Renormalized quasiparticle random phase approximation and double beta decay: A critical analysis of double Fermi transitions

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The proton-neutron monopole Lipkin model, which exhibits some properties that are relevant for those double beta decay ($\beta\beta$) transitions mediated by the Fermi matrix elements, is solved exactly in the proton-neutron two-quasiparticle space. The exact results are compared with the ones obtained by using the quasiparticle random phase approximation (QRPA) and renormalized QRPA (RQRPA) approaches. It is shown that the RQRPA violates the Ikeda sum rule and that this violation may be common to any extension of the QRPA where scattering terms are neglected in the participant one-body operators as well as in the Hamiltonian. This finding underlines the need of additional developments before the RQRPA could be adopted as a reliable tool to compute $\beta\beta$ processes. [S0556-2813(96)06610-1]

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I. INTRODUCTION

The nuclear double β decay could provide evidence on the existence of massive neutrinos and right-handed weak currents [1]. This exciting possibility has attracted much theoretical and experimental work in the last years [2]. In order to extract information about this new physics the data must be complemented with theoretical nuclear matrix elements that are strongly suppressed.

Ten years ago it was realized that the quasiparticle random phase approximation (QRPA), including a particle-particle channel in the residual interaction, can reproduce the experimentally determined two-neutrino double-β decay ($\beta\beta_{2\nu}$) half-lives [3–6]. This step allowed the description of single and double-β decay transitions in many nuclei. However it was soon recognized that the ground state to ground state $\beta\beta_{2\nu}$ transition amplitudes are extremely sensitive to the force parameters, thus limiting the predictive power of the theory. The breakdown of the QRPA approach, for some critical values of the model parameters, made the theoretical description of some cases particularly difficult; i.e., the $\beta\beta_{2\nu}$ decay of ^{100}Mo [3,4,7–9].

Recently the use of a correlated vacuum in the QRPA equation of motion, the so-called renormalized QRPA (RQRPA) [10,11], has been reformulated [12] and applied to the $\beta\beta_{2\nu}$ decay [13]. It was found that the formalism is stable beyond the point where the QRPA collapses. The RQRPA method requires the solution of coupled-nonlinear equations, instead of the usual eigenvalue problem of the QRPA. However the physical consequences of this highly nonlinear behavior, represented by the inclusion of some terms beyond the QRPA order of approximation, have not been explored carefully.

In the present paper we will use a simple solvable model, which is an extension of the Lipkin model [14], to compare the exact, the QRPA, and the RQRPA approaches. It will be shown that if one remains at the leading order of approximation, i.e., by retaining two-quasiparticle terms in the relevant Fermi or Gamow-Teller transition operators as well as in the Hamiltonian, the RQRPA violates the Ikeda sum rule.

II. THE MODEL

The model Hamiltonian [15–17] consists in a single particle, a pairing term for protons and neutrons, and a schematic charge-dependent residual interaction including particle-hole and particle-particle channels. It has been shown in a recent series of papers that this interaction, treated in the framework of the QRPA, is as good as a G matrix constructed from the OBEP Bonn potential in reproducing single- and double-β decay matrix elements [17–19].

The schematic Hamiltonian reads

$$H = H_p + H_n + H_{\rm res}, \tag{1}$$

where

$$H_p = \sum_p e_p a_p^\dagger a_p - G_p S_p^\dagger S_p, \quad H_n = \sum_n e_n a_n^\dagger a_n - G_n S_n^\dagger S_n,$$

$$H_{\rm res} = 2\chi \beta_J^- \cdot \beta_J^+ - 2\kappa P_J^- \cdot P_J^+ . \tag{2}$$

In the above expression the following definitions were introduced:

$$S_p^\dagger = \sum_p a_p^\dagger a_{\bar p}^\dagger/2, \quad S_n^\dagger = \sum_n a_n^\dagger a_{\bar n}^\dagger/2,$$

$$\beta_J^- \cdot \beta_J^+ = \sum_{M=-J}^{J} (-1)^M : \beta_{JM}^- (\beta_{J-M}^-)^\dagger :,$$

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$$P_J^- \cdot P_J^+ = \sum_{M=-J}^{J} (-1)^M : P_{JM}^- (P_{J-M}^-)^\dagger :,$$

$$\beta_{JM}^- = \sum_{i,j} \langle i|\mathcal{O}_{JM}|j\rangle a_i^\dagger a_j ,$$

$$P_{JM}^- = \sum_{i,j} \langle i|\mathcal{O}_{JM}|j\rangle a_i^\dagger a_{\bar{j}}^\dagger, \quad \mathcal{O}_{1M} = \sigma_M \tau^-, \quad \mathcal{O}_{00} = \tau^-, \tag{3}$$

$a_p^\dagger = a_{j_p m_p}^\dagger$ being the particle creation operator and $a_{\bar{p}}^\dagger = (-1)^{j_p - m_p} a_{j_p - m_p}^\dagger$ its time reversal.

