Hartree-Fock-Bogoliubov approximation to relativistic nuclear matter

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We extend the mean-field approximation to relativistic quantum hadrodynamics to include pairing in nuclear matter. This approach permits a simultaneous description of the energy gap in the single-particle spectrum and of the saturation point of nuclear matter. The mean field associated with the pairing correlations has large scalar and vector components that cancel each other resulting in a gap parameter that agrees with the results of nonrelativistic calculations. [S0556-2813(96)03110-X]

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I. INTRODUCTION

The traditional nonrelativistic description of pairing correlations in nuclear physics is made in accordance with the BCS model and has been successful in accounting for many nuclear phenomena [1–10]. It is a simple way of improving the independent-particle approach to the shell model by introducing the short-range correlations associated with the pairing of nucleons in the nuclear medium and responsible for the superfluid phase of the nucleus. Similarly, the long-range correlations associated with collective phenomena in nuclei are well described by the nonrelativistic random phase approximation (RPA) [11].

On the other hand, it is well known that the nonrelativistic approaches using realistic two-body interactions are not able to account for basic phenomena, such as the spin-orbit part of the nucleon-nucleus interaction and the saturation properties of nuclear matter [12–20]. These properties can be well described by a relativistic formulation [21–25] and the success of the mean-field approach to quantum hadrodynamics (QHD) initially developed by Walecka and collaborators [15,26–30], suggests that an enlarged version of the relativistic mean-field approach could also be considered to account for the residual correlations between nucleons. While a consistent relativistic formulation of RPA based on the mean-field approximation to QHD has already been developed [31] the same is not true with respect to the description of pairing correlations. To the best of our knowledge, only a partial attempt toward a relativistic description of pairing has been performed until now in the work of Kucharek and Ring [32]. The present work goes in the direction of developing a consistent relativistic description of the Hartree-Fock-Bogoliubov (HFB) approximation to QHD.

Our formulation [33] uses the algebraic method developed by Gorkov [34] for the description of superconductivity in metals. In the mean-field approximation to QHD the effect of the NN interaction on the single-particle propagator is described in terms of the nucleon self-energies Σ and Δ, where Δ is the the pairing field. In this way, the field Σ represents the vacuum expectation value of the various meson fields exchanged in the NN interaction and describes the long-range particle-hole correlations between the nucleons, while Δ describes the short-range correlations that result from the exchange of these mesons [26,35,36].

We consider the simplest meson exchange model for the NN interactions in which the mesons propagate freely. The values of the coupling constants and of the rest masses of the mesons are adjusted to give the saturation point of nuclear matter, with a binding energy per nucleon of -15.75 MeV, for a Fermi momentum k_F of 1.35 fm^{-1} [30,37]. This defines the usual effective theory for the NN interactions and for the approximate mean fields Σ and Δ [36,38].

The description of pairing correlations in the BCS model [39–41] requires the consideration of the time-reversed baryonic states. Here we modify the usual QHD Lagrangian by introducing an operator ψ^A, related through time reversal to the fermion field operator, ψ. The fermion propagator in the HFB approximation is expressed in terms of $G=\langle T(\psi\overline{\psi})\rangle$ and $\widetilde{G}=\langle T(\psi^A\overline{\psi^A})\rangle$, as well as the anomalous propagators, $F=\langle T(\psi\overline{\psi^A})\rangle$ and $\widetilde{F}=\langle T(\psi^A\overline{\psi})\rangle$, associated with the correlated propagation of the nucleons in the nuclear medium, due to the pairing correlations.

The self-consistency equations are obtained from the Schwinger-Dyson equations for the self-energies Σ and Δ. As an alternative procedure one can also formulate the model by using relativistic Bogoliubov coefficients, in analogy to the nonrelativistic ones [37,42].

The results of these two equivalent algebraic formulations can be compared with calculations of nuclear matter in the literature, in terms of the explicit self-consistency equations for Σ and Δ or in terms of the numerical results for the mean fields and the binding energy per nucleon as a function of the baryon density and momentum [2–4,28,30,32,43]. We obtain a simultaneous description of nuclear matter saturation and of the gap parameter, compatible with traditional nonrelativistic calculations for pairing in nuclear matter [3,4,6].

To obtain numerical results, we make an additional approximation by eliminating the divergent contributions of the negative-energy states of the Dirac sea, analogous to the truncation used in the Hartree or Hartree-Fock (HF) approximations to QHD [30].

We note that the truncation of the negative-energy spectrum can be performed at several points in the calculation. In particular, it is possible to truncate the spectrum at the HF level (''no sea'' approximation), before including the effects of pairing, or at the HFB level (truncation approximation), where both mean field and pairing effects have been taken into account. As the pairing fields are relatively small when compared to the self-energy fields, one might expect the differences to be small. Nonetheless, we find that the pairing fields in the ''no sea'' approximation are much larger than those in the truncation approximation and behave differently as a function of the baryon density.

This paper is organized as follows. In Sec. II we present the model Lagrangians and deduce the self-consistent mean-field equations for Σ and Δ. In Sec. III we give the explicit expressions for these equations, for the energy density and the baryon density. We then introduce the additional truncation approximation and present the numerical results. In Sec. IV, we redefine the model in terms of the Bogoliubov coefficients and present the results of the ''no sea'' approximation. We conclude that the truncation approximation is in better agreement with nonrelativistic results than the ''no sea'' one. Our findings are summarized in Sec. V.

II. THE MEAN-FIELD EQUATIONS FOR Σ AND Δ

In this section we present the Lagrangian of the model and the covariant equations for the mean-field energies Σ and Δ. We use the conventions of Itzykson and Zuber [44], such that for $a^\mu=(a^0,\vec{a})$ and $b^\mu=(b^0,\vec{b})$, the scalar product is given by $a_\mu b^\mu=a^0b^0-\vec{a}\cdot\vec{b}$.

We designate by $\psi(x),\phi(x),V^\mu(x),\vec{\pi}(x),\vec{\rho}^\mu(x)$ the field operators at the point x associated to the nucleons and mesons σ, ω, π, and ρ, respectively. The quantum numbers (J^π,T) for each meson with spin J, intrinsic parity π, and isospin T are

$$\sigma(0^+,0),\quad \omega(1^-,0),\quad \pi(0^-,1),\quad \rho(1^-,1).$$

We designate the effective meson-nucleon coupling constants by g_s,g_v,g_π, and g_ρ, and the respective bare masses by m_s,m_v,m_π, and m_ρ. The nucleon bare mass is M and we assume in the present model that the nucleons and mesons are pointlike. These assumptions are typical of the simplest meson-exchange models of nuclear structure.

The Lagrangian density is given by

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_{\rm int}, \tag{1}$$

where $\mathcal{L}_0$ is the free Lagrangian density

$$\begin{aligned}\mathcal{L}_0(x)=&\overline{\psi}(x)[i\partial\!\!\!/-M]\psi(x)+\frac{1}{2}[\partial_\mu\phi(x)\partial^\mu\phi(x)-m_s^2\phi^2(x)]\\&+\frac{1}{2}m_v^2V_\mu(x)V^\mu(x)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\&+\frac{1}{2}[\partial_\mu\vec{\pi}(x)\partial^\mu\vec{\pi}(x)-m_\pi^2\vec{\pi}^2(x)]\\&+\frac{1}{2}m_\rho^2\vec{\rho}_\mu(x)\cdot\vec{\rho}^\mu(x)-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu},\end{aligned} \tag{2}$$

with vector field tensors

$$\begin{aligned}F_{\mu\nu}&=\partial_\mu V_\nu-\partial_\nu V_\mu,\\ \vec{G}_{\mu\nu}&=\partial_\mu\vec{\rho}_\nu-\partial_\nu\vec{\rho}_\mu,\end{aligned} \tag{3}$$

and the interaction terms are

$$\begin{aligned}\mathcal{L}_{\rm int}(x)=&-g_s\overline{\psi}(x)\phi(x)\psi(x)-g_v\overline{\psi}(x)\gamma_\mu V^\mu(x)\psi(x)\\&-\frac{g_\pi}{2M}\overline{\psi}(x)\gamma_5\gamma_\mu\vec{\tau}\psi(x)\cdot\partial^\mu\vec{\pi}(x)\\&-\frac{1}{2}g_\rho\overline{\psi}(x)\gamma_\mu\vec{\tau}\cdot\vec{\rho}^\mu(x)\psi(x).\end{aligned} \tag{4}$$

We also characterize the NN interactions in nuclear matter by an effective single-particle Lagrangian, $L_{\rm eff}$, given in terms of the self-energies Σ and Δ. In particular, the definition of Δ uses correlated pairs of single-particle states with opposite momenta and spin, in agreement with the original idea of Cooper [39]. Following Gorkov [34] we introduce such pairs by using a modified form of the time-reversed states, which we designate by ψ^A. If we designate the time-reversal operator by $\mathcal{T}$, the time-reversed conjugate of the field ψ, $\psi^{(\mathcal{T})}$, is given by [44]

$$\psi^{(\mathcal{T})}(x)=\mathcal{T}\psi(x)\mathcal{T}^{-1}=B\overline{\psi^T}(\tilde{x})=\gamma_0B\psi^*(\tilde{x}), \tag{5}$$

where

$$\tilde{x}=(-t,\vec{x}),\quad B=\gamma_5C, \tag{6}$$

and C is the charge conjugation operator. Then we define ψ^A as

$$\psi^A(x)=B\otimes\tau_2\psi^{(\mathcal{T})}(\tilde{x})=A\overline{\psi^T}(x), \tag{7}$$

where $A=B\otimes\tau_2$ and

$$\tau_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

is a Pauli matrix operating in the isospin space. We use the following ansatz for the effective single-particle Lagrangian:

$$\int dtL_{\rm eff}=\int d^4xd^4y\Big\{\overline{\psi}(x)[i\partial\!\!\!/-M+\gamma_0\mu]\delta(x-y)\psi(y)-\overline{\psi}(x)\Sigma(x-y)\psi(y)+\frac{1}{2}\overline{\psi}(x)\Delta(x-y)\psi^A(y)+\frac{1}{2}\overline{\psi^A}(x)\overline{\Delta}(x-y)\psi(y)\Big\}, \qquad (8)$$

where $\delta(x-y)$ is a four-dimensional Dirac δ function and μ is the chemical potential to be used as a Lagrange multiplier to fix the average number of particles.

