Triaxiality in quadrupole deformed nuclei

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The intrinsic $E2$ matrix elements $\langle K=2|E2|K=0\rangle$ for 25 deformed nuclei, covering from neodymium to uranium, have been deduced from measured interband $E2$ matrix elements between the ground band and γ band after correcting for the first-order angular momentum dependence of the coupling between the rotation and intrinsic motion. Fairly precise centroids for the triaxiality of the intrinsic $E2$ moments are obtained, and these correlate well with the triaxiality implied by the excitation energies. The strong correlation of the triaxiality derived from the $E2$ properties and level energies provides a quantitative measure of triaxial quadrupole deformation of the nuclear shape for these states. [S0556-2813(96)02611-8]

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The well known γ band, a low-lying predominantly $K^{\pi}=2,2^{+}$ excitation, is a prominent feature of the level spectrum in even-even deformed nuclei. It is called a γ band because its excitation energy and γ-ray decay properties have been interpreted to result from the breaking of the axial symmetry of the quadrupole shape of the ground state [1], that is, the collective γ degree of freedom. This paper addresses the question of the triaxiality of this band.

The triaxiality of the quadrupole shape usually is specified in terms of Bohr's parameters (β,γ), where the quadrupole deformation tensors of a nuclear density contour, in the intrinsic frame, are defined by $\alpha(2,0)=\beta\cos\gamma$ and $\alpha(2,2)=\beta\sin\gamma/\sqrt{2}$. The magnitude of the quadrupole deformation is characterized by β and asymmetry by γ. The general trend of the γ-ray branching ratios for decay of this low-lying 2^{+} γ-band excitation are reproduced roughly by calculations using either a γ-rigid rotor [2] or a rotation-vibration model [3] with asymmetry angles fitted to the experimental excitation energies.

The $E2$ properties in the intrinsic frame can be described in terms of two collective parameters, (Q,δ), where Q specifies the magnitude of the quadrupole deformation and δ the triaxiality [4]. These parameters are defined in terms of the intrinsic-frame $E2$ moments, $E(2,0)=Q\cos\delta$ and $E(2,\pm2)=Q\sin\delta/\sqrt{2}$. Note that we designate δ as the asymmetry angle derived from the $E2$ properties, to differentiate it from the asymmetry angle γ specifying the radial shape of the nucleus. The intrinsic-frame $E2$ parameters Q,δ, can be related directly to the shape parameters β, γ using a model-dependent transition density. A third measure of the triaxiality, designated γ_E, can be derived from the level energies; this is a measure of the asymmetry of the moments of inertia which are influenced by pairing and the microscopic structure. Frequently it is assumed that the centroids of γ, δ, and γ_E are the same which is not necessarily true. The observed qualitative correlation between the excitation energies and the $E2$ data can be attributed to the correlation of the centroids of δ and γ_E. This paper discusses a fairly precise method for extracting the centroids for the triaxiality δ from $E2$ data in quadrupole-deformed nuclei. The $E2$ triaxiality centroids δ, which are related to the triaxiality γ of the nuclear shape, are compared with corresponding γ_E values derived from the excitation energies.

Cline and Flaum [4–6] have developed a generally applicable, model-independent, technique for extracting the expectation values of the intrinsic-frame parameters Q,δ, that relies on use of rotational invariants, and is based on a suggestion by Kumar [7]. The rotational invariance of zero-coupled products of the $E2$ operator is used to relate the expectation values of the zero-coupled products in the intrinsic frame to those evaluated in the laboratory frame. The centroids of the $E2$ asymmetry δ for many nuclei throughout the Periodic Table, determined by this method, correlate with the γ_E values derived from the excitation energies assuming a γ-rigid rotor relationship [4,5]. Although the rotational-invariant method is model independent and is generally applicable, its usefulness is reduced because of appreciable errors in the extracted δ values that result from compounding of the errors from the several products of $E2$ matrix elements involved in evaluating each rotational invariant.

This paper presents a more precise method for extracting δ values from $E2$ data, but the applicability of this method is restricted to nuclei where quadrupole correlations are strong. The method uses band-mixing calculations to extract the intrinsic matrix elements $\langle K=2|E2|K=0\rangle$ and $\langle K=0|E2|K=0\rangle$ for deformed nuclei, from which the asymmetry of quadrupole deformation is determined by the expression

$$\tan\delta=\sqrt{2}\frac{\langle K=2|E2|K=0\rangle}{\langle K=0|E2|K=0\rangle}. \tag{1}$$

These intrinsic matrix elements are determined from the matrix element of the $2_1^{+}\rightarrow0_1^{+}$ transition and the interband $E2$ matrix elements between the ground and γ bands. It is assumed that these interband matrix elements can be correlated by the following equation (Eq. (4-210) in Ref. [1]):

$$\sqrt{B(E2,I_{K'}\rightarrow I_K)}=\langle I_{K'}K'2-2|I_KK\rangle \times\{M_1-M_2[I_K(I_K+1)-I_{K'}(I_{K'}+1)]\}\xi, \tag{2}$$

TABLE I. Extracted intrinsic $E2$ moments (e b) and δ angles.

