# Pion electromagnetic form factor at finite temperature

Chungsik Song and Volker Koch
*Nuclear Science Division, MS 70A-3307, Lawrence Berkeley National Laboratory, Berkeley, California 94720*
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Temperature effects on the electromagnetic couplings of pions in hot hadronic matter are studied with an effective chiral Lagrangian. We show that the Ward-Takahashi identity is satisfied at nonzero temperature in the soft pion limit. The in-medium electromagnetic form factor of the pion is obtained in the timelike region and shown to be reduced in magnitude, especially near the vector-meson resonance region. Finally, we discuss the consequences of this medium effect on dilepton production from hot hadronic matter. [S0556-2813(96)03712-0]

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## I. INTRODUCTION

It is anticipated that there will be a phase transition in quantum chromodynamics (QCD) at very high temperatures. At high temperatures a hadronic system would be in a plasma phase consisting of weakly interacting quarks and gluons, the quark-gluon plasma (QGP), while at low temperatures hadronic matter is described well by mesons and baryons. Chiral symmetry, a symmetry of QCD in the limit of massless quarks, is spontaneously broken in the ground state of QCD as evidenced in the small mass of the pion. At high temperatures, above the phase transition, chiral symmetry is expected to be restored [1], as demonstrated by lattice gauge calculations. The formation and observation of this new phase of hadronic matter comprise the main goal of experiments with high energy nucleus-nucleus collisions [2].

Photon and lepton pair production have been suggested as promising probes to study the properties of hot hadronic matter [3]. The strong temperature dependence of the production rate of these signals makes it possible to discriminate the various states of hadronic matter with different temperatures. Furthermore, they can carry information on hot matter without further distortion since these electromagnetic probes interact very weakly with surrounding particles.

Dilepton production from the low temperature hadronic phase has been considered a possible probe for the chiral phase transition. In hot hadronic matter, even below the phase transition temperature, chiral symmetry is expected to be partially restored; i.e., the magnitude of the order parameter $\langle\overline{q}q\rangle$ is reduced from its vacuum value. As a result the properties of light mesons, in particular the vector mesons, might be modified, and these changes will affect the dilepton spectrum. It has been suggested that the masses of vector mesons would change as the hadronic matter undergoes a phase transition to the chirally symmetric phase [4,5]. If this is the case, one should be able to observe this effect directly through the shift of vector meson peaks in the dilepton spectra from heavy-ion collisions.

A recent study based on partial conservation of axial-vector current (PCAC) and current algebra, however, has shown that up to $T^2$, where $T$ is the temperature, there is no change in vector meson masses but only a mixing between the vector and axial-vector correlators [6]. This result should be satisfied by any models that include the symmetry properties of low energy hadronic physics. Indeed, results from effective chiral Lagrangian approaches [7–9] are consistent with this temperature dependence, and vector meson masses obtained from these models do not change appreciably unless the temperature of the hadronic matter is very close to the critical temperature for the phase transition.

Instead, the mixing of the vector and axial-vector correlation functions at finite temperature implies that the leading effect of the temperature in vector meson properties appears in the coupling of vector mesons to photon, which goes like $T^2$. Recently it has been suggested that this effect will affect the dilepton yield from hot hadronic matter produced in high energy nucleus-nucleus collisions [10]. In hot hadronic matter, the production of dileptons with invariant masses near the $\rho$ resonance is dominated by pion-pion annihilation. According to the vector meson dominance (VMD) assumption [11], two pions in this process form a rho meson that subsequently converts into a virtual photon. The dilepton yield depends, thus, on the pion electromagnetic form factor

$$F_\pi(q^2)=\frac{g_{\rho\pi\pi}g_{\rho\gamma}}{m_\rho^2-q^2-im_\rho\Gamma_\rho}, \tag{1}$$

where $g_{\rho\gamma}$ is the photon–$\rho$-meson coupling constant, $g_{\rho\pi\pi}$ is the pion-$\rho$-meson coupling constant, and $\Gamma_\rho$ is the neutral $\rho$ meson decay width. This form factor has been extensively used in calculating the dilepton emission rate from hadronic matter at finite temperature. In these studies, the form factor has been taken to be independent of temperature. However, the change of the photon-vector meson coupling in the medium indicates that the $F_\pi(q^2)$ is to be modified at finite temperature, and this will affect dilepton production in hot hadronic matter.

In the present paper, we shall study the pion electromagnetic form factor at finite temperature using an effective chiral Lagrangian that includes explicitly the vector mesons and gives also the correct mixing effect at finite temperature. In Sec. II we include details for the Lagrangian we use in the paper. In Sec. III we summarize the modification of pion-photon couplings in hot hadronic matter. We find that the pion-photon coupling is affected in the medium not only because the photon–$\rho$-meson coupling is modified at finite temperature but also because the $\pi$ -$\rho$ coupling as well as the properties of the $\rho$ meson itself is changed in hot hadronic matter. In Sec. IV we study the changes of electromagnetic couplings of pions at low temperatures in the soft pion limit, where pion and photon momenta are assumed to be small compared to vector meson mass. We also calculate the in-medium pion electromagnetic form factor near the vector resonance region. Here we consider the corrections to pion-photon coupling in hot hadronic matter with one-loop diagrams. The one-loop corrections are regarded as the dominant one since the density of most hadrons is small at the temperature we are interested in. In Sec. V we study the effect on the dilepton production rate from $\pi$-$\pi$ annihilation in hot matter, using the temperature-dependent form factor.

## II. HADRONS AT LOW ENERGY: EFFECTIVE FIELD THEORY

There is a general agreement at the present time that quantum chromodynamics (QCD) is the correct theory of strong interactions. Although QCD is simple and elegant in its formulation, the derivation of its physical predictions for low energy phenomena presents, however, arduous difficulties because of long distance QCD effects that are essentially nonperturbative. The theoretical progress has gone through various directions, including lattice simulations, the use of sum rules, and employing effective field theory.

In the effective field theory we regard mesons and baryons as elementary particles and construct a Lagrangian with the symmetry of the fundamental theory. In QCD chiral $SU(N_f)\times SU(N_f)$ symmetry, current algebra and PCAC play an important role to construct an effective Lagrangian. This effective theory can describe all of the couplings in terms of a relatively small number of parameters and is very successful for low energy hadron physics. For example, chiral perturbation theory provides a compact and elegant method for dealing with the interactions of pions at low energies. This approach is reliable, however, only when the internal structure of hadrons, i.e., the quark and gluon content of hadrons, can be neglected.

Another important aspect of hadron physics for the present work is the interaction of hadrons to the photon. This has been remarkably well described by using the vector meson dominance assumption. This assumes that the hadronic components of the vacuum polarization of the photon consist exclusively of the known vector mesons. At energies below 1 GeV the neutral vector mesons $\rho^0$, $\omega$, and $\phi$ play an important role in the electromagnetic interactions of hadrons. The pion electromagnetic form factor is a particularly striking example of the VMD model. The concept of VMD, of course, is purely phenomenological and has not yet been proved from the fundamental theory.

Hidden local symmetry (HLS) is a natural framework for describing the vector mesons in a manner consistent with the chiral symmetry of QCD and vector meson dominance assumption [12]. The HLS Lagrangian yields at the tree level a successful phenomenology for the pions and $\rho$ mesons. Let us start with the $G_{\rm global}\times H_{\rm local}$ ''linear model'' with $G=SU(2)_L\times SU(2)_R$ and $H=SU(2)_V$. It is constructed with two SU(2)-matrix-valued variables $\xi_L(x)$ and $\xi_R(x)$, which transform as $\xi_{L,R}(x)\to\xi'_{L,R}(x)=h(x)\xi_{L,R}g^\dagger_{L,R}$ under $h(x)\in[SU(2)_V]_{\rm local}$ and $g_{L,R}\in[SU(2)_{L,R}]_{\rm global}$.