The symmetries of the effective mean-field Lagrangian under transposition, Hermitian conjugation, and the exchange of dummy variables, x and y, yield the following properties of the mean fields:

$$\Delta(x)=-A^T\Delta^T(-x)A^\dagger \text{ and } \overline{\Delta}(x)=-A\overline{\Delta}^T(-x)A^*, \qquad (9)$$

and

$$\Sigma(x)=\gamma_0\Sigma^\dagger(-x)\gamma_0 \text{ and } \Delta(x)=\gamma_0\overline{\Delta}^\dagger(-x)\gamma_0. \qquad (10)$$

We can put the effective Lagrangian $L_{\rm eff}$ in a more symmetrical form by noting that

$$\int d^4xd^4y\overline{\psi}(x)[(i\partial\!\!\!/-M_0+\gamma_0\mu)\delta(x-y)-\Sigma(x-y)]\psi(y) \qquad (11)$$

$$=\int d^4xd^4y\overline{\psi^A}(x)[(i\partial\!\!\!/+M_0-\gamma_0\mu)\delta(x-y)+\Sigma^A(x-y)]\psi^A(y), \qquad (12)$$

where

$$\Sigma^A(x)=A\Sigma^T(-x)A^\dagger. \qquad (13)$$

The effective Lagrangian can then be rewritten in matrix form as

$$\int dtL_{\rm eff}=\frac{1}{2}\int d^4xd^4x'[\overline{\psi}(x),\overline{\psi^A}(x)]\times\begin{pmatrix}(i\partial\!\!\!/-M+\mu\gamma_0)\delta(x-x')-\Sigma(x-x') & \Delta(x-x')\\ \overline{\Delta}(x-x') & (i\partial\!\!\!/+M-\mu\gamma_0)\delta(x-x')+\Sigma^A(x-x')\end{pmatrix}\begin{pmatrix}\psi(x')\\ \psi^A(x')\end{pmatrix},$$

which immediately yields the following coupled equations of motion for the fields ψ and ψ^A:

$$\int d^4y\begin{pmatrix}(i\partial\!\!\!/-M+\gamma_0\mu)\delta(x-y)-\Sigma(x-y) & \Delta(x-y)\\ \overline{\Delta}(x-y) & (i\partial\!\!\!/+M-\gamma_0\mu)\delta(x-y)+\Sigma^A(x-y)\end{pmatrix}\begin{pmatrix}\psi(y)\\ \psi^A(y)\end{pmatrix}=0. \qquad (14)$$

Defining the generalized baryon field operator as

$$\Psi(x)=\begin{pmatrix}\psi(x)\\ \psi^A(x)\end{pmatrix},$$

one obtains a generalized baryon (quasiparticle) propagator

$$S(x-y)=\begin{pmatrix}G(x-y) & F(x-y)\\ \widetilde{F}(x-y) & \widetilde{G}(x-y)\end{pmatrix}=-i\left\langle\begin{pmatrix}\psi(x)\\ \psi^A(x)\end{pmatrix}(\overline{\psi}(y),\overline{\psi^A}(y))\right\rangle, \qquad (15)$$

where, by $\langle\cdots\rangle$, we mean the time-ordered expectation value in the interacting nuclear matter ground state, $\langle\widetilde{0}|T(\cdots)|\widetilde{0}\rangle$. We assume that the state $|\widetilde{0}\rangle$ contains only nucleons interacting in nuclear matter through the exchange of virtual mesons and contains no real mesons. We also assume that $|\widetilde{0}\rangle$ is symmetric under rotations and translations, so that the propagator S depends only on the difference between the end points of propagation.

We observe that $G(x-y)$ is the usual baryon propagator while $\widetilde{G}(x-y)$ describes the propagation of baryons in time-reversed states. The off-diagonal terms of $S(x-y)$ describe the propagation of correlated baryons and are just the relativistic generalizations of the anomalous propagators defined by Gorkov [34].

The propagator in momentum space satisfies

$$\begin{pmatrix}k\!\!\!/-M+\gamma_0\mu-\Sigma(k) & \Delta(k)\\ \overline{\Delta}(k) & k\!\!\!/+M-\gamma_0\mu+\Sigma^A(k)\end{pmatrix}\times\begin{pmatrix}G(k) & F(k)\\ \widetilde{F}(k) & \widetilde{G}(k)\end{pmatrix}=\begin{pmatrix}\mathbb{1} & 0\\ 0 & \mathbb{1}\end{pmatrix}. \qquad (16)$$

To derive the mean-field equations, we first rewrite the interaction terms of the Lagrangian density $\mathcal{L}_{\rm int}$, using the compact notation [32,33]

TABLE I. Vertices of the meson-nucleon couplings used in the model.

j	σ	ω	ρ	π (p.v.)
$\Gamma_{j\alpha}$	(g_s)	$(g_v\gamma_\mu)$	$(\frac{1}{2}g_\rho\gamma_\mu\vec{\tau})$	$\left(\frac{g_\pi}{2M}\gamma_5\gamma_\mu\partial^\mu\vec{\tau}\right)$

$$\mathcal{L}_{\rm int}(x)=-\sum_j \overline{\psi}(x)\Gamma_{j\alpha}(x)\phi_j^\alpha(x)\psi(x), \qquad (17)$$

where the Greek letters $\alpha,\beta,\ldots$ represent any indices necessary for the correct description of the meson propagation and coupling (Lorentz indices, isospin, etc.). The index j indicates the mesons of the model: σ, ω, π, and ρ and the $\Gamma_j(x)$ designate the respective vertices of the meson-nucleon coupling, given in Table I.

We then rewrite the meson fields ϕ_j in terms of their sources as

$$\phi_j^\alpha(x)=i\int d^4y D_j^{\alpha\beta}(x-y)\overline{\psi}(y)\Gamma_{j\beta}(y)\psi(y), \qquad (18)$$

where $-iD_j^{\alpha\beta}(x-y)$ is the free Feynman propagator of meson j.

Substituting Eq. (18) into Eq. (17), we have

$$\int dtL_{\rm int}=-i\sum_j \int d^4xd^4y\,\overline{\psi}(x)\Gamma_{j\alpha}(x)\psi(x) \times D_j^{\alpha\beta}(x-y)\overline{\psi}(y)\Gamma_{j\beta}(y)\psi(y). \qquad (19)$$

Following Gorkov [34], we then obtain the mean-field contribution of this interaction term by replacing each of the possible pairs of fermion fields by its vacuum expectation value,

$$\begin{aligned}\int dt(L_{\rm int})_{\rm eff}=&\frac{1}{2}\sum_j \int d^4xd^4yD_j^{\alpha\beta}(x-y)\\ &\times\{2\overline{\psi}(x)\Gamma_{j\alpha}(x)\psi(x)\langle\overline{\psi}(y)\Gamma_{j\beta}(y)\psi(y)\rangle\\ &+2\overline{\psi}(x)\Gamma_{j\alpha}(x)\langle\psi(x)\overline{\psi}(y)\rangle\Gamma_{j\beta}(y)\psi(y)\\ &-\overline{\psi}(x)\Gamma_{j\alpha}(x)\langle\psi(x)\psi^T(y)\rangle\Gamma^T_{j\beta}(y)\overline{\psi^T}(y)\\ &-\psi^T(x)\Gamma^T_{j\alpha}(x)\langle\overline{\psi^T}(x)\overline{\psi}(y)\rangle\Gamma_{j\beta}(y)\psi(y)\},\end{aligned} \qquad (20)$$

where $\langle\cdots\rangle$ is again the time-ordered expectation value in the interacting nuclear-matter ground state.