Nucleus	$\langle K=0\|E2\|K=0\rangle$	$\langle K=2\|E2\|K=0\rangle$	$\sqrt{2}\times$ ratio [f]	δ (degree) [a]	δ (degree) [b]
^{150}Nd	1.65	0.215(33)	0.184(28)	10.4(16)	
^{152}Sm	1.86	0.234(15)	0.178(11)	10.1(6)	
^{154}Sm	2.07	0.190(25)	0.130(17)	7.4(10)	
^{156}Gd	2.10	0.238(4) [c]	0.160(3)	9.1(2)	
^{158}Gd	2.23	0.220(20)	0.140(13)	8.0(7)	
^{160}Gd	2.28	0.216(6)	0.134(4)	7.6(2)	
^{160}Dy	2.24	0.253(6)	0.160(4)	9.1(2)	
^{162}Dy	2.31	0.256(7)	0.157(4)	8.9(2)	
^{164}Dy	2.36	0.250(11)	0.150(7)	8.5(4)	
^{166}Er	2.41	0.264(6)	0.155(4)	8.8(2)	~10
^{168}Er	2.43	0.243(8) [c]	0.141(5)	8.0(3)	~9
^{168}Yb	2.40	0.269(29)	0.158(17)	9.0(10)	
^{174}Yb	2.41	0.136(21)	0.080(12)	4.6(7)	
^{176}Hf	2.32	0.241(16)	0.147(10)	8.4(6)	
^{178}Hf	2.17	0.219(11)	0.143(7)	8.1(4)	
^{184}W	1.89	0.255(10) [c]	0.191(7)	10.8(4)	~12
^{186}W	1.88	0.406(17)	0.305(13)	17.0(7)	
^{186}Os	1.67	0.417(11) [c]	0.353(9)	19.4(5)	~21
^{188}Os	1.59	0.401(17) [c]	0.357(15)	19.6(8)	~21
^{190}Os	1.53	0.396(37) [c]	0.366(34)	20.1(17)	~24
^{192}Os	1.46	0.45 [d]	0.436	23.6	~24
^{230}Th	2.84	0.257(25)	0.127(12)	7.2(7)	
^{232}Th	3.03	0.31 [e]	0.144	8.2	
^{234}U	3.30	0.254(12)	0.109(5)	6.2(3)	
^{238}U	3.51	0.267(7)	0.107(3)	6.1(2)	

[a]From the present technique.
[b]From the rotation-invariant technique.
[c]More than the 2^+ state of the γ band were included in the fitting.
[d]From three-band-mixing ($K=0$, 2, and 4) calculation.
[e]From three-band-mixing ($K=0$, 2, and $0'$) calculation.
[f]The ratio of $\langle K=2|E2|K=0\rangle$ to $\langle K=0|E2|K=0\rangle$.

where M_1 and M_2 are the fitted intrinsic matrix elements, $K'=K+2$ and ξ is equal to $\sqrt{2}$ if $K=0$ and equal to 1 otherwise. Equation (2) underlies use of the Mikhailov plot [8]. The applicability of Eq. (2) is based on the assumption that both bands are rotational bands having the same intrinsic deformation. The intrinsic matrix element $\langle K=2|E2|K=0\rangle$ is related to the M_1 and M_2 matrix elements by the following relationship (Eq. (4-211) in Ref. [1]):

$$\langle K'|E2|K\rangle = M_1 + 4(K+1)M_2. \tag{3}$$

The matrix element $\langle K=2|h_{+2}|K=0\rangle$ coupling the $\Delta K=2$ bands can be deduced [1] from the level-energy spacing and the reduced amplitude $\langle K=2|\varepsilon_{+2}|K=0\rangle$ describing the admixture of the two bands. That is

$$\langle K=2|\varepsilon_{+2}|K=0\rangle = \frac{\langle K=2|h_{+2}|K=0\rangle}{E(K=2)-E(K=0)}, \tag{4}$$

where the reduced amplitude is related to the M_2 matrix element derived from the experimental $E2$ data:

$$M_2 = \sqrt{6}\langle K=0|E2|K=0\rangle\langle K=2|\varepsilon_{+2}|K=0\rangle. \tag{5}$$

