Elastic and inelastic scattering of 1.37 GeV α particles from ^{12}C and ^{40,42,44,48}Ca

Ji-feng Liu,[1,2] Yu-shun Zhang,[3,4] Chao-yun Yang,[5] Jun-feng Shen,[6] and B. A. Robson[1]

[1]*Department of Theoretical Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT0200, Australia*

[2]*Department of Physics, Guangxi Normal University, Guilin 541001, People's Republic of China*

[3]*China Center of Advanced Science and Technology, (World Laboratory), Beijing 100080, People's Republic of China*

[4]*Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4-1), Beijing 100039, People's Republic of China*

[5]*Department of Physics, The School of Zhejiang Light Industry, Hangzhou 310015, People's Republic of China*

[6]*Department of Physics, Zhejiang University, Hangzhou 310027, People's Republic of China*

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Elastic and inelastic scattering data of 1.37 GeV α particles on ^{12}C and ^{40,42,44,48}Ca are analyzed within the framework of the Glauber theory. Collective excitations to one-phonon levels are treated using the Tassie model. The effect of the coupling between the elastic and inelastic channels is considered. It is shown that a phase variation of the nucleon-nucleon elastic scattering amplitude leads to a large increase in the calculated differential cross section. The presence of a phase variation leads to a substantial improvement. [S0556-2813(96)00811-4]

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I. INTRODUCTION

Many experimental data and theories [1,2] have already been accumulated about the collision of a low-energy ($E_\alpha \leqslant$ 150 MeV) α particle and a nucleus.

In α-nucleus collisions at intermediate and high energies, Bonin *et al.* measured the elastic scattering on ^{58}Ni, ^{116}Sn, and ^{208}Pb and studied optical potentials with conventional Woods-Saxon (WS) shapes [3]. The elastic and inelastic scattering on ^{12}C of α particles to the lowest 2^+, 3^-, and 0^+ levels of ^{12}C is measured by Chaumeaux *et al.*. at an energy of 1.37 GeV [4]. In 1977, Alkhazov *et al.* measured the elastic and inelastic scattering cross sections of α particles on Ca isotopes at 1.37 GeV using the Saturnes Synchrotron at Saclay [5]. They analyzed the data using the Glauber model [6].

In fact, the Glauber model has been extremely successful in describing high-energy hadron-nucleus scattering data for a variety of hadronic projectiles and targets [7]. Moreover, it has been extended to include composite projectiles at high energies. For composite particle scattering on a nucleus, the multiple scattering picture is clearly less well founded than in the case of proton-nucleus scattering.

The basic theoretical concepts of the Glauber model of composite-particle scattering were developed many years ago [8,9]. However, the aim of the present paper is to report on an analysis of 1.37 GeV α particle scattering on ^{12}C and Ca isotopes which is based on a semiphenomenological approach [10–12]. The nuclear excitation is described in terms of the collective model under the adiabatic approximation [13]. The long-range correlation described by the coupling of the elastic to the (collective) inelastic channels can be treated based on this approach.

The problem is formulated in Sec. II and the results of the calculation are presented and discussed in Sec. III.

II. FORMULATION

According to Glauber's theory, the multiple scattering amplitude between the composite particle systems A and B, i.e., for the $A+B\rightarrow A^*+B^*$ scattering process, is given by [8,9,11]

$$F_{f_i}(\vec{q})=H(\vec{q})\frac{ik}{2\pi}\int d\vec{b}e^{i\vec{q}\cdot\vec{b}}$$
$$\times\int d\vec{x}d\vec{y}\,\psi^*_{A_f}(\vec{x})\psi^*_{B_f}(\vec{y})\Gamma\psi_{A_i}(\vec{x})\psi_{B_i}(\vec{y}), \tag{2.1}$$

where $H(\vec{q})$ is the correction factor for the center of mass [8], k is the momentum of the incident particle system, $\vec{q}$ is the momentum transfer, $\vec{b}$ is the impact parameter, and $\psi^*_{A_f}(\vec{x}),\psi^*_{B_f}(\vec{y})$ and $\psi_{A_i}(\vec{x})\psi_{B_i}(\vec{y})$ are the final and the initial states of the target and the incident particle system, respectively. $\vec{x}$ stands for $\vec{x}_1\cdots\vec{x}_A$, and the coordinate $\vec{y}$ may be defined in the same manner:

$$d\vec{x}=\prod_{i=1}^{A}d\vec{x}_i,\quad d\vec{y}=\prod_{\alpha=1}^{B}d\vec{y}_\alpha.$$

