Spreading width of the isobaric analog state and isospin mixing

Toshio Suzuki*
Department of Physics, College of Humanities and Sciences, Nihon University, Sakurajosui 3-25-40, Setagaya-ku, Tokyo 156, Japan

H. Sagawa†
Center for Mathematical Sciences, The University of Aizu, Aizu-Wakamatsu, Fukushima 965, Japan

G. Colò‡
Dipartimento di Fisica, Università degli Studi, via Celoria 16, 20133 Milano, Italy

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We study a relation between the spreading width of the isobaric analog state and the isospin mixing probability of the corresponding parent state by using the Feshbach projection method. The formula is applied for calculations of the spreading width of several heavy isotopes and compared with available experimental data. Contributions from isovector monopole states are found to be important to explain quantitatively the experimental spreading width. The isospin dependence of the calculated width of several isotopes is also found to be consistent with the experimental observations. [S0556-2813(96)00912-0]

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The question of isospin impurities in nuclei has been a long-standing open question in nuclear physics. While the idea of isospin was proposed by Heisenberg more than 60 years ago, the isospin becomes again a popular subject of nuclear structure, especially due to the recent development of experimental facilities of radioactive beams. It was pointed out theoretically that nuclei near the proton drip line have a much larger isospin impurity than those of stable nuclei because of a favorable isospin geometrical factor [1]. Microscopic calculations predict a large isospin mixing probability up to about 5% in medium heavy nuclei, for example ^{100}Sn. The effect of isospin impurities on the superallowed Fermi β decays has been also studied and a relation of the calculated Fermi transition rates to the Cabbibo-Kobayashi-Maskawa mixing matrix is discussed [2–4].

The discovery of isobaric analog resonances [5] was a highlight of the study of isospin in nuclear physics. One of the important characteristics of the isobaric analog state (IAS) is its narrow width. The width of IAS originates from Coulomb interaction that couples the state to the particle continuum and to other states. Several works have been done to study the width of the IAS [6,7] in relation to the Coulomb matrix elements. There is still no explicit formula which relates the isospin impurity with the width of the IAS. In this paper, we would like to address possible relations between the spreading width of the IAS and the isospin mixing probability in the corresponding parent nucleus.

We first derive a formula for the spreading width of the IAS based on the projection method by Feshbach [8]. The Hamiltonian consists of two parts

$$H = H_0 + H_1, \tag{1}$$

where H_1 is the interaction which violates the isospin, i.e., the isovector part of the Coulomb interaction, the charge symmetry breaking (CSB) and charge independence breaking (CIB) interactions [9], while H_0 conserves isospin. We separate the whole space into two parts, P and Q spaces: The P space consists of the parent state, $|\pi\rangle$, and the IAS, $|\mathrm{IAS}\rangle$,

$$|P\rangle = \{|\pi\rangle, |\mathrm{IAS}\rangle\}. \tag{2}$$

The two states are defined as eigenstates of H_0 with good isospin: $|\pi\rangle = |T,\ T_z = T\rangle$ and $|\mathrm{IAS}\rangle = |T, T-1\rangle$. The other space $Q = 1 - P$ consists of all eigenstates of H_0, except $|\pi\rangle$ and $|\mathrm{IAS}\rangle$, having good isospin. By definition, the two spaces are orthogonal, $PQ = QP = 0$. Since the physical parent state and physical IAS should be eigenstates of the total Hamiltonian H, they have admixtures of different isospins due to H_1. These mixings have been often discussed in relation to the conserved vector current hypothesis and the Cabbibo-Kobayashi-Maskawa unitary matrices in superallowed Fermi transitions [2–4]. For the present purpose, we have to have a good isospin basis to relate the isospin impurity and the width of the IAS as will be discussed later.

The wave function projected onto the P space, $P\Psi$, satisfies the equation

$$[E - PH_{\mathrm{eff}}(E)P]P\Psi = 0, \tag{3}$$

with

$$H_{\mathrm{eff}}(E) = H + H\frac{Q}{E - H_{QQ}}H, \tag{4}$$

where $H_{QQ} = QHQ$. As $PHQ = PH_1Q$ due to $PH_0Q = 0$, P and Q spaces can be connected only by H_1. This is the consequence of defining the P and Q spaces by eigenstates of H_0, not by those of H. It can be shown by theoretical calculations [1,10] that the physical IAS is almost (more than 99%) an eigenstate of H_0 in nuclei near the stable line of mass table.

