Core polarization charges of quadrupole transitions in neutron drip line nuclei

I. Hamamoto

Department of Mathematical Physics, Lund Institute of Technology at University of Lund, Lund, Sweden

H. Sagawa

Center for Mathematical Sciences, University of Aizu, Ikki-machi, Aizu-Wakamatsu, Fukushima 965, Japan

(Received 22 March 1996)

Core polarization charges of quadrupole transitions in light neutron drip line nuclei are studied by using a particle-vibration coupling model with Hartree-Fock single-particle wave functions and random phase approximation response functions in ^{28}O. We obtain very small core polarization charges, $e_{pol}(IS)=0.14$ and $e_{pol}(IV)=0.06$. In addition to the smallness expected in general for light very-neutron-rich nuclei, a considerable amount of further reduction on the e_{pol} values arises from the cancellation between the contributions of low-energy threshold strength and those of giant resonances. It depends crucially on both the property of the one-particle multipole operator and the shell structure around the Fermi level whether the very-low-lying threshold strength contributes to the polarization charges of a given multipole constructively or destructively. [S0556-2813(96)03811-3]

It is pointed out [1–3] that an appreciable amount of transition strength will appear in the very-low-energy region (just above the particle threshold) of neutron drip line nuclei. The nuclei need not be halo nuclei, but the neutron particle-hole (p-h) excitations should be available, in which small angular momenta of particle orbitals are possible combined with neutron hole orbitals with small binding energies [1,2]. According to our knowledge of β-stable nuclei, the core polarization charge is expected to be larger when a strong transition strength is present in the lower-excitation-energy region of the core [4]. The expectation is based on the fact that the static polarization charge is inversely proportional to the excitation energy of the corresponding collective modes. Thus one may imagine that the core polarization charge would be large in neutron drip line nuclei. A careful examination of the core excitations may, however, lead to a conclusion that core polarization charges would be small generally in light very-neutron-rich nuclei. This is because the very small proton core, which cannot contribute anyway so much to the polarization charge, is not efficiently polarized by neutrons. In the present work we study the polarization charge taking an example of $^{28}_{8}$O$_{20}$, of which the exotic behavior of the single-particle response function as well as the random phase approximation (RPA) strength function has been studied in Ref. [2].

In halo nuclei, it is expected that the core polarization induced by halo particles, which have very small binding energies (<1 MeV) and small orbital angular momentum ($\ell=0$ or 1), is weak. This is because those particles spend an appreciable amount of time outside of the core nucleus and, thus, cannot efficiently polarize the core. Although this is an interesting problem in halo nuclei, in the present work we study the feature of core polarization unique in neutron drip line nuclei, which exists even in the absence of halo particles.

Our study of core polarization is based on the self-consistent Hartree-Fock (HF) and the RPA calculation using Skyrme interactions. We estimate collective properties solving the RPA with the Green's function method in the coordinate space, which produces a proper strength function in the continuum [5,6]. The core polarization is estimated by using the particle-vibration coupling model [4,7]. We use the SkM* interaction [8] as a standard interaction in our numerical calculation. We always make sure of our conclusion, checking whether or not it is obtained also by using the SIII or the SG2 interaction.

The unperturbed strength function is defined by

$$S_0(E)\equiv\sum_i |\langle i|Q^\lambda|0\rangle|^2\delta(E-E_i)$$

$$=\frac{1}{\pi}\,\mathrm{Im}\,\mathrm{Tr}(Q^{\lambda\dagger}G_0(E)Q^\lambda), \qquad (1)$$

where G_0 is the noninteracting p-h Green function, while the RPA strength function is obtained by

$$S(E)\equiv\sum_n |\langle n|Q^\lambda|0\rangle|^2\delta(E-E_n)$$

$$=\frac{1}{\pi}\,\mathrm{Im}\,\mathrm{Tr}(Q^{\lambda\dagger}G_{RPA}(E)Q^\lambda), \qquad (2)$$

