Excited halo nuclear state and long range interaction in nuclear reactions

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(Received 13 May 1996)

Based on a three-body formulation of a nuclear reaction, we show that a new type of long-range interaction can exist in the imaginary part of the effective potential for nuclear reactions if the final-state nucleus has an excited neutron-core halo state. The existence of such a long-range interaction can lead to an enhancement of the nuclear cross section. Comparison of the new long-range effective interaction with other previously known ones is given. [S0556-2813(96)00310-X]

PACS number(s): 21.45.+v, 21.30.Fe, 24.10.−i, 24.50.+g

I. INTRODUCTION

Effective potentials play a very important role in quantum scattering theory of many body systems as applied to atomic [1,2] and nuclear reactions [1]. In particular, theoretical derivations of the properties of the effective potential such as the strength and interaction range are essential in understanding the cross sections of many of electron-atom and nuclear reactions. If the interaction range is large, a large enhancement of the cross section may occur for some reactions.

There are some known examples of the long-range effective potentials such as the electric polarization potential and dynamic polarization potential. A long-range potential can arise from a Coulomb interaction as an electric polarization potential in the real part of the effective potential in the elastic-scattering channel at low energies [3–6]. When inelastic channels due to Coulomb excitation of the target nucleus to excited states become open at higher energies, a dynamic polarization potential results from the long-range Coulomb potential. For the case of a quadrupole excitation, the dynamic polarization potential has an imaginary part with an asymptotic behavior of r^{-5} at large distances [7]. For nuclear fusion reactions involving charged nuclei, rearrangement or fusion is involved in the exit channel, and hence it may be reasonable to expect that a long-range interaction may result as an imaginary part of the effective potential due to the long-range Coulomb potential involved in nuclear fusion reactions.

In this paper, based on a three-body formulation of nuclear reactions, we present our theoretical derivation of a new type of long-range interaction in the imaginary part of the effective potential when the final-state nucleus has an excited neutron-core halo state with a small binding energy.

II. THEORETICAL FORMULATION

For simplicity, we consider nuclear reactions $C(d,d)C$ and $C(d,p)A$ where the final-state nucleus A is regarded as a composite of neutron (n) and core (target) nucleus (C), ($A=nC$). To investigate the effective potential for a multichannel system, we consider the three-body system with two open channels

$$(1,2)+3\rightarrow(1,2)+3$$
$$\rightarrow 1+(2,3), \tag{1}$$

where particle labels 1, 2, and 3 refer to p, n, and C, respectively, and (1,2) and (2,3) represent d (deuteron) and $A=(nC)$, respectively. We introduce the Jacobi coordinates given by (i, j, k, cyclic)

$$\vec{r}_{ij}=\vec{r}_i-\vec{r}_j, \quad \vec{\rho}_k=\vec{r}_k-\frac{m_i\vec{r}_i+m_j\vec{r}_j}{m_i+m_j}. \tag{2}$$

For the first (elastic) channel, we define

$$\vec{r}=\vec{r}_{12}=\vec{r}_p-\vec{r}_n, \quad \vec{\rho}=-\vec{r}_c+\frac{m_p\vec{r}_p+m_n\vec{r}_n}{m_p+m_n}, \tag{3}$$

and, for the second (reaction) channel, we have

$$\vec{r}\,'=\vec{r}_{23}=\vec{\rho}-\frac{m_p}{m_p+m_n}\,\vec{r}\approx\vec{\rho}-\frac{1}{2}\,\vec{r}, \tag{4}$$

$$\vec{\rho}\,'=\vec{\rho}_1=\frac{M}{M+m_n}\,\vec{\rho}+\frac{m_n(m_p+m_n+M)}{(m_n+M)(m_p+m_n)}\,\vec{r}, \tag{5}$$

and

$$\vec{r}_{pc}=\vec{r}_{13}=\vec{\rho}+\frac{m_n}{m_p+m_n}\,\vec{r}=\vec{\rho}\,'+\frac{m_n}{M+m_n}\,\vec{r}\,', \tag{6}$$

where M is the rest mass of C.