*Electronic address: suzuki@chs.nihon-u.ac.jp
†Electronic address: sagawa@u-aizu.ac.jp
‡Electronic address: colo@mi.infn.it

The spreading width of the IAS can be expressed as

$$\Gamma_A^{\downarrow}(E) = -2\ \mathrm{Im}\langle \mathrm{IAS}|H_1 Q \frac{1}{E-H_{QQ}} QH_1|\mathrm{IAS}\rangle$$
$$= -2\ \mathrm{Im}\sum_q \frac{|\langle \mathrm{IAS}|H_1|q\rangle|^2}{E-E_q+i\Gamma_q(E)/2}$$
$$= \sum_q \Gamma_q(E)\ \frac{|\langle \mathrm{IAS}|H_1|q\rangle|^2}{(E-E_q)^2+[\Gamma_q(E)/2]^2}\ , \tag{5}$$

where the state $|q\rangle$ belongs to the Q space. Let us assume that the dominant contribution to Eq. (5) comes from couplings to the isovector monopole (IVM) states (denoted by $|M\rangle$) as doorway states, which is the case in many nuclei near closed shells [11]. Then Eq. (5) is rewritten to be

$$\Gamma_A^{\downarrow}(E_A) = \sum_M \Gamma_M(E_A)\ \frac{|\langle \mathrm{IAS}|H_1|M\rangle|^2}{(E_A-E_M)^2+[\Gamma_M(E_A)/2]^2}\ . \tag{6}$$

Note that $\Gamma_M(E_A)$ in Eq. (6) is the width of the IVM state at the energy E_A of the IAS, but not the physical (experimental) width $\Gamma_M(E_M)$ of the IVM state at its excitation energy. We will show later that $\Gamma_M(E_A)$ is much smaller than $\Gamma_M(E_M)$ as is expected from the proportionality to the density of states and from $E_A<E_M$.

Strictly speaking, as the IVM states are eigenstates of H_0, note those of H, there are nondiagonal terms in Eq. (5) in addition to the diagonal ones ($i=j$),

$$\sum_{i\neq j} \langle \mathrm{IAS}|H_1|M_i\rangle\langle M_i|\ \frac{1}{E-H_0} H_1 \frac{1}{E-H_0} + \cdots |M_j\rangle\langle M_j|H_1|\mathrm{IAS}\rangle, \tag{7}$$

where

$$\frac{1}{E-H} = \frac{1}{E-H_0} + \frac{1}{E-H_0} H_1 \frac{1}{E-H}$$
$$= \frac{1}{E-H_0} + \frac{1}{E-H_0} H_1 \frac{1}{E-H_0} + \frac{1}{E-H_0} H_1 \frac{1}{E-H_0} H_1 \frac{1}{E-H_0} + \cdots \tag{8}$$

is used. These terms can be safely neglected since their contributions prove to be small due to cancellation among them, on the one hand, and, on the other hand, due to quite small nondiagonal matrix elements $(H_1)_{ij}=\langle M_i|H_1|M_j\rangle$ for large isospin states compared to the diagonal ones for the isospin-violating interaction. We adopt the calculated energies of IVM states with the two Hamiltonian $H=H_0+H_1$ for the energy denominator of Eqs. (5) and (6). This choice is justified since the nondiagonal matrix elements $(H_1)_{ij}$ are much smaller than the diagonal ones. In fact, the diagonal matrix element $\langle M_i|1/(E-H)|M_i$ is given as

$$\langle M_i|\ \frac{1}{E-H}\ |M_i\rangle$$
$$= \langle M_i|\ \frac{1}{E-H_0} + \frac{1}{E-H_0} H_1 \frac{1}{E-H_0} + \cdots |M_i\rangle$$
$$= \frac{1}{E-E_i} + \frac{1}{E-E_i} (H_1)_{ii} \frac{1}{E-E_i} + \frac{1}{E-E_i} \times \sum_j (H_1)_{ij} \frac{1}{E-E_j} (H_1)_{ji} \frac{1}{E-E_i} + \cdots$$
$$= \frac{1}{E-E_i-(H_1)_{ii}}\ , \tag{9}$$

since the nondiagonal matrix elements $(H_1)_{ij}$ $(i\neq j)$ can be safely neglected in the last step of the modification above.