As mentioned above, the Hamiltonian (1) with $J=1$ provides a reasonable description of the main physics involved in Gamow-Teller transitions. The parameters χ and κ play the role of the renormalization factors g_{ph} and g_{pp} introduced in the literature [3–6].

For the case of $J=0$ the Hamiltonian (1) can be reduced to an isospin scalar if its parameters are selected as

$$e_p = e_n, \quad \chi = 0, \quad G_p = G_n = 4\kappa. \tag{4}$$

If $\chi \neq 0$ the isospin symmetry is broken in the particle-hole channel and for $4\kappa \neq G_p, G_n$ this symmetry is broken in the particle-particle channel. A similar identification of this isospin breaking mechanism can be performed for the case of realistic interactions with renormalized proton-proton (g_{pair}^p), neutron-neutron (g_{pair}^n) and proton-neutron particle-particle (g_{pp}) strengths [3–6]. For the Fermi transitions, such as the ones contributing to the nuclear matrix elements associated with the neutrinoless double-β-decay mode, the value $g_{pp} = 1.0$ is usually adopted, while g_{pair}^p and g_{pair}^n vary from 0.9 to 1.2 to reproduce the observed proton and neutron pairing gaps in medium and heavy mass nuclei. This parametrization, adopted for realistic interactions, provides a first motivation for using the proton-neutron particle-particle strength κ as a parameter independent of G.

In order to construct an exactly solvable model to be used to test the reliability of the RQRPA solutions for the single- and double-β decay observables, we will make a strong approximation. We will consider a single shell with the same angular momentum for protons and neutrons, i.e., $j_p = j_n = j$. Adopting this model space is equivalent to work with the one-level limit of Eq. (1). It will be shown that this model, which is not intended to reproduce actual nuclear properties, does have the qualitative features of a realistic pn-QRPA calculation. Indeed, excitation energies, single- and double-β decay transition amplitudes, and ground state correlations depend on the particle-particle strength parameter κ in the same way as they do in more elaborate calculations with many single-particle levels and with more realistic interactions. The advantage of such a simplification lies in the fact that the proton-neutron excitations can be described within the framework of an exactly solvable model. Particularly, the correspondence between the simplified version (one-shell limit) of Eq. (1) and the ordinary monopole Lipkin model [14] can be established if one chooses the channel with $J=0$ of Hamiltonian (1). Physically, this case will correspond to Fermi-type transitions but we should emphasize the fact that the study of the model and not the adjustment of a given decay channel constitutes the essential aspect of the present discussion.

In the single-shell case, and after performing the Bogolyubov transformations, separately, for protons and neutrons, the first two terms in Eq. (1) become diagonal, i.e.,

$$H_p = \epsilon_p \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p}, \quad H_n = \epsilon_n \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n}, \tag{5}$$

ϵ_p, ϵ_n being the quasiparticle energies and $\alpha_p^\dagger, \alpha_n^\dagger$ the quasiparticle creation operators [20].

The linearized Hamiltonian, neglecting the scattering terms ($\alpha_p^\dagger \alpha_n, \alpha_n^\dagger \alpha_p$) that do not contribute at the QRPA order, reads

$$\begin{aligned} H = {} & \epsilon_p \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p} + \epsilon_n \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n} \\ & + 2\chi(2j+1)[(u_p^2 v_n^2 + v_p^2 u_n^2) \\ & \times A^\dagger \cdot A + u_p v_n v_p u_n A^\dagger \cdot A^\dagger + v_p u_n u_p v_n A \cdot A] \\ & - 2\kappa(2j+1)[(u_p^2 u_n^2 + v_p^2 v_n^2) A^\dagger \cdot A - u_p u_n v_p v_n A^\dagger \cdot A^\dagger \\ & - v_p v_n u_p u_n A \cdot A], \end{aligned} \tag{6}$$

with

$$A^\dagger = [\alpha_p^\dagger \otimes \alpha_n^\dagger]_{M=0}^{J=0}.$$

To show that it is possible to reduce Eq. (6) to an holomorphic version of the Lipkin model we have considered, for simplicity, one single particle orbital for protons and neutrons with the same quasiparticle energies $\epsilon = \epsilon_p = \epsilon_n = (2j+1)G_i/4, \quad i = p,n$ [20].