where $G_{RPA}(E)$ is the RPA response function including the effect of the coupling to the continuum. In Eqs. (1) and (2), Q^λ expresses one-body operators, which are written as

$$Q_\mu^{\lambda=2,\,\tau=0}=\sum_i r_i^2 Y_{2\mu}(\hat{r}_i)$$

for isoscalar quadrupole modes, $\qquad (3)$

$$Q_\mu^{\lambda=2,\,\tau=1}=\sum_i \tau_z r_i^2 Y_{2\mu}(\hat{r}_i)$$

for isovector quadrupole modes. $\qquad (4)$

The RPA response function G_{RPA} behaves in the vicinity of a resonance as

$$\text{Im}[G_{\text{RPA}}(\vec{r},\vec{r}';E_{\text{res}})] \propto \delta\rho(\vec{r})\delta\rho(\vec{r}'), \qquad (5)$$

where $\delta\rho$ is the transition density defined by

$$\delta\rho(\vec{r}) = \langle n | \sum_i \delta(\vec{r}-\vec{r}_i)|0\rangle. \qquad (6)$$

We may use expression (5) also for the threshold strength which does not come from resonances in the usual sense, when the shape of the transition density is practically the same in a certain energy region. We obtain the transition density by integrating over one of the radial coordinates of G_{RPA} as

$$\delta\rho(\vec{r}) = \alpha \int \text{Im}[G_{\text{RPA}}(\vec{r},\vec{r}';E_{\text{res}})]Q^\lambda(\vec{r}')d\vec{r}', \qquad (7)$$

where the normalization factor α is determined from the transition strength $B(\lambda)$ of the resonance,

$$\alpha^2 = B(\lambda)/[\pi S(E_{\text{res}})]^2. \qquad (8)$$

For an isolated resonance, $B(\lambda)$ is the total integrated strength for the operator Q^λ over the resonance.

We use a perturbation theory to calculate the core polarization due to the core excitations by particles. The perturbed single-particle wave function is expressed as

$$|\widetilde{i}\rangle = |i\rangle + \sum_{j,\omega_\lambda} \frac{\langle(j\times\omega_\lambda)i|V_{\text{pv}}|i\rangle}{\varepsilon_i - (\varepsilon_j+\omega_\lambda)}|(j\times\omega_\lambda)i\rangle, \qquad (9)$$

where V_{pv} is the particle-vibration coupling interaction. The reduced transition matrix element for the one-body operator Q^λ is modified as

$$\langle\widetilde{j}\|Q^\lambda\|\widetilde{i}\rangle = \langle j\|Q^\lambda\|i\rangle + \sum_{\omega_\lambda} \frac{2\omega_\lambda}{(\varepsilon_i-\varepsilon_j)^2-\omega_\lambda^2}$$
$$\times\frac{\sqrt{2i+1}\langle(j\times\omega_\lambda)i|V_{\text{pv}}|i\rangle}{\sqrt{2\lambda+1}}\langle\omega_\lambda\|Q^\lambda\|0\rangle. \qquad (10)$$

The particle-vibration coupling V_{pv} is derived from the Skyrme interaction approximating the momentum derivative operators $\mathbf{k}$ and $\mathbf{k}'$ by the Fermi momentum $\mathbf{k_F}$ so that one can use directly the RPA transition densities for the calculation of the matrix element. This approximation was made only here in constructing the particle-vibration coupling. Then, the isoscalar (IS) and the isovector (IV) coupling interactions are expressed as

$$V_{\text{pv}}^\tau(\vec{r}_1-\vec{r}_2) = V_{\text{pv}}^\tau(r)\delta(\vec{r}_1-\vec{r}_2), \qquad (11)$$

where

$$V_{\text{pv}}^{\tau=0}(r) = \{\tfrac{3}{4}t_0 + \tfrac{3}{48}(\alpha+2)(\alpha+1)t_3\rho^\alpha(r)$$
$$+ \tfrac{1}{8}k_F^2[3t_1+t_2(5+4x_2)]\}, \qquad (12)$$

