Effect of nuclear absorption on nucleon transfer probabilities in heavy-ion reactions

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We perform a classical dynamics study of the proton transfer reactions measured for the $^{12}C+{}^{197}Au$ and $^{16}O+{}^{197}Au$ systems. The nuclei are assumed to move along classical trajectories, and the proton transfer probability is considered as a tunneling process around the point of closest approach. The absorption due to the imaginary part of the optical potential also is included. At the highest energies considered, several trajectories contribute to each scattering angle. The contributions associated to the different trajectories are added to obtain the proton transfer probability. We find that, for a properly selected value of the strength parameter in the imaginary part of the optical potential, the theoretical results fit adequately the experimentally measured values. [S0556-2813(96)04212-4]

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I. INTRODUCTION

Heavy-ion induced transfer reactions at large internuclear distances have been a subject of interest in the field of nuclear physics in the last years. One of the features that has drawn special attention is the behavior of the transfer probabilities as a function of the distance of closest approach, which shows an exponential falloff when one or two neutrons are involved in the reaction at large distances [1–3].

The theoretical description has been done in the framework of classical trajectories and tunneling through a potential barrier around the distance of closest approach. Within this semiclassical model, the exponential decay constant is determined by $\kappa=\sqrt{2\mu B_{\rm eff}/\hbar}$, where μ and $B_{\rm eff}$ are the reduced mass and the effective barrier height to be traversed by the transferred particle, and is independent of the beam energy.

One- and two-proton transfer cross sections in the $^{12}C+{}^{197}Au$ and $^{16}O+{}^{197}Au$ systems were studied recently at the TANDAR laboratory at bombarding energies ranging from 56 to 82 MeV and 74 to 110 MeV, respectively [4]. The data were obtained by grouping events in angular bins 4° wide. At each of these energies, angular distributions of the ejectiles have been measured and the differential transfer cross sections as a function of center-of-mass angles were presented as transfer probabilities versus distances of closest approach, $D_{\rm Ruth}$, calculated assuming classical Rutherford trajectories. In this presentation it can be seen that, for the higher distances, the slopes decrease when the beam energy is increased, in contradiction with the semiclassical interpretation of the transfer as a tunneling process described above. In this paper we reconcile the semiclassical picture with the trend of the slopes as a function of the beam energy in the measurements of Ref. [4].

II. THEORY

We assume that the nuclei move along classical trajectories under the influence of the Coulomb and the real part of the nuclear optical potential. For the latter we use a Woods-Saxon shape with radius and strength calculated as in Ref. [5]:

$$R=R_p+R_t+0.29 \text{ fm} \tag{1}$$

with

$$R_i=(1.233A_i^{1/3}-0.98A_i^{-1/3}) \text{ fm} \tag{2}$$

and

$$V_0=16\pi\gamma\overline{R}a \text{ MeV} \tag{3}$$

with

$$\gamma=0.95\left[1-1.8\left(\frac{N_p-Z_p}{A_p}\right)\left(\frac{N_p-Z_p}{A_p}\right)\right] \text{ MeV fm}^{-2} \tag{4}$$

and

$$\overline{R}=\frac{R_pR_t}{R_p+R_t}, \tag{5}$$

where we have taken a diffuseness a = 0.7 fm. In this equation A_p,Z_p, and N_p are the mass, atomic, and neutron numbers for the projectile, whereas A_t,Z_t, and N_t are the corresponding quantities for the target. In this case the actual apsidal distance D is obtained by evaluating numerically the orbit under the influence of both the nuclear and Coulomb potentials. It is well known that when the nuclear force is taken into account, there can be more than one trajectory which leads to a given scattering angle.