Introducing the vector meson $V_\mu$ as the gauge field of the

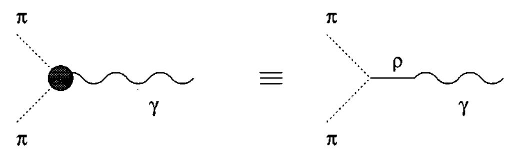


FIG. 1. Vector meson dominance in the pion electromagnetic form factor.

local symmetry and the photon $\mathcal{B}_\mu$ as an external gauge field of the global symmetry, we have the chirally invariant Lagrangian

$$\mathcal{L}=-\frac{f_\pi^2}{4}{\rm tr}[\mathcal{D}_\mu\xi_L\cdot\xi_L^\dagger-\mathcal{D}_\mu\xi_R\cdot\xi_R^\dagger]^2 -\frac{af_\pi^2}{4}{\rm tr}[\mathcal{D}_\mu\xi_L\cdot\xi_L^\dagger+\mathcal{D}_\mu\xi_R\cdot\xi_R^\dagger]^2+\mathcal{L}_{\rm kin}(V_\mu,\mathcal{B}_\mu), \quad (2)$$

where $a$ is an arbitrary constant and $f_\pi=93$MeV is the pion decay constant. The covariant derivative is given by

$$\mathcal{D}_\mu\xi_{L,R}=(\partial_\mu-igV_\mu)\xi_{L,R}+ie\xi_{L,R}\mathcal{B}_\mu\tau_3/2, \quad (3)$$

where $\tau_3$ is the isospin Pauli matrix. The kinetic terms $\mathcal{L}_{\rm kin}(V_\mu,\mathcal{B}_\mu)$ are conventional non-Abelian and Abelian gauge field tensors for vector meson and photon fields, respectively. In order to obtain masses for the pseudoscalar mesons, we introduce an explicit symmetry-breaking term $\mathcal{L}_{\rm SB}(\xi_{L,R})$, which is given by

$$\mathcal{L}_{\rm SB}(\xi_{L,R})=\tfrac{1}{4}f_\pi^2m_\pi^2{\rm tr}(\xi_L\xi_R^\dagger+\xi_R\xi_L^\dagger). \quad (4)$$

In the ''unitary'' gauge

$$\xi_L^\dagger(x)=\xi_R(x)=e^{i\pi(x)/f_\pi}\equiv\xi(x), \quad (5)$$

the effective Lagrangian takes the form

$$\mathcal{L}=-\tfrac{1}{4}(F_{\mu\nu}^V)^2-\tfrac{1}{4}(\partial_\mu\mathcal{B}_\nu-\partial_\nu\mathcal{B}_\mu)^2+\tfrac{1}{4}{\rm tr}(\partial_\mu U\partial^\mu U^\dagger) +\tfrac{1}{2}m_\rho^2V_\mu^2-eg_\rho V_3^\mu\mathcal{B}_\mu+g_{\rho\pi\pi}V^\mu\cdot(\pi\times\partial_\mu\pi) +g_{\gamma\pi\pi}\mathcal{B}^\mu(\pi\times\partial_\mu\pi)_3+\cdots, \quad (6)$$

where $U=\xi^2(x)$ and the parameters are given as

$$m_\rho^2=ag^2f_\pi^2,$$

$$g_{\rho\pi\pi}=\tfrac{1}{2}ag,$$

$$g_{\rho\gamma}=agf_\pi^2,$$

$$g_{\gamma\pi\pi}=(1-\tfrac{1}{2}a)e. \quad (7)$$

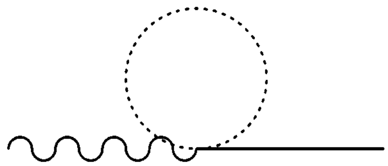

FIG. 2. Correction to the photon–vector-meson coupling at finite temperature. Here and in the following the dotted, solid, and wave lines indicate the pions, vector mesons, and electromagnetic fields, respectively.

With $a=2$, we have the universality of the $\rho$ couplings ($g_{\rho\pi\pi}=g$), the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation $m_\rho^2=2g_{\rho\pi\pi}^2 f_\pi^2$, and the $\rho$ meson dominance of the pion-photon coupling ($g_{\gamma\pi\pi}=0$). In this effective Lagrangian, the pion electromagnetic form factor in free space can be obtained at the tree level from the diagram shown in Fig. 1. One sees that the vector meson dominance appears naturally. A photon converts into a rho meson which interacts with the pion. The resulting pion electromagnetic form factor is exactly the same form as Eq. (1) assumed in VMD.

## III. ELECTROMAGNETIC COUPLING OF PIONS IN THE MEDIUM

At nonzero temperature the couplings of pions to the electromagnetic field is modified by the interaction with particles in the heat bath. These effects can be included by thermal loop corrections to the vertices. In a low temperature pion gas it is possible to expand the medium effect as a series of the power in $T^2/f_\pi^2$. However, we cannot apply the same approximation at the temperatures 100 MeV$<T\leqslant T_c$. Instead, we use the fact that the density of particles is small even at temperatures near $T_c$. In this case we can expand the thermal corrections by the number of loops and include one-loop diagrams as leading terms. We neglect the contributions of vector meson loops in the present calculation, since they are suppressed by Boltzmann factors $\sim e^{-m_V/T}$ with large masses $m_V \gg T$.

### A. Vector and axial-vector mixing

It has been shown that chiral symmetry predicts a mixing between vector and axial-vector current-current correlators at low temperature [6]. This mixing implies that the difference of vector and axial-vector current correlators vanishes with increasing temperature, which is a consequence of the chiral symmetry restoration in hadronic matter at finite temperature [13].

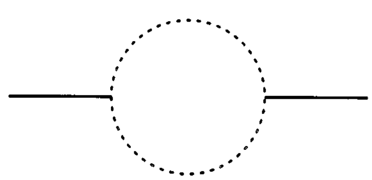

FIG. 3. Vector meson propagator at finite temperature.

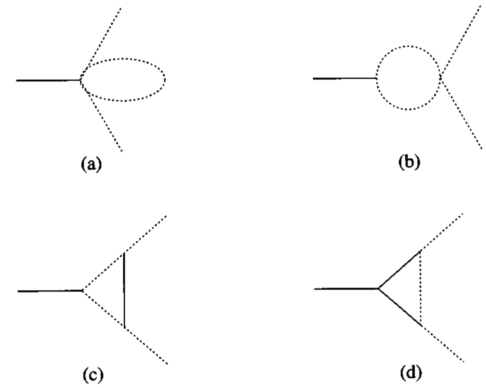

FIG. 4. Correction to the $\pi$-$\pi$-$\rho$ vertex at finite temperature.

The mixing effect in vector and axial-vector correlators at finite temperature has also been studied in the effective Lagrangian approach [7,8]. The interaction with thermal pions as shown in Fig. 2 is responsible for the mixing effect at finite temperature. The effect of the mixing on the pion electromagnetic form factor results in a change of the photon–vector-meson coupling $g_{\gamma\rho}(T)$ at finite temperature:

$$g_{\gamma\rho}(T)=(1+\epsilon)g_{\gamma\rho}(0), \tag{8}$$

where

$$\begin{aligned}\epsilon &= \frac{1}{f_\pi^2} T \sum_{n_l} \int \frac{d^3 l}{(2\pi)^3} \frac{1}{l^2-m_\pi^2} \\ &= -\frac{T^2}{12 f_\pi^2} \quad (\text{in the chiral limit}).\end{aligned} \tag{9}$$

This implies that the vector-photon coupling will be reduced in hot matter due to the mixing of vector and axial-vector currents. Here the sum is over the Matsubara frequency of thermal pions which is given by $\omega_l=2\pi T n_l$ with integer $n_l$.

### B. Vector meson properties

With the vector meson dominance assumption the pion couples to the electromagnetic field only through a rho meson intermediate state. Thus the changes in the rho meson properties in the medium also affect the pion-photon coupling. To include the in-medium rho properties we calculate

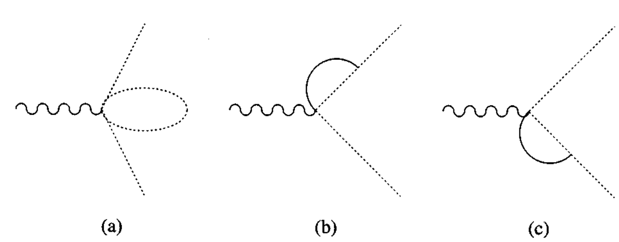

FIG. 5. The $\pi$-$\pi$-$\gamma$ vertex at finite temperature.

the vector meson self-energy with an one-loop diagram in Fig. 3. The explicit form is given by

$$\Pi_\rho^{\mu\nu} = -(\tfrac{1}{2}ag)^2 T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{(2l^\mu-k^\mu)(2l^\nu-k^\nu)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2]}, \tag{10}$$

to leading order, where $k$ is the momentum of vector meson. The in-medium propagator is then given by

$$D_{\mu\nu} = D^0_{\mu\nu} + D^0_{\mu\lambda}(-i\Pi^{\lambda\sigma})D_{\sigma\nu}$$
$$= D^0_{\mu\nu} + D^0_{\mu\lambda}(-i\Pi^{\lambda\sigma})D^0_{\sigma\nu}\cdots, \tag{11}$$

where $D^0_{\mu\nu}$ is the propagator in free space.

