Two mode, finite range distorted wave Born approximation analysis of the ^{9}Be$(p,\alpha)^6$Li reaction at 45 and 50 MeV

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(Received 3 June 1996)

The ^{9}Be$(p,\alpha)^6$Li reaction at 45 and 50 MeV has been analyzed in the finite-range distorted-wave Born approximation (FR-DWBA) formalism including both the direct and the exchange modes, in which all one-step DWBA terms have been treated coherently. The spectroscopic factors are obtained theoretically by making use of the cluster-coupling shell-model wave functions. Numerical calculations show that the inclusion of the coherent contribution of the direct and the exchange modes is of considerable importance to give a reasonable agreement with the experimental data. [S0556-2813(96)03610-2]

PACS number(s): 24.10.Eq, 24.50.+g, 25.40.Hs, 25.55.Hp

Recently several authors have reported the results of an experimental study of the (p,α) reactions in the nuclei ^{9}Be at a proton bombarding energies of 45 [1,2] and 50 MeV [3]. While this reaction for proton bombarding energies below the Coulomb barrier is of interest in studying the mechanism of primordial nucleosynthesis [4,5], our main interest at present will be to obtain the useful information concerning the reaction mechanism itself. At high bombarding energies, (p,α) reactions can be generally described by a direct reaction mechanism. The distorted wave Born approximation (DWBA) is a very successful tool in analysis of direct nuclear reactions.

On the basis of the pictures of the cluster model of the nuclear structure, a finite-range DWBA (FR-DWBA) analysis of the reaction ^{9}Be$(p,\alpha)^6$Li(g.s.) at 45 MeV has been given by DeVries *et al.* [1], suggesting that both the light-particle pickup (LPPU) for forward angles and the heavy-particle pickup (HPPU) for backward angles should be considered in describing the behavior of the experimental angular distributions. Their calculations generally yield acceptable fits to the experiments, while have a tendency to overestimate the absolute values of the experiments for backward angles by three or four times of magnitude, but give rough estimates for the relation between spectroscopic factors of LPPU and HPPU processes. For the same reaction at $18-45$ MeV, Hauser *et al.* [2] have shown that the integrated cross sections of the forward $\sigma(0°-90°)$ and the backward $\sigma(90°-180°)$ are proportional to the spectroscopic strengths for the LPPU and the HPPU processes, respectively. Based on the measurements of the strong rise in forward and backward directions of the angular distributions, they have shown the HPPU process, in which the light-particle knockout (LPKO) is consequently contained, to be an important contribution in describing the experimental data reasonably. Moreover, Gurevich *et al.* [3] have investigated the same reaction at 50 MeV and compared the experimental angular distributions with theoretical ones calculated by the FR-DWBA with the LPPU (triton pickup) in the direct mode and the HPPU (^{5}He pickup) in the exchange mode incoherently. The experimental angular distributions cannot be represented by the resulting calculated ones at whole angles.

In this paper we investigate the reaction ^{9}Be$(p,\alpha)^6$Li(g.s.) at 45 and 50 MeV in the framework of the FR-DWBA including both the direct and the exchange modes, in which all one-step DWBA terms involving transfers of single particles or bound clusters may be treated coherently. Moreover the spectroscopic factors for each mode are obtained theoretically by making use of the cluster-coupling shell-model wave functions [6,7].

Now we briefly explain our FR-DWBA formalism for the ^{9}Be$(p,\alpha)^6$Li(g.s.) reaction. The details of the formalism presented here are discussed more completely in Ref. [8]. The differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\frac{\mu_p\mu_\alpha}{(2\pi\hbar^2)^2}\frac{k_\alpha}{k_p}\frac{1}{(2I_p+1)(2I_{\mathrm{Be}}+1)}\sum_{M_pM_\alpha M_{\mathrm{Be}}M_{\mathrm{Li}}}|M|^2, \tag{1}$$

where M is the total transition amplitude which is the sum of the direct transition amplitude M^D and the exchange one M^E as follows:

$$M=N_D\times M^D+N_E\times M^E \tag{2}$$

with the statistical factors N_D and N_E [9]. The DWBA transition amplitudes M^D for the direct mode, which is described as $p+{}^9\mathrm{Be}(=t+{}^6\mathrm{Li})\rightarrow{}^6\mathrm{Li}+\alpha(=p+t)$, with multi-interactions in the prior form is

$$M^D=\langle\chi_f^D|V_{p\text{-}t}+V_{p\text{-}\mathrm{Li}}-U_{p\text{-}\mathrm{Be}}|\chi_i^D\rangle. \tag{3}$$