Under these approximations the Hamiltonian (6) has the form

$$H = \epsilon C + \lambda_1 A^\dagger A + \lambda_2 (A^\dagger A^\dagger + AA), \tag{7}$$

with

$$\begin{aligned} & C \equiv \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p} + \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n}, \quad \text{and} \\ & \lambda_1 = 2[\chi(u_p^2 v_n^2 + v_p^2 u_n^2) - \kappa(u_p^2 u_n^2 + v_p^2 v_n^2)], \\ & \lambda_2 = 2(\chi + \kappa) u_p v_p u_n v_n . \end{aligned} \tag{8}$$

The operators $\{A, A^\dagger, C\}$ satisfy the SU(2) quasispin algebra [21]

$$[A, A^\dagger] = 1 - C/(2j+1), \quad [C, A^\dagger] = 2A^\dagger. \tag{9}$$

A form similar to Eq. (7) was introduced by Lipkin *et al.* in their original paper [14]. The usual Lipkin model is obtained setting $\lambda_1 = 0$, since this term essentially renormalizes the single-particle energy ϵ. In our case it corresponds to the particular case $Z = N = (2j+1)/2$, which implies $v_p = v_n = u_p = u_n$, and $\chi = \kappa$. However, we shall keep this term in order to be as close as possible to the realistic situation.

III. EXACT SOLUTIONS

To obtain the exact solutions of Eq. (7) we have performed a Holstein-Primakoff mapping [21,22] of this Hamiltonian. It involves the substitution of a pair of fermions by functions of the exact boson operators $b^\dagger$ and b, which fulfill the exact commutation rule $[b,b^\dagger]=1$. The relations between both set of operators are the following:

$$A^\dagger \to b^\dagger\left(1-\frac{b^\dagger b}{(2j+1)}\right)^{1/2}, \quad A\to\left(1-\frac{b^\dagger b}{(2j+1)}\right)^{1/2} b,$$
$$C\to 2b^\dagger b. \tag{10}$$

The exact solutions are obtained using the boson basis

$$|n_b)=\frac{(b^\dagger)^{n_b}}{\sqrt{n_b!}}|), \quad b|)=0. \tag{11}$$

The Hamiltonian (7) has terms that change the number of bosons in two units, thus the exact wave function does not have a definite number of bosons. At the same time, states with odd and even number of bosons are not connected. To construct a decay scheme for double Fermi transitions the exact ground state of the even-even nuclei will be represented by the lowest energy state with an even number of bosons while the 0^+ states of the odd-odd nuclei are those with an odd number of bosons. Spurious states are avoided by limiting the number of bosons to $0\leqslant n_b \leqslant 2j+1$. The physical states are written

$$|\lambda_{\text{even-even}})=\sum_{n_b \text{ even}}^{2j+1} C_{n_b}^{\lambda_e}|n_b), \quad |\lambda_{\text{odd-odd}})=\sum_{n_b \text{ odd}}^{2j+1} C_{n_b}^{\lambda_o}|n_b). \tag{12}$$

It must be clear that what we call ''exact solutions'' are the exact solutions of the Hamiltonian (7), where the terms of the form $\alpha_p^\dagger\alpha_n, \alpha_n^\dagger\alpha_p$ (scattering terms) have been neglected. These exact solutions exhaust any extension of the QRPA intended to diagonalize Hamiltonian (7). However, the solutions of the complete eigenvalue problem of the Hamiltonian (1) span a larger Hilbert space, including states orthogonal to those present in the actual ''exact'' solutions. The operator $\beta^{\pm}$ is sensitive to this truncation of the Hilbert space generated by solving the restricted problem defined by the use of Hamiltonian (7). While at the QRPA level it has no effect, in any extensions beyond QRPA the transition amplitudes exhibit the missed components. Its effects on the Ikeda sum rule are described below.