The modification of the pion-photon vertex due to the change of the vector meson propagator in the medium can be written as

$$\Gamma_\mu^{\rm rho} = -ig_{\rho\gamma}D^0_{\mu\lambda}(-i\Pi^{\lambda\sigma})D^0_{\sigma\nu}g_{\rho\pi\pi}(p_\nu-q_\nu)+\cdots$$
$$= -\frac{1}{4}a^2(ag^2f_\pi^2)^2\frac{g_{\mu\nu}-k_\mu k_\nu/m_\rho^2}{k^2-m_\rho^2}$$
$$\times\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}(2l^\nu-k^\nu)$$
$$\times\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2]}\frac{1}{k^2-m_\rho^2}+\cdots, \tag{12}$$

where $p$ and $q$ are momenta of external pions and $k=p+q$.

### C. Vertex corrections

We consider the thermal effect on the $\pi$-$\pi$-$\rho$ coupling in hot hadronic matter which is given by Fig. 4. Each contribution to the pion-photon coupling is given by

$$\Gamma_\mu^{4(a)} = -(p_\mu-q_\mu)\frac{ag^2f_\pi^2}{k^2-m_\rho^2}\frac{5a}{24}\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2},$$

$$\Gamma_\mu^{4(b)} = -\frac{1}{2}a\left(\frac{3a}{4}-1\right)ag^2f_\pi^2\frac{g_{\mu\nu}-k_\mu k_\nu/m_\rho^2}{k^2-m_\rho^2}\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}(2l^\nu-k^\nu)\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2]},$$

$$\Gamma_\mu^{4(c)} = -\tfrac{1}{2}(\tfrac{1}{2}a)^2(ag^2f_\pi^2)^2\frac{g_{\mu\nu}-k_\mu k_\nu/m_\rho^2}{k^2-m_\rho^2}\frac{1}{2f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\Bigg[(2l^\nu-k^\nu)\frac{(l+p)_\lambda[g^{\lambda\sigma}-(l^\lambda-p^\lambda)(l^\sigma-p^\sigma)/m_\rho^2](l_\sigma-q_\sigma-k_\sigma)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-p)^2-m_\rho^2]}$$
$$-(2l^\nu-k^\nu)\frac{(l_\lambda+q_\lambda)[g^{\lambda\sigma}-(l^\lambda-q^\lambda)(l^\sigma-q^\sigma)/m_\rho^2](l_\sigma-p_\sigma-k_\sigma)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-q)^2-m_\rho^2]}\Bigg],$$

$$\Gamma_\mu^{4(d)} = \frac{1}{2}a(ag^2f_\pi^2)^2\frac{g_{\mu\nu}-k_\mu k_\nu/m_\rho^2}{k^2-m_\rho^2}\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\Big\{[(2l^\nu+p^\nu-q^\nu)g^{\alpha\beta}-(l^\alpha+k^\alpha+q^\alpha)g^{\beta\nu}-(l^\beta-p^\beta-k^\beta)g^{\nu\alpha}]$$
$$\times[g_{\alpha\alpha'}-(l_\alpha-p_\alpha)(l_{\alpha'}-p_{\alpha'})/m_\rho^2][g_{\beta\beta'}-(l_\beta+q_\beta)(l_{\beta'}+q_{\beta'})/m_\rho^2]\frac{(l^{\alpha'}+p^{\alpha'})(l^{\beta'}-q^{\beta'})}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2][(l+q)^2-m_\rho^2]}\Big\}. \tag{13}$$

### D. Direct coupling

In the medium it is possible for pions to couple to the photon fields directly as shown in Fig. 5. The interaction with thermal pions make it possible for pions to couple to the photon, which is forbidden in free space. This implies that strict vector meson dominance is not satisfied at nonzero temperature. Each contribution is given by

$$\Gamma_\mu^{5(a)} = (p_\mu-q_\mu)\frac{5}{3}\left(1-\frac{7a}{8}\right)\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2},$$

$$\Gamma_\mu^{5(b)}+\Gamma_\mu^{5(c)} = -\frac{3a}{4}ag^2f_\pi^2\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\left[\frac{(l^\mu+p^\mu)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}-\frac{(l^\mu+q^\mu)}{(l^2-m_\pi^2)[(l-q)^2-m_\rho^2]}\right]. \tag{14}$$

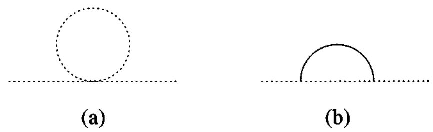


FIG. 6. Pion self-energy at finite temperature.

### E. Pion wave function renormalization

Finally, there is a contribution from the modification of pion properties in hot matter. As pions propagate through the medium they couple to particles in the thermal bath and their properties will be modified. The medium effect on the pion propagator can be included in the self-energy which is defined as the difference between the inverse of the in-medium propagator ($D^{-1}$) and that of the vacuum propagator ($D_0^{-1}$):

$$-i\Pi = D^{-1} - D_0^{-1}. \tag{15}$$

Such a modification renormalizes the pion wave function and affects the strength of $\pi$-$\pi$ annihilation into a vector meson. At zero temperature the wave function renormalization constant can be uniquely defined. At finite temperature, however, it cannot, because Lorentz invariance is broken due to the presence of the heat bath. Thus the self-energy may have separate dependences on the momentum and energy. Here, the wave function renormalization constant will be defined as

$$Z_\pi^{-1} = 1 - \left.\frac{\partial \Pi_\pi}{\partial p_0^2}\right|_{\vec{p}=0,\quad p_0=m_\pi}. \tag{16}$$

We calculate the pion self-energy from the one-loop diagrams shown in Fig. 6. Each contribution is given by

$$\Pi_\pi^{6(a)}(p) = \frac{1}{6f_\pi^2} T \sum_{n_l} \int \frac{d^3 l}{(2\pi)^3}\left[5m_\pi^2 - 4\left(1-\frac{3}{4}a\right)(p^2+l^2)\right] \times \frac{1}{l^2-m_\pi^2},$$

$$\Pi_\pi^{6(b)}(p) = \frac{1}{2}a(ag^2f_\pi^2)\frac{1}{f_\pi^2}T \times \sum_{n_l}\int \frac{d^3l}{(2\pi)^3}\frac{(l+p)^2+(l^2-p^2)^2/m_\rho^2}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}. \tag{17}$$

## IV. PION ELECTROMAGNETIC FORM FACTOR AT $T\neq 0$

### A. Effective charge of pions

First let us calculate the effective charge of a pion in the medium by considering scattering of a photon off the pion. We consider the soft pion limit in which four momenta of pions are assumed to be small compared to the vector meson mass. Actually we approximate that

$$(l-p)^2, (l-q)^2 \ll m_\rho^2, \tag{18}$$

where $l$ is the momentum of pion in a thermal loop and $p$ and $q$ are external pion momenta. Since the momenta in a thermal loop are of the order of the temperature, this approximation should be reasonable for low temperatures and small external pion momenta. We include only thermal pion effects and expand the medium effect as a power series of $T^2/f_\pi^2$.

With this soft pion approximation, we obtain very simple expressions for vertex corrections. For example, we have

$$\Gamma_\mu^{4(c)} \approx -\tfrac{1}{2}(\tfrac{1}{2}a)^2(ag^2f_\pi^2)^2\frac{g_{\mu\nu}-k'_\mu k'_\nu/m_\rho^2}{k'^2-m_\rho^2}\frac{1}{(-m_\rho^2)}\frac{1}{2f_\pi^2}T \times \sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\left[(2l^\nu+k'^\nu) \times \frac{2l\cdot(p'+p)+k'\cdot(p'+p)}{(l^2-m_\pi^2)[(l+k')^2-m_\pi^2]}\right], \tag{19}$$

where we use $p'=-q$ and $k'=(p'-p)=-(p+q)$. The vertex correction is then given by

$$\Gamma_\mu^{\text{vertex}} = -(p'_\mu+p_\mu)\frac{5a}{24f_\pi^2}\frac{ag^2f_\pi^2}{k'^2-m_\rho^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2} + \frac{1}{2}a(ag^2f_\pi^2)\frac{g_{\mu\nu}-k'_\mu k'_\nu/m_\rho^2}{k'^2-m_\rho^2}\frac{1}{f_\pi^2}T \times \sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{(2l^\nu+k'^\nu)}{(l^2-m_\pi^2)[(l+k')^2-m_\pi^2]} \times[(1-\tfrac{1}{2}a)l\cdot(p'+p)+\tfrac{1}{8}ak'\cdot(p'+p)]. \tag{20}$$

where the diagram of Fig. 4(d) is not included since the contribution will be suppressed by the factor of $p^2/m_\rho^2$. With $a=2$ the contributions $\Gamma_\mu^{4(b)}$ and $\Gamma_\mu^{4(c)}$ cancel each other. With the same approximation we also have a simple form for the direct coupling in the medium:

$$\Gamma_\mu^{\text{direct}} = (p'_\mu+p_\mu)\left(\frac{5}{3}-\frac{17a}{24}\right)\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2}. \tag{21}$$