$$V_{\text{pv}}^{\tau=1}(r) = \{-\tfrac{1}{4}t_0(1+2x_0) - \tfrac{1}{24}t_3(1+2x_3)\rho^\alpha(r)$$
$$+ \tfrac{1}{8}k_F^2[-t_1(1+2x_1)+t_2(1+2x_2)]\}, \qquad (13)$$

where $r = \tfrac{1}{2}(r_1+r_2)$. Then the coupling matrix is evaluated to be

$$\langle(j\times\omega_\lambda)i|V_{\text{pv}}^\tau|i\rangle = \frac{1}{\sqrt{2i+1}}\int r^2 dr V_{\text{pv}}^\tau(r)\delta\rho(r)R_j(r)R_i(r)$$
$$\times\langle j\|Y_\lambda\|i\rangle, \qquad (14)$$

where $\delta\rho(r)$ is the radial transition density defined by

$$\delta\rho(\vec{r}) \equiv \delta\rho(r)Y_{\lambda\mu}(\hat{r}) \qquad (15)$$

and $R(r)$ expresses the radial wave functions of single-particle states. In Eqs. (12) and (13), the contributions from the momentum-dependent forces, the t_1 and t_2 terms, are seen to be effectively included by renormalizing the value of t_0. The renormalization factor in the case of the SkM* interaction is 0.94 for the IS interaction and 0.85 for the IV interaction. Thus the effect of the momentum-dependent forces on the coupling interaction is rather small, especially for the IS channel, which makes a dominant contribution to the core polarization charge.

In Fig. 1(a) we show the unperturbed strength function in Eq. (1) and the RPA strength function in Eq. (2) for the IS and IV quadrupole operators. Two very sharp peaks at 27.66 and 32.33 MeV in the dotted line express the proton excitations from the $1p_{1/2}$ and $1p_{3/2}$ orbital at -27.02 and -31.69 MeV to the $1f_{5/2}$ one-particle resonant state at $+0.64$ MeV. Except those two peaks, the unperturbed strength happens to consist almost exclusively of the neutron strength. The proton bound p-h excitations for quadrupole operators lie at 24.7–31.7 MeV, which do not appear in the dotted line. However, because of the coupling to the continuum, the presence of those bound p-h excitations is clearly seen in the RPA strength functions, which are denoted by the solid and dashed lines for the isoscalar and isovector modes, respectively. A peak for RPA IS strength, which may be called IS giant resonance (GR), is seen around 15 MeV, which is appreciably lower than the systematic value [4] for β-stable nuclei, $E_x = 58/A^{1/3}$ MeV = 19.1 MeV. This lower frequency of IS quadrupole GR is typical in nuclei near the neutron drip line. Furthermore, we obtain a substantial strength below 10 MeV, which is the threshold strength coming from the excitation of neutrons with binding energies less than 10 MeV. The large strength below 10 MeV appears also for the RPA IV strength. However, the main contribution to the IV energy-weighted sum rule comes from the strength of peaks at $E_x > 22$ MeV. It is clear that the RPA strength in the very-low-energy region comes from the neutron unperturbed strength in the same energy region. Thus we present the analysis of the unperturbed quadrupole strength.

