couplings are unlikely ever to reach similar accuracy to those for baryon masses. Nonetheless this approach may be able to yield useful information on other couplings whose values are at present not well determined.

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