The total modification of the coupling of pions to the photon field in the medium now can be written as

$$\Gamma_\mu^{\gamma\pi\pi}(T) = \Gamma_\mu^{\text{mixing}}+\Gamma_\mu^{\text{vertex}}+\Gamma_\mu^{\text{direct}}+\Gamma_\mu^{\text{rho}} = (p'_\mu+p_\mu)\left(\frac{1}{4f_\pi^2}-\frac{ag^2f_\pi^2}{k'^2-m_\rho^2}\frac{17}{12f_\pi^2}\right)T \times\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2}+\left(\frac{ag^2f_\pi^2}{k'^2-m_\rho^2}\right)\frac{1}{4f_\pi^2}T \times\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{(2l^\mu+k'^\mu)\rangle k'\cdot(p'+p)}{(l^2-m_\pi^2)[(l+k')^2-m_\pi^2]} -\left(\frac{ag^2f_\pi^2}{k'^2-m_\rho^2}\right)^2\frac{1}{f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}$$

$$\times\left[\frac{(2l_\mu+k'_\mu)l\cdot(p'+p)}{(l^2-m_\pi^2)[(l+k')^2-m_\pi^2]}-\frac{k'_\mu}{m_\rho^2}\frac{k'\cdot(p'+p)}{(l^2-m_\pi^2)}\right], \tag{22}$$

with $a=2$.

In general the change in the vertex function is related to the modification of pion propagator by the Ward-Takahashi (WT) identity [14]

$$(p'_\mu-p_\mu)\Gamma^\mu_{\gamma\pi\pi}(p',p)=\Pi_\pi(p)-\Pi_\pi(p'), \tag{23}$$

where $\Pi_\pi(p)$ is the pion self-energy. This is a consequence of the U(1) gauge symmetry of the theory. At finite temperature we have, from Eq. (22),

$$(p'_\mu-p_\mu)\Gamma^\mu_{\gamma\pi\pi}(p',p;T)=\frac{2}{3f_\pi^2}(p'^2-p^2)T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2} \tag{24}$$

in the limit $k'^2\to 0$. On the mass shell this just shows that the electromagnetic current is conserved at finite temperature. The pion self-energy can also be calculated in the soft pion limit from Eq. (17) and it is given by

$$\Pi_\pi=\frac{1}{6f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2}(5m_\pi^2-4l^2-4p^2). \tag{25}$$

From these two results, i.e., Eqs. (24) and (25), one can easily see that Ward-Takahashi identity is satisfied at finite temperature.

At zero temperature the WT identity implies that the vertex correction in charge renormalization is exactly canceled by the wave function renormalization constant. This is not obvious at finite temperature because of the broken Lorentz invariance in the presence of the heat bath. We must, therefore, be careful when taking limits in order to proceed with the temperature-dependent renormalization of the electric charge. Since at finite temperature a general amplitude $A(k'_0,\vec{k}')$ may have different functional dependences on $k'_0$ and $\vec{k}'$, we should distinguish the limit $\vec{k}'=0,\ k'_0\to 0$ from that of $k'_0=0,\ \vec{k}'\to 0$.

First we consider the limit $\vec{k}'=0,\ k'_0\to 0$. From the WT identity we can show the conservation of the effective charge defined by

$$\Lambda_0(k'_0,k')|_{\vec{k}'=0,\,k'_0\to 0}\equiv-(p'_0+p_0)e^{\mathrm{eff}}, \tag{26}$$

where

$$\Lambda_\mu=-e[(p'_\mu+p_\mu)+\Gamma_\mu]Z_\pi. \tag{27}$$

From Eq. (23) we can get

$$\Gamma_0(k'_0\to 0,\ \vec{k}'=0)=-\partial\Pi_\pi(p)/\partial p_0=2p_0(Z_\pi^{-1}-1), \tag{28}$$

where we use the definition for the wave function renormalization constant Eq. (16) to get the last line. By inserting Eq. (28) into Eq. (27) we can see that the effective charge at finite temperature defined in Eq. (26) is conserved as consequence of the WT identity.

We can show the charge conservation explicitly from the expressions for the $\pi$-$\pi$-$\gamma$ vertex corrections in the medium. The zeroth component of the vertex function to leading order can be written as

$$\Gamma_0^{\mathrm{matt}}(p',p)=(p'_0+p_0)\left[-\frac{5T^2}{3f_\pi^2}g_0(m_\pi^2/T^2)+\frac{T^2}{f_\pi^2}g_0(m_\pi^2/T^2)\right], \tag{29}$$

where

$$g_0(x)=\frac{1}{2\pi^2}\int_0^\infty\frac{y^2dy}{\sqrt{y^2+x^2}(e^{\sqrt{y^2+x^2}}-1)}. \tag{30}$$

The pion wave function renormalization is obtained from Eq. (25) as

$$Z_\pi^{-1}=1+\frac{2}{3f_\pi^2}T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{l^2-m_\pi^2}=1-\frac{2T^2}{3f_\pi^2}g_0(m_\pi^2/T^2). \tag{31}$$

There is an exact cancellation at the leading order of $T^2/f_\pi^2$ between the contributions from the vertex correction and wave function renormalization in the definition of the effective charge. No temperature-dependent correction contributes to the charge renormalization from the vertex function. The photon polarization function is entirely responsible for the charge renormalization.

If we take the other limit, i.e., $\vec{k}'=0$ and $k'_0\to 0$, the WT identity implies that $\Gamma_i$ instead of $\Gamma_0$ is related to the space derivative of the self-energy:

$$\Gamma_i(k'_0=0,k'_j=0,j\neq i;k'_i\to 0)=-\frac{\partial\Pi_\pi(p)}{\partial p_i}. \tag{32}$$

At zero temperature this leads to the same result as that obtained in Eq. (28). It is not true in general at finite temperature. However, in the approximation considered here, the pion self-energy depends only on $p^2$, just as in free space, and we can show the conservation for the effective charge defined by

$$\Lambda_i(k'_0,k')|_{k'_0=0,k'_j=0,j\neq i;k'_i\to 0}\equiv-(p'_i+p_i)e^{\mathrm{eff}}. \tag{33}$$

### B. Pion electromagnetic form factor in the medium

Now we consider the pion electromagnetic form factor for the process in which two pions annihilate into a virtual photon which subsequentially decays into lepton pairs. For simplicity we do the calculation in the rest frame of the virtual photon, i.e., in the frame where lepton pairs move back to back. First we use the results obtained in the soft pion limit for couplings of pions to photon and vector mesons. Even though this result is reliable only near threshold for two-pion annihilation and at low temperatures, this gives a good intuition for the form factor at finite temperature. Moreover, we can do the loop integration exactly in this limit.

We define the pion electromagnetic form factor in the medium as

$$\Gamma_\mu(T)=(p_\mu-q_\mu)F_\pi(T). \tag{34}$$

The in-medium form factor can be written as

$$F_\pi(T)=Z_\pi(T)\left[\frac{g_{\rho\pi\pi}(T)g_{\rho\gamma}(T)}{m_\rho^2-k^2-i\Gamma_\rho m_\rho-\Pi_\rho(T)}+F_\pi^{\mathrm{direct}}(T)\right], \tag{35}$$

where $g_{\rho\gamma}(T)$ and $Z_\pi(T)$ are given by Eq. (8) and (31), respectively. The $g_{\rho\pi\pi}(T)$ is obtained from Eq. (20) as

$$g_{\rho\pi\pi}(T)=g_{\rho\pi\pi}(0)\left(1-\frac{5T^2}{12f_\pi^2}g_0(m_\pi^2/T^2)\right). \tag{36}$$

The last term appears because of the direct coupling of pions to the photon in the medium and is given by

$$F_\pi^{\mathrm{direct}}=-\frac{T^2}{4f_\pi^2}g_0(m_\pi^2/T^2). \tag{37}$$

The $\Pi_\rho(T)$ is the modification due to the in-medium rho meson propagator. In the rest frame of the vector meson the propagator is given by

$$D_{\mu\nu}=\frac{g_{\mu\nu}-k_\mu k_\nu}{k^2}\left(\frac{1}{k^2-m_\rho^2+i\Gamma_\rho m_\rho+\Pi_\rho(T)}\right)+\cdots, \tag{38}$$

where $\Pi_\rho(T)$ can be obtained from rho self-energy in Eq. (10) [29]. The explicit form of $\Pi_\rho(T)$ is given by

$$\Pi_\rho(T)=\frac{g^2}{3\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\frac{|\vec{l}|^2}{\omega^2-k_0^2/4}, \tag{39}$$

where $\omega=\sqrt{|\vec{l}|^2+m_\pi^2}$. Even though the medium effect on the vector meson mass is small, there is an appreciable effect when we include the imaginary part of the rho self-energy. In Fig. 7 we show the effect of vector meson self-energy on the pion electromagnetic form factor at finite temperature. The dashed line is for the result with the real part of the self-energy only and the solid line is that obtained with both real and imaginary parts. We can see that a large effect comes from the modification of the imaginary part of the rho self-energy [15]. We get a comparable result when we restrict ourselves to the leading term in the expansion of the self-energy correction (dot-dashed line). We expect an additional broadening of the vector mesons due to collisions [16], which are, however, not considered here since they are corrections on the two-loop level.

