

Nonlocal effects in a semiclassical WKB approach to sub-barrier nuclear fusion processes

R. Dutt* and T. Sil

Department of Physics, Visva-Bharati University, Santiniketan - 731 235, West Bengal, India

Y. P. Varshni†

Department of Physics, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5

(Received 2 October 1995)

We have shown that the large enhancement of sub-barrier fusion cross sections for heavy-ion collision processes can be partially accounted for by using the variation of the effective mass due to nonlocal effects in WKB theory. Although reasonable agreement of our predicted results for the fusion cross sections and average angular momenta with the observed experimental data for several systems have been observed, it is evident that the nonlocal effects alone cannot explain the entire enhancement as claimed by Galetti *et al.* [Phys. Rev. C **50**, 2136 (1994)]. It is then suggested that nonlocality needs to be supplemented by nuclear deformation and other degrees of freedom. [S0556-2813(96)04806-6]

PACS number(s): 25.70.Jj, 24.10.-i

I. INTRODUCTION

It has been a long standing problem that the experimental sub-barrier nucleus-nucleus fusion cross sections cannot be well accounted for by theoretical predictions based on barrier penetration model (BPM) calculations. The experimental fusion cross sections in heavy-ion reactions at energies near and below the Coulomb barrier are found to be considerably enhanced over the theoretical predictions [1-4]. This enhancement has been attributed to several possible mechanisms such as low energy zero-point vibrations [5,6], static deformations [7], coupling of different channels [8,9], neck formation [6-10], etc. Unfortunately, these calculations are not only plagued with computational complexity but also fail to reproduce the required enhancement, particularly at the incident energies of the projectile much below the Coulomb barrier. It is then realized that more degrees of freedom and energy dependence of the nucleus-nucleus potential have to be judiciously incorporated.

Recently, Galetti and his co-workers [11,12] have proposed an attractive idea that the observed enhancement of the experimental sub-barrier fusion cross sections can be explained by simple BPM calculation considering the nonlocal effect alone. In this approach [12], the nonlocality which simulates the many-body quantum effects [13] manifests only in the sub-barrier region where the reduced mass of the fusing nuclei is not a constant but varies with the so-called nonlocal parameter b as

$$\mu(r, b) = \mu / [1 + (\mu b^2 / 2\hbar^2) |v^{(0)}(\vec{q})|],$$

which can be approximated by the following to a very good degree for small values of b :

$$\begin{aligned} \mu(r; b) &= \mu [1 - \mu b^2 |v^0(R_B)| / (2\hbar^2)], \quad 0 < r < R_B \\ &= \mu, \quad r > R_B. \end{aligned} \quad (1)$$

Here R_B is the location of the Coulomb barrier height. Working with the Hill-Wheeler (HW) parabolic approximation [14] to the Christensen-Winther nuclear potential barrier and invoking Wong's approximation [15], a simple analytic expression for the reduced fusion cross section was obtained [12]

$$\sigma_{\text{red}}(E_{\text{red}}, b) = \left[\frac{1}{(1 - b^2 f/4)} \right] \ln \{ 1 + \exp[2\pi E_{\text{red}}(1 - b^2 f/4)] \}, \quad (2)$$

where $\sigma_{\text{red}} = 2E_{\text{c.m.}} \sigma_F / R_B^2 \hbar \omega_0$ and $E_{\text{red}} = (E_{\text{c.m.}} - v_B) / \hbar \omega_0$. It was claimed that (2) yields substantial enhancement to account for the experimental data for sub-barrier energies for a number of heavy-ion collision processes; the effective nonlocal parameter b was chosen in the range 1.66 to 2.23 fm.

Although the results are quite impressive, it may be noted that there are several shortcomings in their work due to the use of too many simplified assumptions. First, the authors adopted $f=2$ and $b=1.66$ to 2.23 for which the effective mass becomes negative, i.e., unphysical. Secondly, they have taken the viewpoint of Wong in which the variation of the curvature parameter ($\hbar \omega_l$) of the potential with the angular momentum [16] has been ignored. Finally, the replacement of the actual nuclear potential by the HW parabolic potential removes the dependence of the predicted reduced cross sections on the potential parameters which are bound to be different for different systems. More explicitly, expression (2) yields the ratio of the fusion cross section with and without nonlocal effect

$$R = \frac{\sigma_{\text{red}}(b)}{\sigma_{\text{red}}(b=0)} = \frac{\ln \{ 1 + \exp[2\pi E_{\text{red}}(1 - b^2 f/4)] \}}{\ln \{ 1 + \exp[2\pi E_{\text{red}}] \} (1 - b^2 f/4)}, \quad (3)$$

which turns out to be the same for all systems for a given value of E_{red} and b . This becomes clear from Figs. 2-6 of Ref. [12].