The total profile function Γ is given by

$$\Gamma=1-\prod_{i=1}^{A}\prod_{\alpha=1}^{B}[1-\Gamma_{i\alpha}(\vec{b}+\vec{s}_i-\vec{\sigma}_\alpha)], \tag{2.2}$$

where $\vec{s}_i$ and $\vec{\sigma}_\alpha$ are the projections of the particle coordinate $\vec{x}_i$ and $\vec{y}_\alpha$ on the plane perpendicular to $\vec{k}$, respectively, and $\Gamma_{i\alpha}(\vec{b})$ is the two-body profile function. The relation between this function and the two-body scattering amplitude is

$$\Gamma_{i\alpha}(\vec{b})=\frac{1}{2\pi ik'}\int d\vec{q}e^{-i\vec{q}\cdot\vec{b}}f_{i\alpha}(\vec{q}). \tag{2.3}$$

In a high-energy collision, if the spin effect is neglected, we can use the conventional high-energy parametrization of $f_{i\alpha}(\vec{q})$ [10],

$$f_{i\alpha}(\vec{q})=\frac{ik'}{4\pi}\sigma(1-i\epsilon)e^{-aq^2/2}, \tag{2.4}$$

where σ is the total cross section of nucleon-nucleon (NN) scattering, and ϵ is the ratio of the real to the imaginary part of the forward amplitude. Typically, a is taken to be complex:

$$a=\beta^2+i\gamma^2, \tag{2.5}$$

which gives a simple phase variation of the NN amplitude, linear in q^2(or in $t=-\hbar^2q^2$).

In the following we apply Eq. (2.1) to study the elastic and inelastic scattering of 1.37 GeV α particles from collective nuclei. Under the adiabatic approximation, the S matrix may be written as

$$\begin{aligned}S_{fi}(b)=\int &\psi^*_{A_f}(\vec{x}_1\cdots\vec{x}_A)\psi^*_{B_f}(\vec{y}_1\cdots\vec{y}_B)\\ &\times\exp\left(\frac{-i}{\hbar v}\sum_{i=1}^{A}\sum_{\alpha=1}^{4}\int V(\vec{r}-\vec{x}_i+\vec{y}_\alpha)\right)\\ &\times\psi_{A_i}(\vec{x}_1\cdots\vec{x}_A)\psi_{B_i}(\vec{y}_1\cdots\vec{y}_B)\prod_{i=1}^{A}d\vec{x}_i\prod_{\alpha=1}^{4}d\vec{y}_\alpha.\end{aligned} \tag{2.6}$$

At high energy, it is convenient to use the probability density to describe the state, i.e.,

$$\psi^*_{B_f}\psi_{B_i}=\prod_{\alpha=1}^{4}\rho(\vec{y}_\alpha). \tag{2.7}$$

$\vec{f}_{1-t}(q)$ is called the scattering amplitude operator. Under the operation of $\vec{f}_{1-t}(q)$ the ground state $|c,0\rangle$ transits to the collective excited state $|c,f\rangle$. Equation (2.6) may be written as

$$\begin{aligned}S_{fo}(b)=\Bigg\langle c,f\Bigg|\Bigg[\int\prod_{\alpha=1}^{4}\rho(\vec{y}_\alpha)d\vec{y}_\alpha\exp\left(\frac{-i}{\hbar v}\sum_{\alpha=1}^{4}\int_{\mathcal{D}}d\vec{x}dz\right.\\ \left.\times V(\vec{r}-\vec{x}+\vec{y}_\alpha)\right)\Bigg]\Bigg|c,o\Bigg\rangle.\end{aligned} \tag{2.8}$$