A. The initial state

The initial state for both the first and the second channels for reactions (1) can be written as

$$\varphi_i(\vec{r},\vec{\rho})=\varphi_d(\vec{r})F_i(\vec{\rho}), \tag{7}$$

where $\varphi_d(\vec{r})$ is the deuteron wave function, and $F_i(\vec{\rho})$ describes relative motion between d and C in the presence of the Coulomb interaction,

$$\left(-\frac{\hbar^2}{2\mu_i}\,\Delta_\rho-\frac{Ze^2}{\rho}\right)F_i(\vec{\rho})=E_iF_i(\vec{\rho}). \tag{8}$$

The reduced mass μ_i is given by

$$\mu_i = \frac{(m_p + m_n)M}{m_p + m_n + M}, \tag{9}$$

and E_i is related to the total energy E by

$$E = E_i + E_d, \tag{10}$$

where E_d is the deuteron binding energy.

B. The final state

The final state $\psi_f(\vec{r}\,',\vec{\rho}\,')$ in the second channel satisfies the Schröedinger equation

$$H\psi_f = E\psi_f, \tag{11}$$

where the Hamiltonian for $\psi_f(\vec{r}\,',\vec{\rho}\,')$ is given by

$$H = H_0(\vec{\rho}\,') + h + V^c(\vec{r}_{13}) + V^s, \tag{12}$$

where V^c is the Coulomb potential, V^s is a sum of pair-wise strong-interaction potentials, $H_0(\vec{\rho}_1)$ is the kinetic-energy operator of 1 (proton) relative to $(2,3) = (nC)$, and h is the Hamiltonian of the (nC) subsystem

$$h\phi_n(\vec{r}\,') = \varepsilon_n \phi_n(\vec{r}\,'), \tag{13}$$

with $E = E_f + \varepsilon_0$. For low energies, we can expand $\psi_f(\vec{r}\,',\vec{\rho}\,')$ in terms of $\phi_n(\vec{r}\,')$ as

$$\psi_f(\vec{r}\,',\vec{\rho}\,') = \sum_{n=0} \varphi_n(\vec{r}\,') F_n(\vec{\rho}\,'), \tag{14}$$

where $\varphi_n(\vec{r})$ is the solution of Eq. (13), i.e.,

$$\left[-\frac{\hbar^2}{2\mu} \Delta_{r'} + V^s_{nC}(\vec{r}\,') \right] \varphi_n(\vec{r}\,') = \varepsilon_n \varphi_n(\vec{r}\,'), \tag{15}$$

with $\mu = m_n M/(m_n + M)$, and $F_n(\rho)$ is the solution of

$$\left[E - \varepsilon_n + \frac{\hbar^2}{2\mu_f} \Delta_{\rho'} - V^s_{nn}(\rho') - \frac{Ze^2}{\rho'} \right] F_n(\rho') - \sum_{m \neq n} V^s_{nm}(\vec{\rho}\,') F_m(\vec{\rho}\,') - \sum_{n \neq m} V^c_{nm}(\vec{\rho}\,') F_m(\vec{\rho}\,') = 0, \tag{16}$$

with

$$V^s_{nm}(\vec{\rho}\,') = \langle \varphi_n | V^s_{pn} + V^s_{pc} | \varphi_m \rangle, \tag{17}$$

and

$$V^c_{nm}(\vec{\rho}\,') = \left\langle \varphi_n \middle| V^c_{pc} - \frac{Ze^2}{\rho'} \middle| \varphi_m \right\rangle. \tag{18}$$