At this point, it is quite natural to raise the following questions.

*Electronic address: rdutt@vbharat.ernet.in

†Electronic address: ypsvj@acadvm1.uottawa.ca

(i) Can one explain the observed enhancement of the fusion data just from the viewpoint of nonlocality using physically acceptable values of the single parameter b ?

(ii) Is it possible to apply the idea of nonlocality in a semiclassical WKB calculation without invoking HW approximation to a realistic potential barrier for nucleus-nucleus interaction?

(iii) Can one bypass Wong's simplified assumption and reproduce the dynamical dependence of the fusion cross sections considering dependence of barrier height on angular momentum quantum number?

(iv) Is the enhancement due to nonlocality model independent?

In this paper, we attempt to answer these questions. We demonstrate that using the concept of variation of effective mass due to nonlocality in the framework of WKB calculation and using a three-parameter potential [16] that mimics the well known phenomenological "proximity potential" of Blocki *et al.* [17], one may obtain reasonable enhancement of sub-barrier fusion cross sections. Our work exhibits relevant dynamical features of each individual system unlike the work of Galetti *et al.*, yet remaining analytically tractable and simpler than multichannel and other calculations [6,8–10]. We substantiate our claim by explicit results obtained for $^{16}\text{O}+^A\text{Sm}$ ($A=148,150,152,154$), $^{50}\text{Ti}+^{90}\text{Zr}$, and $^{50}\text{Ti}+^{93}\text{Nb}$ systems.

In Sec. II we illustrate the method of inclusion of nonlocality in the expression of nuclear fusion cross section in the barrier penetration model. In Sec. III, we present the results for $^{16}\text{O}+^A\text{Sm}$, $^{50}\text{Ti}+^{90}\text{Zr}$, and $^{50}\text{Ti}+^{93}\text{Nb}$ systems. Concluding remarks are also included in this section.

II. NONLOCAL EFFECTS FOR A NUCLEAR FUSION CROSS SECTION IN THE WKB METHOD

In BPM calculation, the fusion cross section is given by

$$\sigma_F(E,b) = \sum_{l=0}^{\infty} \sigma_l(E,b), \quad (4)$$

where the partial cross section is

$$\sigma_l(E,b) = \pi \lambda^2 (2l+1) T_l(E,b). \quad (5)$$

Here λ is the reduced de Broglie wavelength of the projectile ion and $T_l(E,b)$ is the transmission coefficient of the l th partial wave. In a semiclassical WKB method, the analytic expression for T_l is obtained for a potential barrier $V_l(r)$ [16,18]

$$T_l(E,b) = [1 + \exp(2I)]^{-1}, \quad (6)$$

where

$$I = \int_{r_1}^{r_2} k_l(r) dr = \int_{r_1}^{r_2} \left[\frac{2\mu}{\hbar^2} [V_l(r) - E] \right]^{1/2} dr. \quad (7)$$

The turning points r_1 and r_2 obtained from $v_l(r) = E$ naturally lie on either side of the barrier radius R_B . Consequently one needs to split the integral in (7) into two regions