In Fig. 1(b) we resolve the unperturbed strength expressed by the dotted line in Fig. 1(a), except two sharp proton peaks at 27.66 and 32.33 MeV, into the contributions coming from a given neutron orbital occupied in the ground state. The sum of all strengths in Fig. 1(b) at a given energy is equal to the unperturbed strength in Fig. 1(a) at the same energy, except at 27.66 and 32.33 MeV. Since the binding energies (ϵ_B) of
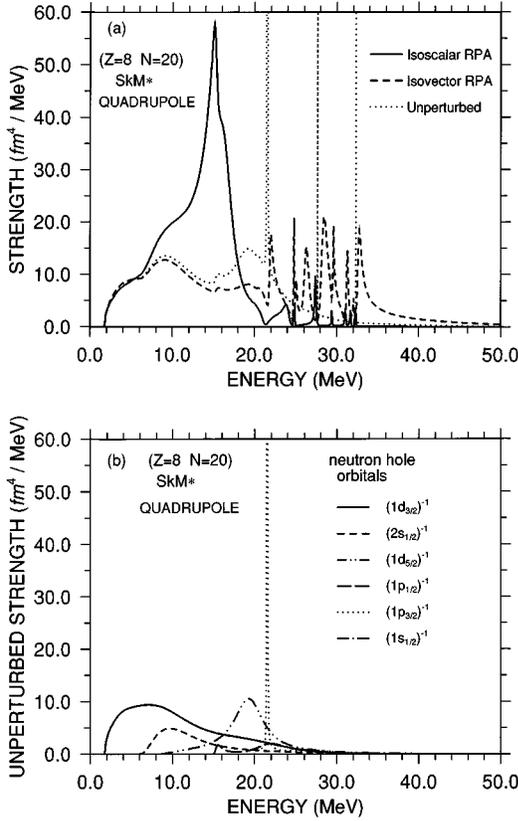
FIG. 1. (a) Unperturbed strength function defined in Eq. (1) and the RPA strength function in Eq. (2) for the isoscalar and the isovector quadrupole mode of $^{28}_{8}O_{20}$ as a function of excitation energy. The unperturbed strength function is the same for the isoscalar and isovector operators. The bound proton p-h excitations at 24.7–31.7 MeV do not appear in the dotted line; however, the presence of those p-h excitations can be recognized as peaks seen in the RPA strength function. (b) Resolving the unperturbed strength in (a), except two very sharp proton peaks at 27.66 and 32.33 MeV, into the components with a given orbital of neutron holes. The sum of all strength at a given energy in the figure is equal to the unperturbed strength in (a) at the same energy. See the text for details.

$1d_{3/2}$, $2s_{1/2}$, $1d_{5/2}$, . . . neutron orbitals are 1.79, 5.99, 8.64, . . . MeV, the corresponding strength starts to appear at respective energies. Above the threshold the strength for neutrons rises as the $\ell+1/2$ power of the available energy [9], namely, as $(E_x-\epsilon_B)^{\ell+1/2}$. Indeed, just above 1.79 MeV

the strength with $(1d_{3/2})^{-1}$ rises as $(E_x-1.79)^{1/2}$ corresponding to $\ell=0$ neutrons. Just above 8.64 MeV the strength with $(1d_{5/2})^{-1}$ rises also as $(E_x-8.64)^{1/2}$, however, in this case the $\ell=0$ neutron strength is so weak as to be recognized in the scale of Fig. 1(b). The main strength in connection with $(1d_{5/2})^{-1}$ appears as the $\ell=4$ neutron strength with a peak around 19.3 MeV, since the neutron $1g_{9/2}$ state is about to become a one-particle resonant state in ^{28}O. The sharp peak at 21.5 MeV comes from the excitation of particles from the $1p_{3/2}$ orbital at -19.93 MeV to the $1f_{7/2}$ one-particle resonant state at $+1.59$ MeV. This is the only neutron unperturbed quadrupole excitation, in which a well-defined one-particle resonant state is available.

The core polarization charges for quadrupole transitions are defined as

$$e_{\rm pol}({\rm IS}) = \frac{1}{2}\left(\frac{\langle \widetilde{j}\|Q^{\lambda=2,\ \tau=0}\|\widetilde{i}\rangle}{\langle j\|Q^{\lambda=2,\ \tau=0}\|i\rangle} - 1\right), \qquad (16)$$

$$e_{\rm pol}({\rm IV}) = \frac{1}{2}\left(\frac{\langle \widetilde{j}\|Q^{\lambda=2,\ \tau=1}\|\widetilde{i}\rangle}{\langle j\|Q^{\lambda=2,\ \tau=1}\|i\rangle} - 1\right). \qquad (17)$$

