

Self-consistent velocity dependent effective interactions

T. Kubo

Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan

H. Sakamoto

Faculty of Engineering, Gifu University, Gifu 501-11, Japan

T. Kammuri*

Department of Physics, Osaka University, Toyonaka 560, Japan

T. Kishimoto†

Institute of Physics, University of Tsukuba, Ibaraki 305, Japan

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The theory of self-consistent effective interactions in nuclei is extended for a system with a velocity dependent mean potential. By means of the field coupling method, we present a general prescription to derive effective interactions which are consistent with the mean potential. For a deformed system with the conventional pairing field, the velocity dependent effective interactions are derived as the multipole pairing interactions in doubly stretched coordinates. They are applied to the microscopic analysis of the giant dipole resonances (GDR's) of $^{148,154}\text{Sm}$, the first excited 2^+ states of Sn isotopes and the first excited 3^- states of Mo isotopes. It is clarified that the interactions play crucial roles in describing the splitting and structure of GDR peaks, in restoring the energy weighted sum rule, and in reducing the values of $B(E\lambda)$. [S0556-2813(96)00511-0]

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I. INTRODUCTION

The understanding of nuclear collective excitations has been one of the most important subjects in the nuclear many body problem [1]. In microscopic analyses of such excitations, separable multipole interactions have been introduced and applied extensively for spherical, deformed, and rotating nuclei. The separable multipole-multipole interactions originate from long-range correlations in particle-hole channels, and the physical meaning of these interactions has been clarified by Mottelson in terms of the core polarization phenomena [2]. On the other hand, the multipole pairing interactions originate from short-range correlations in particle-particle channels, and there are many works concerning the physical meaning of these interactions [1,3-7]. These interactions have been widely used and have been playing crucial roles in the study of nuclear structure, but theoretical foundations for the origin of such effective interactions in both the particle-hole and particle-particle channels have not been established enough from a unified physical picture. In particular, we consider that it is still an open and interesting problem to determine the proper form of the deformation dependence of the pairing interaction. It is the main purpose of the present paper to report a general prescription to derive effective interactions which are consistent with the mean potential, and we will present a unified derivation of the multipole-multipole interactions in particle-hole channels and the multipole pair-

ing interactions in particle-particle channels on an equal footing (preliminary reports on this subject can be seen in Refs. [8,9]).

Since a nucleus can be regarded as a spatially and energetically saturated self-sustained system with a relatively sharp boundary, one can assume that the following condition, which will be called as *nuclear self-consistency* [10,11], is satisfied quite accurately: The shape of the mean potential and that of the density are the same even when the system undergoes collective motions. The concept of nuclear self-consistency, which is much more stringent than the *Hartree self-consistency*, has played important roles not only in describing an equilibrium nuclear shape [12,13] and in deriving effective interactions for a spherical system [1,2], but also in deriving effective interactions for a deformed system [10,11], and higher order effective interactions [11,14,15]. These interactions have been applied successfully to the calculations of the properties of the low-lying vibrational states and high-frequency giant resonances, etc. [10,11,16-23]. It is found that the effective interactions derived by the rigorous application of the nuclear self-consistency are much more reliable than the conventional multipole-multipole interactions [1,2,10,11].

However, it has been pointed out that the spurious velocity dependence in a single-particle mean potential, which violates the Galilean invariance of the system, should be removed [1,3,5-7]. It is well recognized that the velocity dependent terms affect the mass parameters of collective motions and thereby such quantities as the distribution of transition strengths and the absolute value of the energy weighted sum rule, etc., become unreliable [1,3,5-7,24,25]. The realistic nuclear potential in fact contains velocity de-

*Current address: 1-1-34, Minato, Izumisano-shi, Osaka 598, Japan.

†Deceased 14 April 1990.

pendent terms such as the BCS pairing field, $\vec{l} \cdot \vec{s}$ and \vec{l}^2 terms in the Nilsson model, etc. Some parts of them are responsible for violating the Galilean invariance and the classical sum rule, and therefore these symmetries have to be restored properly. From this point of view, Pyatov *et al.* developed a simple and powerful method to introduce additional interactions which restore such broken symmetries in RPA order [5,6]. Since then, symmetry restoring effective interactions have been studied by several authors [5–7,26–30]. In this paper, by investigating the coupling between collective and single-particle degrees of freedom, and by using the concept of *nuclear self-consistency* and *local Galilean invariance* of the system as important guiding principles, we will present a systematic method to derive self-consistent effective interactions in nuclei.

In Sec. II, the field coupling method [1,11] is applied to a spherical system with a velocity dependent potential. In this framework, we derive self-consistent velocity dependent effective interactions by estimating the coupling between the collective displacement of nucleons and the mean field. In Sec. III, we extend the field coupling method to a deformed system and present a simple derivation of the doubly stretched multipole-multipole and multipole-pairing interactions. In Sec. IV, we investigate some fundamental properties of these self-consistent effective interactions. It is shown to be essential to express effective interactions in doubly stretched coordinates for restoring some broken symmetries and also for the natural description of GDR's in deformed nuclei. In Sec. V, we report the results of numerical calculations in RPA of the GDR's of $^{148,154}\text{Sm}$, the first excited 2^+ states of Sn isotopes and the first excited 3^- states of Mo isotopes, by using the self-consistent velocity dependent effective interactions.

II. GENERAL FRAMEWORK OF FIELD COUPLING METHOD

A. Field coupling between collective distortion and mean potential

The method to study collective motion that arises from the action of the field coupling in a system with a degenerate one-particle excitation has been well developed by Bohr and Mottelson [1]. It provides a self-consistent method to construct the relevant effective interaction by identifying the field coupling as the Hartree field of the interaction. Based on the method, which will be referred to as the *field coupling method*, the theory of self-consistent effective interactions in nuclei has been developed [10,11,14]. As a result, it has been shown, for example, that the conventional multipole interaction model must be improved to satisfy the nuclear self-consistency in deformed nuclei, resulting in the doubly stretched multipole interaction model. Here we will extensively apply the method to a system with the velocity dependent mean potential. Some examples of the velocity dependent field coupling were discussed in Ref. [1] in connection with the analysis of the high-frequency quadrupole modes and of the center-of-mass mode.

In a nuclear system, particle motions and collective motions are essentially coupled with each other in order to achieve self-consistency between the potential and the density distribution. If a collective mode excites in a nucleus, the

positions and momenta of nucleons in the system are displaced by the mode accordingly. Then the corresponding change in the nuclear density distribution gives rise to a violation of the self-consistency settled before the displacements are switched on. In this way additional field couplings are induced in the system in order to restore the nuclear self-consistency.

Let us start by considering a 2^λ -pole collective shape oscillation mode of a spherical nucleus. We assume that this mode is characterized by the collective displacement of the nucleonic field variable as

$$\vec{r} \rightarrow \vec{r} + \delta\vec{r}, \quad \delta\vec{r} = - \sum_{\mu} \alpha_{\lambda\mu}^* \vec{\nabla} Q_{\lambda\mu} \quad (\lambda \geq 1), \quad (1)$$

where $\alpha_{\lambda\mu}$ are the collective amplitudes. The collective velocity field associated with this mode is expressed as

$$\vec{v}(\vec{r}) = - \sum_{\mu} \dot{\alpha}_{\lambda\mu}^* \vec{\nabla} Q_{\lambda\mu}, \quad (2)$$

which is irrotational and incompressible. We will require that nucleonic velocities entering into the velocity dependent single-particle potential are to be measured relative to the collective velocity $\vec{v}(\vec{r})$ [1] so that the potential is to be invariant under the local Galilean transformation

$$\vec{p} \rightarrow \vec{p} + \delta\vec{p}, \quad \delta\vec{p} = M\vec{v}(\vec{r}). \quad (3)$$

Then the variation of the average one body potential δV and that of the density distribution $\delta\rho$ produced by the oscillation are determined from the conditions

$$\begin{aligned} V(\vec{r} + \delta\vec{r}, \vec{p} + \delta\vec{p}) &= V_0(\vec{r}, \vec{p}), \\ \rho(\vec{r} + \delta\vec{r}, \vec{p} + \delta\vec{p}) &= \rho_0(\vec{r}, \vec{p}), \end{aligned} \quad (4)$$

where $V_0(\vec{r}, \vec{p})$ and $\rho_0(\vec{r}, \vec{p})$ are the potential and the density at the original equilibrium point in the phase space and are assumed to be spherical, while $V(\vec{r}, \vec{p})$ and $\rho(\vec{r}, \vec{p})$ include the effect of the oscillation. In this paper we assume that these quantities expressed in the phase space are well defined by using some appropriate semiclassical method such as the Wigner transformation [31–34].

The conditions of Eq. (4) provide relations between δV and $\delta\rho$ through the displacement vectors $\delta\vec{r}$ and $\delta\vec{p}$ as

$$\begin{aligned} V(\vec{r}, \vec{p}) &= V_0(\vec{r}, \vec{p}) + \delta V(\vec{r}, \vec{p}), \\ \rho(\vec{r}, \vec{p}) &= \rho_0(\vec{r}, \vec{p}) + \delta\rho(\vec{r}, \vec{p}), \end{aligned} \quad (5)$$

with

$$\begin{aligned} \delta V(\vec{r}, \vec{p}) &= \delta V_r + \delta V_p = - \delta\vec{r} \cdot \vec{\nabla} V_0 - \delta\vec{p} \cdot \vec{\nabla}_p V_0, \\ \delta\rho(\vec{r}, \vec{p}) &= \delta\rho_r + \delta\rho_p = - \delta\vec{r} \cdot \vec{\nabla} \rho_0 - \delta\vec{p} \cdot \vec{\nabla}_p \rho_0, \end{aligned} \quad (6)$$

where notations are defined by

$$\vec{\nabla} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right), \quad \vec{\nabla}_p \equiv \left(\frac{\partial}{\partial p_x}, \frac{\partial}{\partial p_y}, \frac{\partial}{\partial p_z} \right). \quad (7)$$

In Eq. (6), for simplicity, only the leading order terms in $\alpha_{\lambda\mu}$ and $\dot{\alpha}_{\lambda\mu}$ are retained assuming that the vibrational amplitudes are small. Inclusion of the nonlinear field coupling coming from higher order terms in $\alpha_{\lambda\mu}$ has been performed in Ref. [11] resulting in higher order (many body) effective interactions. The δV_r and δV_p can be expressed in the standard form of the field coupling [1] as

$$\delta V_r = \kappa_\lambda \sum_\mu \alpha_{\lambda\mu}^* F_{\lambda\mu}, \quad F_{\lambda\mu} = \frac{1}{\kappa_\lambda} \vec{\nabla} Q_{\lambda\mu} \cdot \vec{\nabla} V_0, \quad (8)$$

$$\delta V_p = \tilde{\kappa}_\lambda \sum_\mu \dot{\alpha}_{\lambda\mu}^* \tilde{F}_{\lambda\mu}, \quad \tilde{F}_{\lambda\mu} = \frac{M}{\tilde{\kappa}_\lambda} \vec{\nabla} Q_{\lambda\mu} \cdot \vec{\nabla}_p V_0. \quad (9)$$