Then the profile function can be expressed by the scattering operator $\hat{f}_{1-t}(\vec{q})$ profile function as

$$\begin{aligned}\Gamma_{(1-t)}(b)&=1-\exp\left(\frac{-i}{\hbar v}\int_D d\vec{x}dzV(\vec{r}-\vec{x})\right)\\ &=\frac{1}{2\pi ik'}\int d\vec{q}'e^{-i\vec{q}'\cdot\vec{b}}\hat{f}_{(1-t)}(\vec{q}'),\end{aligned} \tag{2.9}$$

where $V(\vec{r}-\vec{x})$ is the interaction potential between the free particles and the nucleus A. Substituting Eq. (2.9) into Eq. (2.8) yields

$$\begin{aligned}S_{fo}(b)=\Bigg\langle c,f\Bigg|\Bigg[1-\frac{1}{2\pi ik'}\\ \times\int d\vec{q}'d\vec{y}\rho(\vec{y})e^{-i\vec{q}\cdot(s_y+b)}\cdot\hat{f}_{(1-t)}(\vec{q}')\Bigg]^4\Bigg|c,o\Bigg\rangle.\end{aligned} \tag{2.10}$$

Let the form factor $S_\alpha(q)$ of α particles be

$$S_\alpha(q)=\int d\vec{y}\rho(\vec{y})e^{-i\vec{q}\cdot\vec{y}} \tag{2.11}$$

The S matrix between an α particle and the target nucleus can be rewritten as

$$\begin{aligned}&S_{fo}(b)\\ &=\Bigg\langle c,f\Bigg|\Bigg[1-\frac{1}{2\pi ik'}\int d\vec{q}'d\vec{y}S_\alpha(\vec{y}')\hat{f}_{(1-t)}e^{-i\vec{q}'\cdot\vec{b}}\Bigg]^4\Bigg|c,o\Bigg\rangle\end{aligned} \tag{2.12}$$

and

$$S_\alpha(\vec{q}')=D_1\exp\left(\frac{-q'^2}{4k_1^2}\right)-D_2\exp\left(\frac{-q'^2}{4k_2^2}\right),$$

$$D_1=\frac{k_2^3}{(k_2^3-ck_1^3)},\quad D_2=\frac{ck_1^3}{(k_2^3-ck_1^3)}, \tag{2.13}$$

where c,k_1,k_2 are parameters [14], $\vec{k}'$ is the momentum of the projectile, and q' is the momentum transfer of the projectile.

The correction factor for the center of mass is generally calculated [8] as follows:

$$H(q)=\exp\left\{\frac{1}{6}q^2[\langle r^2_{A_1}\rangle/A_1+\langle r^2_{A_2}\rangle/A_2]\right\}, \tag{2.14}$$

where $\langle r_A^2\rangle^{1/2}$ is the rms radius of the nucleus A.

We consider a nucleus with a set of quadrupole and octupole vibrations. According to Glauber theory $\hat{f}_{(1-t)}(\vec{q}')$ is given by [6]

$$\hat{f}_{(1-t)}(\vec{q}')=\frac{ik'}{2\pi}\int d^2b'e^{i\vec{q}'\cdot\vec{b}'}[\delta_{fo}-e^{i\chi_0(b')}]. \tag{2.15}$$

Let us first concentrate on $\chi_0(b)$, which, as we will see later, describes all the main features of high-energy small-angle scattering for intermediate and heavy mass nuclei. For future convenience we write $\chi_0(b)$ in the form

$$\chi_0(b')=\frac{A}{2\pi k'}\int d^2qe^{-\vec{q}\cdot\vec{b}'}f_{i\alpha}(q)\hat{F}(q) \tag{2.16}$$

and

$$\hat{F}(q)=\int e^{i\vec{q}\cdot\vec{r}}\hat{\rho}(\vec{r})d\vec{r}. \tag{2.17}$$

Let us now be more specific and assume that the nuclei under consideration could be described by the Tassie hydrodynamical model [15], so that the density operator is of the form

$$\hat{\rho}(r)=\rho_0(r)+\sum_{LM}\rho_L(r)[b_{LM}Y_{LM}(\Omega)+b^{\dagger}_{LM}Y^{*}_{LM}(\Omega)], \tag{2.18}$$

where $\rho_0(r)$ is the ground state density, $b^{\dagger}_{LM}$ and b_{LM} are the one-phonon creation and annihilation operators, respectively, and $Y_{LM}(\Omega)$ are the spherical harmonics. Further, the transition density $\rho_L(r)$ is given by

$$\rho_L(r)=N_L r^{L-1}\frac{d}{dr}\rho_0(r), \tag{2.19}$$

where N_L is the transition strength parameter. Then

$$\chi_0(b')=\chi_{00}(b')+\sum_{LM}\chi_{LM}(b')[A_{LM}+A^{\dagger}_{LM}], \tag{2.20}$$