$r_1 < r < R_B$ and $R_B < r < r_2$ where the values of the reduced mass μ are different as in (1) due to nonlocal effect. Thus we get

$$I = \int_{r_1}^{R_B} \left[\frac{2\mu}{\hbar^2} [v_l(r) - E] \right]^{1/2} (1 - b^2 f/2) dr + \int_{R_B}^{r_2} \left[\frac{2\mu}{\hbar^2} [v_l(r) - E] \right]^{1/2} dr \quad (8)$$

in which f is defined same as in Ref. [12]. It is important to indicate that unlike the work of Galetti *et al.*, the value of the parameter b cannot exceed certain maximum value in order to ensure positivity of the effective mass. It is known that fusion cross sections for two colliding nuclei of atomic weight and atomic number, (A_1, Z_1) and (A_2, Z_2) , can be adequately reproduced by using the "proximity potential" due to Blocki *et al.* modified by Vaz and Alexander [1]

$$V_F(r) = 4\pi\gamma\tilde{C}(-3.437)\exp[-(r - C_1 - C_2)/0.75] + \frac{Z_1 Z_2 e^2}{r} + l(l+1)\hbar^2/2\mu r^2, \quad (9)$$

with

$$\gamma = 0.9517 \left[1 - 1.7826 \left(\frac{A - 2Z}{A} \right)^2 \right] \quad (\text{MeV/fm}^2),$$

$$R_i = [1.28A_i^{1/3} - 0.76 + 0.8A_i^{1/3} + \Delta R] \quad (\text{fm}), \quad (10)$$

$$C_i = [R_i - 1/R_i] \quad (\text{fm}),$$

$$\tilde{C} = C_1 C_2 / (C_1 + C_2).$$

Here $A = (A_1 + A_2)$ and $Z = (Z_1 + Z_2)$ are respectively the mass and atomic numbers of the composite system and ΔR is the modification of the effective sharp radius R_i . The surface width is taken to be 1 fm.

Clearly, the integral in (8) cannot be evaluated in closed form if one uses the empirical potential (9). On the other hand, we showed earlier [16] that this potential may be replaced for all practical purposes by an analytical three-parameter potential first suggested by Ahmed [19]

$$V(r) = v_l \left[1 - \left(\frac{1 - \exp\{(r_l - r)/a\}}{1 - c \exp\{(r_l - r)/a\}} \right)^2 \right], \quad (11)$$

with

$$a = \left[\frac{\hbar^2}{2\mu} 4V_l / (\hbar\omega_l) \right]^{1/2} / (1 - c),$$

$$\hbar\omega_l = \left[\frac{\hbar^2}{\mu} \frac{d^2 V_F(r)}{dr^2} \right]_{r=r_l}^{1/2}. \quad (12)$$

Here V_l is the maximum of the barrier height, r_l is the position of the barrier top, and c is a constant. In Ref. [16], we have discussed how the potential (11) can be matched with the empirical potential (9) for the entire range of r by suit-

able choice of parameters. Moreover, the parametrization is done in such a manner that the l dependent centrifugal term is no longer needed as all the parameters v_l , r_l , a , and c are l dependent. The advantage of using this potential is that

a compact analytic expression for the transmission coefficient T_l can be derived.

Substituting (11) in (8) and using standard integrals [20], we finally obtain

$$T_l(E, b) = \left[1 + \exp \left\{ 2\varepsilon \left(\pi(1 - b^2 f/4)(g - 1) \sqrt{V_l/\Delta} + \pi(1 - b^2 f/4) \sqrt{E/\Delta} - \pi(1 - b^2 f/4) \sqrt{(E/\Delta) + (g^2 - 1)(V_l/\Delta)} \right. \right. \right. \\ \left. \left. \left. + \frac{b^2 f}{2} \sqrt{E/\Delta} \sin^{-1}(\sqrt{1 - E/V_l}) - \frac{b^2 f}{2} \sqrt{(E/\Delta) + (g^2 - 1)(V_l/\Delta)} \sin^{-1}(c \sqrt{1 - E/V_l}) \right) \right\} \right]^{-1}, \quad (13)$$

with $\Delta = \hbar^2/2\mu a^2$, $g = 1/c$, and $\varepsilon = \text{sgn}(c - 1)$. One may check that for $b = 0$, (13) reduces to Eq. (3.8) of Ref. [16].

III. RESULTS AND DISCUSSIONS

To make quantitative comparisons with experimental data, we select those systems for which several measured cross sections are available in the far sub-barrier region. We compare our predicted results [computed from (4) and (13)] with the experimental data for the collision of ^{16}O with even isotopes of ^ASm ($A = 148, 150, 152$, and 154) obtained by DiGregorio *et al.* [21]. The comparison is shown graphically in Fig. 1 for the $^{16}\text{O} + ^{148}\text{Sm}$ system while the numerical data for the rest of the systems are presented in Table I. Our WKB results with nonlocal effects are displayed in column 5 and the allowed value of b for the best fit for each system is indicated in column 1. We have used $f = 0.55 \text{ fm}^2$ which implies that the physically acceptable value of b cannot exceed 1.9 fm [see Eq. (8)]. Careful examination of our tabu-

lated results as well as the graphical presentation reveals several encouraging features which need to be focused.