Now we consider that the field couplings δV_r and δV_p are produced as the Hartree field of two-body interactions

$$H_{\text{int}} = \frac{\kappa_\lambda}{2} \sum_\mu F_{\lambda\mu}^\dagger F_{\lambda\mu} + \frac{\tilde{\kappa}_\lambda}{2} \sum_\mu \tilde{F}_{\lambda\mu}^\dagger \tilde{F}_{\lambda\mu} \equiv H_\lambda^{(r)} + H_\lambda^{(p)}, \quad (10)$$

and introduce self-consistency conditions as

$$\alpha_{\lambda\mu}^* = A \langle F_{\lambda\mu}^\dagger \rangle, \quad \dot{\alpha}_{\lambda\mu}^* = A \langle \tilde{F}_{\lambda\mu}^\dagger \rangle. \quad (11)$$

Here the average of a one body operator with respect to the modified ground state $|\ \ \rangle$ corresponding to the density ρ is calculated as

$$A \langle F \rangle = \left\langle \sum_{i=1}^A F(i) \right\rangle = \int F \rho(\vec{r}, \vec{p}) d^3 r d^3 p, \quad (12)$$

while that with respect to the original ground state $|0\rangle$ corresponding to the density ρ_0 is given by

$$A \langle F \rangle_0 = \left\langle \sum_{i=1}^A F(i) \right\rangle_0 = \int F \rho_0(\vec{r}, \vec{p}) d^3 r d^3 p. \quad (13)$$

Equation (11) can be calculated as

$$\begin{aligned} \alpha_{\lambda\mu}^* &= \int F_{\lambda\mu}^\dagger \delta \rho_r d^3 r d^3 p \\ &= \alpha_{\lambda\mu}^* \int F_{\lambda\mu}^\dagger \vec{\nabla} Q_{\lambda\mu} \cdot \vec{\nabla} \rho_0 d^3 r d^3 p \\ &= -\alpha_{\lambda\mu}^* A \langle (\vec{\nabla} F_{\lambda\mu}^\dagger \cdot \vec{\nabla} Q_{\lambda\mu}) \rangle_0, \end{aligned} \quad (14)$$

$$\begin{aligned} \dot{\alpha}_{\lambda\mu}^* &= \int \tilde{F}_{\lambda\mu}^\dagger \delta \rho_p d^3 r d^3 p \\ &= M \dot{\alpha}_{\lambda\mu}^* \int \tilde{F}_{\lambda\mu}^\dagger \vec{\nabla} Q_{\lambda\mu} \cdot \vec{\nabla}_p \rho_0 d^3 r d^3 p \\ &= -M \dot{\alpha}_{\lambda\mu}^* A \langle (\vec{\nabla}_p \tilde{F}_{\lambda\mu}^\dagger \cdot \vec{\nabla} Q_{\lambda\mu}) \rangle_0, \end{aligned} \quad (15)$$

where we have assumed the time reversal invariance of the density ρ_0 which guarantees

$$\int F_{\lambda\mu}^\dagger \delta \rho_p d^3 r d^3 p \propto A \langle \vec{\nabla}_p F_{\lambda\mu}^\dagger \cdot \vec{\nabla} Q_{\lambda\mu} \rangle_0 = 0, \quad (16)$$

$$\int \tilde{F}_{\lambda\mu}^\dagger \delta \rho_r d^3 r d^3 p \propto A \langle \vec{\nabla} \tilde{F}_{\lambda\mu}^\dagger \cdot \vec{\nabla} Q_{\lambda\mu} \rangle_0 = 0. \quad (17)$$

We thus obtain

$$\kappa_\lambda = -A \langle \vec{\nabla} (\vec{\nabla} Q_{\lambda\mu}^\dagger \cdot \vec{\nabla} V_0) \cdot \vec{\nabla} Q_{\lambda\mu} \rangle_0, \quad (18)$$

$$\tilde{\kappa}_\lambda = -M^2 A \langle \vec{\nabla}_p (\vec{\nabla} Q_{\lambda\mu}^\dagger \cdot \vec{\nabla}_p V_0) \cdot \vec{\nabla} Q_{\lambda\mu} \rangle_0. \quad (19)$$

Since $Q_{\lambda\mu}$ does not depend on the momentum \vec{p} , $\tilde{F}_{\lambda\mu}$ of Eq. (9) and $\tilde{\kappa}_\lambda$ of Eq. (19) can be expressed in terms of the Poisson bracket as

$$\tilde{F}_{\lambda\mu} = \frac{M}{\tilde{\kappa}_\lambda} \{Q_{\lambda\mu}, V_0\}, \quad (20)$$

$$\tilde{\kappa}_\lambda = -M^2 \langle \{Q_{\lambda\mu}, \{Q_{\lambda\mu}^\dagger, V_0\}\} \rangle_0. \quad (21)$$

In the above treatment, we have assumed the potential V_0 can be expressed as a sum over individual particles: $V_0 = \sum_{i=1}^A V_0(\vec{r}_i, \vec{p}_i)$. However the monopole pairing potential

$$V_{\text{pair}} = -\frac{\Delta}{2} (P_0^\dagger + P_0), \quad P_0^\dagger \equiv \sum_\gamma c_\gamma^\dagger c_{\tilde{\gamma}}^\dagger = 2 \sum_{\gamma>0} c_\gamma^\dagger c_{\tilde{\gamma}}^\dagger \quad (22)$$

is not expressed in this form. Here c_γ^\dagger creates a nucleon in the state γ , $\tilde{\gamma}$ is a time reversed state of γ , and Δ is a gap parameter. For such a potential we will replace the Poisson brackets in Eqs. (20) and (21) by the corresponding commutation relations as

$$\tilde{F}_{\lambda\mu} = \frac{M}{i\hbar \tilde{\kappa}_\lambda} [Q_{\lambda\mu}, V_0], \quad (23)$$

$$\tilde{\kappa}_\lambda = -\left(\frac{M}{\hbar}\right)^2 \langle [Q_{\lambda\mu}, [V_0, Q_{\lambda\mu}^\dagger]] \rangle_0. \quad (24)$$

Then the corresponding part of the effective interaction becomes

$$H_\lambda^{(p)} = -\frac{1}{2} \sum_\mu \frac{1}{\langle [Q_{\lambda\mu}, [V_0, Q_{\lambda\mu}^\dagger]] \rangle_0} [Q_{\lambda\mu}, V_0]^\dagger [Q_{\lambda\mu}, V_0]. \quad (25)$$

This type of separable effective interactions have been discussed by several authors in connection with the symmetry-restoring treatment of the nuclear Hamiltonian [5–7, 26–30].

B. Two simple examples of self-consistent effective interactions

It is worthwhile to briefly review the derivation of the self-consistent effective interactions in the case of a simple mean potential such as a harmonic oscillator potential [11] or a pairing potential [7].

1. Spherical harmonic oscillator potential

Let us consider a simple situation that the equilibrium potential V_0 is given by the spherical harmonic oscillator potential

$$V_0 = V_{\text{HO}}(r) = \frac{1}{2} M \omega_0^2 r^2. \quad (26)$$

In this case, we do not have to consider the velocity dependent field coupling since the potential does not have any velocity dependence. From Eqs. (8) and (18) we obtain

$$F_{\lambda\mu} = \frac{1}{\kappa_\lambda} M \omega_0^2 \vec{r} \cdot \vec{\nabla} Q_{\lambda\mu} = \frac{1}{\kappa_\lambda} \lambda M \omega_0^2 Q_{\lambda\mu}, \quad (27)$$

$$\begin{aligned} \kappa_\lambda &= -\lambda M \omega_0^2 A \langle \vec{\nabla} Q_{\lambda\mu}^\dagger \cdot \vec{\nabla} Q_{\lambda\mu} \rangle_0 \\ &= -M \omega_0^2 \frac{\lambda^2 (2\lambda + 1)}{4\pi} A \langle r^{2\lambda-2} \rangle_0, \end{aligned} \quad (28)$$

where we used the relations

$$\begin{aligned} \vec{r} \cdot \vec{\nabla} Q_{\lambda\mu} &= \lambda Q_{\lambda\mu}, \quad \Delta Q_{\lambda\mu} = 0, \\ \Delta(Q_{\lambda\mu}^\dagger Q_{\lambda\mu}) &= \frac{2\lambda(2\lambda+1)}{4\pi} r^{2\lambda-2}. \end{aligned} \quad (29)$$

Thus the self-consistent effective interaction coincides with the conventional multipole-multipole interaction

$$H_\lambda^{(r)} = -\frac{1}{2} \chi_\lambda^{\text{self}} \sum_\mu Q_{\lambda\mu}^\dagger Q_{\lambda\mu} \quad (30)$$

with the self-consistent strength

$$\chi_\lambda^{\text{self}} = \frac{4\pi}{2\lambda+1} \frac{M \omega_0^2}{A \langle r^{2\lambda-2} \rangle_0}. \quad (31)$$

2. Pairing potential

Let us introduce a one body operator

$$O = \sum_{\alpha\beta} \langle \alpha | O | \beta \rangle c_\alpha^\dagger c_\beta, \quad (32)$$

whose time reversal property is assumed to be

$$\hat{T} O \hat{T}^{-1} = (-)^T O^\dagger \quad (33)$$

or equivalently

$$\langle \tilde{\alpha} | O | \tilde{\beta} \rangle = \langle \hat{T} \alpha | O | \hat{T} \beta \rangle = (-)^T \langle \beta | O | \alpha \rangle, \quad (34)$$

where \hat{T} denotes the time reversal operator, and $(-)^T$ is a shorthand notation for the time reversal phase and is either +1 or -1. The commutation relation between the operator O and the pairing field V_{pair} is given by

$$[O, V_{\text{pair}}] = \frac{1+(-)^T}{2} \Delta \sum_{\alpha\beta} \langle \alpha | O | \tilde{\beta} \rangle (c_\alpha^\dagger c_\beta^\dagger - c_\beta \bar{c}_\alpha). \quad (35)$$

From this relation, it is easily recognized that the pairing field satisfies the translational invariance but violates the Galilean invariance:

$$[\vec{P}, V_{\text{pair}}] = 0, \quad (36)$$

$$[\vec{R}, V_{\text{pair}}] = \Delta \sum_{\alpha\beta} \langle \alpha | \vec{R} | \tilde{\beta} \rangle (c_\alpha^\dagger c_\beta^\dagger - c_\beta \bar{c}_\alpha) \quad (37)$$

with

$$\vec{P} = \sum_{j=1}^A \vec{p}_j = \sum_{j=1}^A (-i\hbar \vec{\nabla}_j), \quad \vec{R} = \frac{1}{A} \sum_{j=1}^A \vec{r}_j. \quad (38)$$

More generally, if O is an operator depending on coordinate variables, then Eq. (35) means that the monopole pairing field is not invariant under a local Galilean transformation. In this sense the pairing field is considered to be a velocity dependent mean potential [1]. For a system with such a velocity dependent potential, there arises a velocity dependent field coupling as is discussed in Sec. II A.