and

$$\chi_{00}(b')=\frac{A}{k'}\int_0^{\infty}j_0(qb)f_{i\alpha}(q)F_0(q)q\,dq, \tag{2.21}$$

here $A_{LM}=b_{LM}e^{iM\varphi}$, $A^{\dagger}_{LM}=b^{\dagger}_{LM}e^{-iM\varphi}$, and the new operators A_{LM} and $A^{\dagger}_{LM}$ satisfy the same commutation rules as b_{LM} and $b^{\dagger}_{LM}$. A further hypothesis $Z'\simeq 0$ [7] is introduced, so that Y_{LM} has a very simple representation and

$$\chi_{LM}(b)=\begin{cases}0 \quad \text{for } L-M=\text{odd}\\ (-)^M\dfrac{A}{k'}\left(\dfrac{2L+1}{4\pi}\right)^{1/2}\dfrac{[(L-m)!(L+M)!]}{(L-M)!!(L+M)!!}\cdot\displaystyle\int_0^{\infty}J_M(qb)f_{i\alpha}(q)F_L(q)q\,dq \quad \text{for } L-M=\text{even},\end{cases} \tag{2.22}$$

$$F_L(q)=4\pi\int_0^{\infty}j_L(qr)\rho_L(r)r^2dr. \tag{2.23}$$

Substituting Eq. (2.15) into Eq. (2.12) and integrating for $d\vec{q}$, noticing $A_{LM}|0\rangle=0$ and $\langle 0|A^{\dagger}_{LM}=0$, we get

$$\begin{aligned}&\left[1-\frac{1}{2\pi ik'}\int d^2q' S_{\alpha}(q')e^{-i\vec{q}'\cdot\vec{b}}\hat{f}_{(1-t)}(\vec{q}')\right]^4\\ &=\Bigg[1-\int b'db' II(bb')+\int b'db' e^{i\chi_{00}(b')}\\ &\quad\times\exp\Bigg(-\frac{1}{2}\sum_{LM}\chi^2_{LM}(b')+i\sum_{LM}\chi_{LM}(b')A^{\dagger}_{LM}\\ &\quad+i\sum_{LM}\chi_{LM}(b')A_{LM}\Bigg)\cdot II(bb')\Bigg]^4,\end{aligned} \tag{2.24}$$

where

$$II(bb')=2D_1k_1^2e^{-k_1^2(b-b')^2}-2D_2k_2^2e^{-k_2^2(b-b')^2}. \tag{2.25}$$

Then the S matrix for elastic scattering is

$$S_{00}(b)=[a(b)+T(b)]^4, \tag{2.26}$$

$$T^n(b)=t^n(b)\left[\exp\left(-\sum_{LM}\chi^2_{LM}(b')\right)\right]^{n(n-1)/2}, \tag{2.27}$$

$$t(b)=\int b'db' II(bb')e^{i\chi_N(b')}, \tag{2.28}$$

$$a(b)=\int b'db' II(bb'), \tag{2.29}$$

$$\chi_N(b')=\chi_{00}(b')+\chi_{cp}(b'), \tag{2.30}$$

$$\chi_{cp}(b')=\frac{1}{2}i\sum_{LM}\chi^2_{LM}(b'). \tag{2.31}$$

The first term in Eq. (2.30) is the so-called optical limit result and depends only on the ground state density of the target. The second term $\chi_{cp}(b)$ describes the effect of coupling the elastic with the (one-phonon) inelastic channels on the elastic phase in which the target nucleus makes a virtual transition to an excited state and then decays back to the ground state. The S matrix from the ground state (no phonon) to the excited state (N phonons) is

$$S^{(L)}_{fo}(b)=\left\langle N\left|\sum_{j=1}^{4}C_4^j[\widetilde{A}_0(b)+\widetilde{A}_1(b)+\cdots]^j\right|0\right\rangle, \tag{2.32}$$

$$\widetilde{A}_0(b)=\int b'db' e^{i\chi_N(b')}\cdot II(bb'), \tag{2.33}$$

$$\widetilde{A}_1(b)=\int b'db' e^{i\chi_N(b')}II(bb')\left(-i\sum_{LM}\chi_{LM}(b')\right)A^{\dagger}_{LM}. \tag{2.34}$$