An expression similar to Eq. (6) was derived for $\Gamma_A^{\downarrow}$ by Mekjian [11]. The important difference between his and our models is the following. In his model, two states $|\mathrm{IAS}\rangle$ and $|\mathrm{IVM}\rangle$ can be connected by $H=H_0+H_1$ which includes the isospin-invariant nuclear interaction while the isospin-violating term H_1 only has nonzero matrix element in our model. This is the keypoint for treating the isospin-violating part of the interaction consistently to find a relation between the width of the IAS and the isospin impurity of the parent state. Because of smallness of the matrix elements of H_1, the nondiagonal contributions (7) for $\Gamma_A^{\downarrow}$ are negligible in our approach as we pointed out above. He also introduced some *ad hoc* reasoning for neglected couplings of the IAS to IVM states with isospin T and $T+1$. Our model takes into account consistently all isospin multiplets of the IVM, $T-1$, T, and $T+1$, in Eq. (6), without any assumption on their coupling to the IAS.

Using the relation

$$|A\rangle = \frac{1}{\sqrt{2T}} T_-|\pi\rangle, \tag{10}$$

with $T_-=T_x-iT_y=\frac{1}{2}(\tau_x-i\tau_y)$, the spreading width (6) can be rewritten as

$$\Gamma_A^{\downarrow}(E_A) = \sum_i \Gamma^i{}_M(E_A) \frac{1}{2T} \frac{|\langle M_i|H_1T_-|\pi\rangle|^2}{(E_A-E_M^i)^2+(\Gamma_M^i/2)^2}$$
$$\simeq \frac{\Gamma_M}{2T}\sum_i \frac{|\langle M_i|[H_1,T_-]+T_-H_1|\pi\rangle|^2}{(E_A-E_M^i)^2+(\Gamma_M/2)^2} \tag{11}$$

where i labels the three IVM components with isospin $T-1$, and $T+1$, and the Γ_M^i's are assumed to be the same.

We approximate the Coulomb interaction by the one-body potential felt by a proton inside a uniformely charged sphere of radius R [7],

$$V_c = -\frac{Ze^2}{R^3}\sum_i (\tfrac{1}{2}r_i^2-\tfrac{3}{2}R^2)[\tfrac{1}{2}-t_z(i)] \equiv V_{c0}+V_{c1}, \tag{12}$$

and identify the isovector part V_{c1} as H_1. Using the commutator $[t_z, T_-] = -T_-$, we obtain

$$[V_{c1}, T_-] = -\sum_i v_c(i) t_-(i) \equiv -V_c^{(-)}, \tag{13}$$

with $v_c(i) = (Ze^2/R^3)\frac{1}{2} r_i^2$. The width $\Gamma_A^\downarrow$ is now given as

$$\Gamma_A^\downarrow = \Gamma_M \frac{1}{2T} \left\{ \frac{|\langle M;T-1,T-1| - \sqrt{2} V_{-1}^c |\pi\rangle|^2}{(E_A - E_M^{T-1})^2 + (\Gamma_M/2)^2} + \frac{|\langle M;T,T-1| - \sqrt{2} V_{-1}^c |\pi\rangle + \sqrt{2T}\langle M;T,T|V_{c1}|\pi\rangle|^2}{(E_A - E_M^{T})^2 + (\Gamma_M/2)^2} \right.$$
$$\left. + \frac{|\langle M;T+1,T-1| - \sqrt{2} V_{-1}^c |\pi\rangle + \sqrt{2(2T+1)}\langle M;T+1,T|V_{c1}|\pi\rangle|^2}{(E_A - E_M^{T+1})^2 + (\Gamma_M/2)^2} \right\}, \tag{14}$$

where we use the relations

$$T_\pm |T, T_z\rangle = (T \mp T_z)(T \pm T_z + 1) |T, T_z \pm 1\rangle, \tag{15}$$

and $V_{-1}^c = (1/\sqrt{2}) V_c^{(-)}$. The IVM state $|M\rangle$ can be considered as the particle-hole (ph) excitation from both the states $|\pi\rangle$ and $|\mathrm{IAS}\rangle$ in the P space,

$$|M;T+i,T_z\rangle = |\{|P\rangle^T \otimes |\mathrm{ph}^{-1}\rangle^{T=1}\}^{T+i}_{T_z}\rangle. \tag{16}$$