Let us investigate the interaction of Eq. (25) in more detail. By substituting V_{pair} into V_0 and by identifying the original ground state $|0\rangle$ with the BCS vacuum state, the basic quantities in Eqs. (23) and (24) are expressed as

$$[Q_{\lambda\mu}, V_{\text{pair}}] = -\Delta (P_{\lambda\mu}^\dagger - P_{\lambda\mu}^-), \quad (39)$$

$$\langle [Q_{\lambda\mu}, [V_{\text{pair}}, Q_{\lambda\mu}^\dagger]] \rangle_0 = \Delta^2 \sum_{\alpha\beta} \left(\frac{1}{E_\alpha} + \frac{1}{E_\beta} \right) |\langle \alpha | Q_{\lambda\mu} | \beta \rangle|^2, \quad (40)$$

where

$$P_{\lambda\mu}^\dagger \equiv \sum_{\alpha\beta} \langle \alpha | Q_{\lambda\mu} | \beta \rangle c_\alpha^\dagger c_\beta^\dagger, \quad (41)$$

and we have used the familiar relation between the coefficients of Bogoliubov-Valatin transformation and the quasiparticle energy: $u_\alpha v_\alpha = \Delta/2E_\alpha$. Thus the velocity dependent effective interaction of Eq. (25) can be obtained as

$$H_\lambda^{(p)} = -\frac{1}{8} G_\lambda^{\text{self}} \sum_\mu (P_{\lambda\mu}^\dagger - P_{\lambda\mu}^-) (P_{\lambda\mu} - P_{\lambda\mu}^\dagger) \quad (42)$$

with

$$G_\lambda^{\text{self}} \equiv 1 / \sum_{\alpha\beta} \frac{1}{4} \left(\frac{1}{E_\alpha} + \frac{1}{E_\beta} \right) |\langle \alpha | Q_{\lambda\mu} | \beta \rangle|^2. \quad (43)$$

This interaction is a natural extension of the dipole-pairing interaction obtained by Bohr and Mottelson [1] and Pyatov and Salamov [5] to a general 2^λ -pole mode. Equation (43) has the same structure as that of the Belyaev identity [3,4]. Further discussions for this interaction can be found in Refs. [7,20].

III. SELF-CONSISTENT EFFECTIVE INTERACTIONS IN DEFORMED NUCLEI

Now, let us apply the procedure of Sec. II to a deformed system. For simplicity, we assume that the main part of the equilibrium potential is described as a deformed harmonic oscillator with frequencies ω_x , ω_y , and ω_z . In terms of the doubly stretched coordinates [10,11] defined by

$$x'' = \frac{\omega_x}{\omega_0} x, \quad y'' = \frac{\omega_y}{\omega_0} y, \quad z'' = \frac{\omega_z}{\omega_0} z, \quad (44)$$

the deformed harmonic oscillator potential can be expressed in a spherical form as

$$\begin{aligned} V_{\text{HO}}(\vec{r}) &= \frac{1}{2} M (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \\ &= \frac{1}{2} M \omega_0^2 (r'')^2 = V_{\text{HO}}(r''). \end{aligned} \quad (45)$$

As one of the possible and plausible collective shape oscillation modes in the deformed system, we will introduce a doubly stretched 2^λ -pole mode characterized by

$$\vec{r}'' \rightarrow \vec{r}'' + \delta \vec{r}'',$$

$$\delta \vec{r}'' = - \sum_{\mu} \alpha_{\lambda\mu}^* \vec{\nabla}'' Q''_{\lambda\mu} \quad (\lambda \geq 1), \quad (46)$$

with

$$\vec{\nabla}'' \equiv \left(\frac{\partial}{\partial x''}, \frac{\partial}{\partial y''}, \frac{\partial}{\partial z''} \right), \quad Q''_{\lambda\mu} \equiv Q_{\lambda\mu}(\vec{r}''), \quad (47)$$

rather than the conventional 2^λ -pole mode characterized by Eq. (1). In fact, as discussed in Ref. [11] based on the Thomas-Fermi theory, the doubly stretched mode represents great improvements over the conventional mode in the sense that it satisfies (i) the constancy of the Fermi energy (which is equivalent to the saturation condition), (ii) the separation of the center-of-mass motion, (iii) the condition for a fluctuation around the deformed equilibrium shape, and (iv) the self-consistency between the nucleonic density and the potential, etc. The collective velocity field associated with this mode is expressed as

$$\vec{v}''(\vec{r}'') = - \sum_{\mu} \dot{\alpha}_{\lambda\mu}^* \vec{\nabla}'' Q''_{\lambda\mu}, \quad (48)$$

which is a natural extension of the irrotational and incompressible flow of Eq. (2).

Now we will extensively apply the requirement of the local Galilean invariance to the deformed system. Namely, we will require that in terms of the doubly stretched coordinates nucleonic velocities entering into the velocity dependent single-particle potential are to be measured relative to the collective velocity $\vec{v}''(\vec{r}'')$ so that the potential is to be invariant under the transformation

$$\vec{p}'' \rightarrow \vec{p}'' + \delta \vec{p}'', \quad \delta \vec{p}'' = M \vec{v}''(\vec{r}''), \quad (49)$$

where the doubly stretched momenta are introduced as

$$p_x'' = \frac{\omega_0}{\omega_x} p_x, \quad p_y'' = \frac{\omega_0}{\omega_y} p_y, \quad p_z'' = \frac{\omega_0}{\omega_z} p_z. \quad (50)$$

Then by use of the relations $d^3 r = d^3 r''$ and $d^3 p = d^3 p''$, which are equivalent to the volume conservation condition $\omega_x \omega_y \omega_z = \omega_0^3$, we can follow the similar procedure as given in Sec. II to derive self-consistent effective interactions in

deformed nuclei. To do this, in the present case, we should understand that all the \vec{r} , \vec{p} , $\vec{\nabla}$, and $\vec{\nabla}_p$ appearing in Sec. II are to be replaced by the corresponding \vec{r}'' , \vec{p}'' , $\vec{\nabla}''$, and $\vec{\nabla}_p''$ with

$$\vec{\nabla}_p'' \equiv \left(\frac{\partial}{\partial p_x''}, \frac{\partial}{\partial p_y''}, \frac{\partial}{\partial p_z''} \right). \quad (51)$$

For example, the field coupling induced by the collective displacements of Eqs. (46) and (49) becomes

$$\begin{aligned} V(\vec{r}'', \vec{p}'') &= V_0(\vec{r}'', \vec{p}'') + \delta V(\vec{r}'', \vec{p}'') \\ &\equiv V_0'' + \delta V'' \end{aligned} \quad (52)$$

with

$$\begin{aligned} \delta V'' &= \delta V_r'' + \delta V_p'' \\ &= \kappa_\lambda \sum_{\mu} \alpha_{\lambda\mu}^* F''_{\lambda\mu}, + \tilde{\kappa}_\lambda \sum_{\mu} \dot{\alpha}_{\lambda\mu}^* \tilde{F}''_{\lambda\mu}, \end{aligned} \quad (53)$$

$$F''_{\lambda\mu} = \frac{1}{\kappa_\lambda} \vec{\nabla}'' Q''_{\lambda\mu} \cdot \vec{\nabla}'' V_0'', \quad (54)$$

$$\tilde{F}''_{\lambda\mu} = \frac{M}{\tilde{\kappa}_\lambda} \vec{\nabla}'' Q''_{\lambda\mu} \cdot \vec{\nabla}_p'' V_0'' = \frac{M}{\tilde{\kappa}_\lambda} \{Q''_{\lambda\mu}, V_0''\}. \quad (55)$$

From the self-consistency conditions

$$\alpha_{\lambda\mu} = A \langle F''_{\lambda\mu} \rangle, \quad \dot{\alpha}_{\lambda\mu} = A \langle \tilde{F}''_{\lambda\mu} \rangle, \quad (56)$$

the coupling strengths are determined as

$$\kappa_\lambda = -A \langle \vec{\nabla}'' (\vec{\nabla}'' Q''_{\lambda\mu} \cdot \vec{\nabla}'' V_0'') \cdot \vec{\nabla}'' Q''_{\lambda\mu} \rangle_0, \quad (57)$$

$$\begin{aligned} \tilde{\kappa}_\lambda &= -M^2 A \langle \vec{\nabla}_p'' (\vec{\nabla}'' Q''_{\lambda\mu} \cdot \vec{\nabla}_p'' V_0'') \cdot \vec{\nabla}'' Q''_{\lambda\mu} \rangle_0 \\ &= -M^2 \langle \{Q''_{\lambda\mu}, \{Q''_{\lambda\mu}, V_0''\}\} \rangle_0, \end{aligned} \quad (58)$$

and the self-consistent effective interaction to be used in the deformed nucleus is given as

$$\begin{aligned} H_{\text{int}} &= \frac{\kappa_\lambda}{2} \sum_{\mu} F''_{\lambda\mu} \cdot \dagger F''_{\lambda\mu} + \frac{\tilde{\kappa}_\lambda}{2} \sum_{\mu} \tilde{F}''_{\lambda\mu} \cdot \dagger \tilde{F}''_{\lambda\mu} \\ &\equiv H_\lambda^{(r)} + H_\lambda^{(p)}. \end{aligned} \quad (59)$$

For the case of $V_0'' = V_{\text{HO}}(r'')$, the effective interaction becomes

$$H_\lambda^{(r)} = -\frac{1}{2} \chi_\lambda^{\text{self}} \sum_{\mu} Q''_{\lambda\mu} \cdot \dagger Q''_{\lambda\mu} \quad (60)$$

with

$$\chi_\lambda^{\text{self}} = \frac{4\pi}{2\lambda+1} \frac{M\omega_0^2}{A \langle (r'')^{2\lambda-2} \rangle_0}, \quad (61)$$

and in the presence of the pairing field an additional effective interaction is derived as

$$H_\lambda^{(p)} = -\frac{1}{8} G_\lambda^{\text{self}} \sum_\mu (P''_{\lambda\mu}{}^\dagger - P''_{\lambda\mu}) (P''_{\lambda\mu} - P''_{\lambda\mu}{}^\dagger) \quad (62)$$

with

$$G_\lambda^{\text{self}} \equiv 1 / \sum_{\alpha\beta} \frac{1}{4} \left(\frac{1}{E_\alpha} + \frac{1}{E_\beta} \right) |\langle \alpha | Q''_{\lambda\mu} | \beta \rangle|^2. \quad (63)$$

These interactions play crucial roles in restoring broken symmetries of the system in RPA order, the detail of which will be discussed in the next section.