A total of 25 deformed nuclei has been studied in this work, ranging from neodymium to uranium. They are ^{150}Nd [9], ^{152,154}Sm [10,11], ^{156,158,160}Gd [12,13], ^{160,162,164}Dy [13,14], ^{166,168}Er [15,16], ^{168,174}Yb [17,18], ^{176,178}Hf [18], ^{184,186}W [19,20], ^{186,188,190,192}Os [21,22], ^{230,232}Th [23–25], and ^{234,238}U [23,24,26]. The justification for the use of Eq. (2) to correlate the interband $E2$ matrix elements between the ground and γ bands is demonstrated by analyses of the rotational-invariant technique [4–6] applied to ^{168}Er [16], ^{184}W [19], and ^{186,188,190,192}Os [21], which showed an almost constant magnitude and asymmetry for quadrupole deformation in both the ground and γ bands. This is consistent with the interpretation that these bands are rotational bands with approximately equal intrinsic deformation. Cases where the $E2$ data are inconsistent with the linear relationship in Eq. (2), due to mixing with a third state or band mixing, were not included except for ^{192}Os and ^{232}Th, where the three-band-mixing calculations had been done previously [22]. For most cases, only three decay branchings of the $I,K^\pi = 2,2^+$ excitation are involved in the least-squares fit except for ^{156}Gd, ^{166,168}Er, ^{184}W, and ^{186,188,190}Os where the interband $E2$ matrix elements for many members of the $K=2$ band have been measured.

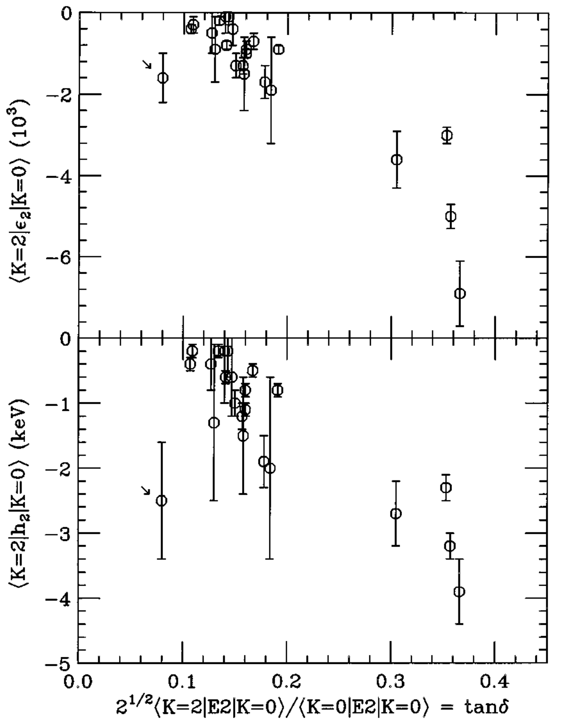

FIG. 1. The reduced amplitude (upper) and coupling matrix element (lower) plotted versus the ratio of intrinsic $E2$ matrix elements which corresponds to $\tan\delta$. The arrow indicates the ^{174}Yb data where the intrinsic $E2$ strength may be missing due to an unusually high excitation energy that allows mixing with two-quasiparticle states.

The extracted intrinsic matrix elements $\langle K=2|E2|K=0\rangle$ for those nuclei and the corresponding δ centroids are listed in Table I. Also included, in the last column of Table I, are the centroids of δ determined using the rotation-invariant technique for some of the nuclei studied; these agree reasonably well with the values obtained in the present work.

Figure 1 shows the mixing amplitudes and mixing matrix elements plotted versus the intrinsic $E2$ matrix element ratio which corresponds to $\tan\delta$. Both the extracted values of the reduced amplitude for the wave function, and the coupling matrix element are small. They correlate with the asymmetry angle δ showing an increase in absolute value with increase in the centroid δ. One notable exception is ^{174}Yb, where the asymmetry is a factor of 2 smaller than that of neighboring nuclei. A possible cause for the small asymmetry in ^{174}Yb, is because the $I,K^{\pi} = 2,2^+$ state has an unusually high excitation energy (1634 keV), allowing mixing with two-quasiparticle states; the resulting strength fragmentation will lead to an underestimate of the intrinsic matrix element $\langle K=2|E2|K=0\rangle$. The $K=2$ purity of the γ band is illustrated by the smallness of the mixing amplitude of the $K=0$ component which, even in the worst case of ^{190}Os, is only 5% for the 2_2^+ state. Note that ^{192}Os is outside of the domain of this discussion, in that the linearity of Eq. (2) is violated.

