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Intermittent and multifractal behaviors of multiplicity distributions in 800 GeV p-nucleus interactions

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We use scaled factorial moments (SFM's) to analyze pseudorapidity fluctuations of nonstatistical origin in p-nucleus interactions at 800 GeV. The SFM's are found to exhibit a power-law dependence on the pseudorapidity interval size. The anomalous dimensions d_q have been calculated up to order 5. The fractional dimensions D_q have been extracted from the slopes of the multifractal plots. Both the multifractal and intermittency approaches have been found to be complementary to each other. The behavior of D_q and d_q with order q indicates a possible self-similar random cascading mechanism for multiparticle production. [S0556-2813(96)04510-4]

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Recently, the observation of large density fluctuations of nonstatistical origin in small regions of phase space, called intermittency [1], has triggered considerable interest, both theoretical and experimental. Bialas and Peschanski [1] in their pioneering work gave an attractive formalism to study these multiplicity fluctuations in terms of noise-suppressed scaled factorial moments. They suggested that if a power-law dependence of scaled factorial moments on the rapidity bin size exists, it is clearly indicative of the presence of intermittency. Several theories have been propounded to explain intermittency, some of which are the formation and decay of jets in a self-similar pattern [1–3], phase transition and the formation of a quark-gluon plasma [4], hadronic-Čerenkov radiation [5], Ising model [6], and multiparticle correlations [7–11]. Intermittent behavior of secondary particles has also been confirmed in several experiments involving different projectiles and targets, namely, e^+e^- [12], μp [13], hadron-hadron [14], hadron-nucleus [15], and nucleus-nucleus [16].

A concept very closely related to intermittency is that of multifractals because self-similarity of the system over a range of scales is a characteristic of fractal geometry. In a multifractal analysis, it has been observed that the well-known G_q moments [17–19] show departure from a predicted linear behavior in a log-log plot of G_q vs bin size. Takagi [20] has argued that this may be due to the fact that the number of points (particles) in experiments does not strictly approach infinity. He has proposed [20] a simple and more attractive multifractal analysis which overcomes this limitation. In an earlier work, following Takagi's approach, we have performed a multifractal analysis for medium energy particles in p-AgBr interactions at 800 GeV [21] and have now extended this analysis to the shower particles (N_s), which contribute the most to the total cross section. In this work, we investigate intermittency and multifractality for the shower particles in p-nucleus interactions at 800 GeV, which is presently the highest energy for fixed targets. Details about the data can be seen in Ref. [22], although the present analysis uses data with larger statistics (3500 events). The anomalous dimensions d_q, which are a measure of the intermittent pattern of nonstatistical fluctuations in the interactions, have been extracted and compared with the fractional dimensions D_q obtained from the corresponding multifractal analysis. The behavior of these dimensions with order q clearly indicates the presence of multifractal geometry, suggestive of a cascading phenomenon in the emission of these shower particles.

The scaled factoral moments (SFM's) offer the most direct approach to investigate fluctuations in high energy multiparticle production processes. The pseudorapidity (η) space of individual events is divided into bins of varying sizes, and the presence of intermittency in such interactions is reflected by the power-law dependence of SFM's upon bin size.

The SFM's can be defined in two ways, viz., the horizontal and the vertical moments [1]. The qth order horizontal and vertical moments are, respectively, defined as

$$\langle F_q\rangle_H = N_{\rm ev}^{-1}\sum_{i=1}^{N_{\rm ev}} M^{-1}\sum_{m=1}^{M} \frac{K_{m,i}(K_{m,i}-1)\cdots(K_{m,i}-q+1)}{(\langle N\rangle/M)^q}, \tag{1a}$$

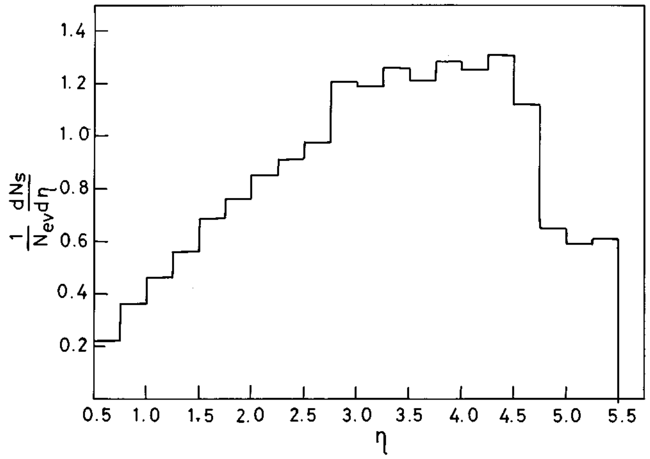

FIG. 1. Pseudorapidity distribution for $0.5\leqslant\eta\leqslant5.5$.