We note that the first term in this expression is a direct Hartree one, the second a Fock exchange one, while the last two, after using the definition of ψ^A to replace the transposed ψ's, can be recognized as pairing terms. Comparing these mean-field contributions to those of the effective quasiparticle Lagrangian, we can express the self-energy and pairing fields in terms of the two-fermion vacuum expectation values as

$$\begin{aligned}\Sigma(x-y)=&-\delta(x-y)\sum_j \Gamma_{j\alpha}(x)\int d^4zD_j^{\alpha\beta}(x-z)\\ &\times\langle\overline{\psi}(z)\Gamma_{j\beta}(z)\psi(z)\rangle-\sum_j \Gamma_{j\alpha}(x)D^{\alpha\beta}(x-y)\\ &\times\langle\psi(x)\overline{\psi}(y)\rangle\Gamma_{j\beta}(y),\end{aligned} \qquad (21)$$

and

$$\Delta(x-y)=-\sum_j \Gamma_{j\alpha}(x)D_j^{\alpha\beta}(x-y)\langle\psi(x)\overline{\psi^A}(y)\rangle A\Gamma^T_\beta(y)A^\dagger, \qquad (22)$$

where the equation for $\overline{\Delta}(x-y)$ can be obtained using the Hermiticity condition of Eq. (10). These expressions become self-consistency equations when we evaluate the expectation values by using their relationship to the generalized baryon propagator, Eq. (15), which is itself a function of the mean fields. We find

$$\begin{aligned}\Sigma(x-y)&\\ =&\delta(x-y)\sum_j \Gamma_{j\alpha}(x)\int d^4zD_j^{\alpha\beta}(x-z){\rm Tr}[\Gamma_{j\beta}(z)G(z-z)]\\ &-\sum_j \Gamma_{j\alpha}(x)D^{\alpha\beta}(x-y)G(x-y)\Gamma_{j\beta}(y),\end{aligned} \qquad (23)$$

and

$$\Delta(x-y)=-\sum_j \Gamma_{j\alpha}(x)D_j^{\alpha\beta}(x-y)F(x-y)A\Gamma^T_{j\beta}(y)A^\dagger. \qquad (24)$$

The equations for Σ and Δ in momentum space are obtained by Fourier transforming the above expressions, giving

$$\begin{aligned}\Sigma(k)=&\sum_j \Gamma_{j\alpha}(0)D_j^{\alpha\beta}(0)\int\frac{d^4q}{(2\pi)^4}{\rm Tr}[\Gamma_{j\beta}(0)G(q)]e^{iq^00^+}\\ &-\sum_j \int\frac{d^4q}{(2\pi)^4}\Gamma_{j\alpha}(q)D_j^{\alpha\beta}(q)G(k-q)\Gamma_{j\beta}(-q),\end{aligned} \qquad (25)$$

and

$$\begin{aligned}\Delta(k)=&-\sum_j \int\frac{d^4q}{(2\pi)^4}\Gamma_{j\alpha}(q)D_j^{\alpha\beta}(q)F(k-q)\\ &\times A\Gamma^T_{j\beta}(-q)A^\dagger,\end{aligned} \qquad (26)$$

respectively.

These expressions are the self-consistency equations of the HFB approximation when we solve them simultaneously with the Dyson equation (16) for the generalized baryon propagator, $S(k)$.

III. THE RELATIVISTIC HFB APPROXIMATION

To obtain explicit equations for Σ and Δ, we first introduce the symmetries of nuclear matter, which permit important simplifications in the structure of the mean fields. We assume nuclear matter to be invariant under rotations, translations, and parity transformations and to have total isospin zero. We also restrict our attention to $^1S_0$ pairing. We can then reduce the form of the model mean fields [30,33], in the nuclear matter rest frame, to the following:

$$\Sigma(k)=\Sigma_s(|\vec{k}|,k^0,\rho_B)-\gamma_0\Sigma_0(|\vec{k}|,k^0,\rho_B)+\vec{\gamma}\cdot\vec{k}\Sigma_v(|\vec{k}|,k^0,\rho_B),$$

and

$$\Delta(k)=[\Delta_s(|\vec{k}|,k^0,\rho_B)-\gamma_0\Delta_0(|\vec{k}|,k^0,\rho_B)-i\gamma_0\vec{\gamma}\cdot\vec{k}\Delta_T(|\vec{k}|,k^0,\rho_B)]\otimes\vec{\tau}\cdot\hat{n}. \tag{27}$$

Although each of the terms in the expression for Δ could, in principle, be an independent isovector, we have simplified the isospin dependence to an overall factor of $\vec{\tau}\cdot\hat{n}$, where the unit vector $\hat{n}$ is arbitrary in direction. Note that this form of the pairing field reduces to the standard one, containing proton-proton and neutron-neutron pairing fields, when $\vec{\tau}\cdot\hat{n}=\tau_2$. The isospin symmetry of nuclear matter, however, allows one to choose arbitrarily the direction in isospin space of the pairing field $\Delta(k)$ without changing its $^1\mathrm{S}_0$ form.

We rewrite the generalized propagator as

$$S(k)=(\omega-\widetilde{\mathcal{H}})^{-1}\begin{pmatrix}\gamma_0 & 0\\ 0 & \gamma_0\end{pmatrix}, \tag{28}$$

where $\omega=k^0$ is the quasinucleon energy and, from Eq. (19),

$$(\omega-\widetilde{\mathcal{H}})=\begin{pmatrix}\gamma_0(\gamma_\mu k^\mu-M+\gamma_0\mu-\Sigma(k)+i\epsilon) & \gamma_0\Delta\\ \Delta^\dagger\gamma_0 & \gamma_0(\gamma_\mu k^\mu+M-\gamma_0\mu+\Sigma^A(k)-i\epsilon)\end{pmatrix}. \tag{29}$$

The symmetries of $\widetilde{\mathcal{H}}$ imply that its eigenvalues are degenerate in spin and isospin. Direct calculation gives the following expression for the determinant of the inverse of $S(k)$:

$$\det[S(k)^{-1}]=\det[\omega-\widetilde{\mathcal{H}}]=[(\omega^2-\alpha)^2-\beta^2]^4, \tag{30}$$

where we have defined

$$\alpha=E_F^{\star 2}+k^{\star 2}+M^{\star 2}+|\Delta_s|^2+|\Delta_0|^2+|\Delta_T|^2,$$

and

$$\beta=2\sqrt{[\mathrm{Re}(\Delta_T\Delta_0^*)]^2+[E_F^\star M^\star+\mathrm{Re}(\Delta_s\Delta_0^*)]^2+[E_F^\star k^\star+\mathrm{Im}(\Delta_s\Delta_T^*)]^2+|M^\star\Delta_T+ik^\star\Delta_0|^2}, \tag{31}$$

in which the asterisk (*) indicating complex conjugation should not be confused with the ''star'' ($\star$) that indicates physical quantities modified by the self-energy. The latter are the effective mass of the nucleon in the nuclear matter

$$M^\star=M+\Sigma_s(k), \tag{32}$$

the effective ''Fermi'' energy

$$E_F^\star=\Sigma_0(k)+\mu,$$

and the effective linear momentum

$$k^\star=[1+\Sigma_v(k)]|\vec{k}|. \tag{33}$$

The 16 roots of the equation

$$\det[\omega-\widetilde{\mathcal{H}}]=0 \tag{34}$$

reduce to four independent solutions given by $\pm\sqrt{\alpha\pm\beta}$, which are degenerate in the spin and the isospin quantum numbers.

We integrate in the upper half plane of the baryon energy in Eqs. (25) and (26) to select the contributions corresponding, in the zero-pairing limit, to the occupied states in the Fermi and Dirac seas. The contributing poles are

$$\omega_-=-\sqrt{\alpha+\beta}\quad\text{and}\quad\omega_+=-\sqrt{\alpha-\beta}, \tag{35}$$

which reduce in the zero-pairing limit and for baryon momenta below the Fermi level, k_F, to the usual HF-values [30]

$$\omega_-=-E_k^\star-E_F^\star\quad\text{and}\quad\omega_+=E_k^\star-E_F^\star, \tag{36}$$

where

$$E_k^\star=\sqrt{k^{\star 2}+M^{\star 2}}. \tag{37}$$

To calculate the baryon density, we use the definition [30]

$$\rho_B=\langle\widetilde{0}|\overline{\psi}(x)\gamma_0\psi(x)|\widetilde{0}\rangle=-i\,\mathrm{Tr}[\gamma_0 G(x-x_+)]. \tag{38}$$

The baryon energy density, relative to the chemical potential, is given by the $\hat{T}^{00}$ component of the energy-momentum tensor. We thus have

$$\hat{H}=\hat{T}^{00}+\mu\hat{N}=-\mathcal{L}'+\mu\hat{N}+\frac{\partial\mathcal{L}'}{\partial(\partial_t\psi)}\partial_t\psi+\frac{\partial\mathcal{L}'}{\partial(\partial_t\psi^A)}\partial_t\psi^A+\sum_j\frac{\partial\mathcal{L}'}{\partial(\partial_t\phi_j^\alpha)}\partial_t\phi_j^\alpha, \quad (39)$$

where $\hat{N}$ is

$$\hat{N}=\overline{\psi(x)}\gamma_0\psi(x). \quad (40)$$

If one neglects the retardation terms in Eq. (39), which are associated with the time derivatives of the meson fields, then the ground state expectation value of $\hat{H}$ can be shown to be

$$\begin{aligned}\epsilon=\langle\hat{H}\rangle=&\frac{i}{2}\lim_{y\to x^+}\mathrm{Tr}[(i\vec{\gamma}\cdot\vec{\partial}-M)G(x-y)\\&+(i\vec{\gamma}\cdot\vec{\partial}+M)\widetilde{G}(x-y)]-\frac{i}{4}\int d^4y\,\mathrm{Tr}[\Sigma(x-y)\\&\times G(y-x)-\overline{\Delta(x-y)}F(y-x)-\Delta(x-y)\widetilde{F}(y-x)\\&-\Sigma^A(x-y)\widetilde{G}(y-x)],\end{aligned} \quad (41)$$

which can be written in momentum space as

$$\begin{aligned}\epsilon=&-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\{[\Sigma(k)+2(\vec{\gamma}\cdot\vec{k}+M)]G(k)\\&-\mathrm{Re}[\overline{\Delta(k)}F(k)]\}.\end{aligned} \quad (42)$$

In the limit of zero pairing, the above expressions reduce to the usual results of the HF approximation to QHD.

