Nuclear fission beyond two-body kinematics

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(Received 17 January 1996)

Disintegration of a heavy nucleus into three charged fragments with the one fragment at rest is possible if a certain relation between the masses and the charges of the moving fragments is fulfilled. A two-arm spectrometer of the angle-velocity-energy correlations has been used at the 1 GeV proton beam to detect such a process. Disintegrations with all three moving fragments have also been observed. The possibility of the identification of the collinear three-body splitting of heavy nuclei at low energies is discussed. [S0556-2813(96)04308-7]

PACS number(s): 24.75.+i, 25.85.Ca, 25.85.Ge

I. INTRODUCTION

Based on the liquid-drop model of the compound nucleus, Meitner and Frisch [1] and independently Bohr [2] proposed to treat nuclear fission as a classic process, which leads to the formation of two massive fragments. Earlier Noddack [3] mentioned the possibility of the formation of several fragments from heavy nuclei bombarded by neutrons. The detection of two massive fragments which separate collinearly seems to confirm unambiguously the two-body character of the nuclear fission reaction, however it is clear that this experimental result is a necessary, but not a sufficient sign of the two-body kinematics. Consistent with the experimental observation of the two collinear fragments one could have considered a more general case, when the classical liquid drop were fissioning into three fragments, the third fragment remaining at rest.

The nuclear fission reaction should be written then as

$$^{M_0}Z_0 \rightarrow {}^{M_1}Z_1 + {}^{M_3}Z_3 + {}^{M_2}Z_2, \quad (1)$$

where M_k and Z_k are the mass numbers and electric charges of the initial fissioning nucleus and of three fragments. The conservation of the mass and the charge leads to

$$M_0 = \sum_{i=1}^{3} M_i \quad \text{and} \quad Z_0 = \sum_{i=1}^{3} Z_i. \quad (2)$$

The fragment $^{M_3}Z_3$, being located between the two others, will not move, if the two repulsive Coulomb forces are equal:

$$\frac{e^2 Z_1 Z_3}{x_{13}^2} = \frac{e^2 Z_2 Z_3}{x_{23}^2}, \quad (3)$$

where x_{13} and x_{23} are the distances between the centers of the fragment charges. As follows from Eq. (3),

$$x_{13} = \sqrt{Z_1/Z_2}\, x_{23}, \quad (4)$$

and this relation should hold for all the time during the fragments separation. This means that the fragment with a larger charge should always be farther from the third one, which does not move. Differentiating the identity (4) in time, one derives the relation for the velocities:

$$\dot{x}_{13} = \sqrt{Z_1/Z_2}\, \dot{x}_{23}. \quad (5)$$

Making use of the momentum balance of the moving fragments as a condition for the third one to be at rest, one obtains from

$$M_1 \dot{x}_{13} = M_2 \dot{x}_{23} \quad (6)$$

a relation

$$M_1/M_2 = \sqrt{Z_2/Z_1}, \quad (7)$$

which requires that the fragment with higher mass has a smaller charge, and vice versa. This corresponds to a charge polarization of the fissioning nucleus and should lead to the appearance of the neutron-deficient fragments, together with their neutron-rich partners. Neutron-deficient fragments have never yet been observed experimentally in the fission processes at low energies. They were observed however in the process of the ^{238}U fission induced by 1 GeV protons [4]. When going from low-energy nuclear fission to the disintegration of heavy nuclei by relativistic protons one should keep in mind that there are some other differences between these processes. Very important features of the disintegrations induced by relativistic protons are (a) the presence of charged particles, accompanying the fragments and (b) the deviation of the folding angle between the two fragments from 180°. Both these facts are well established in experiments on the disintegration of ^{238}U nuclei loaded into nuclear emulsion [5]. Figure 1 shows the experimental dependence of the average folding angle between two massive fragments on the number n of accompanying charged particles for the ^{238}U disintegration induced by 1 GeV protons. We intentionally do not show the corresponding angle for events without accompanying particles, because the event vertex is poorly determined in this case. Nevertheless, at any available multiplicity of charged particles one can find the events with two massive fragments, separating collinearly in nuclear emulsion. So the experiments with 1 GeV protons have shown that there are reasons for studying the collinear three-body disintegration of the ^{238}U nuclei.

