Relationship between ground state and double analog cross sections in pion double charge exchange

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At a pion kinetic energy of 292 MeV, comparison of averages of ground-state (g.s.) and double analog (DIAS) cross sections on isotopes of Ni and Se suggests that the g.s. cross section is comparable to that per nucleon pair for the DIAS. [S0556-2813(96)03412-7]

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Many models of pion-induced double charge exchange (DCX) predict for the double isobaric analog state (DIAS) a forward-angle cross section of the form:

$$\sigma=\left(\frac{A_0}{A}\right)^c T(2T-1)|f_{\rm red}|^2,$$

where A,T are mass number and isospin of the target nucleus, A_0 is a constant, and $f_{\rm red}$ is either independent of A,T or very slowly dependent. Expressions of this type have been tested for data on many nuclei at an incident pion kinetic energy of 292 MeV. Differences in the models are small enough and uncertainties in the data are large enough that meaningful exclusion of models has not been possible.

Nonanalog cross sections to $T-2$ ground states are another matter. They are extremely small at 292 MeV—significantly smaller than predicted in, e.g., seniority (or generalized seniority) models. The purpose of the present note is to point out a previously unrecognized relationship between ground state (g.s.) and DIAS cross sections in two chains of isotopes.

In seniority models of DCX, the DIAS amplitude is a linear combination of two amplitudes A and B (or α and β), only the second of which has a coefficient that depends on N, Z (or T) [apart from the overall $T(2T-1)$ factor]. In these models, the g.s. cross section is proportional to $|B|^2$ (or $|\beta|^2$). In generalized seniority, with only one type of nucleon [i.e., valence neutrons outside a closed proton (and neutron) shell, or proton holes accompanied by a filled neutron shell], the expressions for (π^+,π^-) double charge exchange (DCX) leading to the double isobaric analog state (DIAS) and ground state (g.s.) are [1,2]

$$\sigma_{\rm DIAS}=\frac{n(n-1)}{2}\left|\alpha+\frac{\beta}{n-1}\right|^2,$$

$$\sigma_{\rm g.s.}=\frac{n(n-2)(2\Omega+2-n)}{2(2\Omega+1)(n-2)}|\beta|^2.$$

Here, n is the neutron excess $N-Z$, and is also twice the isospin: $n=2T$. The quantity 2Ω is the number of nucleons (of one type) required for a full shell.

The parameters α and β are related to the A and B of Ref. [2] as

$$A=\alpha+\frac{\beta}{\Omega},$$

$$B=\frac{\Omega-1}{\Omega}\beta.$$

The α,β formalism is somewhat more natural because all the spin-dependent effects are in β, none in α.

Subsequent derivations have demonstrated that the complete generalized seniority expressions are more complicated [3]; i.e., requiring a third amplitude (plus phase) and, for the nonanalog ground states, an additional (unknown) normalization factor, whose magnitude depends on details of nuclear structure.

In every model of DCX, the DIAS cross section contains a factor of $n(n-1)/2$ which multiplies the square of a reduced amplitude that is a very slow function of n—or even independent of n. For the DIAS, it thus seems reasonable to define $\sigma_{\rm red}\equiv\sigma_{\rm DIAS}/[n(n-1)/2]$. The g.s. cross section contains no such $n(n-1)/2$ factor, but is merely $|B|^2$ times the square of a coefficient of order unity that depends slowly on n—reaching a maximum in midshell.

Even in models more general than those with good seniority, or generalized seniority, it is likely that both $\sigma_{\rm red}$ and $\sigma_{\rm g.s.}$ will depend slowly on n.

These formulas have been tested [1,4] in DCX on ^{58,60,62,64}Ni at an incident pion kinetic energy of 292 MeV and a scattering angle of 5°. The quantities α and β (or A and B) are supposed to be independent of N and Z within a shell, but do depend on energy and angle. The only dependence on target mass is an $A_{\rm tgt}^{-C}$ factor, which we suppress for clarity. Relevant cross sections from Ref. [1] are listed in Table I.