Z is the charge of C and μ_f is the reduced mass given by

$$\mu_f = \frac{(m_n + M)m_p}{(m_n + M) + m_p}. \tag{19}$$

To separate $n \neq 0$ contribution, we rewrite Eq. (16) for the $n = 0$ case as

$$\left[-(E - \varepsilon_0) - \frac{\hbar^2}{2\mu_f} \Delta_{\rho'} + V^s_{00}(\vec{\rho}\,') + \frac{Ze^2}{\rho'} \right] F_0(\vec{\rho}\,') + \sum_{m \neq n} V^s_{0m}(\vec{\rho}\,') F_m(\vec{\rho}\,') + \sum_{m \neq 0} V^c_{0m}(\vec{\rho}\,') F_m(\vec{\rho}\,') = 0, \tag{20}$$

and, for $n \neq 0$,

$$\left[-(E - \varepsilon_n) - \frac{\hbar^2}{2\mu_f} \Delta_{\rho'} + \frac{Ze^2}{\rho'} \right] F_n(\vec{\rho}\,') + V^s_{n0}(\vec{\rho}\,') F_0(\vec{\rho}\,') + \sum_{m \neq 0} V^s_{nm}(\vec{\rho}\,') F_m(\vec{\rho}\,') + V^c_{n0}(\vec{\rho}\,') F_0(\vec{\rho}\,') + \sum_{m \neq 0} V^c_{nm}(\vec{\rho}\,') F_m(\vec{\rho}\,') = 0. \tag{21}$$

For large ρ' the expansion of $V^c_{n0}(\rho')$ in terms of the reciprocal powers of ρ' begins with $(\rho')^{-\ell-1}$, where ℓ is the angular momentum characterizing the state n (where $\ell = 0$, V^c_{n0} falls exponentially), and hence the leading term on the left-hand side of Eq. (21) yields for large ρ'

$$\left(E - \varepsilon_n + \frac{\hbar^2}{2\mu_f} \Delta_{\rho'} \right) F_n(\rho') - V^c_{n0} F_0(\rho') = 0, \tag{22}$$

with

$$V^c_{n0}(\rho') \sim \frac{1}{\rho'^{\,\ell+1}}. \tag{23}$$

In Eq. (22), the asymptotic form of $F_0(\rho)$ is given by

$$F_0(\rho') \sim \sin(k_0\rho' - \eta \ln 2k_0\rho' + \delta^c_0 + \delta^s_0), \tag{24}$$

where $k_0 = \sqrt{2\mu_f(E - \varepsilon_0)/\hbar^2}$, $\eta = 1/2k_0 R_f$, $R_f = \hbar^2/2\mu_f Ze^2$, δ^c_0 is the Coulomb phase, and δ^s_0 is the strong-interaction phase shift. Using Eq. (24), we can obtain from Eq. (22) the following result:

$$F_n(\rho') \sim -\frac{V^c_{n0}(\rho') F_0(\rho')}{\varepsilon_n - \varepsilon_0} \tag{25}$$

for $\rho' \to \infty$ which is a well-known result (see, for example, [2]). For the case of large r' and ρ' when $E_n < \varepsilon_n$, we obtain from Eq. (14) the leading term of $\psi_f(\vec{r}\,',\vec{\rho}\,')$ as

$$\psi_f(\vec{r}\,',\vec{\rho}\,') \sim -\frac{\varphi_n(\vec{r}\,') V^c_{n0}(\vec{\rho}\,') F_0(\rho')}{\varepsilon_n - \varepsilon_0}. \tag{26}$$

When the final-state nucleus (nC) has a p-wave excited state φ_n, Eq. (26) can be written as

$$\psi_f(\vec{r}\,',\vec{\rho}\,') \sim -\gamma_n \frac{1}{(\rho')^2} \varphi_n(r') [Y_1(\hat{r}\,') \times Y_1(\hat{\rho}\,')]_{00} F_0(\rho'), \tag{27}$$

where

$$[Y_{\ell_1}(\hat{a})\times Y_{\ell_2}(\hat{b})]_{LM}=\sum_{m_1,m_2} C^{LM}_{l_1m_1,\ell_2m_2}Y_{\ell_1m_1}(\hat{a})Y_{\ell_2m_2}(\hat{b}),$$

and $Y_{\ell m}$ are the spherical harmonics and $C^{LM}_{\ell_1m_1,\ell_2m_2}$ are the Clebsh-Gordan coefficients. A detailed derivation of γ_n is given in Appendix A.