The isotopes of Sm span the transition region from spherical to strongly deformed equilibrium shapes [7] as A increases from 148 to 154. It is clearly seen that the optimum choice of b is the lowest (1.84 fm) for the spherical nucleus ($A = 148$) while its value has to be increased to the maximum allowed value 1.9 fm for more deformed nuclei as for $A = 150, 152$, and 154 . Although the enhancement has been quite significant (nearly 30 times as compared to the same calculation without nonlocality) the disagreement with experimentally observed data becomes more prominent as the deformation of the target nucleus increases. This suggests that there may be a correlation between the concept of nuclear deformation and the presence of nonlocality for which the dynamical origin is yet to be understood. This claim is substantiated by the results for $^{50}\text{Ti} + ^{90}\text{Zr}$ and $^{50}\text{Ti} + ^{93}\text{Nb}$ systems presented in Fig. 2.

It is clear that we have been able to achieve reasonable enhancement for all the systems using a realistic nuclear barrier potential in the WKB method. Quite similar observations were made by Galletti *et al.* who used the Christensen-Winther potential for O+Ni and O+Cu systems. It may then be inferred that perhaps the enhancement caused by the nonlocal effect is model independent.

However, there are certain differences between our approach and the work of Galletti *et al.* It is obvious from our calculation that by simple adjustment of the nonlocal parameter b , it is not possible to explain the observed sub-barrier enhancement. This is in clear contradiction with the claim of Ref. [12] in which the entire enhancement of sub-barrier fusion cross sections of a number of systems have been explained by attributing unphysical values to the nonlocal parameter.

In the far sub-barrier region where $E_{\text{red}} < -1$, we find from the figures of Ref. [12] that their computed fusion cross sections are about 40–60 times larger than the values without b . The occurrence of such a huge overestimation may be due to the following reasons: the application of the HW parabolic approximation introduces an extra enhancement over and above that which arises from the nonlocal effect by narrowing the width of the actual nuclear potential in the region where the bombarding energy is much less than the Coulomb barrier. In this regard, our calculation generates the enhancement purely from the nonlocal effect as we have used an

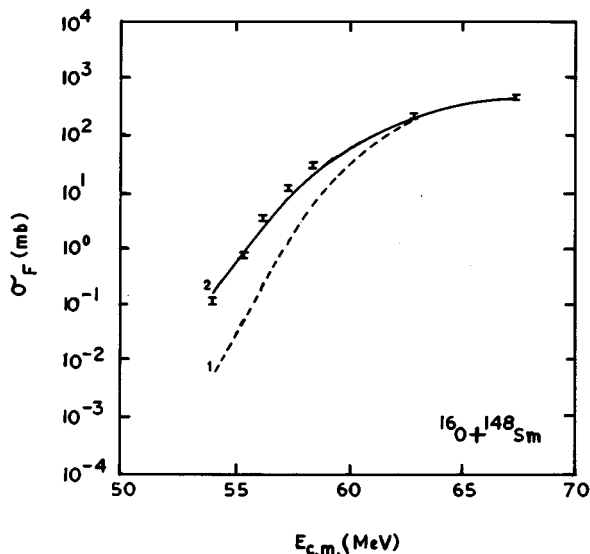


FIG. 1. Plot of fusion cross section for the $^{16}\text{O} + ^{148}\text{Sm}$ system. Curve 1 (dashed) is our WKB predictions without nonlocal effects ($b = 0$); curve 2 (solid) represents the nonlocal predictions for $b = 1.84 \text{ fm}$. The experimental data are shown by dots with error bars.

TABLE I. The calculated and the experimental fusion cross sections are presented for collisions of ^{16}O (projectile) + $^{150,152,154}\text{Sm}$ (target) nuclei for center of mass energy below and above the Coulomb barrier. Columns 4 and 5 consist of WKB results without and with the nonlocal effects. The optimum allowed value of the nonlocal parameter b and the Coulomb barrier height V_B for individual systems are shown in column 1.