To specify the state of a phonon we must also give its total angular momentum L and the Z component M. We thus have

$$S^{(L)}_{fo}(b)=\langle 0|b_{LM}0(b)A^{\dagger}_{LM}|0\rangle\equiv e^{iM\varphi}f^{(L)}_{LM}0(b), \tag{2.35}$$

$$\begin{aligned}0(b)=&(-4+12\widetilde{A}_0-12\widetilde{A}_0^2+4\widetilde{A}_0^3)\int b'db' e^{i\chi_N(b')}\\ &\times[-i\chi_{LM}(b')]II(bb').\end{aligned} \tag{2.36}$$

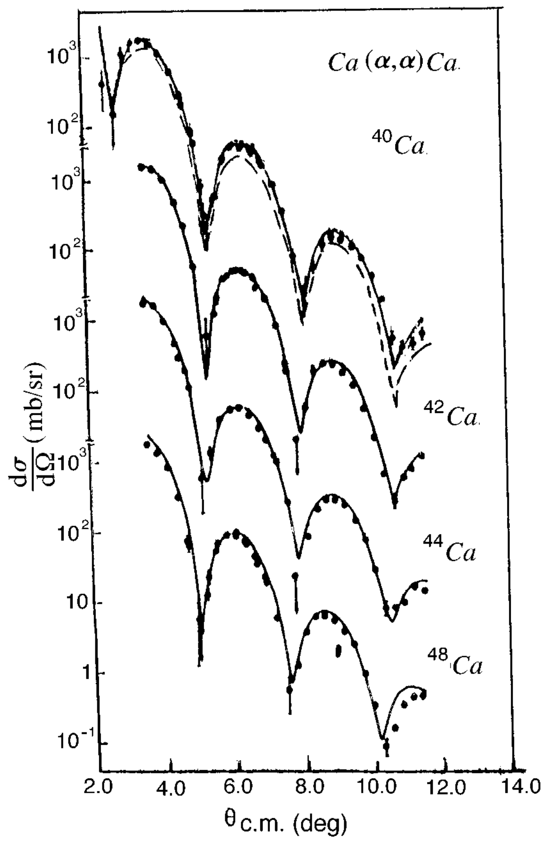

FIG. 1. Elastic scattering of 1.37 GeV α particles from several Ca isotopes. Solid curves show the $\gamma^2 \neq 0$ calculated result. Dashed curve (for ^{40}Ca only) shows the $\gamma^2=0$ calculated result.

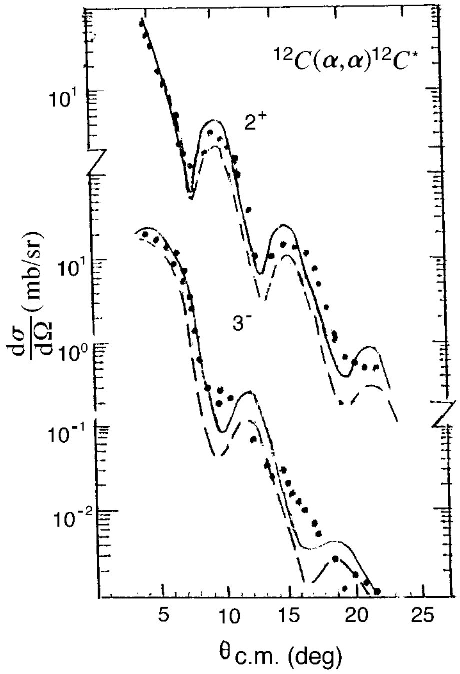

FIG. 3. The differential cross section for the 2^+ and 3^- excited states ^{12}C. The solid curves show the $\gamma^2 \neq 0$ calculated result. Dashed curves show the $\gamma^2=0$ calculated result.

III. RESULTS AND DISCUSSION

In this section the results of a calculation for 1.37 GeV α particle scattering on ^{12}C and ^{40,42,44,48}Ca are presented and compared to experiment. The main inputs needed in the calculations are the NN scattering amplitude and the ground state and transition densities.