IV. QRPA AND RQRPA

The QRPA Hamiltonian H_{QRPA} can be obtained from Eq. (7) by taking the limit $(2j+1)\to\infty$. It is given by

$$H_{\text{QRPA}}=(2\epsilon+\lambda_1)b^\dagger b+\lambda_2\{b^\dagger b^\dagger+bb\}. \tag{13}$$

The QRPA states are generated with the one-phonon operator $O_{\text{QRPA}}^\dagger = XA^\dagger - YA$ acting over the correlated QRPA vacuum $|0\rangle$. The quasiboson approximation assumes the $\langle 0|[A,A^\dagger]|0\rangle=1$, leading to the normalization condition $X^2-Y^2=1$. The QRPA matrix is just a 2×2 one, with submatrices $\mathcal{A}_{\text{QRPA}}=2\epsilon+\lambda_1$ and $\mathcal{B}_{\text{QRPA}}=2\lambda_2$. The eigenenergy is $E_{\text{QRPA}}=[(2\epsilon+\lambda_1)^2-4\lambda_2^2]^{1/2}$. It becomes a pure imaginary number if $2\lambda_2>2\epsilon+\lambda_1$. It means that for this limit the zero-boson component of ground state ceased to be dominant.

In the renormalized QRPA the structure of the ground state is included explicitly [11], in the form

$$|0\rangle=\mathcal{N}e^S|BCS\rangle, \quad S=\frac{cA^\dagger A^\dagger}{2\langle 0|[A,A^\dagger]|0\rangle}. \tag{14}$$

The RQRPA one-phonon state is given by

$$O_{\text{RQRPA}}^\dagger|0\rangle=[\mathcal{X}A^\dagger-\mathcal{Y}A]/\langle 0|[A,A^\dagger]|0\rangle^{1/2}|0\rangle. \tag{15}$$

The condition $O_{\text{RQRPA}}|0\rangle=0$ leads to the value $c=\mathcal{Y}/\mathcal{X}$. After some algebra it is possible to show that $\langle 0|[A,A^\dagger]|0\rangle\equiv D=1-2\mathcal{Y}^2 D/(2j+1)$ [12,13], and that

$$D=\left[1+\frac{2\mathcal{Y}^2}{2j+1}\right]^{-1}. \tag{16}$$

The RQRPA submatrices are $\mathcal{A}_{\text{RQRPA}}=2\epsilon+\lambda_1 D$ and $\mathcal{B}_{\text{RQRPA}}=2\lambda_2 D$. Given $0\leqslant D\leqslant 1$, the presence of D multiplying both λ_1 and λ_2 gives the needed reduction of the residual interaction to avoid the collapse of the QRPA equations [13]. Due to this fact the RQRPA energy E_{RQRPA} is always real. Its value must be obtained by solving simultaneously the nonlinear equations for $E_{\text{RQRPA}}, \mathcal{X}, \mathcal{Y}$, and D, which in the general case will include all possible values of J [13].

V. RESULTS AND DISCUSSION

In the following we will present numerical results that correspond to the model space and parameters

$$j=9/2, \quad Z=4, \quad N=6, \quad \epsilon=1 \text{ MeV}. \tag{17}$$

In order to avoid dealing with small numbers, we now redifine the two parameters

$$\kappa\to\kappa'\equiv(2j+1)\kappa, \quad \chi\to\chi'\equiv(2j+1)\chi. \tag{18}$$

We selected the values $\chi'=0$ or 0.5. The particle-particle strength κ' is kept as a variable.

A. Excitation energies

In Fig. 1 the excitation energies E_1-E_0, E_{QRPA}, and E_{RQRPA} are plotted against κ' for $\chi'=0$ (upper figure) and $\chi'=0.5$ (lower figure). In both cases the collapse of the QRPA is evident at $\kappa'\simeq 1$. The RQRPA excitation energy remains real but it decreases as compared with the exact one. For a relatively large value of κ', ($\kappa'\simeq 2$), the ground state and the first excited one tend to become degenerate.

This Fig. 1 strongly resembles Fig. 1 of Ref. [13] and Fig. 2 of Ref. [8], where the energy of the first excited state is plotted against the particle-particle strength parameter g_{pp}. The curves for the QRPA and the RQRPA are quite similar to those shown here. The advantage of the present simple model is that we can compare them with the exact results, while in the general case with multiple single-particle levels the exact solutions are unknown.