It is clear, however, that the formation of the three fragments of comparable masses in the heavy nucleus disintegration is not necessarily accompanied by collinear separation of the two of them. Assuming that all three fragments should

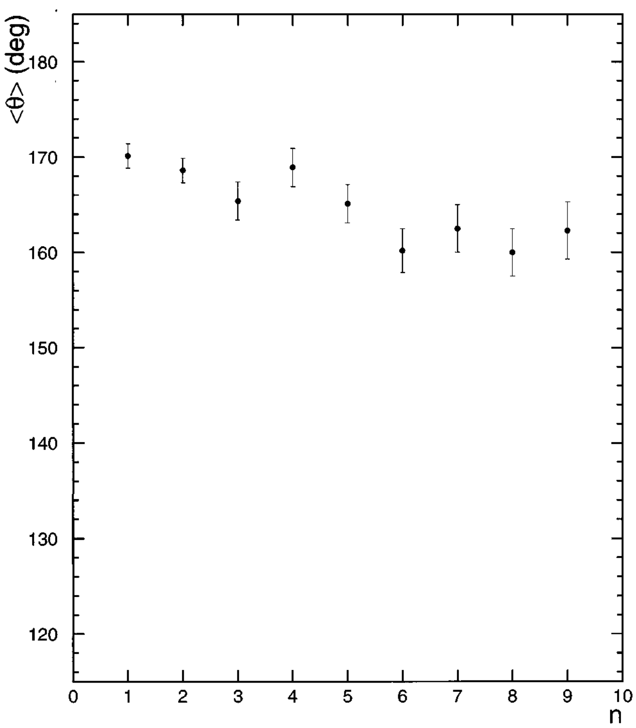

FIG. 1. Experimental dependence of the average folding angle $\langle\theta\rangle$ between the two massive fragments on the number n of accompanying charged particles for ^{238}U disintegration induced by 1 GeV protons. The average folding angle for coplanar ternary fission induced by 1 GeV protons was found to be $119° \pm 1°$.

have similar properties, experimentally one usually tries to detect all of them. In the experiment [6] on the ^{238}U disintegration induced by 1 GeV protons the detection of three-prong events was performed using the backing-free 200 μm layers of nuclear emulsion. In order to improve the efficiency of the detection of three-prong events related to the fragments of comparable masses, the sensitivity threshold of nuclear emulsion was increased up to the values of the ionization losses for the charge $Z=10$. This allowed one to increase considerably the total proton flux at sufficient transparency of the nuclear emulsion layers. For 3.43×10^5 two-prong events 133 three-prong ones were detected, all three tracks belonging to the fragments with comparable masses. After correcting for the scanning efficiency one could find the ratio of the probabilities for ^{238}U fission into three and two massive fragments to be equal to $(4.7\pm0.5)\times10^{-4}$. The total probability for three-prong events is 20 times lower than that for events with two massive fragments and high multiplicity of accompanying charged particles. Among 399 angles between the two of the three tracks it was not found any exceeding the value $\theta=165°$. These quantitative experimental estimates related to the coplanar three-body disintegrations allowed us to choose the geometry for the study of the collinear three-body splitting with the help of the spectrometer of angle-velocity-energy correlations (SAVEC's).

II. KINEMATIC ANALYSIS OF THE TWO MASSIVE FRAGMENTS FROM ^{238}U FISSION INDUCED BY 1-GeV PROTONS WITH THE SAVEC DEVICE

The two-arm time-of-flight SAVEC device comprises a vacuum chamber with two time-of-flight tubes. At the end of each tube at the distance 70 cm a mosaic of eight semiconductor surface-barrier silicon detectors was located. Thin (200 μg/cm^2) target made of ^{238}U or a spontaneously fissioning ^{252}Cf source could be placed in the middle of the chamber. The ^{252}Cf source was used for energy and time calibrations. One of the arms was fixed, the other one could be turned in the horizontal plane that contained a proton beam. An independent time-start setup was located at the fixed arm close to the target for time-of-flight measurements. An experiment was performed at the proton beam of the Gatchina synchrocyclotron with an energy of 1 GeV and an intensity of $(2-5)\times10^{11}$ protons per sec. Two runs, ''collinear'' and ''noncollinear,'' were carried out. In the collinear configuration two massive complementary fragments were detected within narrow cones whose common axis was orthogonal to the proton beam direction. In the noncollinear configuration the movable arm was turned 10° downstream. In both runs kinetic energies E_i and times of flight T_i of the massive complementary fragments were measured simultaneously which allowed us to determine their masses $M_i \sim E_i T_i^2$ and momenta $P_i \sim E_i T_i$. The latter together with appropriate angular resolution made it possible to study event-by-event the vector sum

$$\mathbf{P}=\mathbf{P_1}+\mathbf{P_2}, \tag{8}$$

where $\mathbf{P_1}$ and $\mathbf{P_2}$ are the detected fragment momenta. Momentum distribution was studied for different values of the nucleon losses, defined as the difference between the mass of the target nucleus M_0 and the sum of the measured fragments masses

$$\Delta M = M_0-(M_1+M_2). \tag{9}$$

At first the events with $\Delta M \geqslant 75u$ were considered, which might correspond to a greater extent to the disintegration of the nucleus into three fragments of comparable masses. For these events we analyzed the distributions of the projections of the measured momentum $\mathbf{P}$ to the axis P_a and to the plane orthogonal to the axis, P_p:[1]

$$P^2=P_a^2+P_p^2. \tag{10}$$

The axis for calculating P_a was by the definition orthogonal to the beam direction, crossed the target and was determined by the two central detectors in the mosaics.