A. The truncation method

From previous QHD calculations, one knows that the neglect of terms associated with the pole ω_- gives good results within the relativistic HF approximation [26–30]. Thus one can expect that the same procedure could be successfully applied to the HFB approximation.

We define the *truncation approximation* to the present HFB model as the elimination of the terms associated with ω_- in the self-consistency equations, Eqs. (25) and (26). We shall also assume that the meson masses are fixed and that the coupling constants are adjusted to give the nuclear matter saturation point: a binding energy per nucleon of $BE/A\sim-15.75$ MeV at a Fermi momentum of $k_F\sim1.35$ fm^{-1} [30,37].

1. Numerical results with the truncation approximation

In our numerical evaluations, the masses of the mesons have been fixed at the values usually adopted in the literature and given in Table II [28,30]. We shall designate the various forms of the NN interaction used through explicit indication of the mesons considered in the model.

The coupling constants for the mesons σ and ω are adjusted in the diverse cases. The coupling constant for the pion is fixed at its empirical value of $g_\pi^2=181.0$. The value of the ρ meson coupling constant is not fixed in the literature [21,22,28,32,38,45], so we have used two arbitrary values. One of them (A) is close to the value adopted in Ref. [13] and the other (B) is nearly twice this value and is close to the values adopted in the Bonn potentials [22,24,45]. The two values are

$$A{:}g_\rho^2=25.4,\quad \frac{g_\rho^2}{4\pi}=2.04$$

and

$$B{:}g_\rho^2=53.7,\quad \frac{g_\rho^2}{4\pi}=4.28. \quad (43)$$

The values for the constants g_σ^2 and g_ω^2 for the (σ, ω, π, ρ) model and for the (σ, ω) model are given in Table III for both the HFB approximation and the HF approximation. In both cases the values of g_σ^2 and g_ω^2 are adjusted to fit the saturation point of nuclear matter. We present only results in which the retardation terms in the self-consistency equations have been neglected. Results including the retardation terms are very similar to those shown.

We have introduced, as an additional parameter, a cutoff at $|\vec{k}|=\Lambda$ in the baryon momentum integrals of the self-consistency equations. Such a cutoff is used in many (but not all) nonrelativistic HFB calculations. Here it might be justified as a crude approximation to the nucleon-meson vertex form factors that our calculations do *not* contain.

The values of the components of the self-energy, Σ, as a function of the baryon density for the various models of the NN interaction are very close to the corresponding values in the Hartree-Fock approximation and show almost no variation for increasing values of the cutoff, Λ. The functional dependence of the components Σ_s, Σ_0, and Σ_v of the self-energy with the baryon density for a given cutoff, Λ, is almost identical to that of the Hartree-Fock results. The values of the component Σ_v calculated with interactions which include the pion are more strongly density dependent than those which do not include it, particularly at high baryon densities, while Σ_s and Σ_0 are less model dependent. This result is also found in the HF case [13].

Numerical results for the binding energy per nucleon, BE/A, are shown in Fig. 1 as a function of the Fermi momentum, k_F, for the (σ, ω) and (σ, ω, π, $\rho-A$) models. The binding energy per nucleon, BE/A, of the HFB results are almost identical to the HF ones. The HFB binding energy presents differences with respect to the HF binding energy only for low baryon densities.

The variation in the magnitude of the pairing fields, Δ, for the various types of NN interactions, is quite expressive. To analyze the results for Δ, we use the gap parameter, Δ_G, which is given in Eq. (54) of the next section,

$$\Delta_G=\Delta_0\frac{M^\star}{E_k^\star}-\Delta_s+i\Delta_T\frac{k^\star|\vec{k}|}{E_k^\star}.$$

This form of the gap parameter results from a comparison of the present results with the nonrelativistic ones, as will be explained in Sec. IV. Note that the gap parameter is a result of the partial cancellation between the principal components of the pairing field, Δ_s and Δ_0.

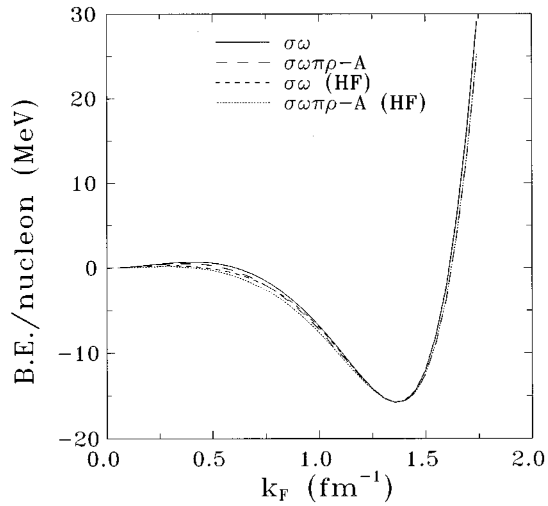

FIG. 1. The binding energy per nucleon as a function of the baryon density in the truncation approximation using a cutoff of 506 MeV/c. The coupling constants have been adjusted to reproduce the saturation point of nuclear matter.

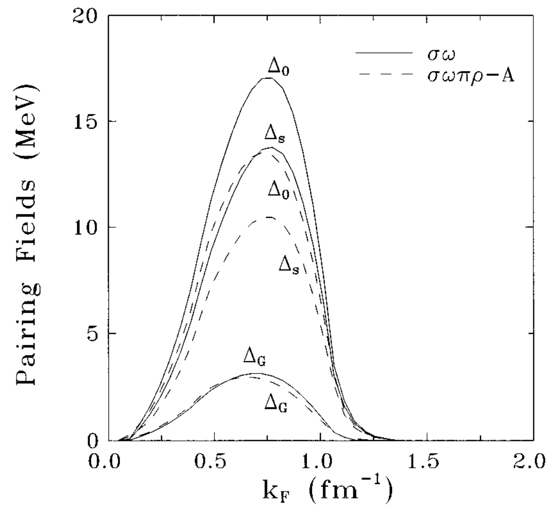

FIG. 2. The two principal components of the pairing field, Δ_s and Δ_0, and the gap parameter, Δ_G, as a function of the baryon density in the truncation approximation for a cutoff of 506 MeV/c. Δ_G is the gap parameter defined in Eq. (76).

The values of the maximum of the gap parameter, $\Delta_{G\max}$, for several values of the cutoff are given in Table IV. Values between 2.4 MeV and 3.1 MeV are obtained for $\Delta_{G\max}$ for a cutoff of 506 MeV/c and a Fermi momenta of $k_F \sim$ 0.7 fm^{-1}. For a cutoff of 410 MeV/c, one obtains slightly larger maxima for Δ_G, between 2.4 MeV and 4.1 MeV in the same density interval. The results for $\Lambda=$ 506 MeV/c, in the (σ, ω, π, $\rho-A$) and (σ, ω) models, are in agreement with the values obtained with nonrelativistic calculations, which estimate a maximum value of Δ_G in nuclear matter between 2.5 MeV and 3.0 MeV for k_F between 0.7 fm^{-1} and 0.8 fm^{-1} [2–4]. For $\Lambda=$ 410 MeV/c, just the (σ, ω, π, $\rho-A$) model and, for $\Lambda=$ 716 MeV/c, just the (σ, ω, π, $\rho-B$) model are in accordance with nonrelativistic calculations.

We thus conclude that, for the models considered, it is possible to find a cutoff between 400 MeV/c and 700 MeV/c such that the maximum of Δ_G in nuclear matter is between the limits indicated by the nonrelativistic calculations.

The results for the components of the field Δ, as a function of the Fermi momentum k_F, are shown in Fig. 2. One notes that the various components of Δ reach a maximum for $k_F \approx$ 0.7 fm^{-1}, which is a characteristic feature of the dependence of the pairing field on k_F. Another important feature appearing in the figures is the relatively small gap parameter Δ_G that results from the partial cancellation of the large components of the pairing field, Δ_s and Δ_0. The component Δ_T is several orders of magnitude smaller than the others and is not shown.