Similarly, M^E for the exchange mode, which is described as $p+{}^9\mathrm{Be}(=\alpha+{}^5\mathrm{He})\rightarrow{}^6\mathrm{Li}(=p+{}^5\mathrm{He})+\alpha$, is

$$M^E=\langle\chi_f^E|V_{p\text{-}\alpha}+V_{p\text{-}\mathrm{He}}-U_{p\text{-}\mathrm{Be}}|\chi_i^E\rangle, \tag{4}$$

where the terms $V_{p\text{-}t}$ and $V_{p\text{-}\mathrm{Li}}$ in Eq. (3) [$V_{p\text{-}\alpha}$ and $V_{p\text{-}\mathrm{He}}$ in Eq. (4)] are responsible for the usual light-particle pickup (LPPU) and heavy-particle knockout (HPKO) [light-particle knockout (LPKO), and heavy-particle pickup (HPPU)]. These interaction potentials for both modes are taken as the real Woods-Saxon form without the Coulomb interaction. Parameters for the transition interactions except for $V_{p\text{-}^5\mathrm{He}}$ given in Table I (middle) were taken to be the same ones as used by several authors [10,11]. Here, as for the interaction $V_{p\text{-}^5\mathrm{He}}$ we derived from the interpolation of the interactions $V_{p\text{-}^6\mathrm{He}}$ and $V_{p\text{-}^3\mathrm{He}}$ [12].

TABLE I. The potential parameters for (top) the bound state, (middle) transition interaction, and (bottom) distorting potential used in the calculation.

Bound state[a]					
System	State	r (fm)	a (fm)	V (MeV)	E_B (MeV)
$t+{}^6$Li	$2p$	2.20	0.90	−57.34	17.69
$t+p$	$1s$	1.18	0.80	−76.14	19.82
$\alpha+{}^5$He	$3s$	1.20	0.40	−126.79	2.53
$p+{}^5$He	$1p$	2.30	0.90	−28.19	4.65
Interaction		r (fm)	a (fm)	V (MeV)	
$V_{p\text{-}t}$[b]		1.39	0.40	−61.40	
$V_{p\text{-Li}}$[c]		1.25	0.60	−59.00	
$V_{p\text{-}\alpha}$[b]		1.40	0.44	−45.00	
$V_{p\text{-He}}$[d]		1.40	0.30	−59.05	
Distortion					
System	E_p (MeV)	V (MeV)	W_V (MeV)	W_S (MeV)	r (fm)	a (fm)	r_I (fm)	a_I (fm)
$p+{}^9$Be[e]	45	−51.6	−7.50	−8.28	1.11	0.57	1.11	0.50
	50	−50.3	−7.50	−8.01	1.11	0.57	1.11	0.50
$\alpha+{}^6$Li[f]	45	−95.0	−3.00	−4.70	0.97	0.70	2.69	0.45
	50	−93.6	−3.56	−4.79	0.97	0.73	2.69	0.48

[a]Reference [3].
[b]Reference [10].
[c]Reference [11].
[d]Reference [12].
[e]Reference [13].
[f]Reference [1].

The χ_i^D and χ_f^D are entrance- and exit-channel wave functions in the direct mode, respectively,

$$\chi_i^D = \Psi^D_{I_{\rm Be}M_{\rm Be}}(\xi_t,\xi_{Li},\mathbf{r}_{t\rm Li})\Psi^D_{I_pM_p}(\xi_p)\varphi_p^{(+)}(\mathbf{k}_p,\mathbf{r}_{p\rm Be}), \quad (5)$$

$$\chi_f^D = \Psi^{D*}_{I_\alpha M_\alpha}(\xi_p,\xi_t,\mathbf{r}_{pt})\Psi^{D*}_{I_{\rm Li}M_{\rm Li}}(\xi_{\rm Li})\varphi_\alpha^{(-)*}(\mathbf{k}_\alpha,\mathbf{r}_{\alpha\,\rm Li}). \quad (6)$$

Similarly, the entrance- and the exit-channel wave functions χ_i^E and χ_f^E in the exchange mode are given. The functions $\Psi^D_{I_{\rm Be}M_{\rm Be}}(\xi_t,\xi_{\rm Li},\mathbf{r}_{t\rm Li})$ and $\Psi^{D*}_{I_\alpha M_\alpha}(\xi_p,\xi_t,\mathbf{r}_{pt})$ are, respectively, the bound state wave functions for ^{9}Be and α in the direct mode. Both of the bound states of the clusters in the target (the residual) nuclei for each mode, which are, respectively, assumed to be $2p$ state in direct and $3s$ state ($1p$ state) in exchange mode for simplicity, have been obtained in terms of a real Woods-Saxon type to give the proper separation energy. The distorted waves $\varphi_p^{(+)}(\mathbf{k}_p,\mathbf{r}_{p\rm Be})$ and $\varphi_\alpha^{(-)*}(\mathbf{k}_\alpha,\mathbf{r}_{\alpha\rm Li})$ are generated by the optical model potential $U_{p\text{-Be}}$ and $U_{\alpha\text{-Li}}$, respectively. As regards the potential $U_{p\text{-Be}}$ in the entrance channel $p+{}^9$Be, we used the energy dependent Woods-Saxon type [13]. In the exit channel $\alpha+{}^6$Li, we used the same potentials as DeVries *et al.* [1] did, whereas parameters at 50 MeV were modified somewhat so as to fit the experimental angular distributions. These parameters are shown in Table I. Hereafter we use the symbols T, R, and C instead of the target ^{9}Be, the residual nuclei ^{6}Li, and the core ^{5}He, respectively.