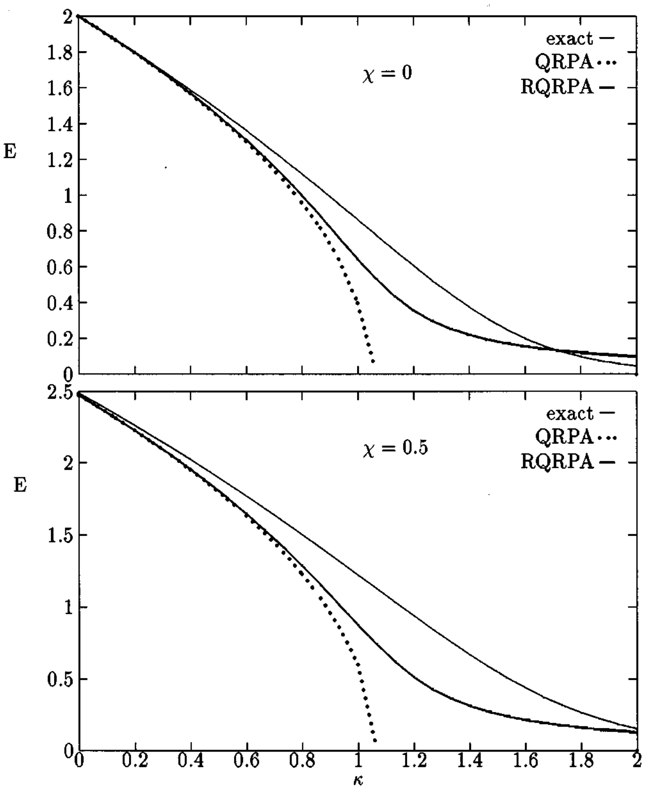

FIG. 1. Excitation energy vs the particle-particle strength κ'. The particle-hole strength is $\chi'=0$ (upper figure) and $\chi'=0.5$. The exact solution is plotted with thin line, the QRPA with dots, and the RQRPA with a thick line.

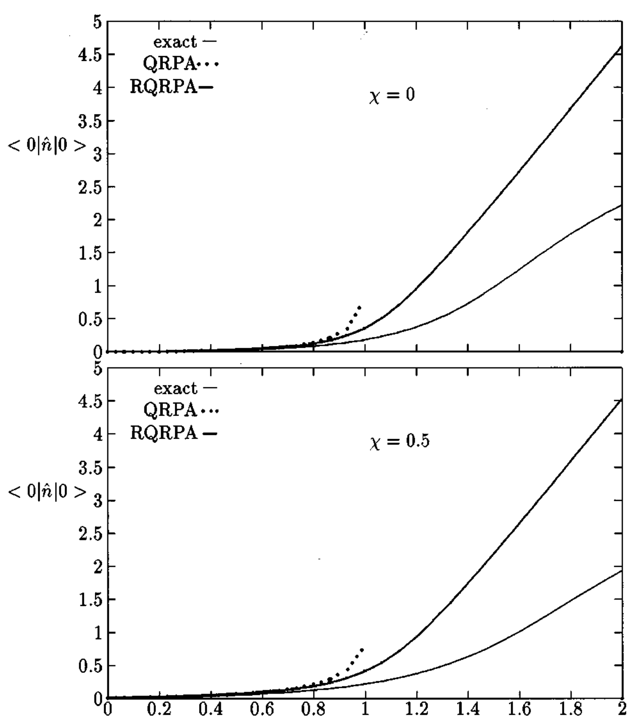

FIG. 2. Number of bosons $\langle 0|\hat{n}|0\rangle$ in the ground state vs κ'. Conventions are the same as in Fig. 1.

B. Boson number

The expectation value of the boson number operator in the ground state measures the difference between the QRPA (or RQRPA) ground states and the BCS vacuum. Its expected value should be half the number of quasiparticle and for the different cases it is written as

$$\langle 0|\hat{n}|0\rangle_{\mathrm{exact}}=\sum_{n\ \mathrm{even}}^{2j+1}|C_n^0|^2 n,\quad \langle 0|\hat{n}|0\rangle_{\mathrm{RPA}}=2Y^2,$$

$$\langle 0|\hat{n}|0\rangle_{\mathrm{RQRPA}}=2\mathcal{Y}^2. \tag{19}$$

Figure 2 shows the behavior of this average number for $\chi'=0$ and 0.5. Up to $\kappa'\simeq 0.7$ the three quantities are nearly indistiguishable. From there on, the QRPA overestimates this correlation, as pointed out long ago [21], and quickly collapses. The RQRPA does not collapse but it shows about twice the exact number of bosons.