The result for pion electromagnetic form factor obtained in the soft pion limit is shown in Fig. 8 for different temperatures. We can see that the form factor is suppressed, particularly near the vector resonance region, as the temperature increases. The reduction of the form factor is mainly due

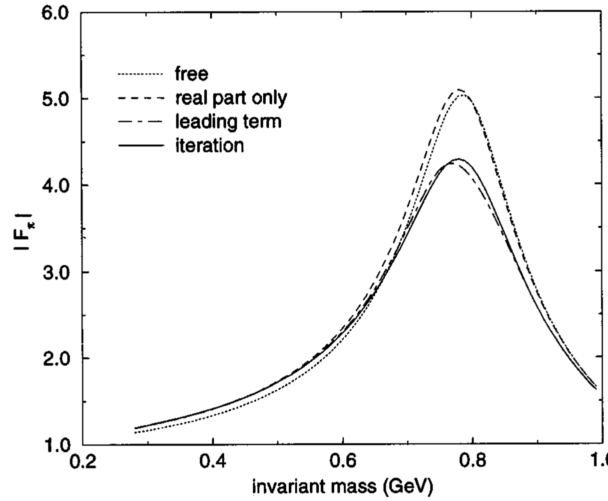


FIG. 7. Pion electromagnetic form factor with the modification in vector meson properties.

to the suppression of photon–$\rho$-meson coupling which comes from the aforementioned mixing effect, and due to the broadening of vector mesons in the medium.

Now we consider the pion form factor near the vector resonance region where the external pion momentum is not negligible compared to the vector meson mass. For this case we cannot simply replace $1/[(l-p)^2-m_\rho^2]$ in the vector propagator by $1/m_\rho^2$. When we include the full propagator for the vector meson we have

$$\Gamma_\mu^{4(b)}\sim T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{l_\mu l\cdot(p-q)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2]}$$
$$\equiv(p_\nu-q_\nu)H^{\mu\nu}(k;T), \tag{40}$$

which has been canceled by $\Gamma_\mu^{4(c)}$ in the soft pion limit. Since $H_{\mu\nu}$ is a second rank tensor, it can be written as

$$H_{\mu\nu}=\alpha A_{\mu\nu}+\beta B_{\mu\nu}+\gamma C_{\mu\nu}+\delta D_{\mu\nu}. \tag{41}$$

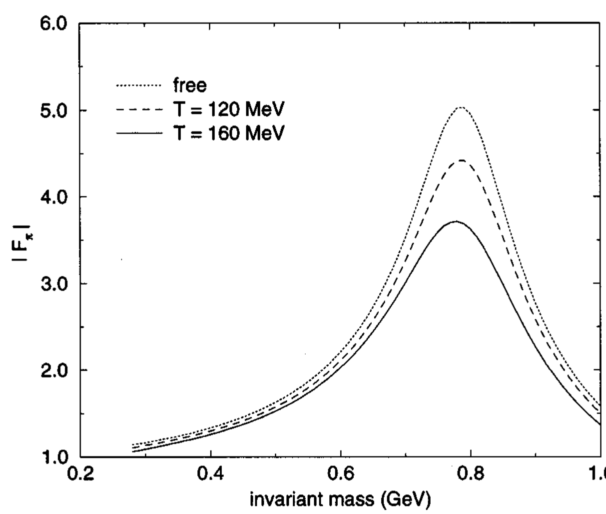


FIG. 8. Pion electromagnetic form factor in soft pion limit.

Here $A$, $B$, $C$, and $D$ are four independent second rank tensors and are given as following [17,18]

$$A_{\mu\nu}=g_{\mu\nu}-\frac{1}{\vec{k}^2}[k_0(n_\mu k_\nu+n_\nu k_\mu)-k_\mu k_\nu-k^2 n_\mu n_\nu],$$

$$B_{\mu\nu}=-\frac{k^2}{\vec{k}^2}\left(n_\mu-\frac{k_0 k_\mu}{k^2}\right)\left(n_\nu-\frac{k_0 k_\nu}{k^2}\right),$$

$$C_{\mu\nu}=-\frac{1}{\sqrt{2}|\vec{k}|}\left[\left(n_\mu-\frac{k_0 k_\mu}{k^2}\right)k_\nu+\left(n_\nu-\frac{k_0 k_\nu}{k^2}\right)k_\mu\right],$$

$$D_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}. \tag{42}$$

where $n_\mu$ specifies the rest frame of hot matter. $\alpha$, $\beta$, $\gamma$, and $\delta$ are given by

$$\delta=\frac{1}{k^2}k^\mu k^\nu H_{\mu\nu},$$

$$\gamma=\frac{\sqrt{2}}{|\vec{k}|}(k^\mu H_{\mu 0}-k_0\delta),$$

$$\beta=-\frac{1}{\vec{k}^2}(k^2 H_{00}-\sqrt{2}|\vec{k}|k_0\gamma-k_0^2\delta),$$

$$\alpha=\tfrac{1}{2}(H^\mu_\mu-\beta-\delta). \tag{43}$$

In the back-to-back frame where $\vec{k}=0$ we have

$$\gamma=0,$$

$$\alpha=\beta. \tag{44}$$

The expression for $\Gamma_\mu$ simplifies even more when we use on the shell condition for pions, i.e., $p^2=q^2=m_\pi^2$,

$$\Gamma_\mu^{4(b)}=(p_\mu-q_\mu)F_\pi(T=0)\frac{1}{f_\pi^2}\alpha(T), \tag{45}$$

and $\alpha(T)$ is obtained from $H_{\mu\nu}$ as

$$\alpha(T)=-\frac{1}{12\pi^2}\int\frac{|\vec{l}|^2 d|\vec{l}|}{\omega(e^{\omega/T}-1)}\frac{|\vec{l}|^2}{\omega^2-k_0^2/4}. \tag{46}$$

The diagram in Fig. 4(c) cannot be calculated exactly with the full propagator of vector mesons. Instead we take a reasonable approximation for the structure of the vertex function $\Gamma_\mu$ and do the calculation for the loop correction. To do this, first, we consider the general form for the pion-photon vertex function at finite temperature which is given by

$$\Gamma_\mu(p,q)=(p_\mu-q_\mu)F+n_\mu G+(p_\mu+q_\mu)G', \tag{47}$$

where $F$, $G$, and $G'$ are scalar functions. When we use the fact that the electromagnetic current should be conserved, that is,

$$(p^\mu+q^\mu)\Gamma_\mu(p,q)=0, \tag{48}$$

we can describe the vertex function only with two independent functions $F$ and $G$ as

$$\Gamma_\mu(p,q)=(p_\mu-q_\mu)F+\left(n_\mu-\frac{k\cdot n}{k^2}k_\mu\right)G, \tag{49}$$

where $k=(p+q)$. In the rest frame of the heat bath, $n_\mu=(1,0,0,0)$, we have

$$\Gamma_0=(p_0-q_0)F-\frac{\vec{k}^2}{k^2}G,$$

$$\Gamma_i=(p_i-q_i)F-\frac{k_0 k_i}{k^2}G. \tag{50}$$

Now we consider the vertex function in the back-to-back frame where $\vec{k}=0$ and $p_0=q_0$. In this frame the vertex function is given by

$$\Gamma_\mu(p,q)=(p_\mu-q_\mu)F, \tag{51}$$

as long as the function $G$ has no singularity at the limit $\vec{k}=0$. We can show explicitly that the function $G$ is regular as $\vec{k}\to 0$ in the soft pion limit. Thus, it is probably safe to assume that the same is true when we include the full propagator for the vector mesons. We therefore write

$$\Gamma_\mu^{4(c)}\approx(p_\mu-q_\mu)F^{4(c)}. \tag{52}$$

With this approximation we can do the loop calculation and the details are given in Appendix A. The modification of the form factor due to the vertex correction is then

$$F_\pi^{4(c)}=F_\pi(T=0)\left\{\frac{g^2}{(p-q)^2}[f_1(p,q;T)+(2m_\pi^2+3k^2/2)g_1(p,q;T)+g_2(p,q;T)]\right\}, \tag{53}$$

where $f_1$ and the $g_i$'s are given in Appendix A.