In Ref. [1], a fit to Ni DIAS cross sections alone gave a value of β that predicted g.s. cross sections significantly larger than observed. It was possible to accommodate both g.s. and DIAS cross sections in a fit, but with a poorer χ^2. Inclusion of g.s. data naturally drove the fit to smaller β, but it also drove the relative phase between α and β to 0°. We might expect α and β to have a relative phase of 0° or 180°, because all the nuclear structure numbers are real, and the pion distortions are common to both α and β.

The feature that DIAS-only fits produce values of β that overpredict g.s. cross sections is a general one, having been observed in a number of $f7/2$ nuclei. The value of φ, the

TABLE I. Ground state and DIAS cross sections at 292 MeV and 5° for isotopes of Ni and Se.

	σ (nb/sr)	
Target	g.s.	DIAS
^{58}Ni [a]		125±13
^{60}Ni [b]	61±23	295±55
^{62}Ni [b]	3±9	471±72
^{64}Ni [b]	41±30	974±172
^{76}Se [c]	14±8	547±73
^{78}Se [c]	6±6	802±81
^{80}Se [c]	17±10	1,000±100
^{82}Se [c]	13±13	1,520±150

[a]Phys. Rev. C **50**, 306 (1994).
[b]Reference [1].
[c]Reference [4].

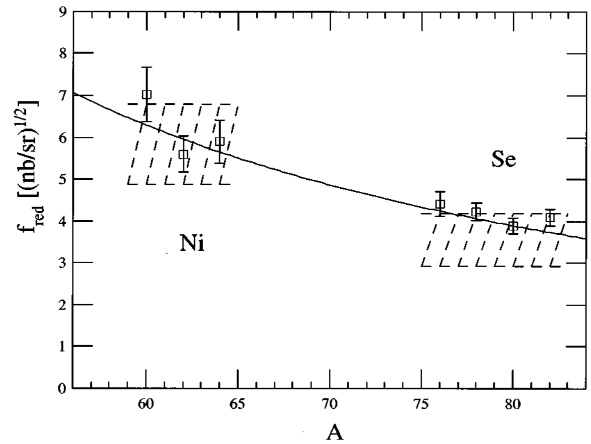

FIG. 1. Points are values of $f_{\mathrm{red}} \equiv [\sigma_{\mathrm{DIAS}}/[T(2T-1)]]^{1/2}$ for isotopes of Ni and Se. Hatched areas are $\overline{\sigma_{\mathrm{g.s.}}}$ for these nuclei. Both are for 292 MeV and 5°. Curve represents an $A^{-5/3}$ distortion factor in the amplitude.

relative phase, is generally poorly determined. In Ref. [4], e.g., a value near 90° was obtained, whereas the fit in Ref. [1] prefers a value near 0°.

Serious sequential calculations have been performed in the seniority model and for several nuclei whose structure is reasonably well described within the nuclear shell model [5]. In every case the predicted nonanalog ground state cross sections at 292 MeV are significantly larger than the experimental ones.

Because both σ_{red} and $\sigma_{\mathrm{g.s.}}$ are slow functions of n, and to improve statistical uncertainties, we choose to average both for ^{60,62,64}Ni. We use $\overline{\sigma} = \Sigma Y_i/\Sigma Q_i$, where Y_i represents the number of counts for isotope i, and Q_i the normalization factor for that isotope that converts counts to cross section. [This is the correct expression for any count rate and approaches the more widely used formula as the number of counts grows large.] These average g.s. cross sections are listed in Table II, along with the averages of $\sigma_{\mathrm{red}} \equiv \sigma_{\mathrm{DIAS}}/[T(2T-1)]$. We note a remarkable fact—viz. the g.s. cross sections are (within uncertainty) equal to the DIAS cross sections for a single pair (i.e., with the pair counting factor divided out).

The only other chain of isotopes for which DCX measurements exist for three or more members (all with $T>1$) is Se, for which cross sections have been published for ^{76,78,80,82}Se; Ref. [6]. These are also listed in Table I. Again, we average $\sigma_{\mathrm{g.s.}}$ and σ_{red}, and we compare them in Table II. As for Ni, we note the remarkable near equality of these two quantities. The fact that both $\sigma_{\mathrm{g.s.}}$ and σ_{red} are smaller in Se than in Ni presumably reflects the A_{tgt}^{-C} distortion factor mentioned earlier.