In Fig. 3, the components of the pairing field as a function of the baryon momentum are shown for the (σ, ω) and (σ, ω, π, $\rho-A$) models, for $k_F=0.68$ fm^{-1}. Note that the gap parameter, Δ_G, changes sign for a baryon momentum of about 2 fm^{-1}.

In the truncation approximation, the dependence of the maximum of Δ_G with the cutoff Λ shows the same general trend in the interaction models considered. The magnitude of the components of the field Δ decrease and reach an absolute minimum for 500 MeV/c $\leqslant \Lambda \leqslant$ 800 MeV/c and then increase for $\Lambda \rightarrow \infty$, as shown in Fig. 4. This behavior is more pronounced in the cases in which the pion is present due to its longer range. For $\Lambda \rightarrow \infty$, the maxima of Δ_G in the (σ, ω, π, $\rho-A$) model and in the (σ, ω) model converge to well-defined asymptotic values.

TABLE II. Values of the masses of the mesons used in the numerical calculations

Meson	σ	ω	π	ρ
Mass (MeV)	550	783	138	770

IV. THE BOGOLIUBOV-VALATIN TRANSFORMATION

In this section, we reformulate the HFB approximation in terms of Bogoliubov coefficients. We develop the ‘‘no sea’’ approximation [13,32,36] and use it in calculations, which we then compare to the results obtained in the truncation approximation.

The method we present was developed by Bogoliubov and collaborators [46–48], and independently by Valatin [49], for the description of pairing correlation among fermions, in the context of the theories of superfluidity and superconductivity. It consists of a description of the fields Σ and Δ of the HFB approximation in terms of the contributions of particles and antiparticles that are generalized to allow for partially occupied single-particle states [50,51]. The method is based upon a canonical transformation in the Fock space

TABLE III. Coupling constants used in the various models.

Model	g_σ^2	g_ω^2(HFB)	g_ω^2(HF)
(σ, ω, π, $\rho-A$)	101.814	121.148	121.258
(σ, ω, π, $\rho-B$)	100.731	120.433	120.533
(σ, ω)	96.392	129.260	129.560

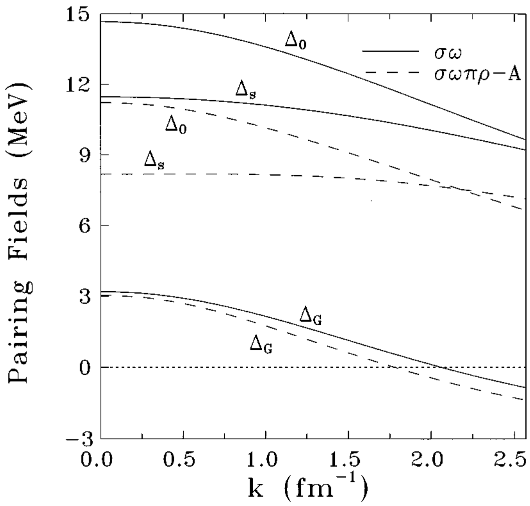

FIG. 3. The two principal components of the pairing field, Δ_s and Δ_0, and the gap parameter, Δ_G, as a function of the baryon momentum,k, for $\rho_B \sim 0.021$ fm^{-3}, in the truncation approximation. The two cases correspond to $\Lambda =$ 506 MeV/c.

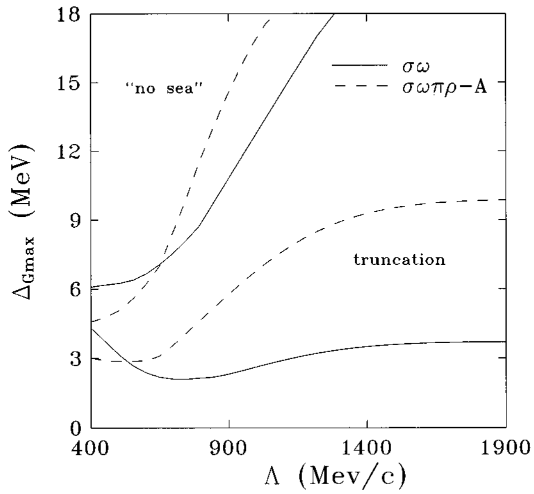

FIG. 4. The maximum gap parameter, $\Delta_{G\max}$, as a function of the cutoff Λ. The curves that saturate at high values of the cutoff were obtained in the truncation approximation while those that extrapolate the upper bound of the figure were obtained in the ''no sea'' approximation.

such that the new operators describe the creation and annihilation of correlated quasiparticles in the interacting ground state $|\tilde{0}\rangle$. One then defines a mean-field model in which the nucleons dressed by the fields Σ and Δ are described by these new operators. The mean-field approximation is given in terms of an approximate ground state that satisfies the equation

$$\alpha_\lambda|\tilde{0}\rangle = 0, \tag{44}$$

where α_λ is the new destruction operator and λ designates the quantum numbers of a one-quasiparticle state in nuclear matter. The Gorkov factorization given in Eq. (20) is a direct result of the above equation and the Wick theorem [37].

In the previous section we saw that the single-particle spectrum of the present model, as well as the self-consistency equations for Σ and Δ, are independent of the nucleon isospin. Therefore, for each type of quasiparticle, one can consider the index λ as designating only the momentum $\vec{k}$ and the spin s.

One can identify the contributions of the different types of states in the present model by comparison with the nonrelativistic case, in which the coefficients of the Bogoliubov transformation, which we designate by A_λ and C_λ, are given by

$$A_{\vec{k}s} = \langle\tilde{0}|a_{\vec{k}s}|\tilde{N}\rangle \quad \text{and} \quad C_{\vec{k}s} = \langle\tilde{0}|a^\dagger_{-\vec{k}-s}|\tilde{M}\rangle. \tag{45}$$

Similarly, the Bogoliubov coefficients for the antinucleons are

$$B_{\vec{k}s} = \langle\tilde{0}|d^\dagger_{-\vec{k}-s}|\tilde{N}\rangle \quad \text{and} \quad D_{\vec{k}s} = \langle\tilde{0}|d_{\vec{k}s}|\tilde{M}\rangle. \tag{46}$$

In these expressions the symbols a_λ and $a^\dagger_\lambda$ are the nucleon annihilation and creation operators, respectively, while the symbols d_λ and $d^\dagger_\lambda$ are the antinucleon ones. All these operators are associated with the one-particle states given by the HF approximation, while $|\tilde{N}\rangle$ and $|\tilde{M}\rangle$ are the intermediate states of nuclear matter with one nucleon more or less than the ground state.

Although the ground state does not have a fixed number of nucleons, the average number of nucleons is fixed as the mean value of the operator $\hat{N}$ given in Eq. (40). The single-particle states and energies are given by the eigenvalues of the zero-temperature thermodynamic potential, $\hat{K} = \hat{H} - \mu\hat{N}$. The states $|\tilde{0}\rangle$, $|\tilde{N}\rangle$, and $|\tilde{M}\rangle$ are eigenstates of $\hat{K}$ and the energy of the quasinucleons is given with respect to the chemical potential. An explicit realization of the relativistic HFB ground state can be derived, just as in the nonrelativistic case [4,42,52] (see Appendix A).

The baryon propagators G and F, defined in Eq. (15), can be rewritten as functions of the Bogoliubov coefficients in an expansion over the intermediate states of nuclear matter, classified in accordance with the types of quasiparticle they contain,

TABLE IV. The maximum of the gap parameter obtained with the truncation approximation for various values of the cutoff Λ.

	$\Delta_{G\max}$			
Model	Λ = 410 MeV/c	Λ = 506 MeV/c	Λ = 601 MeV/c	Λ = 716 MeV/c
(σ, ω, π, $\rho-A$)	2.939	2.802	3.113	3.356
(σ, ω, π, $\rho-B$)	2.404	2.373	2.665	3.072
(σ, ω)	4.066	3.092	2.402	2.058

TABLE V. Vectors of the basis S_u and S_v .

	Spin up	Spin down
Energy $+E_k^\star$	$u_{11}v_{21}$	$u_{12}v_{22}$
Energy $-E_k^\star$	$u_{21}v_{11}$	$u_{22}v_{12}$

$$\begin{aligned} 2iG_{cb} &= \langle\tilde{0}|T(\psi_c\overline{\psi_b})|\tilde{0}\rangle \\ &= \sum_j [\langle\tilde{0}|\psi_c|j\rangle\langle j|\overline{\psi_b}|\tilde{0}\rangle\theta(t_c-t_b)-\langle\tilde{0}|\overline{\psi_b}|j\rangle \\ &\quad\times\langle j|\psi_c|\tilde{0}\rangle\theta(t_b-t_c)], \\ iF_{cb} &= \langle\tilde{0}|T(\psi_c^A\overline{\psi_b})|\tilde{0}\rangle \\ &= \sum_l [\langle\tilde{0}|\psi_c^A|l\rangle\langle l|\overline{\psi_b}|\tilde{0}\rangle\theta(t_c-t_b)-\langle\tilde{0}|\overline{\psi_b}|l\rangle \\ &\quad\times\langle l|\psi_c^A|\tilde{0}\rangle\theta(t_b-t_c)]. \end{aligned} \tag{47}$$

The summations over the intermediate states, $|j\rangle$ and $|l\rangle$, run over all possible quasiparticles of the model.