The proton and neutron core polarization charges are written as $e_{\rm pol}(p) = e_{\rm pol}({\rm IS}) - e_{\rm pol}({\rm IV})$ and $e_{\rm pol}(n) = e_{\rm pol}({\rm IS}) + e_{\rm pol}({\rm IV})$. The calculated results are listed in Table I. Compared with empirical values of β-stable sd-shell nuclei [10], $e_{\rm pol}({\rm IS}) \simeq 0.35$ and $e_{\rm pol}({\rm IV}) \simeq 0.1$, the obtained values in ^{28}O are about a factor of 2 smaller. In β-stable nuclei such as $^{16}_{8}O_8$ or $^{40}_{20}Ca_{20}$ the RPA quadrupole strength is collected into GR, which may have a good isospin, and almost no strength is found for $E_x < 10$ MeV. In contrast, in ^{28}O the very small proton core ($Z/A = 0.29$) is not efficiently polarized by neutrons due to the considerable difference of the radial distribution of protons from that of neutrons. This weak polarization is seen from the fact that the predominant part of the low-lying strength for $E_x < 10$ MeV comes from neutron excitations and that even the "IS" GR around 15 MeV does not collect all higher-lying proton excitation strength. Thus small values of $e_{\rm pol}$ are expected in general. However, a considerable amount of further reduction on $e_{\rm pol}$ arises from an appreciable cancellation between the contributions by the low-lying threshold strength and those by the higher-lying giant resonance. For example, the value of $e_{\rm pol}({\rm IS}) = 0.153$ for the neutron states, $|i\rangle = |f\rangle = |1d_{5/2}\rangle$, is the sum of

TABLE I. Static IS and IV core polarization charges calculated for neutron and proton orbitals in ^{28}O using SkM* force. The $e_{\rm pol}({\rm IS})$ and $e_{\rm pol}({\rm IV})$ values are obtained by using Eqs. (16) and (17) with $\varepsilon_i - \varepsilon_j = 0$. The $B(E2)_{\rm sp}$ value for the transition from the state $|i\rangle$ to $|f\rangle$ is shown for each configuration using the HF wave functions. See the text for details.

		Neutrons			Protons		
f	i	$B(E2)_{\rm sp}$ (e^2 fm^4)	$e_{\rm pol}({\rm IS})$	$e_{\rm pol}({\rm IV})$	$B(E2)_{\rm sp}$ (e^2 fm^4)	$e_{\rm pol}({\rm IS})$	$e_{\rm pol}({\rm IV})$
$1d_{5/2}$	$1d_{5/2}$	15.31	0.153	0.064	12.64	0.163	0.066
$2s_{1/2}$	$1d_{3/2}$	21.75	0.123	0.054	10.98	0.166	0.064
$1d_{3/2}$	$1d_{3/2}$	26.33	0.116	0.053	13.95	0.159	0.066
$2s_{1/2}$	$1d_{5/2}$	13.23	0.147	0.060	8.92	0.172	0.065
$1d_{3/2}$	$1d_{5/2}$	4.697	0.140	0.061	3.48	0.162	0.066
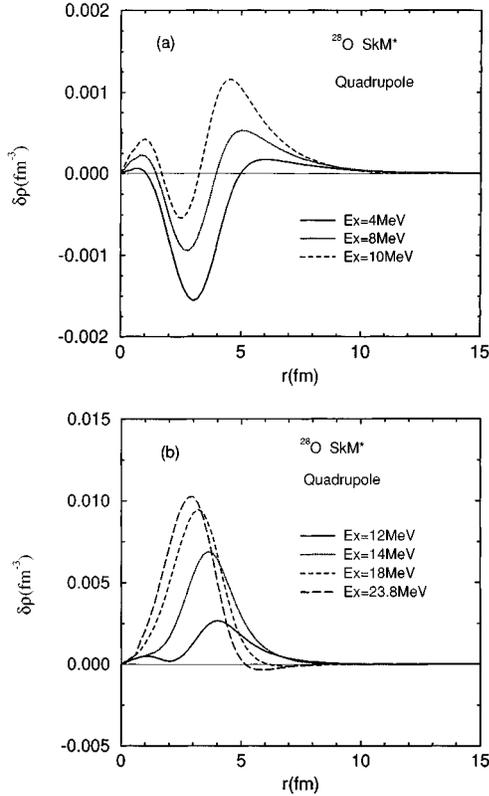
FIG. 2. Radial transition densities of the RPA isoscalar quadrupole excitations of $^{28}_{8}O_{20}$ at various excitation energies E_x as a function of radial coordinate: (a) $E_x \leqslant 10$ MeV and (b) $E_x > 10$ MeV. The absolute magnitude is normalized by using Eq. (7) where $B(\lambda = 2)$ value is obtained integrating $S(E)$ over the energy interval around the peaks. See the text for details.