We should also include the diagram of Fig. 4(d) which has been neglected due to the suppression factor $p^2/m_\rho^2$. We can do the integration with the approximation used for $\Gamma^{4(c)}$ and details are given in Appendix B. Finally, the vertex correction is given by

$$F_\pi^{\text{vertex}}=F_\pi(T=0)\left\{-\frac{5T^2}{12f_\pi^2}g_0(m_\pi^2/T^2)+\frac{1}{f_\pi^2}\alpha(T)+\frac{g^2}{(p-q)^2}[f_1(p,q;T)+(2m_\pi^2+3k^2/2)\times g_1(p,q;T)+g_2(p,q;T)]-2g^2\left(\frac{1}{(p-q)^2}[2h_2(p,q;T)-k^2h_1(p,q;T)]+h_1(p,q;T)+(2m_\pi^2-3k^2/2)h_0(p,q;T)\right)\right\}. \tag{54}$$

The details of the integration for the direct coupling of Fig. 5(b) including the full vector meson propagator are presented in Appendix C. The resulting contribution to the in-medium pion electromagnetic form factor is

$$F_\pi^{\rm direct}(T)=\frac{5T^2}{4f_\pi^2}g_0(m_\pi^2/T^2)-3g^2[A(p;T)+f_0(p;T)], \quad (55)$$

where $A(p;T)$ and $f_0(p;T)$ are given in Appendix C. We should note, however, that the correction to the result obtained in the soft pion approximation is small for this contribution.

The biggest changes from the soft pion result arise from the pion wave function renormalization constant. In the soft pion limit we can see that there is a cancellation between the contribution from the $\pi$-$\rho$ meson loop diagram and $a$-dependent term in $\pi$-tadpole diagrams. The pion wave function correction in hot matter is given by

$$Z_\pi^{-1}=1-\frac{2T^2}{3f_\pi^2}g_0(m_\pi^2/T^2). \quad (56)$$

Thus, in the soft pion limit the pion-tadpole diagram increases the wave function renormalization constant.

When we include the vector meson propagator, the pion self-energy is given by

$$\Pi(p_0,p\to 0)=\frac{c_1}{\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}+\frac{c_2}{\pi^2}\times\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\frac{1}{(p_0^2-m_\rho^2+m_\pi^2)^2-4\omega^2p_0^2}, \quad (57)$$

where the coefficients $c_1$ and $c_2$ are given by

$$c_1=g^2-\frac{1}{12f_\pi^2}[5m_\pi^2+2(p_0^2+m_\pi^2)],$$

$$c_2=-g^2(p_0^2-m_\rho^2+m_\pi^2)\times[2p_0^2-(m_\rho^2-2m_\pi^2)-(p_0^2-m_\pi^2)^2/m_\rho^2]. \quad (58)$$

With the definition in Eq. (16) we have

$$Z_\pi^{-1}=1+\frac{1}{6f_\pi^2}\frac{1}{\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}-\frac{g^2}{\pi^2}(3m_\rho^2-8m_\pi^2)\times\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\frac{1}{(m_\rho^2-2m_\pi^2)^2-4\omega^2m_\pi^2}+\frac{g^2}{\pi^2}\times(m_\rho^2-2m_\pi^2)(m_\rho^2-4m_\pi^2)\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\times\frac{4\omega^2+2(m_\rho^2-2m_\pi^2)}{[(m_\rho^2-2m_\pi^2)^2-4\omega^2m_\pi^2]^2}. \quad (59)$$

The resulting $Z_\pi$ is shown in Fig. 9 together with the result obtained in the soft pion limit. We have an identical result at low temperatures $T<100$ MeV in both cases. However, as the temperature increases the effect from vector mesons becomes important and the $Z_\pi$ begins to drop. This reduction of $Z_\pi$, of course, is simply due to the fact that parts of the pion wave function may now reside in ''rho–thermal-pion'' states with the same quantum numbers as the pion.

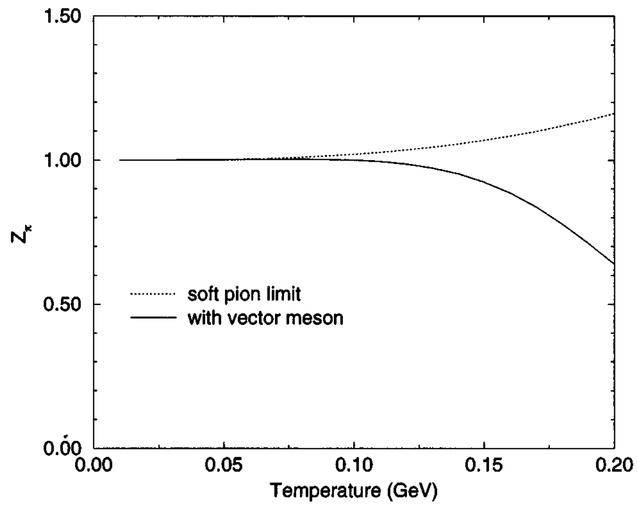


FIG. 9. Pion wave function renormalization constant at finite temperature.

Figure 10 shows the pion form factor around the $\rho$ resonance for different temperatures. The pion electromagnetic form factor is seen to be further reduced near the resonance as the temperature increases. We obtain a reduction of the form factor by 50% at the invariant mass of the virtual photon, $M\sim m_\rho$, when $T=160$ MeV. This result is comparable with that obtained using the QCD sum-rule approach which shows that at $M^2\sim(1\ \text{GeV})^2$ the form factor at $T\sim0.9T_c$ is about half its value at $T=0$ [19]. It is also consistent with that based on perturbative QCD at high $M^2$ [20].

The reduction of the pion electromagnetic form factor at finite temperature is related to chiral symmetry restoration and the deconfinement phase transition in hot hadronic matter. The photon–$\rho$-meson coupling is modified due to the vector axial-vector mixing at finite temperature which has been regarded as a possible signature for the partial restoration of chiral symmetry in hot matter [13]. The resonance width has also been expected to increase in hot hadronic

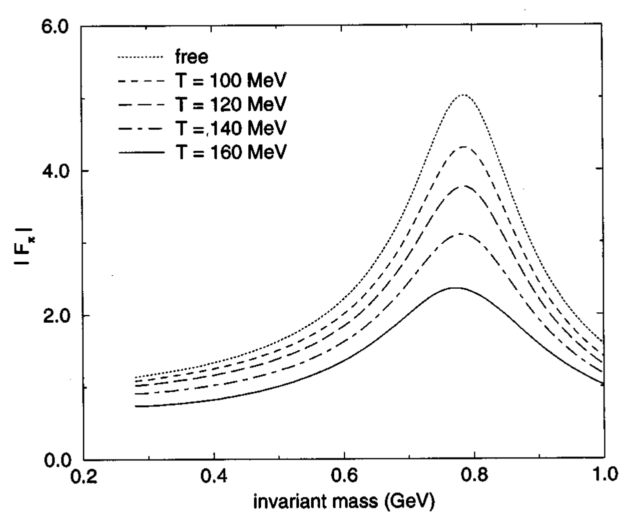


FIG. 10. Pion electromagnetic form factor at finite temperature.

matter as the system undergoes chiral symmetry restoration and the deconfinement phase transition. The vertex corrections, which lead to a reduction of the rho-pion coupling constant at finite temperature, may be related to the recent suggestion that the pion–vector-meson coupling constant vanishes when chiral symmetry is restored in the vector limit [22]. The possible relation between the suppression of the form factor and phase transition in hot hadronic matter also has been suggested via QCD sum rules [19] and the QCD factorization formula [20] to lead to similar suppressions in the pion electromagnetic form factor.

We should note that the form factor is not equal to 1 as the invariant mass approaches zero. This, however, does not contradict the charge conservation discussed in the previous section. Since we are working in the back-to-back frame, the three-momentum of the virtual photon is zero. Therefore, going to the invariant mass zero limit corresponds to the limit $k_0 \to 0$, $\vec{k}=0$. In this limit, the conserved charge derived from the WT identities is related to $\Gamma_0$. The form factor, however, is proportional to the space component of the vertex function, $\Gamma_i$, since we are working in the frame where $p_0 = q_0$ and $\vec{p} = -\vec{q}$. A similar behavior of the form factor also has been observed in dense matter [21].