To expand the fields ψ and ψ^A in terms of the Fock operators of the nucleons, one can use two orthonormal bases in the bispinor space, S_u and S_v , defined by the eigenvectors of

$$\gamma_0\vec{\gamma}\cdot\vec{k}+\gamma_0 M-\mu+\gamma_0\Sigma(k)$$
$$\text{and}\quad \gamma_0\vec{\gamma}\cdot\vec{k}-\gamma_0 M+\mu-\gamma_0\Sigma^A(k), \tag{48}$$

respectively. We designate these as $S_u=\{u_{1\lambda},u_{2\lambda}\}$ and $S_v=\{v_{1\lambda},v_{2\lambda}\}$, with λ corresponding to the values of spin and energy given in Table V. Note that these are the eigenvectors corresponding to the Hartree-Fock mean field alone (no pairing).

The definition of ψ^A and the equation for the baryon propagators imply that

$$v_{i\vec{k}s}=B\overline{u}^T_{i-\vec{k}s}=f_s u_{i-\vec{k}-s}, \quad i=1,2, \tag{49}$$

where f_s is a phase factor. We can thus take the vectors of the bases S_u and S_v in Eq. (48) to be

$$u_{1\vec{k}s}=v_{1-\vec{k}-s}=\eta\begin{pmatrix} E_k^\star+M^\star \\ \vec{\sigma}\cdot\vec{k^\star}\end{pmatrix}\chi_s$$

and

$$u_{2\vec{k}s}=v_{2-\vec{k}-s}=\eta\begin{pmatrix} -\vec{\sigma}\cdot\vec{k^\star} \\ E_k^\star+M^\star\end{pmatrix}\chi_s, \tag{50}$$

where

$$\vec{k^\star}=k^\star\hat{k}=[1+\Sigma_v(k)]\vec{k}, \quad \eta=\frac{1}{\sqrt{2E_k^\star(E_k^\star+M^\star)}}, \tag{51}$$

and χ_s represents the Pauli spinor of the baryon state.

We can expand the propagators of Eq. (47) in terms of these spinors and substitute the latter into the equation they satisfy [the Fourier transform of Eq. (19)]. After simplifying, we obtain an explicit equation for the Bogoliubov coefficients

$$\begin{pmatrix} \omega+E_F^\star-E_k^\star & 0 & -\Delta_G & -\delta \\ 0 & \omega+E_F^\star+E_k^\star & -\delta & -\tilde{\Delta} \\ -\Delta_G^* & -\delta & \omega-E_F^\star+E_k^\star & 0 \\ -\delta & -\tilde{\Delta}^* & 0 & \omega-E_F^\star-E_k^\star \end{pmatrix}\begin{pmatrix} A_{ks} \\ B_{ks} \\ C_{ks} \\ D_{ks}\end{pmatrix}=0, \tag{52}$$

along with the normalization condition

$$|A_{ks}|^2+|B_{ks}|^2+|C_{ks}|^2+|D_{ks}|^2=1, \tag{53}$$

where the quasiparticle energy, ω, is one of the four possible roots, $\pm\sqrt{\alpha\pm\beta}$, and we have defined the following parameters:

$$\begin{aligned} \Delta_G(k) &= \Delta_0\frac{M^\star}{E_k^\star}-\Delta_s+i\Delta_T\frac{k^\star|\vec{k}|}{E_k^\star}, \\ \tilde{\Delta}(k) &= \Delta_0\frac{M^\star}{E_k^\star}+\Delta_s+i\Delta_T\frac{k^\star|\vec{k}|}{E_k^\star}, \\ \delta(k) &= i\Delta_0\frac{k^\star}{E_k^\star}+\Delta_T\frac{M^\star|\vec{k}|}{E_k^\star}. \end{aligned} \tag{54}$$

An analysis of these equations shows that for $\omega=+\sqrt{\alpha-\beta}$, the values of $|A|^2$ and $|C|^2$ are similar to softened step functions of the type $\theta(k-k_F)$ and $\theta(k_F-k)$, respectively, while $|B|^2$ and $|D|^2$ take much smaller values. This solution describes the propagation of particles above the Fermi sea and the propagation of time-reversed holes inside the Fermi sea. In the solution corresponding to $\omega=-\sqrt{\alpha-\beta}$, the roles of $|A|^2$ and $|C|^2$ are reversed. This describes the propagation of holes inside the Fermi sea and the propagation of time-reversed particles above the Fermi sea.

A similar analysis show that the solution $\omega=+\sqrt{\alpha+\beta}$, describes the propagation of time-reversed antiparticles and, finally, the solution $\omega=-\sqrt{\alpha+\beta}$, describes the propagation of antiparticles.

In the expressions of the previous section, the integration over the baryon energy selects the two poles in the upper half

TABLE VI. The maximum of the gap parameter obtained with the ''no sea'' approximation for various values of the cutoff Λ.

Model	$\Delta_{G\max}$ Λ = 410 MeV/c	Λ = 506 MeV/c	Λ = 601 MeV/c	Λ = 716 MeV/c
$(\sigma,\ \omega,\ \pi,\ \rho-A)$	4.825	5.247	7.110	7.821
$(\sigma,\ \omega,\ \pi,\ \rho-B)$	3.938	4.340	6.011	6.697
$(\sigma,\ \omega)$	5.977	6.184	6.956	7.600

plane and given in Eq. (35). The terms associated with the pole ω_- define the contribution associated with the holes in the Dirac sea, which are those responsible for the divergences of the model. In the truncation approximation these divergences were removed by eliminating the terms associated with the pole ω_-. We can see in Eq. (52) that this procedure does not eliminate all the contributions associated with the coefficients B and D, which correspond to the antinucleons of the HF approximation, due to their coupling to the other terms through the component δ of the pairing field. The importance of these contributions will be analyzed in the next section.

A. The ''no sea'' approximation

If one neglects the contribution of δ in Eq. (52), the coefficients B_{ks} and D_{ks} are zero for the ω_+ (Fermi hole) solution while the coefficients A_{ks} and C_{ks} are zero for the ω_- (Dirac hole) solution. The truncation approximation then eliminates the contribution of B_{ks} and D_{ks} together with ω_-. This defines the ''no sea'' approximation [13,32,36] that leads to the following reduced form of Eq. (52) for the Bogoliubov coefficients:

$$\begin{pmatrix} \omega+E_F^\star-E_k^\star & -\Delta_G \\ -\Delta_G^* & \omega-E_F^\star+E_k^\star \end{pmatrix}\begin{pmatrix} A_{ks} \\ C_{ks} \end{pmatrix}=0, \qquad (55)$$

with the normalization condition

$$|A_{ks}|^2+|C_{ks}|^2=1\ . \qquad (56)$$

Recall that, in the truncation approximation, the negative-energy HFB states are discarded. In the ''no sea'' approximation given here, one discards the negative-energy HF states, as well as their coupling to the positive-energy ones through the pairing field $\delta(k)$. The ''no sea'' approximation thus neglects HF-particle–HF-antiparticle correlations that are included in the truncation approximation.

The definition of the pairing field δ, given in Eq. (54), suggests that the ''no sea'' approximation should be almost equivalent to the truncation one at small values of the baryon momentum k, for which the coupling term δ is essentially linear in k and small. For large values of the baryon momentum, however, the coupling between the positive- and negative-energy HF states is on the order of $\Delta_0(k)$ and differences between the two approximations can be expected.

One can obtain a self-consistency equation for Δ_G using Eq. (54), the hypothesis that B_{ks} and D_{ks} are zero and the self-consistency equations for the components of Δ in terms of the Bogoliubov coefficients. With $\Sigma_v=0$, one finds

$$\Delta_G\approx\frac{1}{4\pi^2}\int p^2dp\,v(k,p)\frac{\Delta_G}{\sqrt{(E_F^\star-E_p^\star)^2+|\Delta_G|^2}}, \qquad (57)$$

where the effective gap potential, $v(k,p)$, is given in Appendix B. The equation for Δ_G is formally identical to the nonrelativistic BCS equation. One can thus identify the parameter Δ_G with the usual gap parameter of the BCS theory.

If we neglect the retardation terms in the effective gap potential, Eq. (B1), the result above becomes identical to the gap equation obtained by Ring and Kucharek [32] who also worked in the ''no sea'' approximation. Nevertheless, we note that Ring and Kucharek only partially considered the coupled self-consistency equations for Σ and Δ.