This behavior will affect the Ikeda sum rule, as will be shown below.

C. β and $\beta\beta$ transition amplitudes

The Fermi $\beta^{\pm}$ operators in the quasiparticle basis are

$$\beta^{-}=\sqrt{2j+1}[u_p v_n A^{\dagger}+v_p u_n A-u_p u_n B^{\dagger}+v_p v_n B],$$

$$\beta^{+}=(\beta^{-})^{\dagger},\quad B^{\dagger}=[\alpha_p^{\dagger}\alpha_{\bar{n}}]^{J=0}. \tag{20}$$

Neglecting the scattering terms $B^{\dagger},B$, as was done to obtain Hamiltonian (7), leads to the QRPA order of approximation. In this case the amplitude of the Fermi transitions connecting the ground state $|0)$ of the initial even-even nuclei and the ground and excited states $|0_{\lambda}\rangle$ in the odd-odd nuclei are

$$\langle 0_{\lambda}|\beta^{-}|0)_{\mathrm{exact}}=\sum_{n\ \mathrm{even}}^{2j+1}\sqrt{2j+1}C_n^0 \times\left(u_p v_n C_{n+1}^{*\lambda}\left[\left(1-\frac{n}{2j+1}\right)\times(n+1)\right]^{1/2} +v_p u_n C_{n-1}^{*\lambda}\left[\left(1-\frac{n}{2j+1}\right)n\right]^{1/2}\right),$$

$$\langle 0_1|\beta^{-}|0)_{\mathrm{RPA}}=\sqrt{2j+1}(Xu_p v_n+Yv_p u_n),$$

$$\langle 0_1|\beta^{-}|0)_{\mathrm{RQRPA}}=\sqrt{2j+1}(\mathcal{X}u_p v_n+\mathcal{Y}v_p u_n). \tag{21}$$

Similar expressions hold for β^{+} interchanging the u's and v's. There are several states in the odd-odd nuclei that can be connected to the ground state of the even-even one (five in our example). Only the state with the lowest energy, labeled with $|0_1\rangle$, is described by the QRPA and the RQRPA.

In order to study the $\beta\beta_{2\nu}$ decay amplitudes $M_{2\nu}$ in this simple model we made the approximation $\langle 0_{\mathrm{final}}|\beta^{-}|0_{\lambda}\rangle\approx\langle 0_{\mathrm{initial}}|\beta^{-}|0_{\lambda}\rangle=\langle 0_{\lambda}|\beta^{+}|0\rangle$ [3,4]. In this way, we use the expression

$$M_{2\nu}=\sum_{\lambda}\frac{\langle 0_{\lambda}|\beta^{+}|0\rangle\langle 0_{\lambda}|\beta^{-}|0\rangle}{E_{\lambda}+\Delta}. \tag{22}$$

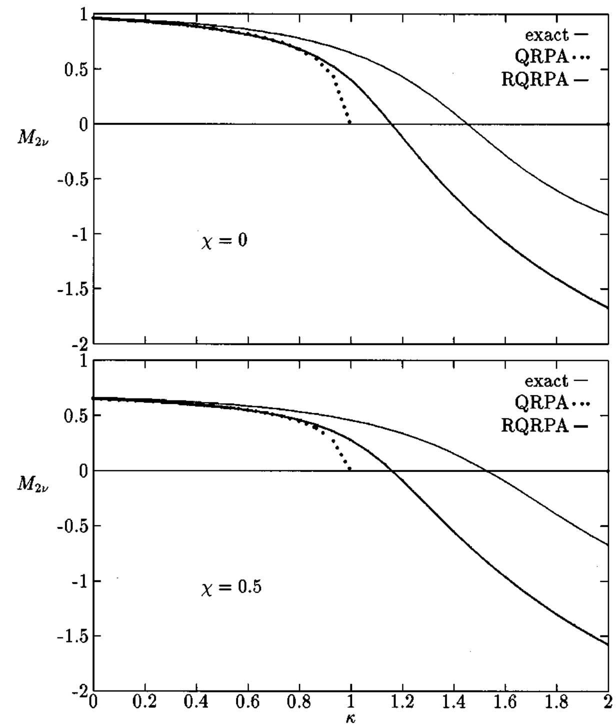

FIG. 3. $\beta\beta_{2\nu}$ transition amplitudes $M_{2\nu}$ vs κ'. Conventions are the same as in Fig. 1.