In Fig. 1, we plot as points the values of $f_{\mathrm{red}} \equiv [\sigma_{\mathrm{DIAS}}/[T(2T-1)]]^{1/2}$ for the isotopes of Ni and Se, and as cross hatching the average value of $\sqrt{\overline{\sigma_{\mathrm{g.s.}}}}$ for these nuclei. The fact that σ_{red} and $\overline{\sigma_{\mathrm{g.s.}}}$ are comparable is apparent.

Many comparisons of DIAS DCX have been performed at 292 MeV. Count rates are highest there, and energy dependence of measured cross sections is weakest. Another favorite testing ground of DCX theories is the Ca isotopes. Data exist for both ^{44}Ca [7] and ^{48}Ca [8,9]. [Remember that $T=1$ (^{42}Ca here) is not relevant for our present purposes.] At 292 MeV, two independent analyses of the same ^{44}Ca data give DIAS cross sections of 0.637±0.102 [7] and 0.587 ±0.106 μb/sr [10]. These correspond to $\sigma_{\mathrm{red}} = 0.102 \pm 0.018$ μb/sr. For ^{48}Ca, the DIAS cross section is 1.746 ±0.290 μb/sr at 292 MeV [8] and 1.950±0.297 μ b/sr at 300 MeV [9], giving an average of 0.0660±0.0074 for σ_{red}. No g.s. counts were observed for either ^{44}Ca or ^{48}Ca. Limits are <0.076 μb/sr for ^{44}Ca [10] and <0.045 μb/sr for ^{48}Ca [8]. Thus the limits on the ratios $\sigma_{\mathrm{g.s.}}/\sigma_{\mathrm{red}}$ of 0.75 ±0.13 for ^{44}Ca and 0.66±0.07 for ^{48}Ca are not inconsistent with the results for Ni and Se.

What is the meaning of the observed near equality of $\sigma_{\mathrm{g.s.}}$ and σ_{red}? Because, in the simplest models, $\sigma_{\mathrm{g.s.}}$ is just $|B|^2$ times a coefficient of order unity, and σ_{red} is dominated by α [$f_{\mathrm{red}} = \alpha + (\beta/n-1)$ in generalized seniority], the present observation suggests approximately equal values of α, β at 292 MeV and 5°. This is rather surprising, because we know that, in this formulation, all the spin-dependent effects are in β, none are in α.

TABLE II. Averages (nb/sr) of $\sigma_{\mathrm{g.s.}}$ and $\sigma_{\mathrm{red}} \equiv \sigma_{\mathrm{DIAS}}/[T(2T-1)]$ for Ni, Se, and Ca at 292 MeV and 5°.

Targets	$\overline{\sigma_{\mathrm{g.s.}}}$	σ_{red}	$\sqrt{\overline{\sigma_{\mathrm{g.s.}}}}$	$\overline{f_{\mathrm{red}}}$
^{60,62,64}Ni	34.02±11.20	35.08±3.50	5.83±0.96	5.92±0.30
^{76,78,80,82}Se	12.6±4.5	16.77±0.89	3.55±0.63	4.10±0.11
^{44,48}Ca	<60	70.7±6.5	<7.7	8.41±0.39

And we know that $(\Delta S=1)^2$ terms in DCX fall off as T_π increases from 140 to 292 MeV, while $(\Delta S=0)^2$ terms increase. It would be very useful to compare $\sigma_{\mathrm{g.s.}}$ and $\sigma_{\mathrm{DIAS}}/[T(2T-1)]$ at a lower pion energy, e.g., 164 MeV, where $\sigma_{\mathrm{g.s.}}$ should be much larger. There, we would expect $\sigma_{\mathrm{g.s.}}/\sigma_{\mathrm{red}}$ to be very much larger than unity. A very limited amount of data [7,8] for ^{44}Ca and ^{48}Ca suggests $\overline{\sigma_{\mathrm{g.s.}}}=131\pm17$ μb/sr and $\sigma_{\mathrm{red}}=14.2\pm2.4$ μb/sr at energies near 164 MeV.

These results suggest that the spin-dependent amplitude, β, is indeed much larger than the spin-independent α at energies near 164 MeV.

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