IV. FUNDAMENTAL PROPERTIES OF SELF-CONSISTENT EFFECTIVE INTERACTIONS

A. Translational invariance of a deformed system

The consistency of the residual interaction with the shell model potential and the method to restore the translational invariance of a nuclear many body system was discussed by Pyatov and Salamov [5] for the case of the spherical oscillator potential. Here we will briefly examine this problem in a deformed system.

Let us consider the oscillator Hamiltonian for the deformed nucleus

$$H_0 = \sum_{i=1}^A \left\{ \frac{p_i^2}{2M} + V_{\text{HO}}(r_i'') \right\}. \quad (64)$$

Since H_0 is a local one body Hamiltonian, it breaks the translational invariance of the system. In fact, we have

$$[H_0, \vec{P}''] = i\hbar AM \omega_0^2 \vec{R}'' \quad (65)$$

with

$$\begin{aligned} \vec{P}'' &= \sum_{j=1}^A \vec{p}_j'' = \sum_{j=1}^A (-i\hbar \vec{\nabla}_j''), \\ \vec{R}'' &= \frac{1}{A} \sum_{j=1}^A \vec{r}_j''. \end{aligned} \quad (66)$$

To recover the translational invariance, we need a counterterm which cancels the above commutation relation. For a spherical nucleus, as is well known, such a counterterm comes from the conventional dipole interaction. Now we will show that for the deformed nucleus, the doubly stretched dipole interaction plays the role to recover the translational invariance of the system.

First of all, for the doubly stretched dipole interaction

$$V_{\lambda=1} = -\frac{\chi_1}{2} \sum_{ij} [Q''_1(i) \cdot Q''_1(j)], \quad (67)$$

we can verify

$$[V_{\lambda=1}, \vec{P}'] = -i\hbar AM \omega_0^2 \frac{\chi_1}{\chi_1^{\text{self}}} \vec{R}''. \quad (68)$$

Then the total Hamiltonian $H = H_0 + V_{\lambda=1}$ satisfies

$$[H, \vec{P}'] = i\hbar AM \omega_0^2 \left(1 - \frac{\chi_1}{\chi_1^{\text{self}}} \right) \vec{R}''. \quad (69)$$

Therefore if the strength χ_1 of the doubly stretched dipole interaction is set equal to its self-consistent value $\chi_1^{\text{self}} = 4\pi M \omega_0^2 / 3A$, we obtain $[H, \vec{P}'] = 0$ exactly. In this case, the total Hamiltonian can be expressed as

$$H = \sum_{i=1}^A \frac{p_i^2}{2M} + \frac{M \omega_0^2}{4A} \sum_{i,j=1}^A |\vec{r}_i'' - \vec{r}_j''|^2, \quad (70)$$

which explicitly guarantees the translational invariance of the deformed system.

B. Restoration of the local Galilean invariance

As is discussed in the previous sections, generally a phenomenological potential, comprising velocity dependent terms such as the \vec{l}^2 term, the $\vec{l} \cdot \vec{s}$ terms and the pairing field, etc., does not commute with arbitrary coordinate operators. If we chose the operator to be the multipole operator, such a situation is expressed from Eq. (23) as

$$[Q''_{\lambda\mu}, V_0(\vec{r}, \vec{p})] = \frac{i\hbar \tilde{\kappa}_\lambda}{M} \tilde{F}''_{\lambda\mu} \neq 0, \quad (71)$$

which means that the potential V_0 violates the local Galilean invariance under the collective multipole oscillation. Here the doubly stretched operators are used for deformed nuclei. For spherical nuclei, of course we can omit the double primes.

Now along the line of the general method of restoring the broken symmetry [5,26], we will show that the self-consistent velocity dependent effective interaction, i.e., $H_\lambda^{(p)}$ of Eq. (59), plays the role to restore the local Galilean invariance of the system under the random phase approximation (RPA). In the RPA order we can verify

$$[Q''_{\lambda\mu}, H_\lambda^{(p)}]_{\text{RPA}} = \frac{\tilde{\kappa}_\lambda}{2} \{ [Q''_{\lambda\mu}, \tilde{F}''_{\lambda\mu}{}^\dagger]_{\text{RPA}}, \tilde{F}''_{\lambda\mu} \}_+, \quad (72)$$

with

$$\begin{aligned} [Q''_{\lambda\mu}, \tilde{F}''_{\lambda\mu}{}^\dagger]_{\text{RPA}} &= \frac{M}{i\hbar \tilde{\kappa}_\lambda} \langle [Q''_{\lambda\mu}, [Q''_{\lambda\mu}{}^\dagger, V_0]] \rangle_0 \\ &= -\frac{i\hbar}{M}. \end{aligned} \quad (73)$$

Thus we obtain

$$[Q''_{\lambda\mu}, V_0 + H_\lambda^{(p)}]_{\text{RPA}} = 0. \quad (74)$$

C. Simple model analysis of GDR of normal nuclei

It is worthwhile to point out that the doubly stretched interaction model is powerful and plausible also for the description of some isovector modes. In confirmation of it, let us briefly review the simple model analysis of the splitting of GDR in an axially symmetric deformed nucleus [11]. The model Hamiltonian is assumed to be

$$H = H_0 + V_{\lambda=1}^{(T=1)}, \quad (75)$$

where H_0 is the deformed oscillator Hamiltonian of Eq. (64) while $V_{\lambda=1}^{(T=1)}$ is a residual isovector dipole interaction. Here we will parametrize the shape of the nuclear potential as

$$\begin{aligned} \omega_{\perp} = \omega_x = \omega_y = \omega_0(\varepsilon)(1 + \varepsilon/3), \\ \omega_z = \omega_0(\varepsilon)(1 - 2\varepsilon/3) \end{aligned} \quad (76)$$

with

$$\omega_0(\varepsilon) = \hat{\omega}_0[1 + \varepsilon^2/9 + O(\varepsilon^3)], \quad \hbar \hat{\omega}_0 \approx 41A^{-1/3} \text{ (MeV)}. \quad (77)$$

For comparison we will introduce two types of isovector dipole interaction, one is an ordinary type and the other is a doubly stretched type. The interaction in the latter case is given by

$$V_{\lambda=1}^{(T=1)} = -\frac{1}{2} \sum_K \chi_{1K}^{(T=1)} (Q''_{1K} \tau_z)^\dagger (Q''_{1K} \tau_z) \quad (78)$$

and we will parametrize the force strength as $\chi_{1K}^{(T=1)} = -\xi \chi_1^{\text{self}}$ where χ_1^{self} is the self-consistent strength of the isoscalar dipole interaction. The typical value of ξ estimated from the symmetry energy term in the mass formula under the Fermi gas approximation is about 3 [1].

Under the RPA, the excitation energy of each K component of the GDR is obtained analytically as

$$\begin{aligned} \Omega_{11} &= \Omega_{\text{GDR}} \left(1 + \frac{\varepsilon}{3(1+\xi)} \right), \\ \Omega_{10} &= \Omega_{\text{GDR}} \left(1 - \frac{2\varepsilon}{3(1+\xi)} \right), \end{aligned} \quad (79)$$

for the ordinary interaction, while

$$\Omega_{11} = \Omega_{\text{GDR}}(1 + \varepsilon/3), \quad \Omega_{10} = \Omega_{\text{GDR}}(1 - 2\varepsilon/3), \quad (80)$$

for the doubly stretched interaction. Here Ω_{GDR} is the resonance energy of a spherical nucleus given by

$$\Omega_{\text{GDR}} = \sqrt{1 + \xi} \omega_0, \quad (81)$$

which, in both cases, is compatible with the experimental systematics of $\Omega_{\text{GDR}} \approx 80A^{-1/3}$ (MeV) if we put $\xi = 3$. For the doubly stretched interaction, the total energy splitting between the $K=0$ and $K=1$ components of the GDR is ε in units of Ω_{GDR} and is independent of ξ , which is consistent with the simple classical geometrical relation of $(\omega_{\perp} - \omega_z)/\omega_0 = \varepsilon$. This is in good agreement with the systematics of the experimental observation [35–37]. On the other hand, for the ordinary interaction, the splitting is too small by a factor of 4 for $\xi = 3$. Thus the doubly stretched interaction model seems much more improved than the ordinary one also for the isovector dipole mode.

D. Simple model analysis of GDR of superconductive nuclei

Effects of the inclusion of the pairing correlation on the properties of giant resonances of superconductive nuclei

TABLE I. $E1$ transition strength distribution for a schematic spherical system with $N=Z=20$. Energies and fractions of the EWSR are shown for some dominant states calculated in the RPA. The dipole-pairing interaction is included for c and e , but not for a , b , and d .

	$\Delta_p = \Delta_n$ (MeV)	E (MeV)	EWSR (%)
a	0.0	24.0	100
b	1.0	21.4 23.6 27.5	8 52 41
c	1.0	24.0	100
d	2.0	20.8 23.6 30.0 51.3	16 24 78 13
e	2.0	23.9	99

have been studied by several authors [5,24,25], and the shifts in excitation energies, the changes in energy weighted sum rule (EWSR), etc., have been observed. However as indicated in Refs. [24,25], some of them seem to be spurious due to the violation of the Galilean invariance of the system. Here, by use of a simple schematic model, we will verify the effects of the pairing correlations on the structure of GDR, and will show that such spurious effects can be remedied by including the dipole pairing interaction.

The model Hamiltonian is assumed to be

$$H = H_0 + V_{\lambda=1}^{(T=1)} + \sum_{\tau} \{V_{\text{pair}} - \lambda \hat{N} + H_{\lambda=1}^{(p)}\}_{\tau}, \quad (82)$$

where H_0 is the oscillator Hamiltonian of Eq. (64), $V_{\lambda=1}^{(T=1)}$ is the isovector dipole interaction of Eq. (78), V_{pair} is the monopole pairing potential of Eq. (22), and $H_{\lambda=1}^{(p)}$ is the dipole pairing interaction of Eq. (62). Here and in the following, the summation index τ is to be taken over the proton and the neutron. The force strength of the isovector dipole interaction is fixed as $\xi = 3$, while that of the dipole pairing interaction is set to be its self-consistent value when it is included. We perform quasiparticle RPA calculations for a schematic model system with $N=Z=20$. It must be noticed that the purpose here is not to compare with the experimental data of ^{40}Ca but to investigate the fundamental properties of the dipole pairing interaction from the purely theoretical point of view.