-0.055 for $E_x < 9$ MeV and $+0.208$ for $E_x > 9$ MeV. And $e_{pol}(IV) = 0.064$ for neutron states, $|i\rangle = |f\rangle = |1d_{5/2}\rangle$, receives -0.035 from $E_x < 15$ MeV and $+0.100$ from $E_x > 15$ MeV. The absolute magnitude is determined by using Eq. (7) where the $B(\lambda = 2)$ value is calculated by integrating $S(E)$ over the peak. Below 20 MeV where the strength function is relatively smooth as a function of energy, the response is cut by a 2-MeV energy interval and integrated to obtain the $B(\lambda = 2)$ value at the corresponding energy. The IS quadrupole transition densities at various energies are shown in Fig. 2. A clear difference is seen between the transition densities below 10 MeV and near the GR around 15 MeV. The former has two radial nodes and a large tail, while the latter shows a typical Tassie-type radial dependence as a collective surface mode. The unique shape of radial transition densities of the low-energy threshold strength, which is quite different from that of collective modes such as the giant resonance, leads to the destructive contribution to the polarization charge. It is interesting that the presence of the low-energy transition strength does not always lead to a large polarization charge, which would be expected from the result of β-stable nuclei. Instead, the presence may even lead to a cancellation of the contribution to the polarization charge by the giant resonance, depending on the shape of the transition density of the threshold strength.

The presence of two nodes in the radial transition densities for $E_x < 10$ MeV comes from the neutron excitations,

$1d_{3/2} \rightarrow s_{1/2}$, in which the wave function of $s_{1/2}$ with a small positive one-particle energy has to be orthogonal to that of the occupied $ns_{1/2}$ state with $n = 1$ and 2. That means that inside of the nucleus the radial dependence of the wave function of $s_{1/2}$ would be similar to that of $3s_{1/2}$. Now, using Eqs. (10), (14), (16), and (17), in the short-range limit of effective interactions [setting $V^{\tau}_{pv}(r)$ in Eq. (11) to be r independent] the contribution from a p-h excitation to the IS and IV static quadrupole polarization charges is negative, if the sign of the product

$$\int r^2 dr R_p(r) R_h(r) R_j(r) R_i(r) \qquad (18)$$

and

$$\int r^4 dr R_p(r) R_h(r) \qquad (19)$$

is negative. Since the behavior of radial wave functions inside of the nucleus plays a role (namely, halo particles are not considered at present), harmonic oscillator wave functions may be used for getting the sign. Taking $|i\rangle = |j\rangle = |1d\rangle$, the sign of the above product, in which $|particle\rangle = |3s\rangle$ and $|hole\rangle = |1d\rangle$, is negative. More generally, taking $|i\rangle = |j\rangle = |hole\rangle$, the sign of the product of expressions (18) and (19) is negative when $|particle\rangle = |n+1, \ell\rangle$ and $|hole\rangle = |n, \ell\rangle$ or when $|particle\rangle = |n+2, \ell-2\rangle$ and $|hole\rangle = |n, L\ell\rangle$. Thus, in the case of ^{28}O, we see that the product has a negative sign also for another low-lying threshold strength, $1d_{3/2} \rightarrow d_j$, where the wave function of d_j has to be orthogonal to the occupied $1d_j$ state. In contrast, the sign of the product of expressions (18) and (19) is generally positive when the number of the radial node of the particle and the hole orbitals is the same or when the particle and the hole orbitals belong to the same major shell in the harmonic oscillator. We have confirmed that the statements on the sign of the product described above are valid also for realistic finite-well potentials.