The matrix elements in Eq. (14) are expressed by the reduced matrix element $\tilde{v}_c = (1/\sqrt{3})\langle (\mathrm{ph}^{-1})^{T=1} || v_c || 0\rangle$ to be

$$\langle M;T+i,T-1|V_{-1}^c|T,T\rangle = (TT1-1|T+iT-1)\tilde{v}_c \tag{17}$$

and

$$\langle M;T,T|V_c|T,T\rangle = (TT10|TT)\tilde{v}_c. \tag{18}$$

The spreading width $\Gamma_A^\downarrow$ is then expressed as

$$\Gamma_A^\downarrow = \Gamma_M \tilde{v}_c^2 \frac{1}{T} \left\{ \left(\frac{2T-1}{2T+1}\right) \frac{1}{[(E_A - E_M^{T-1})^2 + (\Gamma_M/2)^2]} \right.$$
$$+ \frac{(T-1)^2}{T+1} \frac{1}{[(E_A - E_M^{T})^2 + (\Gamma_M/2)^2]}$$
$$\left. + \frac{4T^2}{(2T+1)(T+1)} \frac{1}{[(E_A - E_M^{T+1})^2 + (\Gamma_M/2)^2]} \right\}. \tag{19}$$

Note in Eq. (19) that the largest contribution to $\Gamma_A^\downarrow$ comes from the second term on the right-hand side (RHS), namely, from the IVM state $|M;T,T-1\rangle$ in stable nuclei with $T \gg 1$. This is different from Ref. [11] in which only the $|M;T-1,T-1\rangle$ state is considered as the IVM. We can express the isospin dependence of the energy denominator $\Delta E_M^{T'} = E_M^{T'} - E_A$ by using an isovector potential [12]

$$\Delta E_M^{T'} = \hbar\omega + \frac{V_1}{A} \vec{t} \cdot \vec{T}_c, \tag{20}$$

where $\hbar\omega$ and $\vec{t}$ are the excitation energy and isospin of the IVM state, respectively, T_c is the isospin of the core, and V_1 is the symmetry potential coefficient.

A perturbation formula is often used to obtain the isospin mixing probability in the parent nucleus due to the coupling to the IVM state [7]. The mixing amplitude of the parent state $|\pi\rangle = |T,T\rangle$ can be given also by the reduced matrix element $\tilde{v}_c$ as

$$\alpha_{\pi,M}^{T+1} = \frac{1}{E_\pi - E_M^{T+1}} \langle M;T+1,T|V_c|T,T\rangle$$
$$= -\frac{1}{\Delta E_{M\pi}} (TT10|T+1T)\tilde{v}_c$$
$$= -\frac{1}{\Delta E_{M\pi}} \frac{1}{\sqrt{T+1}} \tilde{v}_c, \tag{21}$$

where $\Delta E_{M\pi} = E_M^{T+1} - E_\pi$. Comparing Eqs. (19) and (21), we can obtain a relation between $\Gamma_A^\downarrow$ and $\alpha_{\pi,M}^{T+1}$ as

$$\Gamma_A^\downarrow = \Gamma_M (\alpha_{\pi,M}^{T+1})^2 \frac{T+1}{T} \left\{ \left(\frac{2T-1}{2T+1}\right) \frac{1}{(\Delta E_M^{T-1})^2} \right.$$
$$+ \frac{(T-1)^2}{T+1} \frac{1}{(\Delta E_M^{T})^2} + \frac{4T^2}{(2T+1)(T+1)} \frac{1}{(\Delta E_M^{T+1})^2} \Big\}$$
$$\times (\Delta E_{M\pi})^2. \tag{22}$$

In the case of superallowed Fermi β decay, the isospin mixing yields the second-order effects in the Fermi transition probability between the parent state and the IAS [2]. On the other hand, formula (22) shows a direct relation between $\Gamma_A^\downarrow$ and the mixing probability $(\alpha_{\pi,M}^{T+1})^2$ with some coefficients.

The isospin mixing probability $(\alpha_{\pi M}^{T+1})^2$ is estimated by using the energy-weighted sum rule for the IVM states [7],

TABLE I. Widths $\Gamma_M(E)$ in ^{208}Bi at the corresponding energies of the main RPA IVM states and at E_A (second column). All values are in MeV.