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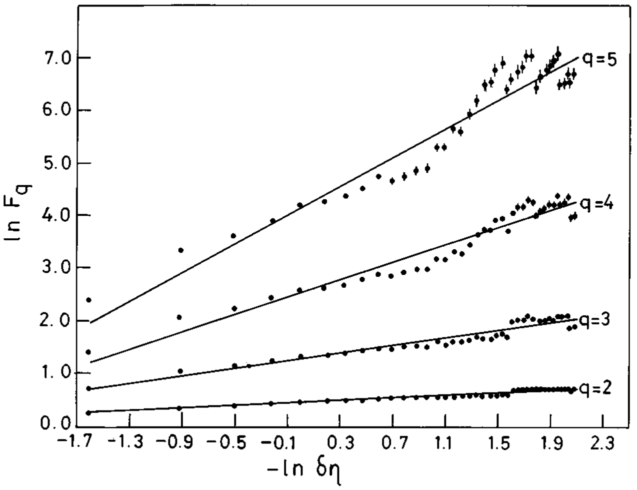

FIG. 2. Plot of $\ln F_q$ vs $-\ln\delta\eta$ for $q=2$–5. Solid lines indicate least squares fit in the linear region.

$$\langle F_q\rangle_v = M^{-1}\sum_{m=1}^{M} N_{\rm ev}^{-1}\sum_{i=1}^{N_{\rm ev}} \frac{K_{m,i}(K_{m,i}-1)\cdots(K_{m,i}-q+1)}{\langle K_m\rangle^q}, \tag{1b}$$

where

$$\langle K_m\rangle^q = N_{\rm ev}^{-1}\sum_{i=1}^{N_{\rm ev}} K_{m,i} \tag{2}$$

is the average content of mth bin of size $\delta\eta$ over the ensemble of events, M is the number of bins into which the pseudorapidity window is divided, N is the multiplicity in the total η interval, and $N_{\rm ev}$ is the number of events in the sample. The two definitions [(1a) and (1b)] become identical if the single-particle η distribution is flat. However, if the distribution is not flat, one should either consider the vertically averaged moments or apply a correction factor [23] to the horizontal moments. The vertical analysis, where normalization is done locally in each bin, is a particularly simple way of analyzing the SFM's. In the present work, we have studied the vertical factorial moments, which for convenience we will denote by F_q instead of $\langle F_q\rangle_v$ in the following. A log-log plot of F_q versus $1/\delta\eta$ yields the intermittency index

$$\phi_q = -\partial \ln F_q/\partial \ln\delta\eta, \tag{3}$$

which is related to the anomalous dimension d_q through the relation

$$d_q = \phi_q/(q-1). \tag{4}$$

TABLE I. Values of slopes ϕ_q from least squares fits of Eq. (3) to the data. The errors (in parentheses) are standard.

Order	ϕ_q
2	0.121 (0.005)
3	0.364 (0.018)
4	0.828 (0.035)
5	1.364 (0.063)

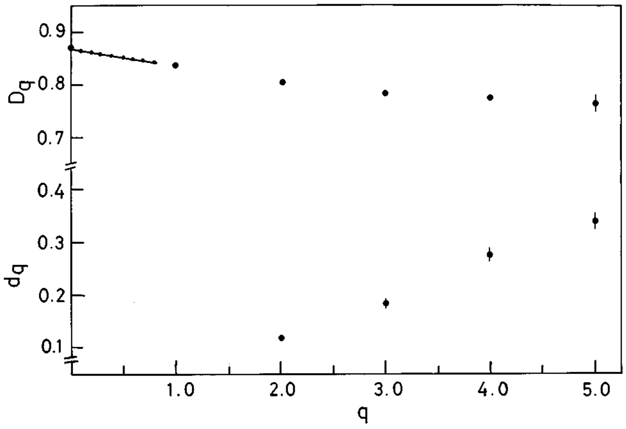

FIG. 3. Distribution of fractional dimensions D_q and anomalous dimensions d_q as a function of q. Solid line indicates least squares fit for $0<q<0.9$.