The strongest motive for considering the ''no sea'' approximation lies in the similarity of the algebraic expressions with the nonrelativistic ones. The latter have produced a good description of pairing in nuclear matter [2–4]. The nonrelativistic limit of Eq. (55) yields

$$(E_F^\star)_{\rm NR}=E_F^\star-M=\frac{k_F^2}{2M} \quad \text{and} \quad V_{\rm NR}=\Sigma_s-\frac{E_k^\star}{M}\Sigma_0 \qquad (58)$$

where $V_{\rm NR}$ is the nonrelativistic nucleon-nucleon interaction potential [15]. Thus, in this limit, Eq. (55) corresponds closely to the original formulations of Gorkov, Bogoliubov, and Valatin [34,46–49] of the description of pairing correlations.

B. Numerical results of the ''no sea'' approximation

The results for the different NN interactions, designated in accordance with the notation defined in the previous section, are shown in Table VI. They were obtained with a procedure analogous to the one used in the truncation method. The meson masses have been fixed at the values given in Table II and the coupling constants have been adjusted to fit the saturation point of nuclear matter. The values of the coupling constants obtained for the ''no sea'' approximation are identical to those used in the truncation approximation due to the small values of the pairing field Δ at the saturation point for the different NN interactions considered.

The values for the components of the self-energy Σ in the ''no sea'' approximation are very close to the corresponding ones obtained with the truncation method. In both approximations the results for the self-energy are close to the those of the HF approximation.

The results for the components of the pairing field as a function of the baryon density in the ''no sea'' approximation are given in Fig. 5 for the $(\sigma,\ \omega)$ and the $(\sigma,\ \omega,\ \pi,\ \rho-A)$ models. One observes a similarity in form with the

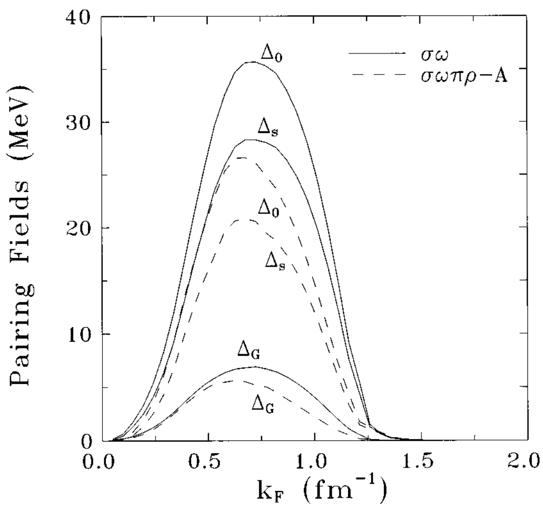

FIG. 5. Pairing fields as a function of the baryon density in the ''no sea'' approximation using a cutoff of 506 MeV/c. The dashed curves correspond to the (σ,ω,π,ρ) model while the solid ones correspond to the (σ,ω) model.

results of the truncation approximation but a difference in magnitude of the resulting gap parameter. The results as a function of the baryon momentum are given in the Fig. 6, also for the (σ, ω) and (σ, ω, π, $\rho-A$) models, for $k_F=0.68$ fm^{-1}. One again notes a similarity in form but a difference in magnitude when comparing the ''no sea'' results with the truncation ones.

One observes in Fig. 4 that the maximum value of the gap parameter in the ''no sea'' approximation is always larger than that in the truncation approximation. The difference between the two values grows as the value of the cutoff is increased. These differences can be explained in terms of the coupling of positive- and negative-energy HF states through the pairing field δ that is neglected in the ''no sea'' approximation. The contribution of this interaction reduces the values of the pairing fields in the truncation approximation below those of the ''no sea'' one. Because the coupling first increases and then decreases slowly with the baryon momentum [in proportion to $\Delta_0(k)$], its contribution grows and the difference between the two approximations increases as the cutoff is raised and higher momentum states are included.

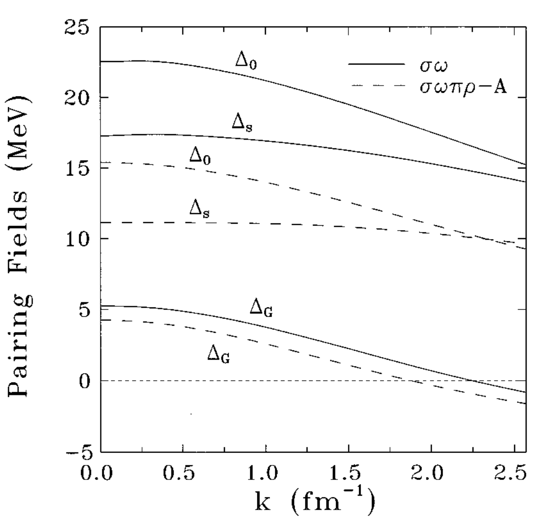

FIG. 6. The two principal components of the pairing field, Δ_s and Δ_0, and the gap parameter, Δ_G, as a function of the baryon momentum, k, for $\rho_B \sim 0.021$ fm^{-3}, in the ''no sea'' approximation. The two cases correspond to $\Lambda=$ 506 MeV/c.

The values of the gap parameter obtained in the ''no sea'' approximation are systematically larger than nonrelativistic ones, except at values of the cutoff Λ smaller than 300 MeV/c. The values of the maxima of the gap parameter in the ''no sea'' approximation are all larger than 3.9 MeV. Furthermore, these values systematically increase for increasing cutoffs, as shown in Fig. 4. We thus conclude that the truncation approximation of the previous section provides results in better agreement with the nonrelativistic ones than those of the ''no sea'' approximation.

The results we have obtained are qualitatively similar to those of Kucharek and Ring [32], who also found large values for the relativistic gap parameter in comparison with the nonrelativistic ones. Kucharek and Ring also obtained decreasing values of the gap parameter for decreasing cutoffs, but the threshold value of the cutoff below which they obtained good agreement with the nonrelativistic results was $\Lambda \leqslant$ 700 MeV/c, instead of the 300 MeV/c that we found.

We believe the different threshold cutoffs to be due to the use of the Hartree approximation by Kucharek and Ring in their evaluation of the self-energy Σ. The coupling constants in the Hartree approximation are significantly different from the values obtained in the HF and the HFB approximations given in Table II . As the pairing field depends quite strongly on the NN interaction used, the Δ field obtained with the coupling constants of the Hartree approximation can show large differences with respect to the results obtained in the HF or the HFB approximations.

The use of the Hartree or the HF approximation to the Σ field is not, strictly speaking, consistent with the pairing correlations of the HFB approximation. Nonetheless, the very similar values obtained for the coupling constants and for the baryon self-energy, Σ, in the HF and the HFB approximations, show the HF approximation to Σ to be an excellent estimate of its HFB value. The use of the Hartree approximation to Σ in a calculation that includes pairing correlations seems to us to be inadequate, if the same coupling constants are to be used in the calculation of the self-energy and pairing fields.

V. CONCLUSIONS

In this work we have presented a relativistic formulation of the HFB approximation. We have used the mean fields of the self-energy, Σ, and of the pairing correlations, Δ, to describe correlated nucleons in nuclear matter at zero temperature. Our formulation followed the algebraic method originally developed by Gorkov [34] for the description of superconductivity in metals, in agreement with the BCS model [39–41] and with the approach developed by Bogoliubov *et al.* [46–48].

We use a Lagrangian formalism for nucleons interacting through the exchange of mesons and an effective Lagrangian involving the self-energy and the pairing fields. The correlated baryon states are described by a generalized baryon field $\Psi=(\psi,\psi^A)$ involving the fields associated with two baryon states that are time-reversal conjugates of one another. The general equations of motion for Ψ are derived from the Lagrangians of the model and a generalized baryon propagator is defined involving the usual baryon propagators of the type $\langle \psi\bar{\psi}\rangle$, and $\langle \psi^A\overline{\psi^A}\rangle$, as well as anomalous propagators of the type $\langle \psi^A\bar{\psi}\rangle$, and $\langle \psi\overline{\psi^A}\rangle$. As a consequence, the self-energy of the generalized baryons also involves the pairing field.

The self-consistency equations of the HFB approximation to QHD are deduced using simplified algebraic structures for the fields Σ and Δ in Lorentz and isospin spaces, in accordance with the symmetries of nuclear matter. This permits an explicit definition of the baryon propagators in terms of the components of Σ and Δ. One finds that the most important components of these fields are the scalar ones (Σ_s and Δ_s) and the vector zero-component ones (Σ_0 and Δ_0), which take large values. The partial cancellation of these large components yield the usual nonrelativistic results both for the nucleon-nucleon potential, $V_{\mathrm{NR}}=\Sigma_s-E_k^\star\Sigma_0/M^\star$, and for the gap parameter of the BCS model, $\Delta_G=M^\star\Delta_0/E_k^\star-\Delta_s$. The remaining components, Σ_v and Δ_T, are several orders of magnitude smaller than the principal ones, while the components Σ_T and Δ_v are identically zero here.