According to the modern comprehension of the interaction mechanism of relativistic protons with heavy nuclei, the formation of the massive fragments takes place as a result of the fission of the residual nucleus after the cascade-evaporation stage of the reaction. This means that all randomly emitted particles are responsible for the recoil momentum $\mathbf{P}$ of the fissioning nucleus. In the chosen experimental geometry the axial component of this vector P_a corresponds to the imbalance of the momenta of the fragments, while the planar component P_p results in their noncollinearity. Both effects can be investigated in the condi-

[1]The notions ''axis'' and ''plane'' we use here, though very unusual for the beam experiment, are quite suitable for the description of the collinear separation of the fission fragments.

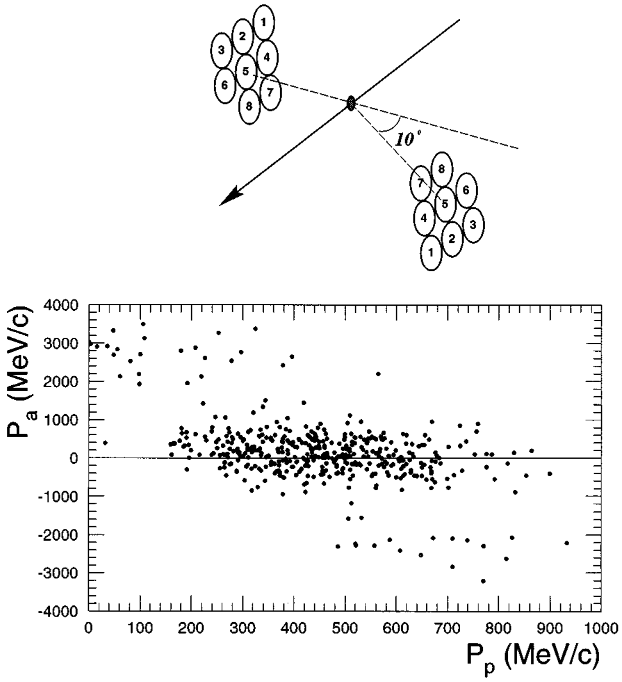

FIG. 2. Correlation plot (P_p, P_a) for events with $\Delta M \geqslant 75u$ for the noncollinear experiment. The scheme of the experiment is shown in the upper part of the picture.

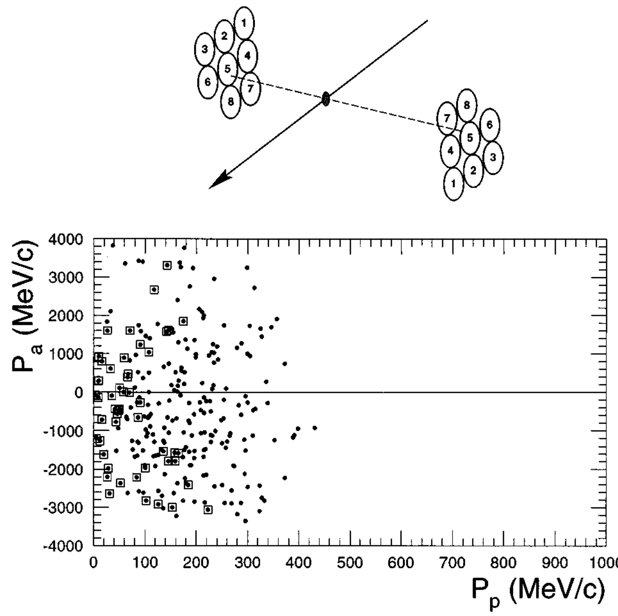

FIG. 3. Correlation plot (P_p, P_a) for events with $\Delta M \geqslant 75u$ for the collinear experiment. The scheme of the experiment is shown in the upper part of the picture. Squares mark the events in which the complementary fragments hit the detectors with identical numbers.

tions of the experiments performed taking into account the kinematic characteristics of the detected fragments as well as of the accompanying particles, which form here the nucleon losses or the missing mass. The folding angle range amounted to $173° \leqslant \theta \leqslant 180°$ for the collinear experiment and to $165° \leqslant \theta \leqslant 175°$ for the noncollinear one, so both experiments together covered the folding angle range $165° \leqslant \theta \leqslant 180°$.