The cluster expansion for the target nucleus $T(=t+R)$ is given by

$$\Psi^D_{I_TM_T}(\xi_t,\xi_R,\mathbf{r}_{tR}) = \sum_{[\delta_T]} \theta_{T(t,R)}(I_t l_1 M_t m_1|j_1\mu_1) \times (j_1 I_R \mu_1 M_R|I_T M_T)\Psi^D_{I_tM_t}(\xi_t) \times \Psi^D_{I_RM_R}(\xi_R)u^D_{n_1l_1}(r_{tR})Y_{l_1m_1}(\hat{r}_{tR}), \quad (7)$$

where the expansion coefficient $\theta_{T(t,R)}$ is fractional parentage coefficient (cfp), whose value is calculated in the L-S coupling scheme according to the Appendix of Ref. [6]. Similarly, the functions $\Psi^D_{I_\alpha M_\alpha}(\xi_p,\xi_t,\mathbf{r}_{pt})$, $\Psi^E_{I_TM_T}(\xi_\alpha,\xi_C,\mathbf{r}_{\alpha C})$, and $\Psi^E_{I_RM_R}(\xi_p,\xi_C,\mathbf{r}_{pC})$—the second and the third term of which represent the bound states of the target T and the residual nuclei R in the exchange mode—are expanded in terms of the corresponding fractional parentage coefficients $\theta_{\alpha(p,t)}$, $\theta_{T(\alpha,C)}$, and $\theta_{R(p,C)}$, respectively. Therefore the spectroscopic amplitudes for the direct and the exchange mode S^D and S^E are, respectively, expressed by $N_D\cdot\theta_{T(t,R)}\cdot\theta_{\alpha(p,t)}$ and $N_E\cdot\theta_{T(\alpha,C)}\cdot\theta_{R(p,C)}$. Our results for the spectroscopic amplitudes $S^D=0.245$ and $S^E=-0.406$ are consistent with the values of Refs. [14–16], respectively.

We calculated the differential cross sections for the ^{9}Be$(p,\alpha)^6$Li(g.s.) reaction at 45 and 50 MeV in terms of our FR-DWBA. In Figs. 1 and 2, the calculated angular distributions are shown together with the experimental data at both energies. The solid line is the calculated result with the total amplitude M in Eq. (2) with S^D and S^E as mentioned just above. Also, the dotted line and the dashed line are the cal-

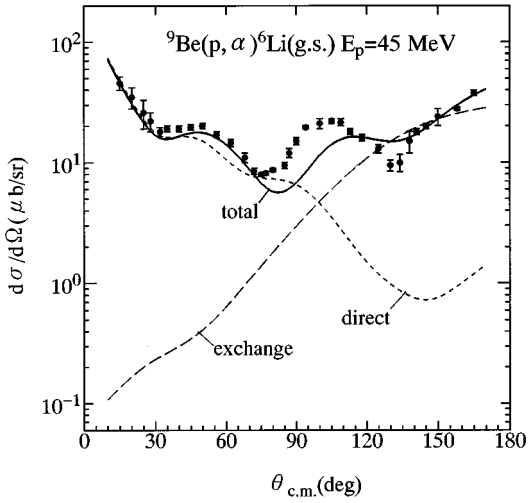

FIG. 1. Angular distribution for the reaction $^9\mathrm{Be}(p,\alpha)^6\mathrm{Li}$(g.s.) at 45 MeV. The solid, dotted, and dashed curves are the calculated results for the total, direct, and exchange modes, respectively. The experimental data are taken from Refs. [1,2].

culated results of the direct mode and the exchange mode, which are characterized by the amplitudes M^D in Eq. (3) and M^E in Eq. (4), respectively. In these figures, the contribution of the exchange mode to the angular distributions at backward angles is found to be remarkable. In particular, comparing with the results by DeVries *et al.* [1], Hauser *et al.* [2], at 45 MeV and Gurevich *et al.* [3] at 50 MeV, our results of including the two knockout processes denoted by HPKO and LPKO, namely ^{6}Li knockout in the direct mode and α knockout in the exchange mode, and the interference between the each mode considerably improved the fit to the backward angles for E_p=45 MeV and to the whole angles for E_p=50 MeV.