The self-consistency equations in the HFB approximation contain two contributions, corresponding to those of the Dirac and the Fermi seas. The contributions from the negative-energy states are divergent, as in the Hartree-Fock case, and the numerical calculations have been performed using two different procedures for the elimination of these contributions: the truncation and the ‘‘no sea’’ approximations. In the truncation approximation, the negative-energy HFB states are discarded. In the ‘‘no sea’’ approximation, one discards the negative-energy HF states, as well as their coupling to the positive-energy ones through the pairing field. The ‘‘no sea’’ approximation thus neglects HF-particle–HF antiparticle correlations that are included in the truncation approximation.

The relative smallness of the pairing fields obtained in nonrelativistic calculations, when compared to the self-energy fields obtained in mean-field approaches to QHD, suggests that the HFB results should consist of the usual HF results for the baryon self-energy along with a relatively small value for the field Δ, which should not depend importantly on the procedure used to evaluate it. Nonetheless, the results obtained in the truncation and the ‘‘no sea’’ approximations are quite different and show different behaviors for increasing values of the cutoff in the baryon momentum integrals.

The pairing fields in the truncation approximation show an initial decrease with an absolute minimum for 500 MeV/$c \leqslant \Lambda \leqslant$ 800 MeV/c, followed by an increase to an asymptotic value for increasing cutoffs, while the ‘‘no sea’’ pairing results are larger and systematically increase with the cutoff. The differences in the two results can be understood in terms of the coupling of the positive- and negative-energy HF states that is neglected in the ‘‘no sea’’ approximation. Its contribution is always negative, reducing the values of the pairing fields in the truncation approximation below those of the ‘‘no sea’’ one. Because this coupling tends to increase with the momentum, its contribution grows and the difference between the two approximations increases as the cutoff is raised and higher momentum states are included.

We have compared the results of these two procedures with the results of nonrelativistic calculations for the pairing fields, which show [2–4] a maximum for Δ_G in nuclear matter of about 3.0 MeV for a Fermi momentum $k_F\sim 0.8$ fm^{-1}. This maximum of Δ_G could only be obtained with the ‘‘no sea’’ approximation by using a value of the momentum cutoff smaller than 300 MeV/c. Such a cutoff is small in comparison with the usual values. The values of the cutoff obtained with the truncation approximation are larger and in better agreement with other estimates [22,36,53,54]. As the derivation of the ‘‘no sea’’ procedure also involves more drastic approximations than the truncation one, we conclude that the truncation approximation is better suited than the ‘‘no sea’’ one for the evalution of the pairing fields within the HFB approximation.

The total HFB energy per nucleon as a function of the Fermi momentum is extremely close to the HF one, except at low baryon densities. The functional dependence of the self-energy field Σ on the baryon density is almost independent of the cutoff and close to the HF results [13,27,28,30]. The values of the coupling constants for all meson exchanges considered are also very close to the values obtained in the HF evaluations. Thus one can conclude that the HF result for the field Σ would serve as a good approximation to its actual value in the evaluation of the Δ field. Such a prescription would be analogous to the use of the nonrelativistic HF mean-field wave functions in the determination of the single-particle spectrum using the BCS equations [8–10].

To summarize, we have obtained two basic results in this work. First, we have derived a consistent relativistic formulation for the HFB approximation to QHD that yields results in good agreement with the traditional nonrelativistic calculations. Second, we have characterized the truncation approximation as a better method, in comparison with the ‘‘no sea’’ procedure, for obtaining approximate results within the mean-field approximation to QHD.

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APPENDIX A: EXPLICIT DEFINITION OF THE INTERACTING GROUND STATE

Using the definitions and properties of the Bogoliubov coefficients of Sec. IV, we assume that the ground state has the form

$$|\tilde{0}\rangle=\prod_\lambda \,[m_\lambda+n_\lambda a_\lambda^\dagger a_{-\lambda}^\dagger+o_\lambda(a_\lambda^\dagger d_{-\lambda}^\dagger-d_\lambda^\dagger a_{-\lambda}^\dagger)+p_\lambda d_\lambda^\dagger d_{-\lambda}^\dagger + q_\lambda a_\lambda^\dagger a_{-\lambda}^\dagger d_\lambda^\dagger d_{-\lambda}^\dagger]|0\rangle,$$

and define the quasiparticle operators as

$$\alpha_\lambda(\omega_-)=A_{1\lambda}a_\lambda+B_{1\lambda}d^\dagger_{-\lambda}+C_{1\lambda}a^\dagger_{-\lambda}+D_{1\lambda}d_\lambda,$$

$$\beta_\lambda(\omega_-)=-C_{1\lambda}a^\dagger_\lambda-D_{1\lambda}d_{-\lambda}+A_{1\lambda}a_{-\lambda}+B_{1\lambda}d^\dagger_\lambda,$$

$$\gamma_\lambda(\omega_+)=A_{2\lambda}a_\lambda+B_{2\lambda}d^\dagger_{-\lambda}+C_{2\lambda}a^\dagger_{-\lambda}+D_{2\lambda}d_\lambda,$$

$$\delta_\lambda(\omega_+)=-C_{2\lambda}a^\dagger_\lambda-D_{2\lambda}d_{-\lambda}+A_{2\lambda}a_{-\lambda}+B_{2\lambda}d^\dagger_\lambda,$$

where the indices 1 and 2 refer to the poles ω_- and ω_+ given in Eq. (35). Then the usual anticommutation conditions,

$$\{\alpha^\dagger_\lambda,\alpha_\lambda\}=1,$$

$$\{\beta^\dagger_\lambda,\beta_\lambda\}=1,$$

$$\{\gamma^\dagger_\lambda,\gamma_\lambda\}=1,$$

and

$$\{\delta^\dagger_\lambda,\delta_\lambda\}=1,$$

with all the other anticommutators being equal to zero, as well as the conditions that define $|\tilde{0}\rangle$ as the vacuum for the new Fock operators,

$$\alpha_\lambda|\tilde{0}\rangle=\beta_\lambda|\tilde{0}\rangle=\gamma_\lambda|\tilde{0}\rangle=\delta_\lambda|\tilde{0}\rangle=0,$$

give the following results:

$$m_\lambda=A_{1\lambda}D_{2\lambda}-A_{2\lambda}D_{1\lambda},\quad n_\lambda=C_{1\lambda}D_{2\lambda}-C_{2\lambda}D_{1\lambda},$$

$$o_\lambda=B_{1\lambda}D_{2\lambda}-B_{2\lambda}D_{1\lambda},\quad p_\lambda=A_{1\lambda}B_{2\lambda}-A_{2\lambda}B_{1\lambda},$$

and

$$q_\lambda=B_{1\lambda}C_{2\lambda}-B_{2\lambda}C_{1\lambda},$$

as well as the normalization properties of the Bogoliubov coefficients of Sec. IV.

APPENDIX B: THE EFFECTIVE GAP POTENTIAL

In the gap equation, Eq. (57), we use the following definitions:

$$v(k,p)=v_\sigma(k,p)+v_\omega(k,p)+v_\pi(k,p)+v_\rho(k,p) \quad \text{(B1)}$$

and

$$v_\sigma(k,p)=\frac{-g_s^2}{2E_p^\star E_k^\star}\left[1+\frac{2M_k^\star M_p^\star+2E_p^\star E_k^\star+H_{\sigma+}(k,p)}{4kp}\times\theta_{\sigma+}(k,p)\right],$$

$$v_\omega(k,p)=g_v^2\frac{2E_p^\star E_k^\star-M_k^\star M_p^\star}{2E_p^\star E_k^\star kp}\theta_{\omega+}(k,p),$$

$$v_\pi(k,p)=\left(\frac{g_\pi^2}{2M}\right)^2\frac{1}{E_p^\star E_k^\star}\Bigg\{\Bigg[M_k^\star M_p^\star-E_p^\star E_k^\star+\frac{1}{2}(H_{\pi+}(k,p)-m_\pi^2)\Bigg]-\frac{\theta_{\pi+}(k,p)}{4kp}\Bigg[m_\pi^2E_p^\star E_k^\star-2M_k^\star M_p^\star\times\left(k^2+p^2+\frac{m_\pi^2}{2}-H_{\pi+}(k,p)\right)+2k^2p^2+\frac{1}{2}H_{\pi+}(k,p)(m_\pi^2-H_{\pi+}(k,p))\Bigg]\Bigg\},$$

$$v_\rho(k,p)=\frac{g_\rho^2}{4}\frac{2E_p^\star E_k^\star-M_k^\star M_p^\star}{2E_p^\star E_k^\star kp}\theta_{\rho+}(k,p). \quad \text{(B2)}$$

The functions H_{j+} and θ_{j+} are defined as

$$H_{j+}(k,p)=k^2+p^2+m_j^2-[\omega_+(p)-\omega_+(k)]^2, \quad \text{(B3)}$$

where

$$\omega_+(k)=\sqrt{(E_F^\star-E_k^\star)^2+|\Delta_G|^2} \quad \text{(B4)}$$

and

$$\theta_{j+}=\ln\left|\frac{H_{j+}(k,p)+2kp}{H_{j+}(k,p)-2kp}\right| \quad \text{with } j=\sigma,\omega,\pi,\rho. \quad \text{(B5)}$$

We have used $k=|\vec{k}|$ and $p=|\vec{q}|$ here.

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