$\Gamma_M(E)$	At its energy	At the IAS ($E_A = 18.8$)
$E = 39.0$	6.00	1.79
$E = 45.7$	5.34	0.41
$E = 50.7$	4.90	0.16

TABLE II. Spreading widths of the IAS, $\Gamma_A^{\downarrow}$, in Sn, Sb, I, and Bi isotopes. T denotes isospin, and $P_{\rm IM}$ given by Eq. (24) is the isospin mixing probability without the geometrical factor $(T+1)$. $\Gamma_A^{\downarrow}$ calculated by using Eq. (25) with $\Gamma_M(E_A)$ given are shown with experimental values taken from Ref. [15].

			$\Gamma_A^{\downarrow}$ (keV)		
^{A}Sn	T	$P_{\rm IM}$	$\Gamma_M=500$ keV	$\Gamma_M=700$ keV	Expt.
^{111}Sn	11/2	0.039 264	20.5	28.7	17±3
^{113}Sn	13/2	0.039 733	20.9	29.2	22±10
^{115}Sn	15/2	0.040 201	21.2	29.7	22±8
^{117}Sn	17/2	0.040 665	21.6	30.3	22±8
^{119}Sn	19/2	0.041 129	22.0	30.8	36±9
^{121}Sn	21/2	0.041 588	22.4	31.3	40±8
^{123}Sn	23/2	0.042 045	22.7	31.8	39±8
			$\Gamma_A^{\downarrow}$ (keV)		
^{A}Sb	T	$P_{\rm IM}$	$\Gamma_M=600$ keV	$\Gamma_M=700$ keV	Expt.
^{113}Sb	11/2	0.041 338	25.8	30.2	17±5
^{115}Sb	13/2	0.041 826	26.3	30.7	22±5
^{117}Sb	15/2	0.042 310	26.8	31.2	30±5
^{119}Sb	17/2	0.042 790	27.2	31.8	29±5
^{121}Sb	19/2	0.043 268	27.7	32.3	32±5
^{123}Sb	21/2	0.043 743	28.2	32.9	34±5
				$\Gamma_A^{\downarrow}$ (keV)	
^{A}I	T	$P_{\rm IM}$		$\Gamma_M=500$ keV	Expt.
^{125}I	19/2	0.047 751		25.4	23±5
^{127}I	21/2	0.048 261		25.8	24±5
^{129}I	23/2	0.048 766		26.2	28±5
				$\Gamma_A^{\downarrow}$ (keV)	
^{A}Bi	T	$P_{\rm IM}$		$\Gamma_M=450$ keV	Expt.
^{207}Bi	41/2	0.163 925		78.2	78±8
^{209}Bi	43/2	0.164 979		79.0	75±5

$$(\alpha_{\pi M}^{T+1})^2=\frac{1}{T+1}P_{\rm IM}. \tag{23}$$

with the value

$$P_{\rm IM}=Z^2A^{2/3}\times 6.8\times 10^{-7}. \tag{24}$$

$\Gamma_A^{\downarrow}$ then becomes

$$\Gamma_A^{\downarrow}=\Gamma_M(E_A)P_{\rm IM}\frac{1}{T}\left\{\left(\frac{2T-1}{2T+1}\right)\frac{1}{[1-(V_1/A\hbar\omega)(T+1)]^2}+\frac{(T-1)^2}{T+1}\frac{1}{[1-(V_1/A\hbar\omega)]^2}+\frac{4T^2}{(T+1)(2T+1)}\frac{1}{[1+(V_1/A\hbar\omega)T]^2}\right\}, \tag{25}$$

where Eq. (20) has been explicitly used.

As was pointed out before, $\Gamma_M(E_A)$ is the width of the IVM state at the energy of the IAS. It is not possible to obtain experimental information on $\Gamma_M(E_A)$, while there is very little experimental information on the value of the width $\Gamma_M(E_M)$ of the IVM. In order to estimate the value of $\Gamma_M(E_A)$, this quantity has been numerically calculated in ^{208}Bi, according to the formalism developed in Ref. [13]. First, a self-consistent discrete random phase approximation (RPA) calculation of the IVM strength has been performed by employing the Skyrme interaction SIII. Discrete states at positive energy are obtained by diagonalizing the Hartree-Fock mean field on a harmonic oscillator basis ($\hbar\omega=6.2$ MeV, $N_{\rm shell}=15$). The particle-hole basis has been checked to be large enough so that the results exhaust more than 95% of the IVM energy-weighted sum rule. Three main peaks around the excitation energies $E=39.0$, 45.7, and 50.7 MeV collect together most of the monopole strength and their wave functions are labeled, respectively, $|M_1\rangle$, $|M_2\rangle$, and $|M_3\rangle$. The total width of the state $|M_i\rangle$ lying at energy E is determined as

$$\Gamma_M(E)=-2\ {\rm Im}\langle M_i|W^{\uparrow}(E)+W^{\downarrow}(E)|M_i\rangle. \tag{26}$$