C. *T* matrix and cross section

The T matrix for the second channel $C(d,p)A$ $[A=(nC)]$ of reaction (1) can be written in two equivalent forms [1]:

$$T_{fi}=\langle\psi_f^{(-)}|W_1|\varphi_i\rangle=\langle\phi_f|W_2|\psi_i^{(+)}\rangle, \tag{28}$$

where

$$W_1=V^s_{pc}+V^s_{nc}+V^c_{pc}-\frac{Ze^2}{\rho}, \tag{29}$$

and

$$W_2=V^s_{pn}+V^s_{pc}+V^c_{pc}-\frac{Ze^2}{\rho'}. \tag{30}$$

$\psi^{(\pm)}_{i,f}$ and $\phi^{(\pm)}_{i,f}$ are related by

$$|\psi^{(\pm)}_{i,f}\rangle=\lim_{\varepsilon\to 0}\pm i\varepsilon\,\frac{1}{E-H\pm i\varepsilon}\,|\phi^{(\pm)}_{i,f}\rangle, \tag{31}$$

where H is the total Hamiltonian.

The T matrix given by Eq. (28) can be written as

$$T_{fi}=\int \psi_f^{(-)}(\vec{r},\vec{\rho})W_1(\vec{r},\vec{\rho})\varphi_d(\vec{r})F_i(\vec{\rho})d\vec{r}\;d\vec{\rho}, \tag{32}$$

which can be rewritten as

$$T_{fi}=\int g(\vec{\rho}\,')F_i(\vec{\rho}\,')d\vec{\rho}, \tag{33}$$

where

$$g(\vec{\rho})=\int \psi_f^{(-)}(\vec{r}\,',\vec{\rho}\,')W_1(\vec{r},\vec{\rho})\varphi_d(\vec{r})d\vec{r}. \tag{34}$$

Since $W_1\sim(V^c_{pc}-Ze^2/\rho)$ for large ρ, and

$$\left(V^c_{pc}-\frac{Ze^2}{\rho}\right)\sim Ze^2\,\frac{2}{3}\,\pi\,\frac{1}{\rho^2}\,r[Y_1(\hat{\rho})\times Y_1(\hat{r})]_{00}+\cdots\;, \tag{35}$$

for $\rho>r$, we obtain from Eq. (34)

$$g(\vec{\rho})\sim\lambda\widetilde{F}_0(\rho)\varphi_n(\rho)\,\frac{1}{\rho^4}, \tag{36}$$

where $\widetilde{F}_0(\rho)\sim\cos(k_0\rho-\eta\ln 2k_0\rho+\delta^c_0+\delta^s_0)$ for large ρ, and $\varphi_n(\rho)$ is the p-wave excited-state wave function [see Eq. (A5) in Appendix A]. Equation (36) shows that, if the binding energy of the p-wave bound state is small, $g(\rho)$ has a long interaction range, i.e.,

$$g(\vec{\rho})\sim\lambda\;\cos(k_0\rho-\eta\ln 2k_0\rho+\delta^c_0+\delta^s_0)\cdot\frac{1}{\rho^5}. \tag{37}$$

Since the cross section is proportional to $|T_{fi}|^2$, we have

$$\sigma\propto\left|\int F_i(\rho)g(\rho)d\rho\right|^2. \tag{38}$$

Using the optical theorem, we can write, for orbital momentum $\ell=0$,

$$\mathrm{Im}f=\frac{k}{4\pi}\,(\sigma_e+\sigma), \tag{39}$$

where f is the elastic nuclear scattering amplitude, σ_e is the elastic cross section, and σ is the reaction cross section. Using low-energy relations $\sigma_e\sim e^{-4\pi\eta}/k^2$ and $\sigma\sim e^{-2\pi\eta}/k^2$, we obtain $\mathrm{Im}f\approx(k/4\pi)\sigma$. In terms of the effective two-body t matrix, T, the elastic amplitude f can be written as