The predicted differential cross sections at 45 MeV associated with each term, i.e., $V_{p\text{-}t}$ ($V_{p\text{-}\alpha}$), $V_{p\text{-Li}}-U_{p\text{-Be}}$ ($V_{p\text{-He}}-U_{p\text{-Be}}$) and all of the terms for the direct (exchange) mode, respectively, are illustrated in Fig. 3. As mentioned above in Figs. 1 and 2, the interference effect denoted by the solid line in this figure has been found to be large for these energies. We cannot reproduce the experimental data without any one term $V_{x\text{-}y}$ in M^D and M^E. In conclusion from these figures, we see that the inclusion of the coherent contribution of the direct and the exchange-mode with all one-step DWBA terms is of considerable importance to give a reasonable agreement with the experimental data.

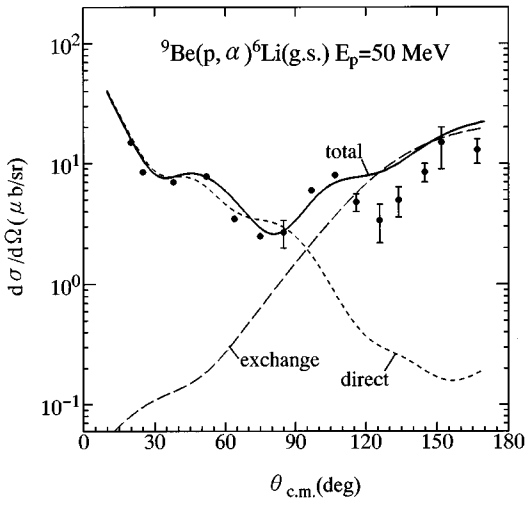

FIG. 2. Same as Fig. 1 but for E_p=50 MeV. The experimental data are taken from Ref. [3].

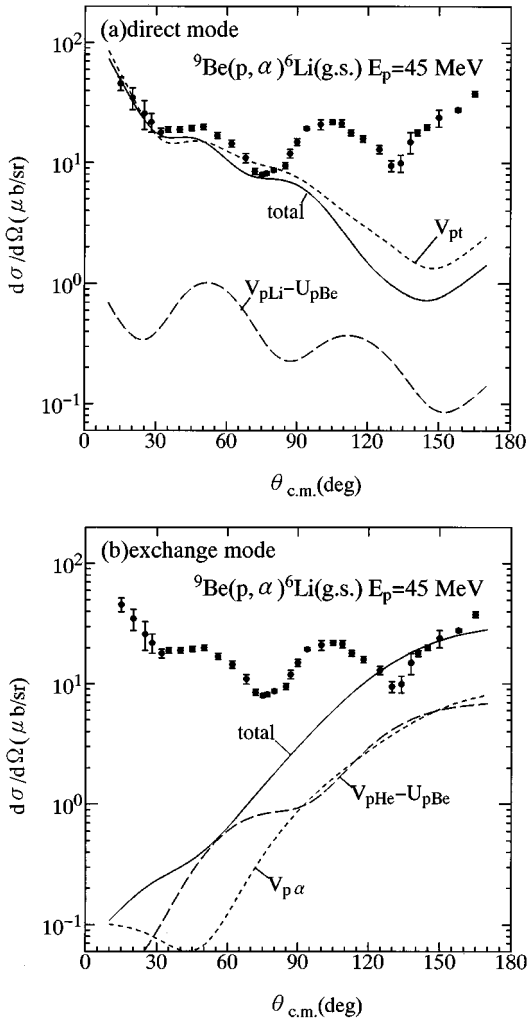

FIG. 3. Decomposition of the calculated differential cross section for the reaction $^9\mathrm{Be}(p,\alpha)^6\mathrm{Li}$(g.s.) at 45 MeV into the direct and exchange mode contributions: (a) the triton pickup term $V_{p\text{-}t}$ (dotted curve) and the other term $V_{p\text{-Li}}-U_{p\text{-Be}}$ (dashed curve) in the direct mode, and (b) the α knockout term $V_{p\text{-}\alpha}$ (dotted curve) and the other term $V_{p\text{-He}}-U_{p\text{-Be}}$ (dashed curve) in the exchange mode, respectively.

In summary, we have investigated the reaction $^9\mathrm{Be}(p,\alpha)^6\mathrm{Li}$(g.s.) at 45 and 50 MeV in the framework of the FR-DWBA including both the direct and the exchange modes, in which all one-step DWBA terms have been treated coherently. The spectroscopic factors for each mode are obtained theoretically by making use of the cluster-coupling shell-model wave functions. Numerical calculations show that the inclusion of the coherent contribution of the direct and the exchange mode, in which all one-step DWBA terms are included, is of considerable importance to give a reasonable agreement with the experimental data.

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