Table I shows our results on the electric dipole strength distributions calculated by assuming the system to be spherical. To study the effect of the pairing correlation, we artificially change the value of Δ . For the simplest case of $\Delta = 0$ (a), the GDR is located at 24.0 MeV, and the energy weighted $E1$ transition strength of this state exhausts its classical sum rule value of

$$S(E1)_{\text{class}} = \frac{9}{4\pi} \frac{\hbar^2}{2M} \frac{NZ}{A} e^2. \quad (83)$$

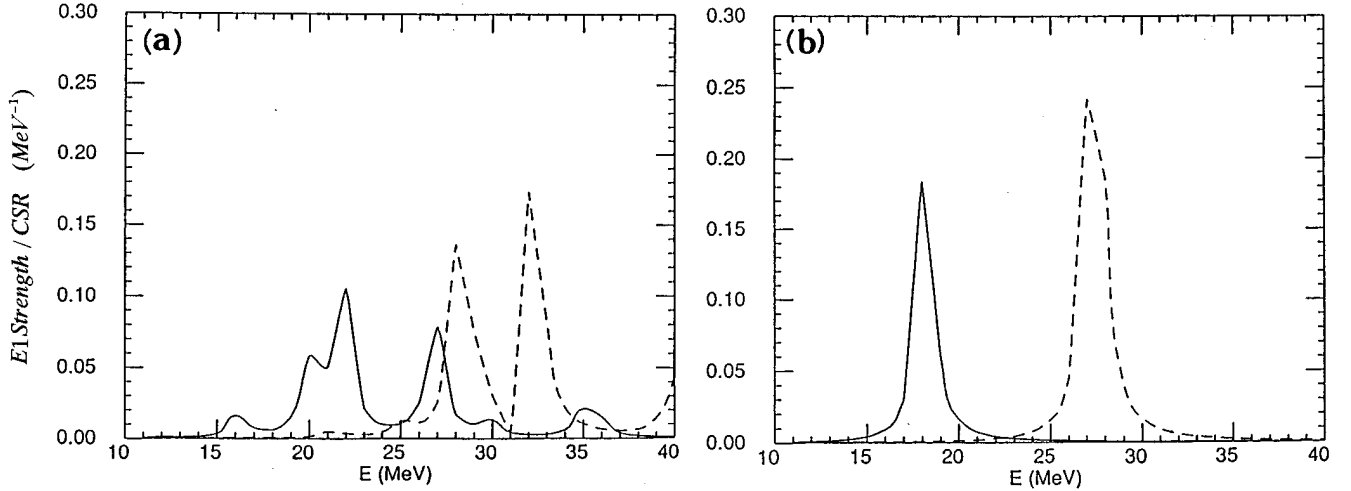


FIG. 1. $E1$ transition strength distribution for a schematic model system of $N=Z=20$ with axially symmetric deformation of $\beta=0.4$ calculated in RPA. The continuous strength function, representing the strength per unit energy, is given in units of MeV^{-1} relative to the classical sum rule value (CSR). In the model Hamiltonian of Eq. (82), the pairing gap is fixed as $\Delta=2.0$ MeV both for protons and for neutrons. $K=0$ and $K=1$ modes are shown by the solid and the dashed curves, respectively. (a) and (b) correspond to the results obtained with and without the inclusion of the dipole-pairing interaction, respectively.

When the pairing gap is set to be $\Delta = 1.0$ MeV (*b*), reflecting the situation that the quasiparticle states are no more degenerate, the GDR splits in spite of the assumption that the system is spherical. There are mainly three components of the GDR (21.4, 23.6, and 27.5 MeV). The center of the GDR shifts upward about 1 MeV compared to the case of $\Delta=0$, and the EWSR is overestimated by about 10% relative to the classical one. In order to remedy this situation, we switch on the dipole pairing interaction (*c*). In this case, the $E1$ transition strength concentrates again on a single state at about 24.0 MeV, and the EWSR recovers to its classical value. The fact that the pairing correlation destroys the order in the structure of the GDR can further be emphasized by increasing Δ to 2.0 MeV, though the value is not realistic. In this hypothetical situation (*d*), the transition strength splits into mainly four components at 20.8, 23.6, 30.0, and 51.3 MeV, and the EWSR is overestimated by about 36%. Even in such an extreme situation, if we additionally include the dipole pairing interaction (*e*), the spurious effect of the pairing interaction can be removed and the $E1$ strength is concentrated on a single state at 23.9 MeV to recover the EWSR.

Figure 1 shows the corresponding results obtained by assuming that the model system is deformed to be axially symmetric shape of $\beta=0.4$. The continuous strength function, representing the transition strength per unit energy, is constructed by using the Lorentzian weight function

$$\rho(\omega - \omega_\nu) = \frac{2}{\pi} \frac{\Gamma \omega^2}{(\omega^2 - \omega_\nu^2)^2 + \Gamma^2 \omega^2}. \quad (84)$$

In the present model calculations, we choose the width to be $\Gamma=1.0$ MeV only for the sake of not wiping out the fine structure of the resonance. Because of the deformation of the system, the GDR splits into $K=0$ and $K=1$ components. For the case of Fig. 1(a), the GDR shows rather complicated structure. The reason for it can be traced to the violation of the Galilean invariance for the pairing field and we can

eliminate such a spurious effect by restoring the broken invariance. In fact, if we additionally include the dipole pairing interaction, the structure of the resonance becomes simpler both for $K=0$ and $K=1$ modes [Fig. 1(b)]. Furthermore, the total energy splitting between the $K=0$ and $K=1$ modes, relative to the average energy of these modes, becomes approximately equal to β which is consistent to the simple classical geometrical relation explained before.

V. NUMERICAL RESULTS

In this section, we report some characteristic results of numerical calculations in RPA of the GDR's of $^{148,154}\text{Sm}$, the first excited 2^+ states of Sn isotopes and the first excited 3^- states of Mo isotopes. It should be noted here that our present calculations contain essentially only one free parameter in the following sense; the single-particle bases are constructed from the Nilsson + BCS model with standard parameters; the strengths of the velocity dependent effective interactions such as the multipole pairing interactions are fixed to be their self-consistent values when they are included; only the strengths of the multipole-multipole interactions are adjusted under the condition that a common value of ξ , χ_2 , and χ_3 is to be adopted for all the isotopes of Sm, Sn, and Mo, respectively.

Since the positions of bandhead states are very sensitive to the choice of the single-particle energies as are generally observed in the RPA calculation of vibrational states in deformed nuclei, one can improve the fit to experiments by adjusting the single-particle energies, and further improvement can be obtained by slightly varying the strengths of the effective interactions around the vicinity of the predicted self-consistent values for each isotope. The fit to experiment we obtained is insufficient and an improved fit could have been obtained if these parameters were treated as adjustable as well. We have not done so in the present work, because the purpose of the present numerical investigation is to see

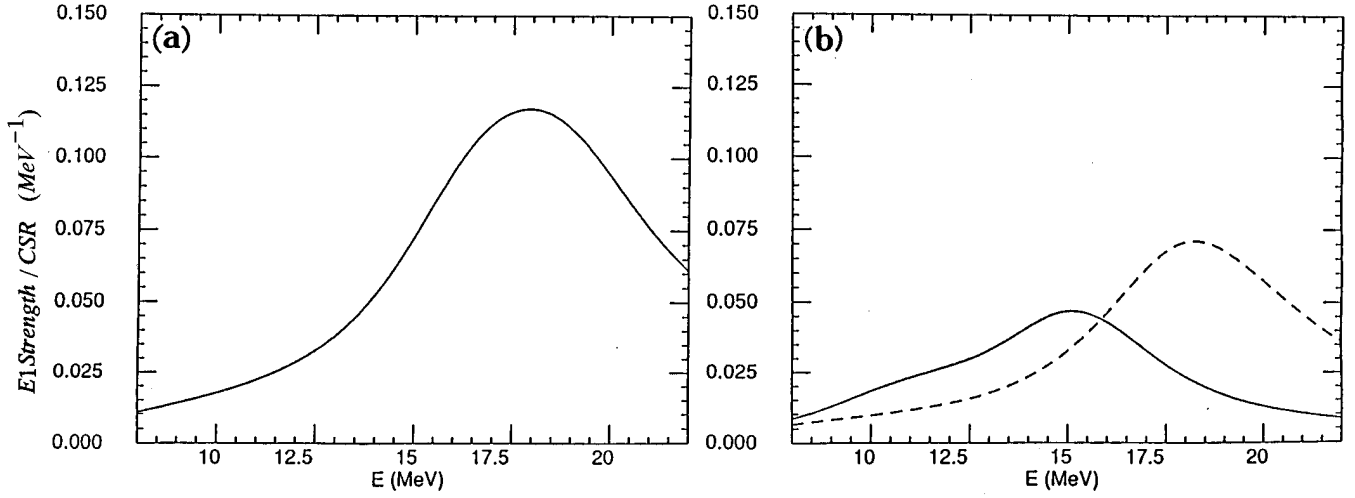


FIG. 2. $E1$ transition strength functions of (a) ^{148}Sm and (b) ^{154}Sm are given in units of MeV^{-1} relative to the classical sum rule value (CSR). The model Hamiltonian is same as that of Eq. (85) except that the dipole-pairing interaction is not included here. In (b), solid and dashed curves correspond to the $K=0$ and $K=1$ modes, respectively.

first to what extent our theory works without much playing around with parameters and to provide understandings rather than precision tools for the fitting of experimental data.

A. $E1$ strength distributions of $^{148,154}\text{Sm}$

We here present the results of realistic calculations on the $E1$ strength distributions of $^{148,154}\text{Sm}$ in the quasiparticle RPA. The model Hamiltonian is assumed to be

$$H = H_0 + V_{\text{Nil}} + V_{\lambda=1}^{(T=1)} + \sum_{\tau} \{V_{\text{pair}} - \lambda \hat{N} + H_{\lambda=1}^{(p)}\}_{\tau}, \quad (85)$$

$$V_{\text{Nil}} = \sum_{i=1}^A [-\kappa \hbar \hat{\omega}_0 \{2(\vec{l} \cdot \vec{s}) + \mu(\vec{l}^2 - \langle \vec{l}^2 \rangle)\}]_i, \quad (86)$$

which is essentially same as Eq. (82) except that the Nilsson potential V_{Nil} is additionally included.

We will study the effects of two kinds of velocity dependent interactions to restore the Galilean invariance of the system; the one arising from V_{pair} and the other from the velocity dependent part of V_{Nil} . The former, the dipole pairing interaction $H_{\lambda=1}^{(p)}$, is given by Eq. (62), while the latter, $H_{\lambda=1}^{(\text{Nil})}$, is given by

$$H_{\lambda=1}^{(\text{Nil})} = \frac{1}{2} \sum_{\mu} \kappa_1 \eta_{1\mu}^{\dagger} \eta_{1\mu} \quad (87)$$

with

$$\eta_{1\mu} = [Q''_{1\mu}, V_{\text{Nil}}], \quad (88)$$

$$\kappa_1 = -1 / \langle [Q''_{1\mu}, [V_{\text{Nil}}, Q''_{1\mu}^{\dagger}]] \rangle_0. \quad (89)$$

Although it is known that there exists a term associated with the coordinate distortion of the spin-orbit potential given by Eq. (6-70) of Ref. [1], we will not consider its effects in order to concentrate our present analysis on the effects of the self-consistent velocity dependent interactions. We will also

neglect isovector corrections coming from the Nilsson potential to the isovector dipole-dipole interaction because of the same reason.