$$f\propto\langle F_i|T|F_i\rangle,$$

and hence σ as

$$\sigma\propto\langle F_i|\mathrm{Im}T|F_i\rangle. \tag{40}$$

The above formula and Eq. (38) for σ show that the imaginary part of the elastic effective two-body t matrix, $T(r,r')$, in the coordinate representation, is separable, i.e.,

$$\mathrm{Im}T(r,r')\sim g(r)g(r'). \tag{41}$$

Since $T(r,r')$ is the solution of the Lippmann-Schwinger equation, we can prove that (i) for the case of very low energies, the imaginary part of the effective two-body elastic T matrix is separable, and hence the imaginary part of the effective (optical) potential is also separable, and (ii) for the case in which we have an excited p-wave ''halo'' state as a closed channel in the final state, $g(r)$ has the asymptotic behavior, Eqs. (36) and (37) (see Appendix B for a detailed derivation of λ).

III. HALO NUCLEAR STATES

There are many known examples of one-neutron and two-neutron halo nuclei. We describe below some known examples of one-neutron (nC) and one-proton (pC) halo nuclei.

Halo nuclei [8], i.e., loosely bound systems whose wave functions will spread out to distances far away from the binding potential [9], are by now well established on the neutron drip line [10,11]. Reference [9] gives several examples of halo states in light nuclei: ^{11}Be, ^{11}Be*, ^{17}F, ^{17}F*, ^{21}Na*, and ^{25}Ne*. The excitation energy E_x (in MeV, with respect to the ground state, $E_x=0$), separation energy E_s (in keV), configuration (for E_s), partial wave ℓ of the state, and the rms (in fm, $\langle r^2\rangle^{1/2}$) for these halo nuclei are (E_x, E_s, configuration, ℓ, $\langle r\rangle^{1/2}$)$={}^{11}$Be[0(g.s.), 503, $n+{}^{10}$Be, 0, 5.90 or 6.58], ^{11}Be*(0.32, 183, $n+{}^{10}$Be, 1, 5.35), ^{17}F [0(g.s.), 600, $p+{}^{16}$O, 2, 3.9], ^{17}F*(0.5, 105, $p+{}^{16}$O, 0, 4.4 or 5.74), ^{21}Na*(2.42, 7, $p+{}^{20}$Ne, 0, 4.25 or 5.65), and ^{25}Ne*(4.07, 90, $n+{}^{24}$Ne, 2, 5.26), respectively [8]. Note that a halo nucleus may exist as an excited state. The ground-state halo nuclei such as ^{11}Be(g.s.) are of course better documented than excited halo states. One interesting question is how a halo structure depends on value of the angular momentum. Riisager *et al.* [10] investigated the behavior of the moments $\langle r^n\rangle$ of the wave function as the neutron separation energy decreases towards zero and found the moment to be finite provided $n<(2\ell-1)$. Therefore if a halo is defined as divergent second moment of r as a function of separation energy, then this occurs only for s and p states. Similar observations have been made by Sagawa [12], who notes that states with $\ell>1$ are not likely to form a halo.

At present, we cannot prove nor rule out theoretically the existence of such one-neutron halo excited states for heavier nuclei. Therefore, it is important to investigate both theoretically and experimentally possible existence of halo excited states in both light and heavy nuclei.

IV. COMPARISON WITH OTHER LONG-RANGE INTERACTIONS

The new long-range interaction, given by Eq. (36), in the imaginary part of the effective potential is similar to the dynamical polarization potential which has an imaginary part with an asymptotic behavior of r^{-5} at large distances [7], which originates from a different mechanism involving Coulomb excitation of the target nucleus.

The interaction range of the imaginary part of the effective potential in the elastic channel for nuclear fusion reactions at low energies has not been investigated extensively, although there are some well-known examples of the real part of the effective potential with a long finite interaction range. One example is the polarization potential, $V^{\rm pol}$, due to electric polarizability of the target.