The operators $W^{\uparrow,\downarrow}$ are defined in Ref. [13], where also the detailed calculation of their matrix elements is sketched. $W^{\uparrow}$ couples the discrete RPA states with nuclear configurations in which one particle is in the continuum; therefore, the imaginary part of its diagonal matrix elements is intended to reproduce the escape width of the RPA states. $W^{\downarrow}$ couples the discrete RPA states with doorway states made up with particle-hole configurations plus a low-lying collective vibra-

tion, and its presence in Eq. (26) should ensure that also the spreading width is taken into account. The widths defined by Eq. (26) for the three states $|M_i\rangle$ $(i=1,\ldots,3)$ are shown in Table I. The second column displays the widths $\Gamma_M(E_M)$ of each state at its own energy. They are estimated to be about 5 MeV, which looks reasonable in comparisons with other nuclei: $\Gamma_M(E_M)=10-15$ MeV in ^{16}O and ^{40}Ca [14]. The widths $\Gamma_M(E_A)$ of each state at the energy of the IAS are in the last column of Table I: They are rather state dependent, anyhow much smaller than $\Gamma_M(E_M)$. This is due to the fact that $\Gamma_M(E)$ depends on the density of states at the energy E, and E_A is much smaller than E_M. The average of the three values $\Gamma_M(E_A)$ is about 500 keV with the weight of the transition strengths (which are not shown). We therefore used the values $\Gamma_M(E_A)=500-700$ keV for the following analysis of the spreading width of the IAS.

The spreading widths of the IAS calculated according to Eq. (25) are tabulated in Table II for Sn, Sb, In, and Bi isotopes and compared with the experimental data [15]. The adopted $\Gamma_M(E_A)$ value is also shown in the upper line of each table. V_1 and $\hbar\omega$ in Eq. (20) are taken to be 100 and 30 MeV (28 MeV in the case for ^{208}Bi), respectively [10,16]. The calculated widths show in general good agreement with the empirical values. The experimental width increases as the isospin T increases for the Sn, Sb, and In isotopes. The isotope dependence of the calculated values is consistent with the empirical one, although the rate of calculated increase is smaller than that of the empirical increase. We should notice that the calculated increase is due to the contribution of the second term in Eq. (25) [from the IVM state $|M;T,T_z=T,T-1\rangle$ in Eq. (14)]. If there were only the contribution from the first term (from the IVM state $|M;T-1,T-1\rangle$) in Eq. (25), which is the case in Ref. [11], the width itself is one order of magnitude smaller than the values in Table II and the calculated values decrease as the function of the isospin T, contradicting the empirical tendency. The main configurations of the IVM state, $|M;T,T-1\rangle$, are those with ph excitations built on the IAS. Important contributions, thus, come from coupling to the 2p-2h states top of the IAS, which are not included in the previous approach [7,11].

In summary, we derived a relation between the spreading width of the IAS, $\Gamma_A^{\downarrow}$, and the isospin mixing probability of the corresponding parent state. It is pointed out that the model basis should have good isospin in order to obtain the explicit relation between the width $\Gamma_A^{\downarrow}$ and the isospin mixing in the ground states. The dominant contribution to the width $\Gamma_A^{\downarrow}$ comes from the coupling with the IVM states $|M;T,T-1\rangle$ and $|M;T,T\rangle$. With the use of a model calculation of the isospin mixing probability, the values $\Gamma_A^{\downarrow}$ are obtained for the Sn, Sb, In, and Bi isotopes and show quantitatively reasonable agreement with empirical data. The calculated increase of the width as the isospin increases for each isotope is consistent with the experimental observations while the increase rate of experiments is larger than that of the calculations. The present analysis based on formula (25) suggest the intimate connection between the spreading width of the IAS and the isospin impurity. It would be quite interesting to study experimentally the width $\Gamma_A^{\downarrow}$ of nuclei near the proton drip line since the isospin mixing in these nuclei is expected one order of magnitude larger than those of stable nuclei so that the width of the IAS will be also much larger than that of stable nuclei.

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