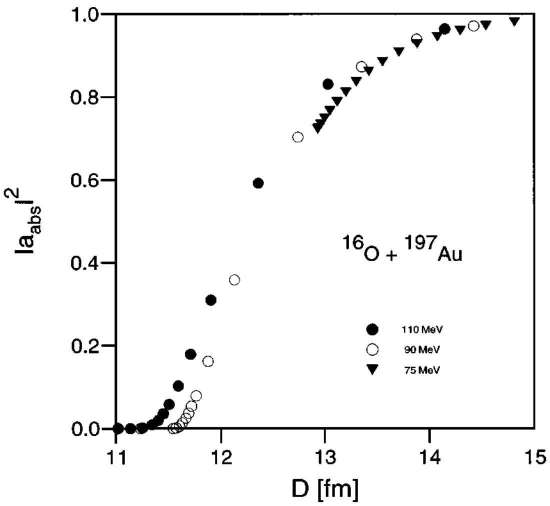

FIG. 1. The survival probability $|a_{\mathrm{abs}}|^2$ as a function of the apsidal distance D for the $^{16}O+^{197}Au$ system at $E_{\mathrm{lab}}=$ 75, 90, and 110 MeV. The cutoff in the curve at $E_{\mathrm{lab}}=$ 75 MeV corresponds to the distance of closest approach for this energy.

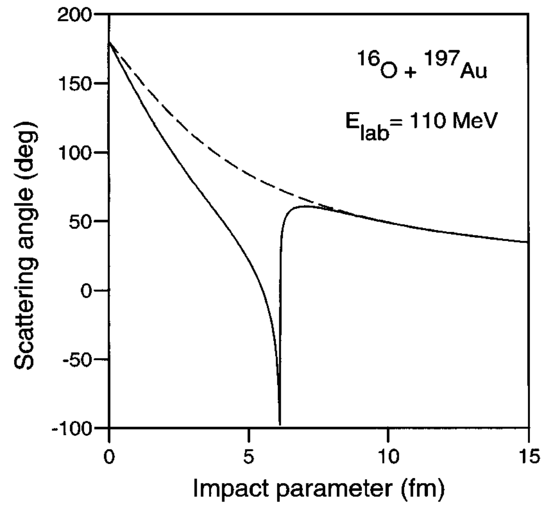

FIG. 2. Deflection function for the $^{16}O+^{197}Au$ collision at the beam energy of 110 MeV. The solid line is obtained when both the Coulomb and the nuclear interactions are taken into account and the dashed line is obtained when only the Coulomb potential is considered.

We consider the absorption due to the imaginary part of the nuclear optical potential between the colliding partners. The probability amplitude for survival from absorption a_{abs} can be calculated as [6]

$$a_{\mathrm{abs}}=\exp\left(\frac{-1}{\hbar}\int_{-\infty}^{+\infty}W(t)dt\right), \tag{6}$$

where W is the imaginary part of the nucleus-nucleus potential. For transfer at large distances, W can be approximated by their exponential tail

$$W(t)=W(r(t))=W_0e^{[R-r(t)]/a}, \tag{7}$$

being W_0 the strength of W. To calculate a_{abs} from Eq. (6), the internuclear distance $r(t)$ was approximated by their expansion around the point of closest approach up to second order in the time, as done in Ref. [7], which gives

$$a_{\mathrm{abs}}=\exp\left[\frac{-W_0}{\hbar}\sqrt{\frac{2\pi a}{\ddot{r}_0}}\exp\left(\frac{(R-D)}{a}\right)\right], \tag{8}$$

where $\ddot{r}_0$ is the acceleration at the point of closest approach. The bombarding energy dependence of this probability is shown in Fig. 1, where $|a_{\mathrm{abs}}(\theta)|^2$ is plotted as a function of the actual apsidal distance D for three different energies for the $^{16}O+^{197}Au$ system.