The effective potential V for scattering of a charged projectile from a target with an extended charge distribution can be written as

$$V=V^S+V^C+V^{\rm pol}, \tag{42}$$

where V^C and V^S are Coulomb and strong interactions, respectively. The polarization potential in the adiabatic approximation is given for $r\gg R_B$ by

$$V^{\rm pol}(r)\approx\frac{a_e e^2}{2R_B r^4}, \tag{43}$$

where a_e is the electric polarizability of the target and R_B is the Bohr radius of the target plus the projectile, $R_B=\hbar^2/(2\mu Z_a Z_b e^2)$. If we denote the S factor, S, corresponding to the case of no polarization potential ($V^{\rm pol}=0$, and $V=V^S+V^C$) and the polarization S factor, S_p, for the case including $V^{\rm pol}(V=V^S+V^C+V^{\rm pol})$, perturbation calculations [3–6] yield $|S_p/S-1|<10^{-3}$, and show that $V^{\rm pol}$ has negligible effects for fusion reactions involved in stellar nucleosynthesis. Even though $V^{\rm pol}$ has a long range, it is the real part of the effective potential, and hence it does not contribute to the enhancement of the reaction cross section which is due to a finite-range interaction in the imaginary part of the effective potential.

Using the observed value of the deuteron polarization $\tilde{a}=a_e/R_B=0.63$ fm^3 [13], we can show that $V^{\rm pol}(R_B)\approx 10$ eV. To obtain an upper bound of $|S_p/S-1|$, we can use the following expression:

$$\left|\frac{S_p(E)}{S(E)}-1\right|<|\exp(2\pi\{\eta[E+V_{\rm pol}(R_B)]-\eta(E)\})-1|. \tag{44}$$

For the $d+d$ reaction with $E=2$ keV, Eq. (44) yields $|S_p(E)/S(E)-1|<10^{-2}$. In laboratory beam experiments, the electron screening effect needs to be taken into account using the screening energy, which is larger than $V^{\rm pol}(R_B)\approx 10$ eV. Therefore, the effect of the polarization potential may be negligible for low-energy fusion reactions. This example shows that the contribution of the real part of the nonlocal effective potential to the reaction cross section behaves drastically different from that of the imaginary part of the effective potential.

V. SUMMARY AND CONCLUSIONS

Using a rigorous derivation, we have shown that a new type of finite long-range interaction exists in the imaginary part of the effective nuclear interaction in the elastic channel (ENIEC) for a nuclear reaction if an excited halo nuclear state exists in one of the final-state nuclei. We have obtained a separable form factor for the imaginary part of ENIEC which at large distances behaves as $\cos(k_0\rho-\eta\ln 2k_0\rho+\delta_0^c+\delta_0^s)\varphi_n(\rho)/\rho^4$ [see Eq. (36)], where k_0, η, δ, and $\varphi_n(\rho)$ are the final-state wave number, the Sommerfeld parameter, the phase shift, and the wave function for the excited p-wave halo nuclear state, respectively. This new finite long-range interaction and can lead to a large enhancement of the reaction cross section at low energies.

APPENDIX A

In this appendix, we give a detailed derivation of γ_n in Eq. (27) which will be used to our estimate for λ in Eq. (36). We start with

$$V_{n0}^c(\rho')=\left\langle\varphi_n\left|V_{pc}^c-\frac{Ze^2}{\rho'}\right|\varphi_i\right\rangle$$
$$=\int\varphi_n(r')\tilde{V}(r',\rho')\varphi_0(r')dr', \tag{A1}$$

where (for p wave)

$$\widetilde{V}(\rho',r')=-\frac{Ze^2}{\sqrt{3}}\begin{cases}\frac{1}{\rho'}\left(\frac{m_n}{m_n+M}\right)\left(\frac{r'}{\rho'}\right), & \rho'>\frac{m_n}{m_n+M}r',\\ \frac{1}{r'}\left(\frac{m_n+M}{m_n}\right)^2\left(\frac{\rho'}{r'}\right), & \rho'<\frac{m_n}{m_n+M}r'.\end{cases} \tag{A2}$$