System	E_{lab} (MeV)	E_{red} (MeV)	$\sigma_{\text{WKB}}(E, b=0)$ (mb)	$\sigma_{\text{WKB}}(E, b)$ (mb)	σ_{expt} (mb)
$^{16}\text{O} + ^{150}\text{Sm}$	60.0	-1.24	0.01	0.29	0.472 ± 0.047
$V_B = 59.65$ MeV	61.2	-0.99	0.06	1.04	2.22 ± 0.22
$R_B = 11.17$ fm	62.5	-0.72	0.40	3.70	7.75 ± 0.8
$\hbar\omega_0 = 4.386$ MeV	63.8	-0.46	2.37	11.64	20.2 ± 2.0
$b = 1.9$ fm	65.0	-0.21	10.88	28.29	38.4 ± 3.8
	70.0	0.82	221.95	221.95	243.0 ± 24.0
	75.0	1.85	461.39	461.39	440.0 ± 44.0
$^{16}\text{O} + ^{152}\text{Sm}$	59.9	-1.27	0.008	0.25	1.06 ± 0.11
$V_B = 59.77$ MeV	61.2	-1.00	0.06	0.98	4.4 ± 0.44
$R_B = 11.14$ fm	62.4	-0.75	0.32	3.21	11.7 ± 1.2
$\hbar\omega_0 = 4.392$ MeV	63.7	-0.49	1.94	10.25	24.4 ± 2.4
$b = 1.9$ fm	64.9	-0.24	9.07	25.86	43.9 ± 4.4
	70.0	0.81	218.37	218.37	213.0 ± 21.0
	75.0	1.84	456.97	456.97	462.0 ± 46.0
$^{16}\text{O} + ^{154}\text{Sm}$	60.0	-1.23	0.011	0.30	2.21 ± 0.22
$V_B = 59.75$ MeV	61.3	-0.96	0.07	1.19	6.24 ± 0.62
$R_B = 11.15$ fm	62.5	-0.71	0.42	3.84	15.3 ± 1.5
$\hbar\omega_0 = 4.388$ MeV	63.8	-0.45	2.51	12.02	29.4 ± 2.9
$b = 1.9$ fm	65.0	-0.20	11.44	29.59	55.8 ± 5.6
	70.1	0.85	228.91	228.91	235.0 ± 24.0
	75.1	1.87	466.49	466.49	430.0 ± 43.0

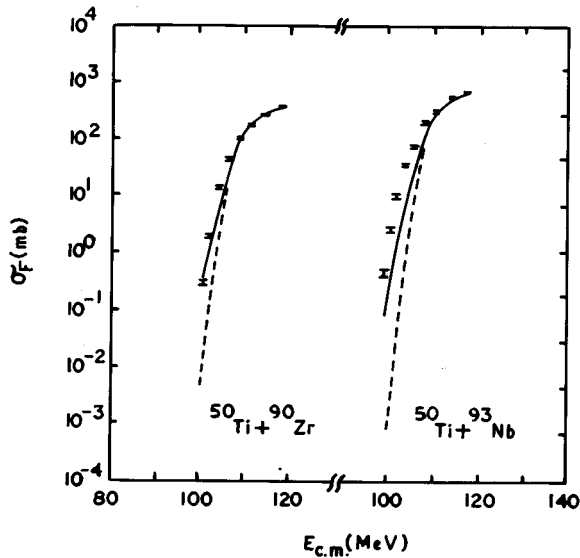


FIG. 2. Plot of fusion cross section for $^{50}\text{Ti} + ^{90}\text{Zr}$ and $^{50}\text{Ti} + ^{93}\text{Nb}$ systems. Curve with dashed line is our WKB predictions without nonlocal effects ($b=0$); curve with solid line represents the nonlocal predictions for $b=1.9$ fm (maximum allowed value for these two systems). The experimental data taken from Ref. [22] are shown by dots with error bars.