One of the advantages of SAVEC is that it can detect not only the random momenta resulted from the cascade-evaporation stage but also the large momentum of the third massive fragment in case of its formation. This can be seen in Fig. 2, where the correlation plot (P_a, P_p) for the noncollinear experiment is shown. The folding angle between the fragments did not exceed 175° in this configuration. The majority of the events shown in Fig. 2 satisfies the hypothesis of the random recoil momenta with relatively small axial projections P_a, though some events have P_a as large as 2000 MeV/c and even higher, testifying to the presence of a third participant of the splitting. Three-body kinematics becomes more evident if one compares these data with those of the collinear experiment, shown in Fig. 3. Here the folding angle between the two massive fragments was not smaller than 173°. The two-dimensional distributions are not normalized, so the statistics of Fig. 2 corresponds to the five times larger statistics than that shown in Fig. 3. Comparing Figs. 2 and 3 one should come to the conclusion that the collinear experiment does not see at all the events with the random recoil momenta, which form the majority of the events in the noncollinear experiment. This means that the character of the process of heavy nucleus splitting registered in the collinear experiment differs drastically from what is observed in the noncollinear one.

The schemes of both experiments are shown in the upper parts of Figs. 2 and 3. The detectors in each mosaic were numbered so that the pair with identical numbers in the collinear experiment corresponded to maximum collinearity, i.e., minimal deviation from 180°. In the calibration experiment with the ^{252}Cf source 64% of all events of spontaneous fission were registered by detectors with identical numbers. Squares in Fig. 3 denote events in which the fragments hit the detectors with identical numbers. The total amount of these events reaches 20% of all events detected by the whole mosaics. This is important information for the investigation of the mechanism of the complicated many-body reaction of nuclear disintegration.

III. COLLINEAR TRIPARTITION OF ^{238}U NUCLEI INDUCED BY RELATIVISTIC PROTONS

The experiments show that the observed deviation of the detected massive fragments from the two-body kinematics of ^{238}U fission induced by 1 GeV protons is caused by the missing mass motion. In the noncollinear experiment this deviation is not large and for the majority of events can be explained by the recoil momentum of the accompanying particles. However, some of the events have remarkably large recoil momenta and the number of such events increases when going to the collinear experiment geometry. For this reason one should analyze quantitatively the kinematics of the separation of the two massive fragments for all the events of the collinear experiment. The most important thing here is the change of the kinematics with the increasing missing mass ΔM.

Analyzing the sum of the large number $N \gg 1$ of random vectors and its projections to a fixed plane and to the axis orthogonal to that plane, one can obtain for these quantities Maxwellian, Rayleigh's, and Gaussian distributions, respectively. All these distributions are determined by a single dis-

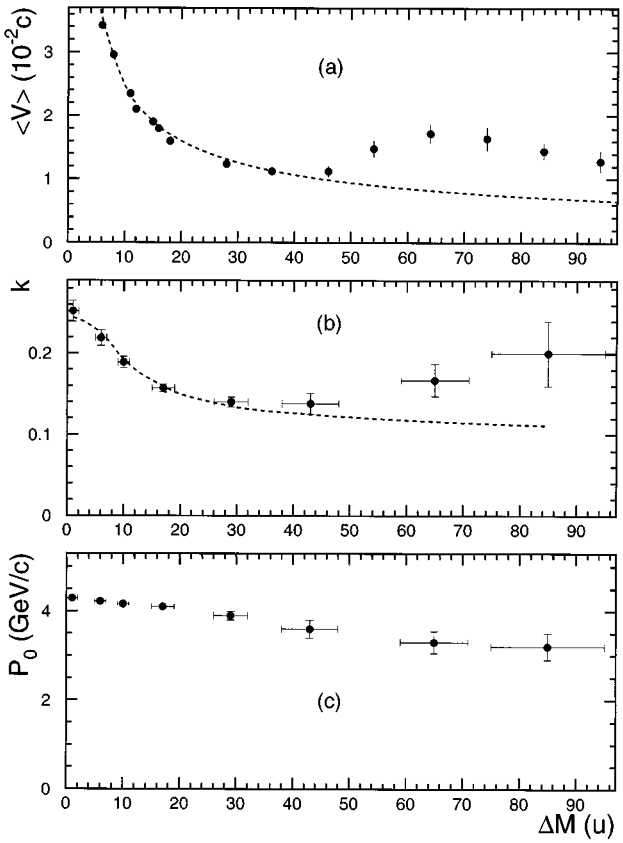

FIG. 4. Missing mass dependence in the collinear experiment for (a) average axial component of the missing mass velocity $\langle V\rangle$; dashed line, random vector summation with $\sigma_{\rm in}=208$ MeV/c, $q=126$ MeV/c; (b) collinearity coefficient k; dashed line, random vector summation with $\sigma_{\rm in}=208$ MeV/c, $q=126$ MeV/c, $\theta_2=177.8°$; (c) average momentum P_0 of the detected fragments.