For the NN parameters we take the values at 344 MeV as given by Alberi *et al.* [16] and which were found to give satisfactory results for the elastic d-d scattering at the incident deuteron laboratory momentum 1.75 GeV/c. The parameter values are

$$\sigma_{pp}=27 \text{ mb}, \quad \beta_{pp}^2=0.44 \ (\text{GeV}/c)^{-2}, \quad \epsilon_{pp}=0.6,$$

$$\sigma_{np}=34 \text{ mb}, \quad \beta_{np}^2=2.0 \ (\text{GeV}/c)^{-2}, \quad \epsilon_{np}=0.$$

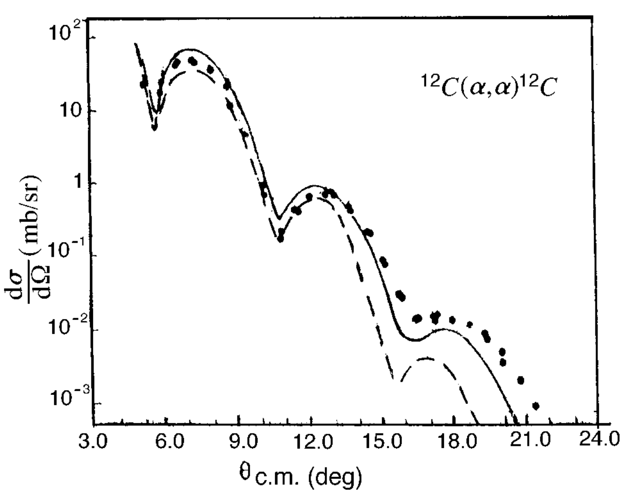

FIG. 2. Elastic scattering of 1.37 GeV α particles on ^{12}C. The solid curves shows the $\gamma^2 \neq 0$ calculated result. Dashed curves show the $\gamma^2=0$ calculated results.

In the calculations we have used the average values of the neutron and proton parameters in the NN scattering amplitude. However, it has been verified that the predictions of the average parameter values are not significantly different from the nonaveraged ones. The parameter γ^2 leads to an overall phase factor $e^{-i\gamma^2 q^2/2}$ which cannot be obtained directly from NN scattering measurements. It will be treated as a free parameter in N-N collisions [10]. It will be fixed in nuclear collisions and hence will be independent of the nuclei involved in the collision, provided that the kinetic energies per nucleon are the same in all cases. Thus the same value of γ^2 will be used in describing all nucleus-nucleus measurements at a given kinetic energy per nucleon. We take

$$\gamma^2=10.5 \ (\text{GeV}/c)^{-2}.$$

The nuclear density is represented by the Fermi distribution as follows:

$$\rho_0(r)=\rho_0/(1+e^{(r-c'/a')}).$$

The density parameters of ^{12}C and ^{40,42,44,48}Ca used in the calculations are given in Table I.

The strength parameter N_L in Eq. (2.19) can be determined from the strength of EL transitions

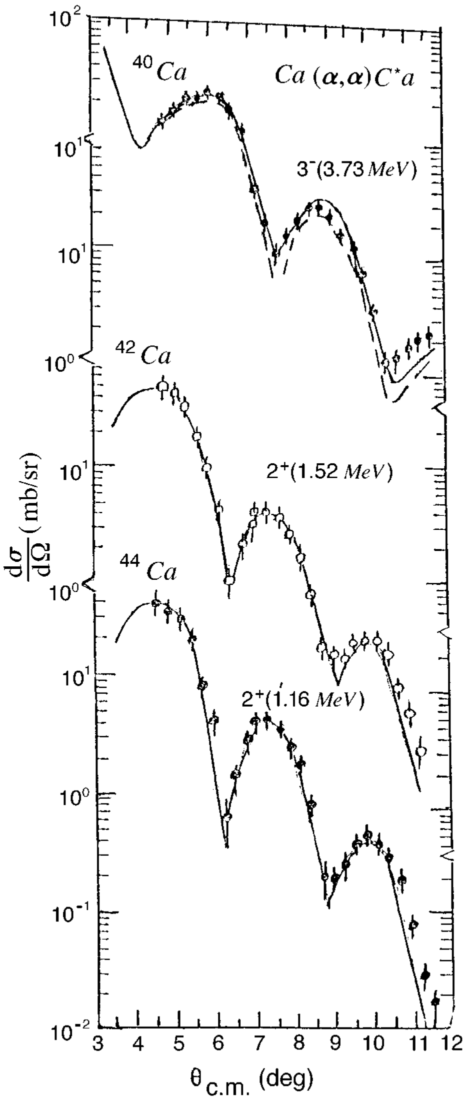

FIG. 4. The differential cross section for the 2^+ and 3^- excited states in ^{40,42,44}Ca. The solid curves show the $\gamma \neq 0$ calculated result. Dashed curves show the $\gamma^2 = 0$ calculated result.

$$B^{(p)}(\mathrm{EL}) = (2L+1)\left[\int_0^\infty r^{L+2} \rho_L^p(r) dr \right]^2 .$$

Much less is known about the neutron transition matrix element $B^{(n)}$. In the case of $N=Z$ nuclei ^{12}C and ^{40,42,44,48}Ca, the neutron and proton transition densities are assumed to be identical. Thus, the strength parameters of neutrons and protons are equal. The parameters used in the calculations are listed in Table I.