We selected $\Delta = 0.5$ MeV. The sum runs over all the odd-odd states, which are only one in the QRPA and RQRPA.

Figure 3 shows $M_{2\nu}$ as a function of κ' for $\chi' = 0$ and 0.5. Their behavior is very similar to that encountered in realistic calculations [3,5,6,13], including its cancellation near the collapse in the QRPA description. The success of the RQRPA in extending this curve far beyond the value of κ' where the QRPA collapsed is clearly seen. By the other side, compared with the exact values of $M_{2\nu}$, the RQRPA performance is poor. The overestimation of the ground-state correlation causes a premature cancellation of the $\beta\beta_{2\nu}$ transition amplitude, as compared with the exact result.

D. Ikeda Sum Rule

There is an additional point that must be mentioned. The Ikeda sum rule

$$S^- - S^+ = N - Z, \quad S^{\pm} = \sum_{\lambda} |\langle 0_\lambda | \beta^{\pm} | 0 \rangle|^2, \tag{23}$$

must be fulfilled in any model where the Hilbert space for the odd-odd nuclei includes all the states that can be connected to the ground state via the β decay.

The behavior of $S^- - S^+$ against κ' is exhibited in Fig. 4. As is well known, in the QRPA this sum rule is always fulfilled, irrespective of how large the ground-state correlations are. It is seen in the figure as a dotted straight line, interrupted where the QRPA collapses. The RQRPA strongly violates the sum rule by nearly 50% at large values of κ. The exact solution of Eq. (7) also violates the sum rule but in a smaller amount. The origin of this problem can be attributed to the adopted structure of both the Hamiltonian and the transition operators. As was mentioned above neglecting the scattering terms in the operators H, β^-, and β^+ provides a plausible explanation of this failure. At the QRPA level these terms play no role, but when ground-state correlations are explicitly taken into account they become relevant and cannot be neglected, neither in the transition operators nor in the Hamiltonian.

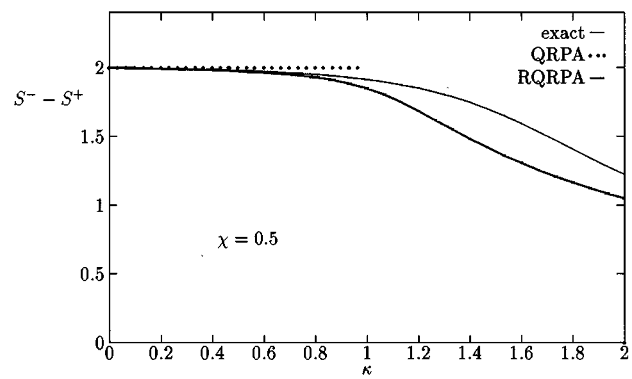

FIG. 4. Ikeda sum rule vs κ'. Conventions are the same as in Fig. 1, for $\chi' = 0.5$ only.

VI. CONCLUSIONS

We have presented a solvable model that is holomorphic to the Lipkin model and used it to study the QRPA and RQRPA approaches. The case of double β decay transitions of the Fermi type has been discussed. We have found that the excitation energies and double β decay amplitudes, for these transitions, have the same qualitative behavior found in realistic calculations. It is shown that, as expected, the RQRPA does not collapse as the QRPA description does when renormalized particle-particle correlations are included. However, the apparent stability of the RQRPA is hindered by the fact that it strongly overestimates the effect of ground-state correlations. This tendency is reflected by the premature change in the sign of $M_{2\nu}$, as compared to the exact solution, and by the violation of the Ikeda sum rule. In our opinion it implies that if one wants to include explicitly the so-called ground-state correlations in the fashion of [11] one should also exceed the two-quasiparticle order. The usually neglected one-quasiparticle terms are included in the RQRPA when the exact commutation relations for $A^\dagger$ and A are used but they should be also included in the transition operators and in the Hamiltonian. In our exact model, the inclusion of these terms implies the use of the SO(5) algebra [22,23] instead of the present SU(2) one. Details about the work in progress will be published elsewhere [24].

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