The probability for tunneling through the transfer potential barrier was determined similarly to Ref. [8]. Denoting by $U(r)$ the potential which actuates over the transferred nucleon and R_B the position where the potential barrier reaches its maximum, $U_B=U(R_B)$, and being B_E the binding energy in the donor nucleus for that nucleon, we use the WKB approximation when $U_B+B_E>0$,

$$P_{\mathrm{tun}}=|a_{\mathrm{tun}}|^2=(1+e^S)^{-1} \tag{9}$$

in which

$$S=2\int_{R_1}^{R_2}\left(\frac{2\mu}{\hbar}[U(r)+B_E]\right)^{1/2}dr. \tag{10}$$

In the region $U_B+B_E<0$ the potential barrier relevant to the transfer process can be approximated by an inverted parabola, allowing us the use of the analytic expression of Hill-Wheeler [9] for the tunneling probability

$$P_{\mathrm{tun}}=\{1+\exp[2\pi/\hbar\omega(U_B+B_E)]\}^{-1} \tag{11}$$

with

$$\hbar\omega=\left(-\frac{\hbar^2}{\mu}\frac{d^2U(R_B)}{dr^2}\right)^{1/2}. \tag{12}$$

We have taken

$$U(r)=U_1(r)+U_2(D-r), \tag{13}$$

$$U_i(r)=U_{C_i}(r)+U_{N_i}(r), \tag{14}$$

where the subscripts 1 and 2 refer to the donor and acceptor cores, respectively, D is the distance of closest approach between them, and r is the spatial coordinate of the transferred particle with respect to the donor core. U_{C_i} is the Coulomb potential and U_{N_i} the nuclear potential generated by the core i over the particle. For the proton case to be considered below, we use the prescription of Ref. [5] and take the Coulomb potential as that generated by a charged sphere of radius $R_c=1.25(A_p-1)^{1/3}$ fm and the nuclear part as a Saxon-Woods potential with radius parameter $r_0=1.2$ fm, diffuseness $a_u=0.63$ fm, and depths

$$U_{0_i}=\left[51+33\frac{N_p-Z_p+1}{A_p-1}\right]\ \mathrm{MeV}. \tag{15}$$

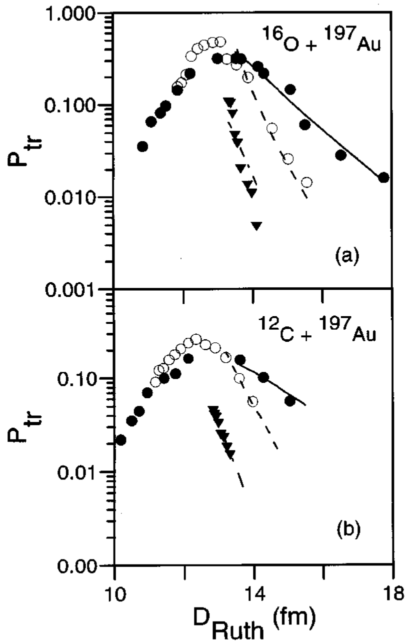

FIG. 3. One-proton stripping probability as a function of D_{Ruth} for (a) the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ collision at beam energies of 75, 90, and 110 MeV and (b) the $^{12}\mathrm{C}+{}^{197}\mathrm{Au}$ collision at beam energies of 57, 70, and 82 MeV. Full circles are the experimental data of Ref. [4] for $E_{\mathrm{lab}}=$ 110 MeV (82 MeV), open circles for 92 MeV (70 MeV), and triangles for 75 MeV (57 MeV) for the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ ($^{12}\mathrm{C}+{}^{197}\mathrm{Au}$) system. Lines are the theoretical results of this work for the corresponding energies.

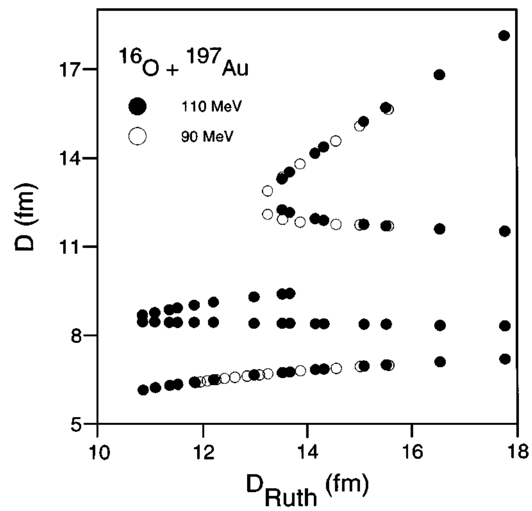

FIG. 4. Actual apsidal distance D as a function of D_{Ruth} for the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ collision. Open circles are for 90 MeV and solid rhombs for 110 MeV of beam energy. See text for details.