persion parameter σ^2. Taking into account the finite instrumental resolution, one can write

$$\sigma^2=\sigma_{\rm in}^2+Nq^2/3, \tag{11}$$

where $\sigma_{\rm in}^2$ is the instrumental dispersion, q^2 is the mean square of the N random momenta to be summed, $N=\Delta M/m+1$ and m is a nucleon mass. Figure 4(a) shows the ΔM dependence of the average axial component of the missing mass velocity $\langle V\rangle$, observed in the collinear experiment for 22 000 events of the ^{238}U splitting by 1 GeV protons. Approximating this dependence by

$$\langle V\rangle=\sqrt{2/\pi}(\sigma_{\rm in}^2+Nq^2/3)^{1/2}/(\Delta M+m), \tag{12}$$

we obtained for the events with nucleon losses $\Delta M\leqslant 45\ u$

$$\sigma_{\rm in}=208\pm 11\ \text{MeV}/c \quad \text{and} \quad q=126\pm 3\ \text{MeV}/c.$$

Trying to describe with the same formula (12) the data for $\Delta M>75u$ one obtains the value

$$q=286\pm 16\ \text{MeV}/c.$$

Assuming such a large value of q, one obtains the unreasonably large total kinetic energy of the randomly emitted nucleons which form the missing mass. This circumstance led us to the conclusion that here we observe the collective motion of the missing mass as a whole. It was the first important result of the study of three-body ^{238}U splitting by 1 GeV protons in the collinear experiment. The obtained threshold value $\Delta M=(45\pm 5)u$ is related with the SAVEC sensitivity.

Turning to the investigation of the folding-angle distributions of the two massive detected fragments, one can see that these distributions yield important information about the projection of the missing mass momentum to the plane, perpendicular to the separation axis. As it was mentioned, fragments hitting the detectors with identical numbers in the collinear experiment had the minimal deviation from collinearity, i.e., the folding angles closest to 180°. Introducing the collinearity coefficient k as the fraction of events registered by the detectors with identical numbers, one has

$$k=Y(\theta_2)/Y(\theta_1). \tag{13}$$

The analytic expression for the collinearity coefficient k was obtained using the normalized distribution in the folding angle between the massive detected fragments in the case of the random momenta of nucleons forming the missing mass:

$$f(\theta)=\begin{cases} S\sin\theta\exp[-S(1+\cos\theta)], & 0<\theta\leqslant 180° \\ \exp(-2S), & \theta=0. \end{cases} \tag{14}$$

This folding-angle distribution is the corollary to the Rayleigh's recoil momentum distribution. The dimensionless parameter S is determined by the average momentum of the fragments detected P_0 and the dispersion σ^2, Eq. (11):

$$S=P_0^2/(\sigma_{\rm in}^2+Nq^2/3). \tag{15}$$

Integrating Eq. (14) from θ_1 or θ_2 to 180°, one obtains for the collinearity coefficient

$$k=\{1-\exp[-S(1+\cos\theta_2)]\}/\{1-\exp[-S(1+\cos\theta_1)]\}. \tag{16}$$

In the calibration experiment with a ^{252}Cf source the value P_0 was found to be 4575 ± 50 MeV/c and $k=0.64\pm 0.02$ for the angles $\theta_1=173°$ and $\theta_2=178.8°$, which were characteristic for the SAVEC for small size of the source of the fragments. These data allowed us to determine the parameter $S_{\rm Cf}=4660\pm 250$ and the momentum dispersion

$$\sigma_{\rm Cf}=67\pm 2\ \text{MeV}/c.$$

In the collinear experiment on the ^{238}U splitting by 1 GeV protons the dimensions of the beam spot on the target were larger than those of the ^{252}Cf source, so the angular parameter θ_2 of the SAVEC was changed, making the angular resolution worse.