To fix the parameter ξ for the strength of the isovector dipole-dipole interaction in this mass region, we first calculate the case of ^{148}Sm by assuming its shape to be spherical. The resultant value of $\xi=2.7$ is adopted also for ^{154}Sm whose quadrupole deformation parameter is assumed and fixed as $\beta=0.35$ in the following calculations. All other strengths of the interactions are fixed just as the self-consistent values which restore the Galilean invariance of the system. It should be noticed here that the doubly stretched interactions are used for the deformed nucleus of ^{154}Sm . For the model space, we retain all the Nilsson single-particle states with $2 \leq N_{\text{osc}} \leq 7$ for protons and $3 \leq N_{\text{osc}} \leq 8$ for neutrons. The Nilsson parameters are taken from Ref. [13]. By using experimental binding energies of Ref. [38], the pairing gap parameters are determined from the even-odd mass differences as $\Delta_n=1.01$ MeV, $\Delta_p=1.36$ MeV for ^{148}Sm and $\Delta_n=1.07$ MeV, $\Delta_p=0.86$ MeV for ^{154}Sm . We use the Lorentzian distribution of Eq. (84) to reproduce the resonance width. We choose Γ (in units of MeV) as 5.10 for ^{148}Sm , and 3.25, 5.25 for the $K=0, 1$ modes of ^{154}Sm , respectively.

Figures 2(a) and 2(b) show the $E1$ strength functions for

TABLE II. Calculated $E1$ EWSR values of $^{148,154}\text{Sm}$. Strengths integrated over the excitation energies from 5.5 MeV to 30.0 MeV are given in units of % relative to the classical sum rule value. The model Hamiltonian is given by Eq. (85). The columns a and b correspond to the results obtained without and with the inclusion of the dipole-pairing interaction $H_{\lambda=1}^{(p)}$, respectively, while in c the interaction $H_{\lambda=1}^{(\text{Nil})}$ is also included in addition to $H_{\lambda=1}^{(p)}$.

Nucleus	a	b	c
^{148}Sm	116.8	85.9	86.2
^{154}Sm ($K=0$)	41.1	31.3	30.2
^{154}Sm ($K=1$)	64.7	56.0	56.9

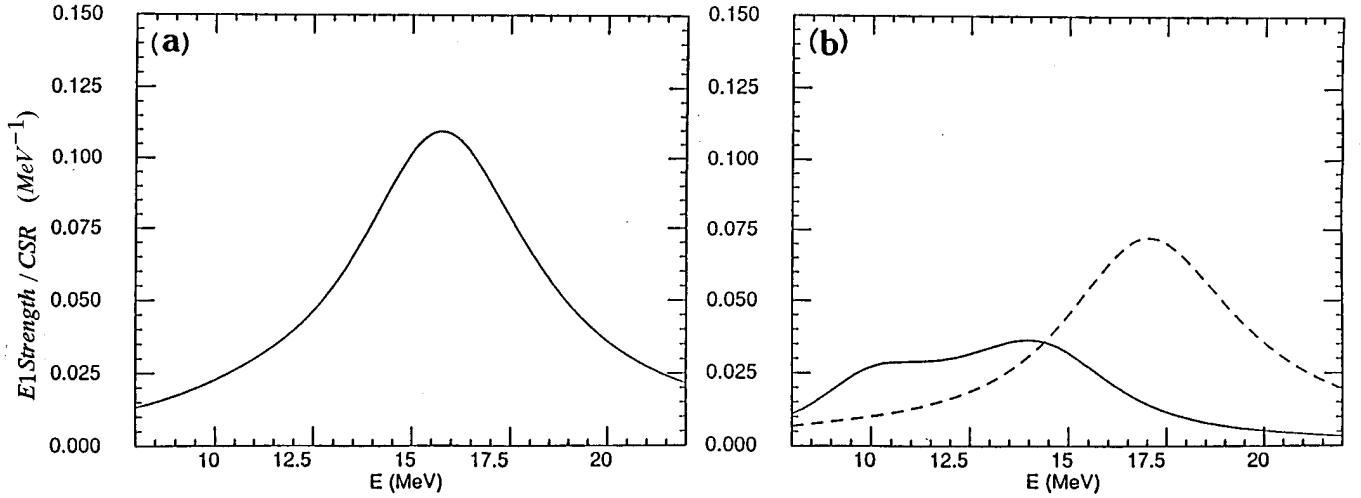


FIG. 3. Same as Fig. 2, except that the dipole-pairing interaction is included.

^{148}Sm and ^{154}Sm , respectively, taking the model Hamiltonian of Eq. (85) but switching off the dipole-pairing interaction. Here we see that even if we use the doubly stretched interaction, the calculated splitting between $K=0$ and $K=1$ resonances of ^{154}Sm is too small. Furthermore, as can be seen from column *a* of Table II, the EWSR values exceed the classical values for both nuclei. As stated repeatedly, these difficulties stem from the mixture of spurious states arising from the broken Galilean invariance of V_{pair} and V_{Nil} . In the following we show the results obtained by restoring the broken symmetry in two steps.

First, we study the effect of $H_{\lambda=1}^{(p)}$. The column *b* of Table II shows that the sum rule values approach the classical values. However, from Fig. 3, we see that the centers of the resonances shift to lower excitation energies and the structure in the lower peak region reveal unnatural shape. Finally we take into account $H_{\lambda=1}^{(\text{Nil})}$ so that the Galilean invariance of the Hamiltonian is restored. As can be seen from Fig. 4 and column *c* of Table II, the unnatural resonance structure disappears and the EWSR values keep close to the classical limit. We note that the final results of Figs.4(a) and

4(b) agree quite well with the Lorentzian distribution of Figs.4(c) and 4(d) which fit the experimental data of photo-neutron cross section [35].

B. The first excited 2^+ states in Sn isotopes

We now consider the effects of the self-consistent quadrupole pairing interaction on the excitation energies $E(2_1^+)$ and the $E2$ transition probabilities $B(E2)$ of Sn isotopes within the quasiparticle RPA. The model Hamiltonian is assumed to be

$$H = H_0 + V_{\text{Nil}} + V_{\lambda=2} + \sum_{\tau} \{V_{\text{pair}} - \lambda \hat{N} + H_{\lambda=2}^{(p)}\}_{\tau}, \quad (90)$$

where $V_{\lambda=2}$ and $H_{\lambda=2}^{(p)}$ are the quadrupole-quadrupole interaction and the quadrupole pairing interaction, respectively.

The single-particle model space is spanned by all the Nilsson states with $2 \leq N_{\text{osc}} \leq 6$ for protons and $2 \leq N_{\text{osc}} \leq 7$ for neutrons. The Nilsson parameters are taken from Ref. [13], and the deformation is set to be zero. To investigate the

TABLE III. Energies of the first excited 2^+ states and $E2$ transition probabilities in Sn isotopes are given in units of MeV and $B(E2)_{\text{sp}}$, respectively. Results of calculations obtained without and with the inclusion of the quadrupole-pairing interaction $H_{\lambda=2}^{(p)}$ are compared with experimental data. The pairing gaps adopted in the calculations are taken from the experimental even-odd mass differences for *a*, while those for protons are set to be zero for *b*.

<i>N</i>	<i>a</i>				<i>b</i>				Expt.	
	Without $H_{\lambda=2}^{(p)}$		With $H_{\lambda=2}^{(p)}$		Without $H_{\lambda=2}^{(p)}$		With $H_{\lambda=2}^{(p)}$			
	$E(2^+)$	$B(E2)$	$E(2^+)$	$B(E2)$	$E(2^+)$	$B(E2)$	$E(2^+)$	$B(E2)$	$E(2^+)$	$B(E2)$
62	1.72	18.7	1.57	13.3	1.66	15.7	1.57	10.7	1.26	16.2
64	1.72	19.6	1.57	14.1	1.68	15.9	1.58	10.8	1.30	15.3
66	1.41	19.8	1.19	17.1	1.37	15.7	1.34	10.6	1.29	12.9
68	1.23	23.9	1.03	20.4	1.20	19.0	1.21	12.2	1.23	12.7
70	1.20	24.8	1.00	21.2	1.16	20.0	1.18	12.7	1.17	11.6
72	1.22	22.7	1.13	15.0	1.17	18.5	1.18	11.9	1.14	10.9
74	1.35	18.0	1.27	12.9	1.31	14.9	1.27	10.0	1.13	9.3