The proton transfer probability for a certain scattering angle was obtained by adding the contributions associated to the different trajectories which lead to the same angle, i.e.,

$$P_{\mathrm{tr}}(\theta)=\left|\sum a_{\mathrm{tun}}(\theta)a_{\mathrm{abs}}(\theta)e^{-i\Phi(\theta)}\right|^2. \qquad (16)$$

In the expression above, $\Phi(\theta)$ stands for the total phase (Coulomb plus nuclear) associated to each trajectory. They were calculated in the WKB approximation as described in Ref. [10]. This prescription contrasts with the one employed by Kim *et al.* [11], which does not consider the absorption due to the imaginary part of the optical potential.

The interference between the contributing trajectories gives rise to an oscillation pattern in the calculated angular distributions, which depends strongly on the system, bombardment energy, and angular range under consideration. In the case of the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ system at 110 MeV, in the angular region where the amplitude of the oscillations is maximum, their period is about 5 deg. At lower energies the oscillations become much smaller, and are not noticeable below 85 MeV. When taking into account the width of the experimental angular bins (4 deg in our case), the averaging of the theoretical transfer amplitudes almost completely obliterates the oscillations, and results in a transfer probability that approximately coincides with that obtained from the incoherent sum of the contributions,

$$P_{\mathrm{tr}}(\theta)\approx\sum P_{\mathrm{tun}}(\theta)|a_{\mathrm{abs}}(\theta)|^2, \qquad (17)$$

which thus is employed in the remaining calculations presented in this work.

III. RESULTS AND DISCUSSION

Now we consider the one-proton stripping channel in the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ system, for which the binding energy is $B_E=12.128$ MeV. The transfer probabilities are calculated as described in the preceding paragraph for beam energies of 75, 90, and 110 MeV. At the beam energy of 75 MeV, which is close to the Coulomb barrier in this case, the deflection function is essentially unchanged with respect to that calculated by consideration of only the Coulomb potential, but the situation is not the same at the highest energies. As an illustration, we show in Fig. 2 the deflection function for this system at 110 MeV. We can see that when the nuclear potential is taken into account, this function is very different from the corresponding one in the case of Rutherford scattering. Several trajectories can contribute to the same scattering angle, and to each of them corresponds a different distance of closest approach. We note that negative angles cannot be disentangled experimentally from the positive ones. The only free parameter in our calculation is the depth of the imaginary part of the optical potential, which in this case took the value of $W_0=33.89$ MeV.

In Fig. 3 the experimental data of Ref. [4] for the transfer probability are plotted as a function of D_{Ruth}, the distance of closest approach calculated for a Rutherford trajectory at the given angle for the $^{16}\mathrm{O}+{}^{197}\mathrm{Au}$ and $^{12}\mathrm{C}+{}^{197}\mathrm{Au}$ systems [Figs. 3(a) and 3(b), respectively]. In view of the previous considerations about the influence of the nuclear potential in the trajectory, we have to keep in mind that D_{Ruth} is not the true distance of closest approach, it is only a parametrization of the scattering angle. Also shown are the theoretical results calculated as described above and scaled by appropriate factors. We found that the theoretical expectations are in good agreement with the experimentally measured. In particular, we see that the semiclassical calculation can reproduce the energy dependence of the slopes at the larger distances. A similar agreement is found for the $^{12}C+^{197}Au$ system ($B_E=$ 15.957 MeV) using the prescription for the optical model parameters given in Sec. II, and the same value of W_0, see Fig. 3(b).

This resolution of the apparent discrepancy between the experimental data and the semiclassical model is a consequence of the modification of the classical trajectories with respect to the Rutherford orbits due to the nuclear interaction (including absorption). This can be seen clearly in Fig. 4, where D, the true distance of closest approach, is plotted as a function of D_{Ruth} for the two highest energies considered above. At these energies this function is multivalued; several trajectories correspond to each scattering angle. In our calculation of the transfer probabilities, only those trajectories with D greater than 11 fm contribute to the scattering amplitude. The other three trajectories correspond to a complete absorption of the projectile.