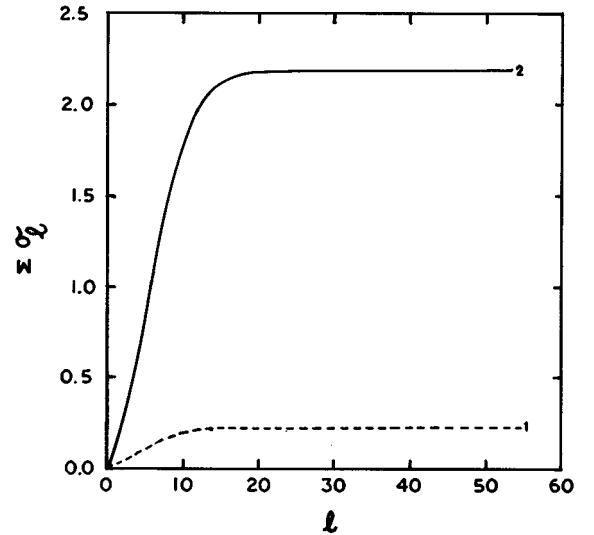


FIG. 3. Plot of $\sum \sigma_l$ versus l for the $^{16}\text{O} + ^{148}\text{Sm}$ system for energy $E_{\text{c.m.}} = 56.40$ MeV. Curve 1 (dashed) is our WKB predictions without nonlocal effects and curve 2 (solid) represents the nonlocal predictions for $b=1.84$ fm.

TABLE II. Theoretical and experimental average angular momenta $\langle l \rangle$ and $\langle l^2 \rangle$ for fusion of the $^{16}\text{O}+^{152}\text{Sm}$ system for different energies are presented. The Coulomb barrier height of the system is $V_B=59.77$ MeV.

E_{lab} (MeV)	$E_{\text{c.m.}}$ (MeV)	$\langle l \rangle$		Expt. (Ref. [23])	$\langle l^2 \rangle$		Expt. (Ref. [23])
		Theoretical			Theoretical		
		$b=0$ (fm)	$b=1.9$ (fm)		$b=0$ (fm)	$b=1.9$ (fm)	
60.0	54.3	6.1	7.3	7.0	49.3	69.1	60
62.5	56.6	6.2	7.6	8.5	50.9	75.9	90
65.0	58.8	6.5	8.5	11.5	58.5	91.2	160
70.0	63.3	11.7	11.7	15.5	159.7	159.7	280
80.0	72.4	21.2	21.2	25.5	509.9	509.9	770

analytic potential whose profile matches almost identically with the true fusion potential barrier over the entire region of tunneling (see Fig. 2 of Ref. [16]).

To study the nonlocal effect on the individual contribution of each partial wave to the fusion cross section, we display a plot of $\sum \sigma_l$ versus l in Fig. 3 for the $^{16}\text{O}+^{148}\text{Sm}$ system for the center of mass energy $E_{\text{c.m.}}=56.40$ MeV which is well below the Coulomb height $V_B=59.94$ MeV. It is clearly seen that there is a cutoff value of l beyond which no contribution is added to the cross section. This cutoff value also shifts to the higher side when the parameter b is taken into account. Nonlocality also boosts each partial wave contribution in such a way that we obtain the required enhancement by a factor of 10 making the theoretical prediction, $\sigma_F=2.19$ mB, quite close to the experimentally measured value $\sigma_{\text{expt}}=(3.13 \pm 0.31)$ mb.

Experimental investigations of the angular momentum distribution leading to fusion provide important information that is complementary to the study of cross sections. A real test of any heavy-ion fusion model is that it should not only explain sub-barrier fusion enhancement but also predict correct compound nucleus spin distributions [6,23]. It may be worthwhile to examine how well various moments of spin distribution $\langle l^n \rangle$ can be accounted from the point of view of the present nonlocal approach. Just for a check, we compute $\langle l \rangle$ and $\langle l^2 \rangle$ for the $^{16}\text{O}+^{152}\text{Sm}$ system with and without nonlocal effects for different incident energies using

$$\langle l \rangle = \sum l \sigma_l / \sigma_F, \quad (14a)$$

$$\langle l^2 \rangle = \sum l^2 \sigma_l / \sigma_F. \quad (14b)$$

Our predicted results have been compared in Table II with the experimental data obtained from graphs of Ref. [24]. It is observed that the inclusion of nonlocality in BPM calculation gives improvement to the predicted values of $\langle l \rangle$ and $\langle l^2 \rangle$. However, the required enhancement cannot be achieved keeping the value of b within its physical domain.