Figure 4(b) shows the experimental missing mass dependence of the collinearity coefficient k. As seen from Fig. 4(b), the experimental dependence $k(\Delta M)$ reveals nonmonotonous behavior, similar to that we saw in Fig. 4(a) for the average axial velocity $\langle V\rangle$, the peculiarity occurring approximately at the same ΔM in both cases. Such a dependence of the collinearity coefficient on the missing mass can be considered as an additional experimental result, indicating the change in the mechanism of the ^{238}U splitting process when going from small to high nucleon losses. This second result relates to the motion of the missing mass in the plane, perpendicular to the separation axis of the two massive detected fragments. In order to obtain quantitative characteristics of the splitting process the first five points of the experimental $k(\Delta M)$ function were used to estimate from Eq. (16) the angular parameter θ_2 in the collinear run with reduced resolution, fixing the values $\sigma_{\mathrm{in}}=208\pm11$ MeV/c and $q=126\pm3$ MeV/c. We found

$$\theta_2=177.8°\pm0.1°,$$

the χ^2 value being 0.3 per degree of freedom. This θ_2 value as well as the values $k(\Delta M\geqslant75u)=0.20\pm0.04$ and $P_0=3200\pm300$ MeV/c [see Fig. 4(c)] were used to obtain the rms random momentum

$$q=20\pm25\ \text{MeV}/c.$$

Thus it turned out that for large nucleon losses the transverse motion of the missing mass is practically absent, since the experimental q value is compatible with zero.

All the above-mentioned arguments can be summarized as follows. We have tried to consider the imbalance of the fragment momenta and their noncollinearity as a result of the random recoil momentum of the accompanying particles. We found that for small nucleon losses the experimental data corroborate this point of view. However, for large nucleon losses ($\Delta M\geqslant45u$) the recoil momentum in question becomes rather peculiar. Its axial component becomes much larger than the expected value, and for large ΔM values it requires a too high $q\sim300$ MeV/c to explain the experimental data. At the same time the planar component of the recoil momentum at large ΔM decreases and becomes compatible with zero. One can explain both these peculiarities suggesting that we deal here with the collinear tripartition of heavy nuclei, induced by relativistic protons.

One should bear in mind that it is necessary to compensate the momentum of the beam particle to provide the collinear motion of two massive fragments. This compensation can be performed by emitting several cascade particles, including the incident one, to the forward hemisphere. If ν cascade particles participate in such process, the nucleus might get excited, the maximum possible excitation energy being determined by the formula

$$E_{\mathrm{ex}}=m\nu+T-\sqrt{(m\nu)^2+2mT+T^2}, \tag{17}$$

where T is the kinetic energy of the incident proton. If two cascade particles are emitted, the excitation energy can reach the value 344 MeV for a 1 GeV incident proton. Such energy, being concentrated inside the third fragment, may be high enough to cause its disintegration which is very much alike the multifragmentation process observed in heavy ion reactions [7]. Thus it turns out that both fission and multifragmentation coexist in the reaction of the ^{238}U disintegration induced by 1 GeV protons [8]. In such a reaction only one of the three fragments undergoes multifragmentation, namely that which is located between the two others. The excitation energy which is necessary for multifragmentation is transferred to the disintegrating fragment as a result of the interaction of the relativistic proton with the target nucleus. If the excitation energy is not high enough the fragment remains stable. Anyway, stable or unstable, the excited fragment manifests itself as a single object which moves with small kinetic energy and does not disturb the collinear separation of the two other fragments and their momentum balance. There can be numerous decay modes of the unstable fragment, however it is hardly probable that several intermediate mass fragments (IMF's) be formed as a result of its decay. What is more probable is the formation of a single IMF accompanied by several single- or double-charged particles. This fact is corroborated by experiments with nuclear emulsion [9]. It is almost impossible to observe the stable third fragment with the small kinetic energy within the most of the conventional experimental schemes. It can be revealed however by analyzing the kinematics of the missing mass motion.

IV. TENTATIVE DESCRIPTION OF THE COLLINEAR TRIPARTITION PROCESS

The collinear tripartition is a disintegration of a heavy nucleus into two fission fragments together with an extremely neutron-rich neck in between. The third inner fragment is formed by the double rupture of the neck. The axial momentum imbalance of the two massive detected fragments arises due to the momentum of the third inner fragment. Despite the large value of this momentum, the kinetic energy of the third fragment turns out to be lower as a rule than the energies of the detected fragments, due to its large mass. In several cases this energy was found to be zero. This means that the third fragment is at rest, the two others being detected in the experiment as moving. These splitting events look like usual binary fission of a heavy nucleus into two detected fragments. In order to have a tentative description of the collinear tripartition process it is necessary to determine the electric charges of all three fragments. In the previous paper [10] an attempt was made to obtain the charge of the third fragment Z_3 from the experimental data on the nucleon composition of the missing mass in nuclear reactions with the formation of the ^{149}Tb nuclide. We came then to the conclusion that the third fragment should have a neutron excess. One may use, however, quite general considerations about the instability of heavy nuclei for this purpose. It is well known that the $^{238}_{92}$U nucleus undergoing successive α and β decays is transformed into $^{206}_{82}$Pb. The multistep process of the ^{238}U decay finally results in the loss of the nucleon mass equivalent to the nucleus $^{32}_{10}$Ne, which has the ratio $Z/M=10/32=0.3125$. This value was chosen as a hypothetical one for the missing mass in the events of ^{238}U splitting by 1 GeV protons. The missing mass value is measured, so the ratio Z/M chosen allows one to obtain the charge of the third fragment Z_3, the charges Z_1 and Z_2 being known according to Eq. (7). This procedure allows us to describe the observed events of the collinear tripartition using chemical symbols. The results of the analysis are given in Table I for four events (selected as examples) in which the third fragment is at rest. A small uncertain value ν denotes several cascade nucleons, compensating the momentum of the incident proton.