Using the argument similar to the one described above, we come to a conclusion that the destructive contribution from the very-low-energy threshold strength to the polarization charge would not occur in the dipole or the octupole case in ^{28}O. This conclusion was confirmed also by performing numerical calculations for those excitation modes.

We note that the neutron particle orbitals giving rise to considerable threshold strength in the very-low-energy region must have low orbital angular momentum (possibly, $l = 0$ or 1). Then, it is seen that in spherical closed-shell nuclei near the neutron drip line the possible very-low-lying threshold strength has always a destructive contribution to the quadrupole polarization charge.

As seen from the above explanation using Eq. (18), the destructive contribution from the threshold strength to quadrupole polarization charges is related to the very-short-range character of effective interactions. It remains interesting to see how the destructive contribution will be modified for interactions with a reasonable finite range.

In the recent shell model analysis of measured quadrupole moments in ^{14}B and ^{15}B [11], small core polarization charges are suggested in agreement with the values obtained in the present work. However, the binding energies of the

least-bound neutrons as well as the distribution of the whole transition strength as a function of excitation energy vary considerably, as the mass number changes along the neutron drip line. Thus caution should be taken before making a direct comparison of our values with the analysis of the data in [11].

Making a careful examination of the RPA strength for $E_x < 6$ MeV in Fig. 1(a), we observe the fact that the RPA IS (IV) quadrupole strength is slightly weaker (stronger) than the unperturbed strength at the same energy, instead of being stronger (weaker). This fact, which is unexpected from a simple schematic model, comes from exactly the same origin as the one described above for the destructive contribution to the quadrupole polarization charges from the low-lying threshold strength.

In conclusion, using a particle-vibration coupling together with the HF wave functions and the RPA response functions in $^{28}_{8}O_{20}$, we have obtained the quadrupole polarization charges, which are much smaller than those known for β-stable nuclei. Besides the smallness expected in general for light very neutron-rich nuclei, a considerable amount of further reduction on e_{pol} values arises from an appreciable cancellation between the contributions from low-energy threshold strength and those from giant resonances. In the case of octupole or dipole polarization charges in ^{28}O the same kind of cancellation mechanism does not occur. From the result obtained in the present paper we might say that the idea of using a fixed polarization charge for a given major shell in the shell model calculation may be questioned for neutron drip line nuclei. The conclusions drawn in the present paper may not be trivially applicable for nuclei which have halo particles, although a similar cancellation mechanism might occur in halo nuclei. The polarization charge in halo nuclei should be separately studied.

---

[1] I. Hamamoto, H. Sagawa, and X. Z. Zhang, Phys. Rev. C **53**, 765 (1996).

[2] I. Hamamoto and H. Sagawa, Phys. Rev. C **53**, R1492 (1996).

[3] F. Ghielmetti, G. Colo, E. Vigezzi, P. F. Bortignon, and R. A. Broglia (private communication).

[4] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II.

[5] S. Shlomo and G. F. Bertsch, Nucl. Phys. **A243**, 507 (1975).

[6] N. Van Giai and H. Sagawa, Nucl. Phys. **A371**, 1 (1981).

[7] H. Sagawa and B. A. Brown, Nucl. Phys. **A430**, 84 (1984).

[8] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Hakansson, Nucl. Phys. **A386**, 79 (1982).

[9] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952), Chap. VIII.

[10] B. A. Brown, R. Radhi, and B. H. Wildenthal, Phys. Rep. **101**, 313 (1983).

[11] K. Izumi, K. Asahi, H. Ueno, H. Okuno, H. Sato, K. Nagata, Y. Hori, M. Adachi, N. Aoki, A. Yoshida, G. Liu, N. Fukunishi, and M. Ishihara, Phys. Lett. B **366**, 51 (1996).