Using Eqs. (A1) and (A2), it is straightforward to show that, for $\rho'\to\infty$,

$$V^c_{n0}(\rho')\approx-\frac{Ze^2}{\sqrt{3}\rho'^2}\frac{m_n}{m_n+M}\int_0^\infty\varphi_0(r')\varphi_n(r')r'dr', \tag{A3}$$

where

$$\varphi_0(r)\approx\frac{A}{r_c^{1/2}}e^{-\kappa r},\quad r>r_c,\quad A=\sqrt{2\kappa r_c}, \tag{A4}$$

$$\varphi_n(r)\approx\left(\frac{N}{r_c}\right)^{1/2}\left(1+\frac{1}{\beta r}\right)e^{-\beta r},\quad r>r_c, \tag{A5}$$

$$\kappa=\sqrt{2\mu\frac{|\varepsilon_0|}{\hbar^2}},\quad \beta=\sqrt{2\mu\frac{|\varepsilon_n|}{\hbar^2}},$$

and r_c is the nuclear interaction radius. If β is smaller than κ ($\beta<\kappa$) and $N\approx\beta^2r_c^2$, one can write to a very good approximation

$$\int_0^\infty\varphi_0(r')\varphi_n(r')r'dr'\approx\frac{A}{\kappa}. \tag{A6}$$

Combining Eqs. (A6) and (A3) with Eq. (27), we obtain

$$\gamma_n\approx-\frac{Ze^2}{\sqrt{3}}\frac{m_n}{m_n+M}\left(\frac{A}{\kappa}\right)\frac{1}{\varepsilon_n-\varepsilon_0}. \tag{A7}$$

APPENDIX B

In this appendix, we give an estimate of λ in Eq. (36). From Eqs. (27) and (34) we have for large ρ

$$g(\rho)=\int\psi_f^{(-)}(\vec{r}',\vec{\rho}')W_1(\vec{r},\vec{\rho})\varphi_d(r)d\vec{r}\,d\hat{\rho}$$
$$\approx-\frac{2}{3}\pi\frac{Ze^2}{\rho^2}\gamma_n\int\frac{F_0(\rho')}{\rho'^2}\varphi_n(r')[Y_1(\hat{r}')\times Y_1(\hat{\rho}')]_{00}$$
$$\times[Y_1(\hat{\rho})\times Y_1(\hat{r})]_{00}\varphi_d(r)r^3dr\,d\hat{\rho}\,d\hat{r}, \tag{B1}$$

where

$$\varphi_d(r)\approx Ne^{-\kappa_d r}, \tag{B2}$$

with

$$N^2\approx\frac{\kappa_d^3}{\pi},\quad \kappa_d=\sqrt{\frac{2\mu_{pn}E_d}{\hbar^2}}. \tag{B3}$$

Using the expansion of $F_0(\rho')$ valid for $\rho>1/2r$,

$$F_0(\rho')\approx F_0\left(\left|\vec{\rho}+\frac{1}{2}\vec{r}\right|\right)$$
$$\approx F_0(\rho)+\frac{2\pi}{\sqrt{3}}k_0r\widetilde{F}_0(\rho)[Y_1(\hat{r})Y_1(\hat{\rho})]_{00}+\cdots, \tag{B4}$$

we obtain for $m_n/M<1$ and $\rho\to\infty$

$$g(\rho)\approx\lambda\widetilde{F}_0(\rho)\varphi_n(\rho)\cdot\frac{1}{\rho^4}, \tag{B5}$$

where

$$\lambda\approx-\frac{8Ze^2\gamma_nk_0\sqrt{\kappa_d\pi}}{\kappa_d^4}. \tag{B6}$$

If β is extremely small, we can rewrite Eq. (B5) as

$$g(\rho)\approx\lambda\sqrt{r_c}\widetilde{F}_0(\rho)\cdot\frac{1}{\rho^5}, \tag{B7}$$

where $F_0(\rho)$ in Eqs. (B4), (B5), and (B7) has the asymptotic form (for large ρ) given by

$$\widetilde{F}_0(\rho)\approx\cos(k_0\rho-\eta\ln2k_0\rho+\delta_0^c+\delta_0^s). \tag{B8}$$

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