## V. DILEPTON EMISSION FROM PION-PION ANNIHILATION

In this section we consider the effect of the medium corrections, especially the in-medium pion electromagnetic form factor, on the dilepton production from hot hadronic matter. This has been of interest because of recent dilepton measurements at SPS-energy heavy-ion collisions. Experiments measured by the CERES collaboration at the CERN/SPS show a significant enhancement of dileptons over a hadronic cocktail in the invariant mass region 200 MeV$<M<$1500 MeV in the S+Au collision at 200 GeV/nucleon [23]. On the other hand, in proton-induced reactions such as the 450 GeV $p$-Be and $p$-Au collisions, the low mass dilepton spectra can be satisfactorily explained by dileptons from hadron decays. The enhancement seen in the CERES experiment is for dileptons at central rapidity where the charge particle density is high. In another experiment by the HELIOS-3 Collaboration [24], dileptons at forward rapidities were measured, where the charge particle density is low, and the enhancement was found to be smaller. Suggestions have thus been made that the excess dileptons seen in these experiments are from pion-pion annihilation, $\pi^+\pi^- \to e^+e^-$. However, model calculations which have taken this channel into account can at best reach the lower end of the sum of statistical and systematic errors of the CERES data in the low invariant mass region. For the HELIOS data, which unfortunately do not show a systematic error, there is still a disagreement by a factor of $\sim$1.5 around an invariant mass of 500 MeV [25–27].

It is, therefore, interesting to see to which extent the in-medium correction modifies the dilepton production. Here we will concentrate on the pion annihilation channel. With modified pion properties in the medium the production rate of dileptons with vanishing three-momentum can be written as [28]

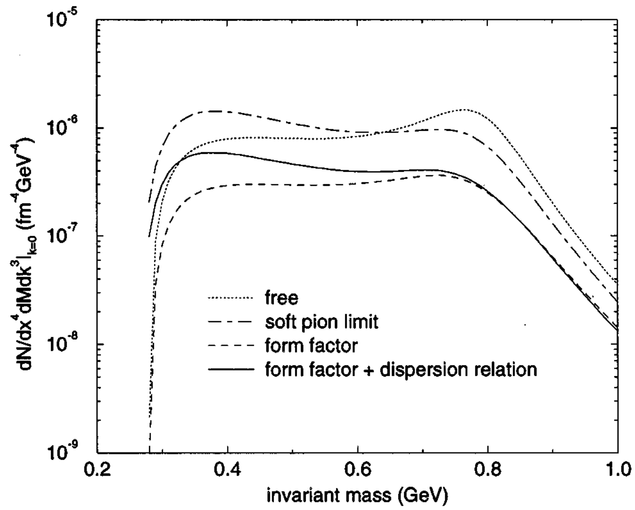


FIG. 11. Dileptons from two-pion annihilation at finite temperature.

$$\left.\frac{d^4R}{d^3k\,dM}\right|_{\vec{k}=0} = \frac{\alpha^2}{3(2\pi)^4}\frac{|F_\pi(M,T)|^2}{(e^{\omega/T}-1)^2}\sum_{|\vec{p}|}\frac{|\vec{p}|^4}{\omega^4}\left|\frac{d\omega}{d|\vec{p}|}\right|^{-1}, \qquad (60)$$

where $M$ is the dilepton invariant mass. The momentum and energy of the pion are denoted by $|\vec{p}|$ and $\omega$, respectively, and are related by its dispersion relation in the medium. The last factor takes into account this effect. The sum over $|\vec{p}|$'s is restricted by $\omega(|\vec{p}|)=M/2$. We also include the in-medium form factor obtained in the previous section.

The pion dispersion relation at finite temperature is determined from the equation

$$p_0^2 - \vec{p}^{\,2} - m_\pi^2 - \Pi_\pi(p_0,\vec{p}) = 0, \qquad (61)$$

where $\Pi_\pi(p_0,\vec{p})$ is given in Eq. (17). Since the pion self-energy also depends on the momentum and energy of pion, the above equation should be solved self-consistently. The real part of the equation determines the dispersion relation of the pion in the medium, while the imaginary part is related to the absorptive properties of pions in hot matter. We have shown that the pion mass, which is defined as the pole position of the propagator, decreases with temperature and the dispersion curve is softened in the low momentum region at finite temperature [29,30].

With these medium effects on the pion dispersion relation and the form factor we get the dilepton production rate as shown in Fig. 11 for $T=$ 160 MeV. The result obtained with the modified pion form factor (dashed line) is compared with that calculated using the form factor in free space (dotted line). Since the production rate is proportional to the square of the form factor, we obtain a larger reduction with temperature in the dilepton production rate than in the form factor above. Near the $\rho$ meson resonance we have $dR[F_\pi(M,T)] \sim (1/2)^2\, dR[F_\pi(M,0)]$ at $T=160$ MeV, and the dilepton production rate is reduced by almost a factor of 4. Finally, when we include the effect from the dispersion relation of pions we obtain the solid line. There are two prominent effects due to changes of the pion dispersion relation in hot hadronic matter. First, the threshold of dilepton production from pion-pion annihilation is lowered because of the reduction of the pion mass, $m_\pi^*(T)$, at finite temperature. Second, the dilepton production rate is enhanced in the invariant mass region, $2m_\pi^*(T) < M < m_\rho$, and shows a maximum at $M \sim 350$ MeV, which is due to the softening of the pion dispersion relation in medium. The latter, however, does not have any effect on dileptons with invariant masses near the vector meson resonance. For comparison we also show the result using the form factor obtained in the soft pion limit as the dot-dashed line.

We see that there is a significant suppression near the vector resonance region. Since the production rate around the two-pion threshold, $M \sim 2m_\pi$, is not changed due to the additional effect of the modified dispersion relation, we have a relative enhancement at the low invariant mass region, which is in qualitative agreement with the CERES data. However, in order to compare with experiment, we need to include properly the expansion dynamics of hot matter that is formed in high energy nucleus-nucleus collisions as well as the contribution from all other channels and the experimental acceptance.

In a recent work we have included these medium effects on dilepton production from hot hadronic matter in a hadronic transport model [27]. In this calculation the result obtained from the calculation in the soft pion limit has been used in order to generate the largest possible effect of the medium correction in the low invariant mass region. We found that in the total spectrum the in-medium effect is hardly visible, especially in the interesting low invariance mass region. This is simply due to the fact that pion annihilation contributes less than 1/3 to the total yield in this region and even an enhancement of a factor of 2 would increase the total spectrum by less than 30%.

These medium effects, however, might be observable in the dilepton spectrum from the mixed phase. If there is a phase transition and the hadronic system goes through a long-lived mixed phase before freeze-out, the most important contribution to dilepton production would come from the hadronic component of the mixed phase at $T_c = 160$–$180$ MeV. In this case our results imply a significant suppression in the production rate of lepton pairs with invariant mass near the vector meson mass. Moreover, when we assume that the mixed phase expands very slowly [31] and, hence, produces more dileptons the effect is more significant and may induce a large suppression due to the modification of the form factor.

## VI. SUMMARY

In summary, we have studied the pion electromagnetic form factor in hot hadronic matter using an effective Lagrangian with the vector mesons. In this model pions couple to the photon only through vector mesons, according to the vector meson dominance assumption.

We have considered leading corrections for the pion-photon coupling at finite temperature. We have shown that in the soft pion limit the Ward-Takahashi identity is satisfied. While the WT identity implies charge conservation at zero temperature, it is not straightforward in the medium. We have considered two different limits and define an effective charge separately. We could show that this effective charge is conserved in each case.

We furthermore have studied the pion electromagnetic form factor in the timelike region at nonzero temperature. We could show that there is a reduction in the magnitude of the form factor, which can be understood in terms of the partial restoration of chiral symmetry and the deconfinement transition in hot hadronic matter. The reduction in the electromagnetic form factor leads to a suppression of dilepton production from two-pion annihilation in hot matter. However, this suppression is hardly visible in the full spectrum because of the contribution from other channels. Only when the $\pi$-$\pi$ contribution is dominant, for example, in the long-lived mixed phase, may one be able to observe a medium effect on the pion electromagnetic form factor.

We expect various observable consequences of these medium effects on the electromagnetic couplings of pions and vector mesons. For example, the ratio $N_\phi/(N_\rho + N_\omega)$ of the produced vector mesons, which is extracted from dilepton measurements, would be modified [7,32]. Our results show that the number of dileptons from rho-meson decay will be suppressed because of the reduced coupling to the photon in the medium. For the $\omega$ meson, on the other hand, there is no such effect. Also, thermal corrections to the phi decay should be small, since these involve kaons. Consequently, the ratio of dileptons from phi decay over those from omega and rho decay would show an enhancement even if the actual particle ratios are unchanged. Therefore, when extracting the particle ratios from the dilepton yields one should not conclude an enhancement of phi mesons before the corrections to the rho-photon couplings have been properly taken into account. In this context it is of interest to extend present calculations to the SU(3) limit and to study the temperature dependence of the photon-phi meson coupling. It is also very interesting to study the form factor needed in $K$-$\bar{K}$ annihilation. This will be relevant to the double phi meson peak in the dilepton spectrum, which has recently been suggested as a possible signal for the phase transition in hot matter [33]. The second phi peak in the dilepton spectrum is from the decay of phi mesons in the mixed phase, which have reduced masses as a result of partial restoration of chiral symmetry.