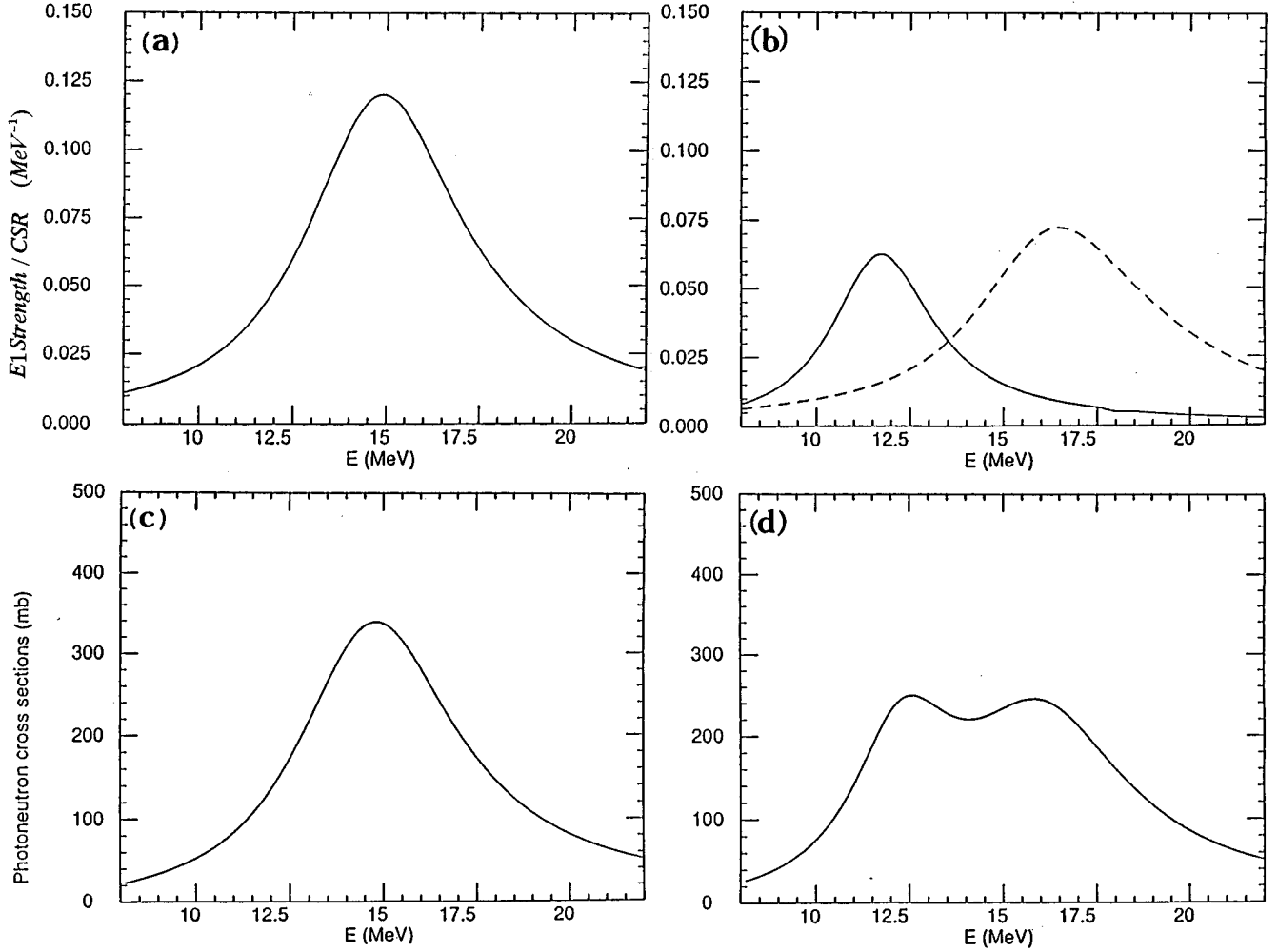


FIG. 4. Same as Fig.3 for (a) and (b), except that the additional interaction $H_{\lambda=1}^{(Nil)}$ is included. For comparison, Lorentzian distributions which fit the experimental data of the photoneutron cross section [35] are given in (c) and (d) for ^{148}Sm and ^{154}Sm , respectively. The experimental Lorentz line parameters for the mathematical expression $\sigma_L(E) = \sum_i \sigma_i (E\Gamma_i)^2 / [E^2 - E_i^2]^2 + (E\Gamma_i)^2$ are taken from Ref. [35] as $E_1 = 14.8 \pm 0.1$, $\Gamma_1 = 5.1 \pm 0.2$, $\sigma_1 = 339 \pm 12$ for the best single line fit of ^{148}Sm and $E_1 = 12.35 \pm 0.10$, $\Gamma_1 = 3.35 \pm 0.15$, $\sigma_1 = 192 \pm 10$, $E_2 = 16.1 \pm 0.1$, $\Gamma_2 = 5.25 \pm 0.20$, $\sigma_2 = 204 \pm 10$ for the best two-line fit of ^{154}Sm , respectively. Here E 's and Γ 's are given in units of MeV while σ 's are given in units of mb.

effect of the quadrupole pairing force on the $B(E2; 0_g \rightarrow 2_1^+)$ value, the strength of the quadrupole pairing force, G_2 , is fixed to be its self-consistent value when it is included, while that of the quadrupole interaction, χ_2 , is used as an adjustable parameter to reproduce experimental $E(2_1^+)$ of $^{116,118,120}\text{Sn}$ [39].

First, we adopt the pairing gaps determined from the even-odd mass differences. The adopted value of χ_2 for the best fit obtained without (with) the inclusion of the quadrupole-pairing interaction, $H_{\lambda=2}^{(p)}$, is 0.92 (0.90) in units of χ_2^{self} . The calculated $E(2_1^+)$ and $B(E2)$ are plotted in Figs. 5(a) and 5(b), respectively, and corresponding numerical values are given in column (a) of Table III. These results show that $H_{\lambda=2}^{(p)}$ is necessary and important in reproducing experimental $E(2_1^+)$ and $B(E2)$ values simultaneously.

Second, we have also performed numerical calculations by fixing the proton energy gap to be zero. This is because the proton shell is closed in Sn. In this case, the adopted value of χ_2 for the best fit obtained without (with) the inclusion of $H_{\lambda=2}^{(p)}$ is 0.940 (0.965) in units of χ_2^{self} . The calcu-

lated values of $E(2_1^+)$ and $B(E2)$ are plotted in Figs. 6(a) and 6(b), respectively, and corresponding numerical values are given in column b of Table III, where we see better agreements with experimental data compared to the case with $\Delta_p \neq 0$.

C. The first excited 3^- states in Mo isotopes

Here we will study the effects of the self-consistent octupole pairing interaction, $H_{\lambda=3}^{(p)}$, in the quasiparticle RPA calculations of the excitation energies $E(3_1^-)$ and the $E3$ transition probabilities of the first excited 3^- states in Mo isotopes. The model Hamiltonian is assumed to be

$$H = H_0 + V_{\text{Nil}} + V_{\lambda=3} + \sum_{\tau} \{V_{\text{pair}} - \lambda \hat{N} + H_{\lambda=3}^{(p)}\}_{\tau}, \quad (91)$$

where $V_{\lambda=3}$ and $H_{\lambda=3}^{(p)}$ are the octupole-octupole interaction and the octupole pairing interaction, respectively.

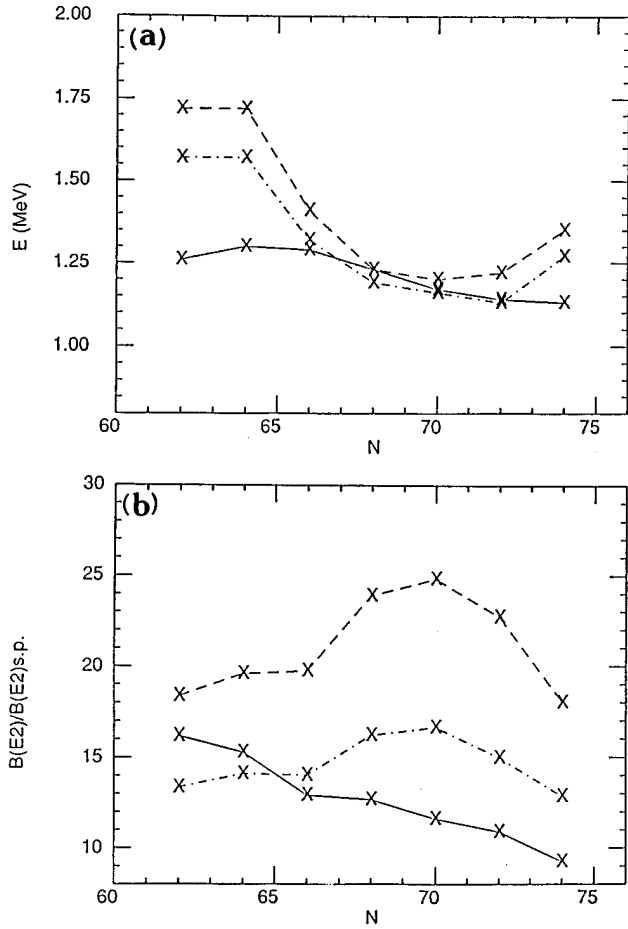


FIG. 5. (a) Excitation energies of the first 2^+ states and (b) $E2$ transition probabilities in Sn isotopes. Results of calculations with and without the inclusion of the quadrupole pairing interaction are shown by dot-dashed and dashed lines, respectively. Solid lines correspond to the experimental data. In the calculations, pairing gaps are determined from the odd-even mass differences both for protons and for neutrons.

The single-particle model space is spanned by all the Nilsson states with $0 \leq N_{\text{osc}} \leq 7$ for protons and $0 \leq N_{\text{osc}} \leq 9$ for neutrons. Energy gaps are determined from the experimental even-odd mass differences. To investigate the effect of the octupole pairing force on the $B(E3; 0_g \rightarrow 3_1^-)$ value, the strength of the octupole pairing force, G_3 , is fixed to be its self-consistent value when it is included, while that of the octupole interaction, χ_3 , is used as an adjustable parameter to reproduce the experimental data of $E(3_1^-)$ in $^{94,96,98,100}\text{Mo}$ [40]. The adopted value of χ_3 for the best fit obtained without (with) the inclusion of the octupole-pairing interaction, $H_{\lambda=3}^{(p)}$, is 0.95 (0.97) in units of χ_3^{self} .

The calculated $E(3_1^-)$ and $B(E3)$ from the ground state to the 3_1^- state are plotted in Figs. 7(a) and 7(b), respectively, and corresponding numerical values are given in Table IV. Here we see that the calculated $B(E3)$ values are reduced and improved systematically by the effects of $H_{\lambda=3}^{(p)}$, although the fit to experimental data is insufficient.

VI. SUMMARY

We have extensively applied the prescription to derive self-consistent effective interactions, needed for the unified

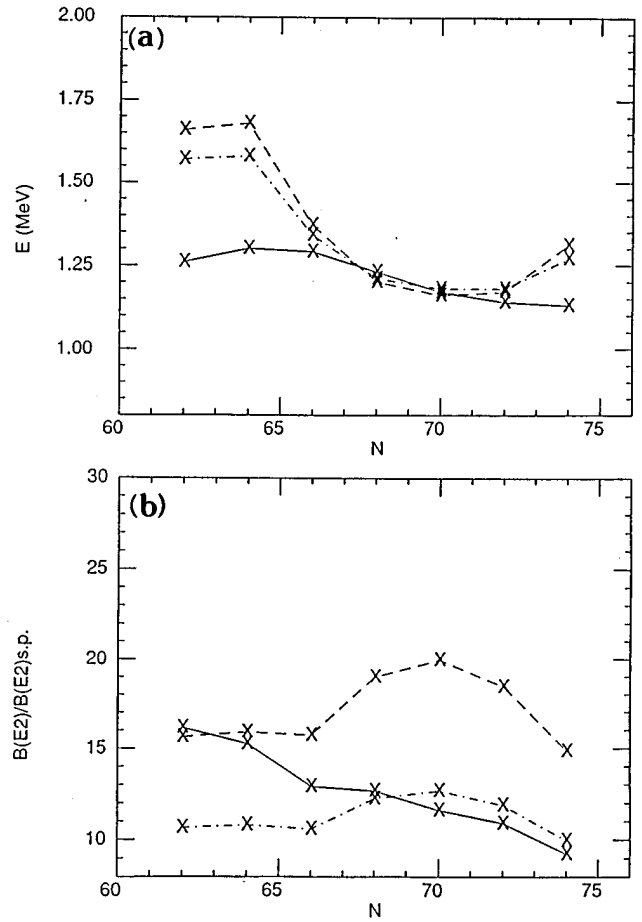


FIG. 6. Same as Fig. 5, except that the pairing gaps for protons are set to be $\Delta_p = 0$ in the calculations.

description of the collective motions of atomic nuclei, especially to the system with velocity dependent mean potentials. As the guiding principles for this purpose we have imposed the conditions of nuclear self-consistency [10,11] and local Galilean invariance [1,3,5–7] of the system, implemented with the simple and transparent field coupling method developed by Bohr and Mottelson [1].