The intrinsic-frame $E2$ centroids (Q,δ), which are experimental observables, can be related to the model-dependent shape parameters (β,γ) within a collective model and using the adiabatic approximation. The reverse process is more difficult in that it involves knowing the radial dependence and fluctuation widths of β and γ [6]. However, for the deformed nuclei considered here, it is expected that the centroids of δ and γ are comparable. Note that the $E2$ asymmetry centroid δ does not differentiate the fluctuation amplitude for dynamic triaxial motion from a possible static quadrupole potential energy minimum at $\gamma\neq 0$.

A measure of the triaxiality γ_E, relating the moments of inertia, can be derived from the excitation energy of the $I,K^{\pi} = 2,2^+$ state. Although the triaxiality is expected to be a dynamic, rather than a static effect, the extreme rigid triaxial rotor model can be used to obtain a crude estimate of the centroid of γ_E using the relation between the excitation energies of the 2_1^+ and 2_2^+ states,

$$\frac{E_{2^+(K=2)}}{E_{2^+(K=0)}} = \frac{1+\sqrt{1-(8/9)\sin^2 3\gamma_E}}{1-\sqrt{1-(8/9)\sin^2 3\gamma_E}}. \tag{6}$$

Table II lists those excitation energies and the corresponding estimate of the γ_E centroids for all nuclei studied.

TABLE II. Excitation energies in keV for the first and second 2^+ states of deformed nuclei and the extracted γ_E values.

Nucleus	$E(2^+_{K=0})$	$E(2^+_{K=2})$	Ratio [a]	γ_E (degree)
^{150}Nd	130.2	1061.9	8.2	13.8
^{152}Sm	121.8	1085.9	8.9	13.2
^{154}Sm	82.0	1440.4	17.6	9.6
^{156}Gd	89.0	1154.1	13.0	11.0
^{158}Gd	79.5	1187.1	14.9	10.4
^{160}Gd	75.3	988.2	13.1	11.0
^{160}Dy	86.8	966.2	11.1	12.0
^{162}Dy	80.7	888.2	11.0	12.0
^{164}Dy	73.4	761.8	10.4	12.4
^{166}Er	80.6	785.9	9.8	12.6
^{168}Er	79.8	821.2	10.3	12.4
^{168}Yb	87.7	983.9	11.2	11.8
^{174}Yb	76.5	1634.	21.4	8.7
^{176}Hf	88.4	1341.3	15.2	10.2
^{178}Hf	93.2	1174.6	12.6	11.2
^{184}W	111.2	903.3	8.1	14.0
^{186}W	122.6	737.9	6.0	16.0
^{186}Os	137	767	5.60	16.4
^{188}Os	155	633	4.08	19.2
^{190}Os	187	558	2.98	22.8
^{192}Os	206	489	2.37	25.4
^{230}Th	53.2	781.0	14.7	10.4
^{232}Th	49.4	785.5	15.9	10.0
^{234}U	43.5	926.7	21.3	8.7
^{238}U	44.9	1060	23.6	8.3

[a]The ratio of $E(2^+_{K=2})$ to $E(2^+_{K=0})$.

To have a better understanding of the systematics of triaxiality in quadrupole deformed nuclei, the $E2$ data and the excitation-energy ratio for the $I,K^{\pi} = 2,2^+$ excitation and the first 2^+ state are plotted against each other in Fig. 2. Figure 2(a) shows that there is a strong correlation between them. Figure 2(b) shows the strong correlation of the $E2$ δ

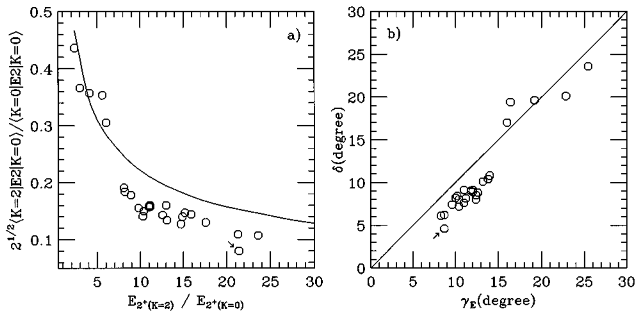

FIG. 2. The asymmetry of quadrupole deformation derived from the intrinsic $E2$ matrix elements vs that derived from the excitation-energy ratio between the $I,K^\pi=2,2^+$ and the first 2^+ states. The solid curve is the assumed correlation based on a static triaxial quadrupole shape. The arrow indicates the ^{174}Yb data where the intrinsic $E2$ strength may be missing (see caption for Fig. 1).