One can check the proposed mechanism of the nuclear reaction, comparing the neutron excess in the third neutron-rich fragment with the number of neutrons registered by the

TABLE I. Selected events of the collinear tripartition of the ^{238}U nucleus induced by 1 GeV protons.

E_1+E_2, MeV	Tentative nuclear rection	D_3, fm
186±4	^{78}Rb + $^{75-\nu}$Cr + ^{86}Ga + ν $Z_1=37$ $Z_3=24$ $Z_2=31$	34.2±0.7
145±3	^{67}As + $^{98-\nu}$Ga + ^{74}Ni + ν $Z_1=33$ $Z_3=31$ $Z_2=28$	46.9±1.0
156±3	^{65}Ge + $^{104-\nu}$As + ^{70}Co + ν $Z_1=32$ $Z_3=33$ $Z_2=27$	47.5±0.9
137±3	^{63}Co + $^{115-\nu}$Kr + ^{61}Cu + ν $Z_1=27$ $Z_3=36$ $Z_2=29$	50.6±1.0

neutron spectrometer ORION in the experiments on the ^{238}U splitting induced by protons and ^{3}He nuclei [11]. The probability of the observation of the emission of a large number of neutrons turns out to be in reasonable agreement with the probability of the formation of the third unstable fragment as a source of neutrons and charged particles. For the events given in Table I, the maximum number of extra neutrons can be easily obtained by comparison of the third fragment with the nearest stable isotope:

$$^{75-\nu}\,\mathrm{Cr}+\nu\rightarrow{}^{54}\,\mathrm{Cr}+p+20n,$$

$$^{98-\nu}\,\mathrm{Ga}+\nu\rightarrow{}^{71}\,\mathrm{Ga}+p+26n,$$

$$^{104-\nu}\,\mathrm{As}+\nu\rightarrow{}^{75}\,\mathrm{As}+p+28n,$$

$$^{115-\nu}\,\mathrm{Kr}+\nu\rightarrow{}^{86}\,\mathrm{Kr}+p+28n.$$

In order to calculate the distance between the outer fragments in the last column of Table I one should obtain the Coulomb energy of the three collinearly located charges. Using Eq. (4) one obtains

$$U_3=e^2Z_1Z_3/x_{13}+e^2Z_2Z_3/x_{23}+e^2Z_1Z_2/(x_{13}+x_{23}). \tag{18}$$

Expressing all the denominators via

$$D_3=x_{13}+x_{23}, \tag{19}$$

one finally has

$$U_3=e^2[Z_1Z_2+Z_3(\sqrt{Z_1}+\sqrt{Z_2})^2]/D_3. \tag{20}$$

Equating this expression to the measured total kinetic energy of the two fragments one can obtain the D_3 value.

The obtained D_3 values for the three-fragment collinear disintegration of the ^{238}U nucleus, being dispersed considerably, exceed the double of the interfragment distance for the same nucleus, undergoing the binary fission at lower excitation energies [12]. Since the three-body collinear disintegration includes the binary fission as a particular case, it would be very important to observe this process at a low energy. One should expect that the three-fragment collinear disintegration might be interpreted in some experiments as a binary fission. The main purpose of the analysis to follow is to consider these experiments from the standpoint of the three-body kinematics.