The experimental data have also been analyzed in terms of multifractals. Following Takagi [20], we have used $\langle n\ln n\rangle/\langle n\rangle$ and $\ln\langle n^q\rangle$ moments of multiplicity distributions in limited intervals of pseudorapidity, where n is the multiplicity in a single interval of the η space. They are plotted against increasing $\ln\langle n\rangle$, i.e., increasing interval size. In case of nonstatistical, self-similar density fluctuations (multifractility), this should yield a straight line behavior. Fractional dimensions D_q for $q=1,2,\ldots$ are determined from the slopes

$$D_q = (B_q-1)/(q-1), \quad q=2,3,\ldots, \tag{5}$$

$$D_1 = B_1, \tag{6}$$

where B_q is the slope of $\ln\langle n^q\rangle$ vs $\ln\langle n\rangle$ and B_1 is the slope of $\langle n\ln n\rangle/\langle n\rangle$ vs $\ln\langle n\rangle$.

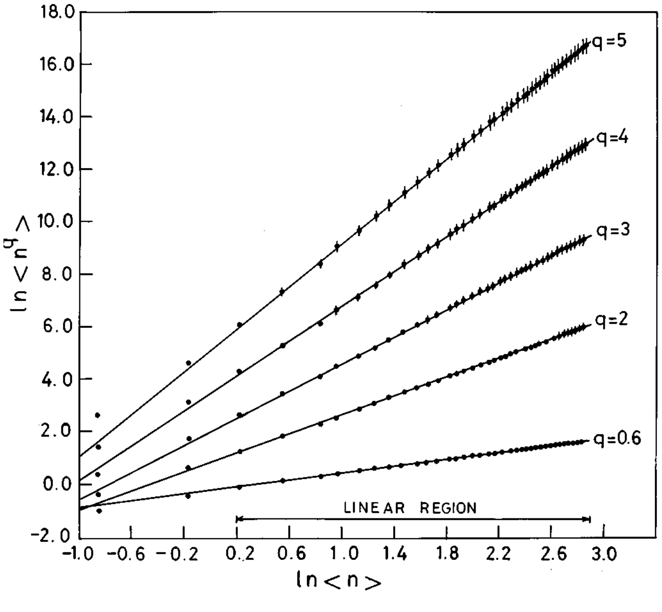

FIG. 4. Plot of $\ln\langle n^q\rangle$ vs $\ln\langle n\rangle$ for $q=0.6$ and $q=2$–5. Solid lines indicate least squares fits in the linear region.

TABLE II. Values of slopes B_q and intercepts A_q from least squares fits of Eq. (7) to the data. The errors (in parentheses) are standard.

Order	B_q	A_q
2	1.809 (0.002)	0.838 (0.005)
3	2.580 (0.004)	2.029 (0.008)
4	3.331 (0.005)	3.477 (0.011)
5	4.071 (0.057)	5.123 (0.013)

We started with the analysis of data in the central region in the pseudorapidity range $0.5 \leqslant \eta \leqslant 5.5$. The η distribution in this interval is shown in Fig. 1. We have varied $\delta\eta$ from 5.0 to 0.125 by increasing M from 1 to 40. In order to extract most of the statistically significant information, SFM's up to order 5 were calculated. Figure 2 exhibits the dependence of F_q on $\delta\eta$ in a log-log plot which shows a linear behavior. From the least squares fits to this plot, the values of the slopes ϕ_q were obtained and are given in Table I. The anomalous dimensions d_q were determined using Eq. (4) and Fig. 3 shows the dependence of d_q on order q. This behavior clearly signals nonstatistical, self-similar density fluctuations in the production of shower particles. Another remarkable result can be drawn from the scaling relation $d_q = q(d_2/2)$, which holds to a good degree of approximation. We can conclude that all of the statistically significant information is already present in the second-order moments [24]. Higher order moments do not contribute much to new information.