To summarize, we have performed a WKB barrier penetration calculation for heavy-ion systems at sub and near Coulomb barrier energies. Use is made of nonlocal effects to study to what extent the observed enhancement of the experimental data can be explained. Interesting extensions of this scheme to other aspects of heavy-ion collision processes may be possible in the foreseeable future with the availability of new data from more versatile colliders. More detailed calculations in this direction are obviously needed to pin down the possible connection between the nonlocal effects and other conventional nuclear degrees of freedom.

ACKNOWLEDGMENTS

Two of us (R.D. and T.S.) wish to acknowledge the financial support received from the Department of Atomic Energy and the Council for Scientific and Industrial Research, Govt. of India. This work was also supported in part by a research grant from the Natural Sciences and Engineering Research Council of Canada to one of the authors (Y.P.V.).

- [1] L. C. Vaz, J. M. Alexander, and G. R. Satchler, Phys. Rep. **69**, 373 (1981).
[2] S. G. Steadman, *Fusion Reactions below the Coulomb Barrier*, Lecture Notes in Physics Vol. 219 (Springer-Verlag, Heidelberg, 1985).
[3] M. Beckerman, Rep. Prog. Phys. **51**, 1047 (1988).
[4] A. B. Balantekin, J. R. Bennett, and S. Kuyucak, Phys. Rev. C **49**, 1079 (1994); S. Kailas and P. Singh, *ibid.* **50**, 1230 (1994), and see references therein.
[5] H. Esbensen, Nucl. Phys. **A352**, 147 (1981).
[6] Drazen Vorkapic, Phys. Rev. C **49**, 2812 (1994).
[7] R. G. Stokstad *et al.*, Phys. Rev. Lett. **41**, 465 (1978), and see references therein.
[8] C. H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. **A405**, 381 (1983); **A407**, 221 (1983); **A432**, 495 (1985).
[9] J. A. Christley, M. A. Nagarajan, and I. J. Thompson, J. Phys. G **17**, L163 (1991); T. Izumoto, T. Udagawa, and B. T. Kim, Phys. Rev. C **51**, 761 (1995).
[10] J. Randrup, Nucl. Phys. **A383**, 468 (1982).
[11] D. Galetti, S. S. Mizrahi, and B. M. Pimental, Phys. Scr. **27**, 72 (1983); D. Galetti and M. A. Candido Ribeiro, *ibid.* **45**, 417 (1992).

- [12] D. Galetti and M. A. Candido Ribeiro, *Phys. Rev. C* **50**, 2136 (1994).
- [13] L. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971).
- [14] D. L. Hill and J. A. Wheeler, *Phys. Rev.* **89**, 1102 (1953).
- [15] C. Y. Wong, *Phys. Rev. Lett.* **31**, 766 (1973).
- [16] T. Sil, R. Dutt, and Y. P. Varshni, *Phys. Rev. C* **50**, 2458 (1994).
- [17] J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, *Ann. Phys. (N.Y.)* **105**, 427 (1977).
- [18] N. Fröman and P. O. Fröman, *JWKB Approximation: Contributions to the Theory* (North-Holland, Amsterdam, 1965).
- [19] Z. Ahmed, *Phys. Rev. A* **47**, 4761 (1993).
- [20] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, New York, 1965).
- [21] D. E. DiGregorio *et al.*, *Phys. Rev. C* **39**, 516 (1989).
- [22] P. H. Stelson *et al.*, *Phys. Rev. C* **41**, 1584 (1990).
- [23] R. Vandenbosch, T. Murakami, C. C. Sahm, D. D. Leach, A. Ray, and M. J. Morphy, *Phys. Rev. Lett.* **56**, 1234 (1986); C. H. Dasso, S. Landowne, and G. Pollaroio, *Phys. Lett. B* **217**, 25 (1989); A. K. Mohanty, S. V. S. Sastry, S. K. Kataria, and V. S. Ramamurthy, *Phys. Rev. C* **46**, 2012 (1992).
- [24] A. H. Wuosmaa *et al.*, *Phys. Lett. B* **263**, 23 (1991).