V. COLLINEAR TRIPARTITION OF HEAVY NUCLEI AT LOW ENERGY

According to Eq. (7) the only possibility to observe the process of the collinear three-body fission without the nucleon polarization of the fissioning nucleus is determined by the two conditions:

$$M_1=M_2 \quad \text{and} \quad Z_1=Z_2.$$

This is a strictly symmetric fission, when the masses and charges of the two fragments are equal. Denoting the initial distance between the outer charges as D_{3S} one obtains for the Coulomb energy of the three charged bodies

$$U_{3S}=\frac{e^2}{D_{3S}}[(Z_0-Z_3)^2/4+2Z_3(Z_0-Z_3)]$$
$$=\frac{e^2}{D_{3S}}[Z_0^2/4+(3/2)Z_0Z_3-(7/4)Z_3^2].$$

It can be measured experimentally as an average total kinetic energy (TKE) of the two detected fragments:

$$\langle\mathrm{TKE}\rangle=\frac{e^2}{D_{3S}}[Z_0^2/4+(3/2)Z_0Z_3-(7/4)Z_3^2]. \tag{21}$$

From Eq. (21) it is clear that $\langle\mathrm{TKE}\rangle$ of the two symmetric fragments in the process of the collinear tripartition is a function of the third charge Z_3. It is easy to show that for the binary fission, when $Z_3=0$, $\langle\mathrm{TKE}\rangle$ is always smaller than that for the collinear tripartition if

$$Z_3<\frac{6}{7}Z_0. \tag{22}$$

The maximum $\langle\mathrm{TKE}\rangle$ is reached at

$$Z_3=\frac{3}{7}Z_0.$$

Thus in the low-energy experiment strictly symmetric fragments with the enhanced average TKE could have been the sign of the collinear tripartition. It is known that both these features are observed in the spontaneous fission of the isotope $^{258}_{100}$Fm [13], as well as of some other heavier nuclei [14]. It should be noted that the symmetric mass distribution of the fragments observed in the experiment was obtained from the fact of the equality of measured kinetic energies

$$E_1=E_2.$$

It is clear however that this equality does not allow one to determine the fragments masses unambiguously. This just means that

$$M_1=M_2.$$

Usually, while assuming the two-body kinematics, one assumes additionally the formation of the two symmetric fragments such as $_{50}$Sn with mass numbers close to the double-magic value $A=132=50+82$. In this case $Z_3=0$. However there is no experimental ground for such a choice. Anyway experimental data do not contradict any other choice of $Z_3 \neq 0$ within the range (22).

The important experimental result is also the partition of the TKE spectrum into two components which have different average values. In particular, for the spontaneous $^{258}_{100}$Fm fission these values are equal to 232 and 205 MeV, respectively. The most conventional explanation for such an experimental result is at present the assumption about compact and elongated forms of the fissioning nucleus, splitting into the same fragments. In this picture the compact form is responsible for the large $\langle$TKE$\rangle$ values, while the elongated one is for the smaller. In the collinear tripartition the $\langle$TKE$\rangle$ value depends on Z_3. For this reason the two observed $\langle$TKE$\rangle$ values might correspond to the two different Z_3 values and, therefore, to the two different symmetric fragments registered. The initial deformations D_{3S} in these two cases may be different enough so that the process with the larger $\langle$TKE$\rangle$ may have the larger deformation. One cannot exclude, however, that the deformations are equal in both cases. Then to describe self-consistently the experimental data [13,14] one needs two kinds of the decay process with $Z_3 = 0$ and $Z_3 = 2$ or 3.

VI. CONCLUSIONS

The picture of the three-body splitting considered here is very simplified. It would be more correct to consider the slowly moving third fragment with the kinetic energy much smaller than that of its partners. However the main condition contained in Eq. (7) remains valid as well as the conclusion that the two-body kinematics in the spontaneous fission of the heaviest of the investigated nuclei is not a well-established experimental fact. The possibility of the collinear three-body decay with the third slowly moving fragment should be checked in the experiment by measuring the nucleon composition of the fragments formed.

It is just the right time to quote the statement of Wilkins *et al.* [15] who measured the masses and kinetic energies of the coincident heavy fragments emitted from the ^{238}U target bombarded with 11.5 GeV protons: ‘‘The origin of abundant low-mass symmetric fragments with equal and opposite laboratory momenta is difficult to explain using the conventional picture of the intranuclear cascade followed by an evaporation-fission competition. A large fraction of the target mass is missing, but does not cause a momentum imbalance, thus eliminating a ternary fission mechanism.’’ Contrary to the last statement we think that in this experiment the collinear tripartition processes in the ^{238}U nuclei were detected.

It is important that the collinear tripartition is a more general case of the decay, which includes the traditional binary fission as a particular case when $Z_3 = 0$. For small $Z_3 \neq 0$ the strictly symmetric fragments with enhanced $\langle$TKE$\rangle$ appear. When all the three charges are comparable, as, e.g., in the reactions shown in Table I, the process of the collinear three-body splitting of the heavy nucleus, induced by relativistic protons, takes place. As Z_3 approaches its limit Z_0 the process turns into the double α decay and two-proton radioactivity.

In any case, the string ternary decay of heavy nuclei may have a certain significance in nuclear physics.

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