The nuclear self-consistency requires that the shape of the mean potential and that of the density are the same even when the system undergoes collective motions, while the lo-

TABLE IV. Energies of the first excited 3^- states and $E3$ transition probabilities in Mo isotopes are given in units of MeV and $B(E3)_{\text{sp}}$, respectively. Results of calculations obtained without and with the inclusion of the octupole-pairing interaction $H_{\lambda=3}^{(p)}$ are compared with experimental data. The pairing gaps adopted in the calculations are taken from the experimental even-odd mass differences.

N	Without $H_{\lambda=3}^{(p)}$		With $H_{\lambda=3}^{(p)}$		Expt.	
	$E(3^-)$	$B(E3)$	$E(3^-)$	$B(E3)$	$E(3^-)$	$B(E3)$
52	2.74	51.2	2.66	44.3	2.53	17
54	2.36	53.4	2.32	44.6	2.23	24
56	1.93	58.6	1.97	47.3	2.02	33
58	1.91	60.2	1.92	47.8	1.91	32

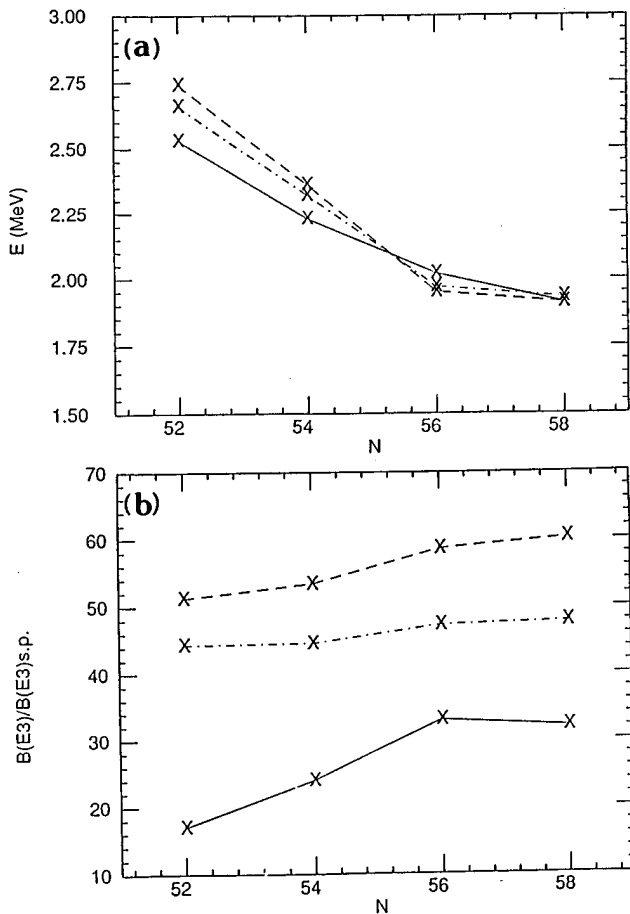


FIG. 7. (a) Excitation energies of the first 3^- states and (b) $E3$ transition probabilities in Mo isotopes. Results of calculations with and without the inclusion of the octupole pairing interaction are shown by dot-dashed and dashed lines, respectively. Solid lines correspond to the experimental data.

cal Galilean invariance requires that the nucleonic velocities entering into the velocity dependent single-particle potential should be expressed relative to the local collective flow. In the field coupling method, the coupling between the particle motion and the collective field is identified, within the Hartree approximation, as the averaged one body field of the effective interaction which we look for, and the coupling strength of it can be fixed by the above conditions.

For the multipole collective shape oscillation modes in the harmonic oscillator potential with the monopole pairing correlation, we have derived the multipole-multipole interactions for particle-hole channel and the multipole-pairing in-

teractions for particle-particle channel from the unified physical picture. In the case of deformed nuclei, it is shown that these interactions must be expressed in terms of the doubly stretched coordinates so as to guarantee the conditions of nuclear self-consistency and local Galilean invariance of the system.

The origin of the doubly stretched multipole-multipole interactions have already been clarified and they have found many successful applications [10,11,16–23], while the origin of the doubly stretched multipole pairing interactions are clarified in this paper on the same footing. We have applied the doubly stretched multipole pairing interactions to the analyses of some collective states in Sm, Sn, and Mo isotopes by means of RPA, and for the dipole mode we have also tested the velocity dependent effective interaction arising from the Nilsson potential. We have seen the effects of such velocity dependent effective interactions in the recovery of the classical $E1$ sum rule for GDR's of $^{148,154}\text{Sm}$, in the systematic reduction of the $E2$ transition probabilities for the first excited 2^+ states of Sn isotopes, and also in the systematic reduction of $E3$ transition probabilities for the first excited 3^- states of Mo isotopes. It should be noted here that recently the doubly stretched quadrupole pairing interaction was successfully applied also to the microscopic analysis of identical bands in superdeformed nuclei, and it was shown that the doubly stretched quadrupole pairing interaction has several advantages compared to the nonstretched and stretched ones [41].

In summary, it is clarified that the self-consistent velocity dependent effective interactions play crucial roles to recover the local Galilean invariance and eliminate various unphysical effects arising from the spurious velocity dependence of the mean potential. For rotating nuclei, we can apply similar prescription in order to find the proper effective interactions which faithfully take into account the effects of the collective rotation. Results of it will be reported in a separate paper.

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[1] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II, Chap. 6.
 [2] B. R. Mottelson, *Nikko Summer School Lectures* (NORDITA, Copenhagen, 1967), Pub. 288.
 [3] S. T. Belyaev, *Sov. J. Nucl. Phys.* **4**, 671 (1967).
 [4] I. Hamamoto, *Nucl. Phys.* **A232**, 445 (1974).
 [5] N. I. Pyatov and D. I. Salamov, *Nukleonika* **22**, 127 (1977).

[6] N. I. Pyatov, S. I. Gabrakov, and D. I. Salamov, *Sov. J. Nucl. Phys.* **26**, 139 (1977).
 [7] H. Sakamoto and T. Kishimoto, *Phys. Lett. B* **245**, 321 (1990).
 [8] T. Kishimoto, H. Sakamoto, S.-I. Kinouchi, T. Kubo, and T. Kammuri, in *Proceedings of the Texas A&M Symposium on Hot Nuclei*, edited by S. Shlomo, R. P. Schmitt, and J. B. Natowitz (World Scientific, Singapore, 1988), p. 89.

- [9] S.-I. Kinouchi, H. Sakamoto, T. Kubo, and T. Kishimoto, in *Nuclear Collective Motion and Nuclear Reaction Dynamics*, edited by K.-I. Kubo, M. Ichimura, M. Ishihara, and S. Yamaji (World Scientific, Singapore, 1989), p. 111.
- [10] T. Kishimoto, J. M. Moss, D. H. Youngblood, J. D. Bronson, C. M. Rozsa, D. R. Brown, and A. D. Bacher, *Phys. Rev. Lett.* **35**, 552 (1975).
- [11] H. Sakamoto and T. Kishimoto, *Nucl. Phys.* **A501**, 205 (1989); **A501**, 242 (1989).
- [12] S. G. Nilsson, *Mat. Fys. Medd. Dan. Vid. Selsk.* **29**, No. 16 (1955).
- [13] S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I.-L. Lamm, P. Möller, and B. Nilsson, *Nucl. Phys.* **A131**, 1 (1969).
- [14] T. Kishimoto, T. Tamura, and T. Kammuri, *Prog. Theor. Phys. Suppl.* **74&75**, 170 (1983).
- [15] E. R. Marshalek, *Phys. Rev. Lett.* **51**, 1534 (1983); *Phys. Rev. C* **29**, 640 (1984); *Phys. Lett. B* **244**, 1 (1990).
- [16] T. Kishimoto and T. Tamura, *Nucl. Phys.* **A192**, 246 (1972).
- [17] T. Kishimoto and T. Tamura, *Nucl. Phys.* **A270**, 317 (1976).
- [18] T. Tamura, K. J. Weeks, and T. Kishimoto, *Phys. Rev. C* **20**, 307 (1979); *Nucl. Phys.* **A347**, 359 (1980).
- [19] H. Sakamoto and T. Kishimoto, *Nucl. Phys.* **A486**, 1 (1988).
- [20] H. Sakamoto and T. Kishimoto, *Nucl. Phys.* **A528**, 73 (1991).
- [21] M. Matsuo and K. Matsuyanagi, *Prog. Theor. Phys.* **78**, 591 (1987).
- [22] H. Aiba, *Prog. Theor. Phys.* **84**, 908 (1990).
- [23] S. Mizutori, Y. R. Shimizu and K. Matsuyanagi, *Prog. Theor. Phys.* **83**, 666 (1990); **85**, 559 (1991).
- [24] J. M. Arias, M. Gallardo, and J. Gómez-Camacho, *Nucl. Phys.* **A528**, 144 (1991).
- [25] O. Civitarese, A. G. Dumrauf, M. Reboiro, P. Ring, and M. M. Sharma, *Phys. Rev. C* **43**, 2622 (1991).
- [26] S. Cwiok, J. Kvasil, and B. Choriev, *J. Phys. G* **10**, 903 (1984).
- [27] R. Nojarov and A. Faessler, *Nucl. Phys.* **A484**, 1 (1988).
- [28] O. Civitarese and M. C. Licciardo, *Phys. Rev. C* **38**, 967 (1988); **39**, 1550 (1989).
- [29] O. Civitarese, A. Faessler, and M. C. Licciardo, *Nucl. Phys.* **A542**, 221 (1992).
- [30] R. Nojarov, A. Faessler, and M. Dingfelder, *Phys. Rev. C* **51**, 2449 (1995).
- [31] E. Wigner, *Phys. Rev.* **40**, 749 (1932).
- [32] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
- [33] R. Bengtsson and P. Schuck, *Phys. Lett.* **89B**, 321 (1980).
- [34] M. Di Toro and V. M. Kolomietz, *Z. Phys. A* **328**, 285 (1987).
- [35] P. Carlos, H. Beil, R. Bergere, A. Lepretre, A. de Miniac, and A. Veyssiere, *Nucl. Phys.* **A225**, 171 (1974).
- [36] M. Danos, *Nucl. Phys.* **5**, 23 (1958).
- [37] K. Okamoto, *Phys. Rev.* **110**, 143 (1958).
- [38] A. H. Wapstra and G. Audi, *Nucl. Phys.* **A432**, 1 (1985).
- [39] C. M. Lederer and V. S. Shirley, *Table of Isotopes*, 7th ed. (Wiley, New York, 1978).
- [40] J. Barrette, M. Barrette, A. Boutard, R. Haroutunian, G. Lamoureux, and S. Monaro, *Phys. Rev. C* **6**, 1339 (1972).
- [41] W. Satuła and R. Wyss, *Phys. Rev. C* **50**, 2888 (1994).