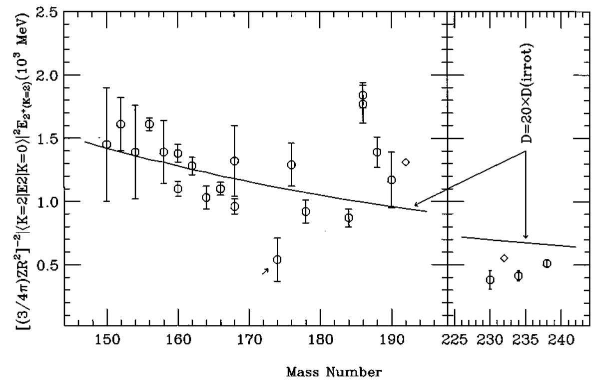

FIG. 3. The products of the vibration amplitude and frequency of the $I,K^\pi=2,2^+$ excitation plotted against the mass number. The solid curve is resulted from a mass parameter of 20 times of the mass parameter, D(irrot), for a surface vibration in the liquid drop model. The symbol (◇) indicates the data derived from band-mixing calculations and no error was assigned. See the caption of Fig. 1 for the arrow indicator.

centroid, versus the γ_E centroid derived using the extreme rigid-rotor model. The anomalous case of ^{174}Yb is marked in Fig. 2 by the arrow. The correlation, shown in Fig. 2, is strong evidence for the existence of triaxial quadrupole deformation. This correlation is moderately well described by the extreme γ-rigid rotor model which ignores dynamic shape effects. The systematic deviation, for the most strongly deformed nuclei, is not unexpected considering the extreme model used to estimate γ_E. Note that pairing correlation effects [3] are too small to account for this systematic deviation.

Within the framework of the γ vibrator model, the $I,K^\pi=2,2^+$ excitation is due to vibrational motion which breaks the axial symmetry for quadrupole deformation of the ground state. The intrinsic matrix element $\langle K=2|E2|K=0\rangle$ (a measure of the vibration amplitude) is related to the excitation energy (a measure of the vibration frequency) by the following equation (Eq. (6-92) in Ref. [1]):

$$\langle K=2|E2|K=0\rangle=\left(\frac{3}{4\pi}ZR^2\right)\sqrt{\frac{\hbar^2}{2DE_{2^+(K=2)}}}=\left(\frac{3}{4\pi}ZR^2\right)\sqrt{\frac{E_{2^+(K=2)}}{2C}}, \tag{7}$$

where Z is the atomic number, $R=1.2A^{1/3}$ fm, D is the mass parameter, and C is the restoring force parameter. The product of the intrinsic moment and the excitation energy for the $I,K^\pi=2,2^+$ state, which can be interpreted as the vibrator mass parameter $\hbar^2/2D$, is plotted against the mass number in Fig. 3. A mass parameter that is 20 times D(irrot) (see Eq. (6A-31) in Ref. [1]), for a surface vibration in the liquid drop model, is consistent with the data. The quantity, D(irrot)/D, also is a measure of the classical oscillator strength implying that the γ-vibration motion in deformed nuclei carries about 5% of the classical $E2$ oscillator strength.

Recent random-phase approximation (RPA) calculations [27], for the γ-vibrational states in the strongly deformed nuclei, predict that the low-lying γ-vibrational mode carries about 10% of the classical $E2$ oscillator strength. In these calculations, the parameters for the quadrupole-quadrupole interaction were fixed so that the excitation energy of the $I,K^\pi=2,2^+$ state is reproduced. This predicted $E2$ strength is twice the observed strength.

In summary, the determination of the intrinsic matrix element $\langle K=2|E2|K=0\rangle$ for 25 deformed nuclei has been achieved from the interband matrix elements between the ground and γ bands after correcting for the first order angular momentum dependence of the coupling between the rotational and intrinsic motion. These provided a fairly precise study of the triaxiality angle δ characterizing the centroid of the $E2$ moments in the intrinsic frame for deformed nuclei ranging from neodymium to uranium. The $E2$ triaxiality centroids, δ, correlate well with γ_E centroids, derived from the excitation energy using the extreme rigid triaxial rotor model, demonstrating quantitatively that the root mean square shape is triaxially deformed. The $E2$ centroids δ provide a good measure of the triaxiality of the nuclear shape for these states but are not sensitive to dynamic shape fluctuations. Under the assumption of a γ vibrator, the relationship between the intrinsic matrix element and excitation energy of the $I,K^\pi=2,2^+$ state implies that the γ-vibrational strength in deformed nuclei accounts for about 5% of the classical $E2$ oscillator strength.

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