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## APPENDIX A: VERTEX CORRECTION $\Gamma_\mu^{4(c)}$

When we assume that

$$\Gamma_\mu \equiv (p_\mu - q_\mu) F, \tag{A1}$$

$F$ is given by

$$F=\frac{1}{(p-q)^2}(p^\mu-q^\mu)\Gamma_\mu . \tag{A2}$$

For the $\Gamma_\mu^{4(c)}$ we have

$$\Gamma_\mu^{4(c)}=(p_\mu-q_\mu)\frac{1}{2}g^2F_\pi(T=0)\frac{1}{(p-q)^2}\overline{F^{4(c)}}, \tag{A3}$$

where $F_\pi(T=0)$ is the pion form factor in free space and

$$\begin{aligned}\overline{F^{4(c)}}=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\Bigg\{&\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}\\&+\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l+q)^2-m_\rho^2]}+(2m_\pi^2-3k^2/2)\\&\times\frac{2l\cdot(p-q)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-p)^2-m_\rho^2]}\\&+\frac{2[l\cdot(p-q)]^2}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-p)^2-m_\rho^2]}\Bigg\}\\\equiv{}& f_1(p,q;T)+f_1'(p,q;T)+2(2m_\pi^2-3k^2/2)g_1(p,q;T)\\&+2g_2(p,q;T).\end{aligned} \tag{A4}$$

The loop integration can be done in a back-to-back frame where $p_0=q_0$ and $\vec{p}=-\vec{q}$.

(i) The functions $f_1$ and $f_1'$ are given by

$$\begin{aligned}f_1(p,q;T)&=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}\\&=\frac{1}{8\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\\&\quad\times\left(4-\frac{A_+}{2|\vec{p}||\vec{l}|}L_+-\frac{A_-}{2|\vec{p}||\vec{l}|}L_-\right),\end{aligned}$$

$$\begin{aligned}f_1'(p,q;T)&=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l+q)^2-m_\rho^2]}\\&=f_1(p,q;T),\end{aligned} \tag{A5}$$

where

$$L_\pm=\ln\left[\frac{A_\pm+2|\vec{p}||\vec{l}|}{A_\pm-2|\vec{p}||\vec{l}|}\right], \tag{A6}$$

with

$$A_\pm=(m_\rho^2-2m_\pi^2)\pm2\omega p_0. \tag{A7}$$

(ii) The functions $g_n$'s can also be obtained and are given by

$$\begin{aligned}g_1(p,q;T)&=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\\&\quad\times\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-p)^2-m_\rho^2]}\\&=\frac{1}{4\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\Bigg\{\frac{1}{4p_0}\left(\frac{1}{p_0+\omega}+\frac{1}{p_0-\omega}\right)\\&\quad\times\left(2-\frac{A_+}{2|\vec{p}||\vec{l}|}L_+\right)+\frac{1}{4p_0}\frac{1}{p_0+\omega}\\&\quad\times\left(\frac{A_+}{2|\vec{p}||\vec{l}|}L_+-\frac{A_-}{2|\vec{p}||\vec{l}|}L_-\right)\Bigg\}\end{aligned} \tag{A8}$$

and

$$\begin{aligned}g_2(p,q;T)&=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{[l\cdot(p-q)]^2}{(l^2-m_\pi^2)[(l-k)^2-m_\pi^2][(l-p)^2-m_\rho^2]}\\&=\frac{1}{4\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\Bigg\{\frac{A_+}{4p_0}\left(\frac{1}{p_0+\omega}+\frac{1}{p_0-\omega}\right)\left(2-\frac{A_+}{2|\vec{p}||\vec{l}|}L_+\right)-\frac{1}{4p_0}\frac{1}{p_0+\omega}\Bigg[A_+\left(2-\frac{A_+}{2|\vec{p}||\vec{l}|}L_+\right)\\&\qquad-A_-\left(2-\frac{A_-}{2|\vec{p}||\vec{l}|}L_-\right)\Bigg]\Bigg\}.\end{aligned} \tag{A9}$$

## APPENDIX B: VERTEX CORRECTION $\Gamma_\mu^{4(d)}$

With the same approximation we can also calculate the contribution from the diagram of Fig. 4(d) in the back-to-back frame:

$$\Gamma^{4(d)}=(p_\mu-q_\mu)2g^2F_\pi(T=0)\overline{F^{4(d)}}, \tag{B1}$$

where

$$\overline{F^{4(d)}}=-T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\left\{\frac{1}{(p-q)^2}\frac{2[l\cdot(p-q)]^2-(k^2+4p\cdot q)l\cdot(p-q)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2][(l+q)^2-m_\rho^2]}+\frac{l\cdot(p-q)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2][(l+q)^2-m_\rho^2]}\right.$$
$$\left.+(2m_\pi^2-3k^2/2)\frac{1}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2][(l+q)^2-m_\rho^2]}\right\}$$
$$\equiv\frac{2}{(p-q)^2}h_2(p,q;T)+\left[1-\frac{k^2+4p\cdot q}{(p-q)^2}\right]h_1(p,q;T)+(2m_\pi^2-3k^2/2)h_0(p,q;T). \tag{B2}$$

The functions $h_n$'s are defined as

$$h_n=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{[l\cdot(p-q)]^n}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2][(l+q)^2-m_\rho^2]}. \tag{B3}$$

Each integration has been done and is given by

$$h_0=\frac{1}{32\pi^2p_0|\vec{p}|}\int\frac{|\vec{l}|d|\vec{l}|}{\omega^2(e^{\omega/T}-1)}(L_+-L_-),$$

$$h_1=-f_0(p;T)-(m_\rho^2-2m_\pi^2)h_0(p,q;T),$$

$$h_2=(m_\rho^2-2m_\pi^2)[f_0(p;T)+(m_\rho^2-2m_\pi^2)h_0(p,q;T)]$$
$$-f_1(p,q;T), \tag{B4}$$

where $L_\pm$ and $f_1$ are given in Appendix A and $f_0(p;T)$ is defined by

$$f_0(p;T)=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{1}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}$$
$$=\frac{1}{8\pi^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}\frac{1}{2|\vec{p}||\vec{l}|}(L_++L_-). \tag{B5}$$

## APPENDIX C: DIRECT COUPLING IN THE MEDIUM $\Gamma_\mu^{5(b)}$

For the direct coupling of pions to photon fields in the medium we have

$$\Gamma_\mu^{5(b)}=-3g^2T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\left[\frac{(l^\mu+p^\mu)}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}\right]. \tag{C1}$$

It can be written as

$$\Gamma_\mu^{5(b)}=-3g^2[g_\mu(p;T)+p_\mu f_0(p;T)], \tag{C2}$$

where $f_0(p;T)$ is given in Appendix B and

$$g_\mu(p;T)=T\sum_{n_l}\int\frac{d^3l}{(2\pi)^3}\frac{l_\mu}{(l^2-m_\pi^2)[(l-p)^2-m_\rho^2]}. \tag{C3}$$

For $g_\mu(p;T)$ it can be written in general as

$$g_\mu(p;T)=A(p;T)p_\mu+B(p;T)p_0n_\mu, \tag{C4}$$

with

$$A=-\frac{1}{|\vec{p}|^2}(p^\mu g_\mu-p^0g_0),$$

$$B=g_0/p_0-A. \tag{C5}$$

In a back-to-back frame we have

$$A(p;T)=-\frac{1}{16\pi^2|\vec{p}|^2}\int\frac{|\vec{l}|^2d|\vec{l}|}{\omega(e^{\omega/T}-1)}$$
$$\times\left[4-\frac{(m_\rho^2-2m_\pi^2)}{2|\vec{p}||\vec{l}|}(L_++L_-)\right.$$
$$\left.-\frac{p_0\omega}{|\vec{p}||\vec{l}|}(L_+-L_-)\right],$$

$$B(p;T)=\frac{1}{8\pi^2p_0}\int\frac{|\vec{l}|^2d|\vec{l}|}{(e^{\omega/T}-1)}\frac{1}{2|\vec{p}||\vec{l}|}(L_+-L_-)-A. \tag{C6}$$

In the same way we have

$$\Gamma_\mu^{5(c)}=3g^2[g_\mu(q;T)+q_\mu f_0(q;T)], \tag{C7}$$

with

$$g_\mu(q;T)=A(q;T)q_\mu+B(q;T)q_0n_\mu. \tag{C8}$$

Since $p_0=q_0$ and $|\vec{p}|=|\vec{q}|$ in the frame we are working, $f_0(p;T)=f_0(q;T)$, $A(p;T)=A(q;T)$, and $B(p;T)=B(q;T)$. Thus

$$\Gamma_\mu^{5(b)}+\Gamma_\mu^{5(c)}=-3g^2(p_\mu-q_\mu)[A(p;T)+f_0(p;T)]. \tag{C9}$$

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