Using the parameters above, we calculate the elastic α-^{42,44,48}Ca scattering at 1.37 GeV. The results are shown by the solid curves in Fig. 1. It is seen that the data are very nicely reproduced. Thus the value of γ^2 that fits the ^{40}Ca data nicely accounts equally well for the data on the neighboring nuclei. This implies that the effective NN amplitude nearly saturates at the energy under consideration.

Our most important result is presented in Fig. 2; experimental data are compared to our calculation for the elastic α-^{12}C scattering at 1.37 GeV. The data are fairly well reproduced. Although the agreement with the data in this case is not as good as for the Ca isotopes, yet it is very satisfying to find that the height of the first maximum and the positions of the minima are predicted fairly accurately. As a matter of fact the present calculations provide considerable improvement over the RP model results [7,13] and we are unaware of any realistic parameter-free calculation which accounts for the data so well.

TABLE I. Parameters used in the calculations.

Nuclei	C'(fm)	a'(fm)	$\langle r^2 \rangle^{1/2}$	Refs.	$B(E2)$ [19] (fm)^4	$B(E3)$ [20] (fm)^6
α			1.696	[17]		
^{12}C	2.320	0.420	2.472	[18]	41.5	610.8
^{40}Ca	3.661	0.594	3.490	[5]	96	20400
^{42}Ca	3.627	0.641	3.540	[5]	420	9100
^{44}Ca	3.655	0.625	3.550	[5]	470	5600
^{48}Ca	3.837	0.550	3.580	[5]	84	8300

The results for the $2^+[\alpha(1.37\ \mathrm{GeV}) + {}^{12}\mathrm{C}]$, $3^-[\alpha(1.37\ \mathrm{GeV}) + {}^{12}\mathrm{C}]$ and the $2^+[\alpha(1.37\ \mathrm{GeV}) + {}^{42,44}\mathrm{Ca}]$, $3^-[\alpha(1.37\ \mathrm{GeV}) + {}^{40}\mathrm{Ca}]$ angular distributions are shown in Figs. 3 and 4, respectively, as solid curves. It is seen that the data are fairly well reproduced except $3^-[\alpha(1.37\ \mathrm{GeV}) + {}^{12}\mathrm{C}]$ in Fig. 3.

Next we study the effect of including the long-range correlation $\gamma^2 = 0$; i.e., the coupling between the elastic and the low-lying inelastic channels is dominantly collective. This is achieved by evaluating χ_{cp} as given by Eq. (2.31) only for the 2^+ and 3^- states and substituting it in Eqs. (2.26) and (2.35). The result is shown by the dashed curves in Figs. 1–4. The situation seems to improve in two respects: First, the theoretical cross sections are now closer to the experimental results for the Ca isotopes and at smaller angles for ^{12}C; second, the positions of the calculated second and third minima are now very close to the positions of the corresponding experimental minima. However, the calculated large-angle cross sections are low for the ^{12}C. The effect of the coupling on the inelastic cross section (not shown) is found to be small.

In summary, the essential feature of the presently proposed method is the use of a phase variation of the nucleon-nucleon elastic scattering amplitude which agrees with the empirical amplitude at low q's at the appropriate energy, and its large-q behavior is left adjustable in terms of one free parameter. This amplitude, when calibrated on ^{40}Ca for 1.37 GeV α scattering, not only reproduces the data on the other Ca isotopes very nicely but also gives a fairly good account of the ^{12}C data.

The effect of the phase variation is to eliminate minima or to make them shallower and to generally increase cross sections even at the momentum transfers where no minima originally occurred [19,20].

Franco and Yin have suggested that the phase of the N-N scattering amplitude should vary with the momentum transfer. So far the physical origin of this phase variation has not yet been settled. This phase modifies the ratio of the real part to the imaginary part of the forward amplitude and makes the diffraction pattern shallower.

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