Following Takagi [20], we reduced size $\Delta\eta$ of the η space ($0.5 \leqslant \eta \leqslant 5.5$) symmetrically in steps of 0.05 from both ends. Thus the largest interval had size $\Delta\eta = 5.0$ and was decreased in 50 such steps until it became $\Delta\eta = 0.1$. We calculated the quantity $\ln\langle n^q\rangle$ where $q = 2$–5 for each of the intervals and studied its dependence on $\ln\langle n\rangle$. These plots are shown in Fig. 4 along with least squares fits for the linear region. As can be seen, all the moments show a remarkably linear behavior with resolution according to the relation

$$\ln\langle n^q\rangle = A_q + B_q \ln\langle n\rangle. \tag{7}$$

Table II lists the slopes B_q and intercepts A_q. The fractional dimensions D_q have been obtained using Eq. (5), and their dependence on order q is illustrated in Fig. 3. This behavior clearly favors multifractality and self-similar cascading in the present interactions.

We have found that a simple statistical model with a multinomial distribution [21] gives a trivial result $B_q = q$ or $D_q = 1$. Hence the value of $1 - D_q$, which is a measure of nonstatistical fluctuations in the interaction processes, receives no contribution from the background.

In order to calculate the fractal dimension D_0 of the set, we have extended the above analysis to q values ($0 < q < 0.9$). $\ln\langle n^q\rangle$ vs $\ln\langle n\rangle$ plots for these q values were studied, and Fig. 4 shows one of the plots corresponding to $q = 0.6$. From the slopes B_q [Eq. (7)], we determined D_q using Eq. (5), which are shown in Fig. 3. The D_q vs q behavior is found to satisfy the linear relation

$$D_q = a + bq; \quad q = 0.1, \ldots, 0.8. \tag{8}$$

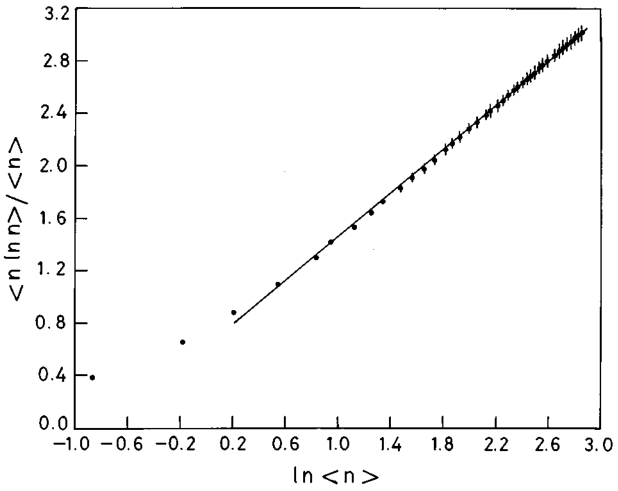

FIG. 5. Plot of $\langle n \ln n\rangle/\langle n\rangle$ vs $\ln\langle n\rangle$. Solid lines indicate least squares fits in the linear region.

The least squares fit is indicated by the solid line in Fig. 3. The values of the intercept (a) and slope (b) parameters are 0.868 and -2.999×10^{-2}, respectively. The standard errors are found to be very small as the fit is extremely good. Intercept a of Eq. (8) yields the fractal dimension D_0 also shown in Fig. 3. The information dimension D_1 was extracted using Eq. (6). Figure 5 shows the linear behavior of $\langle n \ln n\rangle/\langle n\rangle$ vs $\ln\langle n\rangle$. The value of D_1, which is equal to the slope of Fig. 5, is included in Fig. 3. From the behaviors of D_q and d_q with increasing q, it is observed that the following relation [25]

$$D_q + d_q = 1 \tag{9}$$

holds to a good approximation. Hence we can conclude that the multifractal and intermittency approaches are complementary to each other and this result is in agreement with the results of other authors [22,26].

In the intermittency analysis, we observe the linear behavior of $\ln F_q$ as a function of $-\ln\delta\eta$. The observed q dependence of anomalous fractal dimensions, d_q, shows that (a) self-similar cascading in the underlying multiparticle dynamics and (b) the dominant contribution to nonstatistical fluctuations come from the second order moment.

Linear relations are found to hold between $\ln\langle n^q\rangle$ and $\ln\langle n\rangle$ as well as between $\langle n \ln n\rangle/\langle n\rangle$ and $\ln\langle n\rangle$. This clearly points to the existence of fractality in the emission of these fast produced shower particles. The plot of fractional dimension D_q vs q for $q < 1$ shows a linear relation, which can be extrapolated to $q = 0$ to give a value of fractal dimension D_0 close to unity. For $q > 1$, D_q does not vary linearly with q, suggesting a cascading mechanism in the present interactions.

The intermittency and multifractal approaches used to analyze multiplicity fluctuations are found to be complementary to each other.

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