Figure 4 also shows that the true distances of closest approach for the reactions at $E_{\mathrm{lab}}=90$ MeV and 110 MeV are very similar for $D_{\mathrm{Ruth}}>14$ fm. This indicates that the tunneling probabilities are, correspondingly, very similar. However, since the absorption depends on the bombarding energy through the acceleration at the point of closest approach $\ddot{r}_{\bigcirc}$, Eq. (8), the proton transfer probabilities P_{tr} are different in these cases. It also should be pointed out that absorption is more important in the branch of trajectories for which $D\sim$ 11.5 fm, leading to the paradoxical result that the largest transfer probabilities correspond to the largest values of D. This could explain the slope dependence on the beam energy as it was observed experimentally.

IV. CONCLUSIONS

The semiclassical model of nucleon transfer reactions can be reconciled with the energy dependence of the slopes of one-proton transfer probability versus D_{Ruth} measured by Tomasi *et al.* [4]. The fundamental point is that, as a result of the nuclear interaction, the deflection function is very different from the case of pure Rutherford scattering. Several trajectories correspond to a given experimental angle, each of them with a distinct distance of closest approach. Moreover there are some trajectories having different absorption at the same actual apsidal distances, which strongly influences the transfer probabilities. This behavior determines the energy dependence of the slopes.

It would be interesting to perform experiments similar to those of Ref. [4], but with better angular resolution, in order to measure oscillations in the angular distribution. As we have seen in the present work, these oscillations are averaged out when grouping over large angular bins.

According to our estimates, an angular resolution of $\approx 0.5°$ (i.e., approximately one-tenth of the calculated period) should be sufficient to observe these oscillations at energies well above the barrier. The resulting data should provide a more stringent test of the models employed to describe these reaction processes.

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[1] R.B. Roberts, S.B. Gazes, J.E. Mason, M. Satteson, S.G. Teichmann, L.L. Lee, J.F. Liang, J.C. Mahon, and R.J. Vojtech, Phys. Rev. C **47**, 1831 (1993).

[2] K.E. Rehm, B.G. Glagola, W. Kutschera, F.L.H. Wolfs, and A.H. Wuosmaa, Phys. Rev. C **47**, 2731 (1993).

[3] D. Tomasi, J.O. Fernández Niello, A.O. Macchiavelli, A.J. Pacheco, J.E. Testoni, D. Abriola, O.A. Capurro, D.E. Di Gregorio, M. di Tada, C.P. Massolo, and F. Penayo, Phys. Rev. C **48**, 2840 (1993).

[4] D. Tomasi, J.O. Fernández Niello, A.J. Pacheco, D. Abriola, J.E. Testoni, A.O. Macchiavelli, O.A. Capurro, D.E. Di Gregorio, M. di Tada, G.V. Martí, and I. Urteaga, Phys. Rev. C **54**, 1282 (1996).

[5] R.A. Broglia and A. Winther, *Heavy Ion Reactions* (Addison-Wesley, Reading, MA, 1991).

[6] M.W. Guidry, R.W. Kincaid, and R. Donangelo, Phys. Lett. **150B**, 265 (1985).

[7] L.F. Canto, R. Donangelo, R.S. Nikam, and P. Ring, Phys. Lett. B **192**, 4 (1987).

[8] L.C. Vaz and J.M. Alexander, in *Proceedings of the International Conference Held at the MIT*, Cambridge, Massachusetts, 1984, edited by S.G. Steadman (Springer-Verlag, Berlin, 1985), p. 288.

[9] D.L. Hill and J.A. Wheeler, Phys. Rev. **89**, 1102 (1953).

[10] E. Vigezzi and A. Winther, Ann. Phys. (N.Y.) **192**, 432 (1989).

[11] H.J. Kim, J. Gomez del Campo, M.M. Hindi, D. Shapira, and P.H. Stelson, Phys. Rev. C **38**, 2081 (1988).