Inclusive (p,p') reactions on nuclei in the mass range 115 to 181 at incident energies from 120 to 200 MeV

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Double differential cross sections have been measured for inclusive (p,p') reactions on ^{115}In, ^{167}Er, ^{173}Yb, and ^{181}Ta at incident proton energies of 120, 150, 175, and 200 MeV and for ^{141}Pr at 120 and 200 MeV. These targets were chosen to investigate a possible mass dependence of the effective interaction strength when comparisons are made with multistep direct calculations based on the Feshbach, Kerman, and Koonin theory. The inclusion of two-nucleon emission is also investigated. The multiparticle emission contributes significantly at low emission energies and forward angles, but it is shown that the energy dependence of the strength of the effective interaction must also be taken into account to explain the discrepancy in this region. [S0556-2813(96)00809-6]

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I. INTRODUCTION

In previous papers [1–3], pre-equilibrium (p,p') cross-section measurements and multistep direct calculations were compared for ^{90}Zr (80 to 200 MeV), ^{58}Ni, ^{100}Mo, ^{197}Au (100 to 200 MeV), ^{89}Y, ^{92}Mo, ^{94}Mo, ^{96}Mo, and ^{98}Mo (120 to 200 MeV). Angular distributions were calculated with the multistep direct reaction code of Bonetti and Chiesa [4], hereafter referred to as the Milan code, based on the statistical multistep direct reaction theory of Feshbach, Kerman, and Koonin (FKK) [5]. In these studies it was shown that the FKK theory gives reasonably good results with only one free parameter V_0, the strength of the simple effective interaction used, a Yukawa potential of 1 fm range. However, an unexpected mass dependence was seen in the extracted V_0 values. For several nuclei with $89 \leqslant A \leqslant 100$, similar V_0 values were obtained [3], but for ^{58}Ni much higher values and for ^{197}Au much lower values were extracted [2]. The targets in the present study, ^{115}In, ^{141}Pr, ^{167}Er, ^{173}Yb, and ^{181}Ta, were chosen to investigate the mass dependence in the region between mass 100 and mass 197.

Some important calculational improvements were made to the multistep direct reaction (MSD) code used [4] in order to obtain as reliable V_0 values as possible. Spurious values could result, for example, from the use of too few particle-hole exciton configurations [6], or restrictions on the number of configurations in the multistep part of the calculation, which may lead to a false mass dependence.

Although good agreement between the FKK theory and experiment has been obtained on the whole, systematic deviations at very high and very low emission energies have been noted [2,3]. It has long been recognized that multiple pre-equilibrium emission processes are important in inelastic reactions at incident energies in the 100 MeV region, and it has been suggested [2] that the discrepancies could be associated with such processes. The importance of multinucleon emission is expected to grow with increasing incident energy, and for the relatively high energies used in this study it is essential to investigate this contribution.

Because the original FKK theory does not include the possibility of the emission of two or more nucleons from a particular exciton stage ($np-nh$ excitation, with an $2n$ the exciton number), and the inclusive measurements do not exclude the possibility of a detected nucleon being accompanied by another, there is a need to extend the basic theory at energies where multiple emission is possible. The quantum-mechanical extension of the FKK theory has been provided by Ciangaru [7], but the implementation is not straightforward. Recently, Chadwick *et al.* [8] have shown how multiple pre-equilibrium emission (two-nucleon emission) can be approximated in the multistep direct calculations. This method makes use of distorted wave Born approximation (DWBA) matrix elements already calculated for the primary (single-nucleon) emission and accordingly, is relatively simple to implement. In this work the contribution of two-nucleon emission to the cross section is calculated using the method of Chadwick and co-workers. The angular dependence of the multiple emission is investigated as well.

The theoretical work in the present study is also important

TABLE I. Target thicknesses in mg cm^{-2}.

^{115}In	^{141}Pr	^{167}Er	^{173}Yb	^{181}Ta
3.44	3.74	2.50	1.63	3.47

from the point of view of studies of the Gamow-Teller strength distribution in nuclei [9] and medical applications of proton-nucleus interactions, e.g., proton radiotherapy. In order to extract the true Gamow-Teller strength in charge-exchange reactions involving high excitations, it is necessary to perform a background subtraction in the continuum region, and it is, therefore, vital to be able to predict the multistep direct contribution as accurately as possible. The methodology of making such predictions is also applied to dose rate calculations involving nuclei of biological importance. In the present study it will be shown that two important ingredients that must be considered in such predictions are multiparticle emission and the energy dependence of the strength of the two-body interaction.

In Sec. II the experimental details are discussed. The inclusion of the multiple pre-equilibrium emission in the FKK theory is discussed in Sec. III A. In Sec. III B important features of the theoretical calculations are discussed. Section IV A consists of a comparison between the theoretical and experimental angular distributions of the continuum spectra at selected emission energies, and in Sec. IV B systematic trends of the effective interaction strength V_0 (dependence on mass and energy) are investigated. Finally, in Sec. V, a summary of the main conclusions is given.

II. EXPERIMENTAL DETAILS

The experiment was carried out at the cyclotron facility of the National Accelerator Centre, Faure. Accounts of the equipment and experimental technique have been presented in Refs. [10–12]. The targets used and target thicknesses are summarized in Table I. Target thicknesses were determined by comparing the measured energy loss of alpha particles from a ^{228}Th source with calculated values using stopping power tables of Ziegler [13]. The absolute thickness determination is accurate to within 8% due to uncertainties in the energy-loss calculation, and relative errors are less than 3%. The target uniformity is typically $\approx$ 1%/mm. The overall systematic error in the cross-section data is considered to be less than 10%.

Corrections for the reaction tail and efficiency of the NaI detectors followed the procedure described by Green *et al.* [14], where they assume that the reaction tail increases linearly from zero at zero energy to a maximum at the full energy of the detected particle. This assumption was checked in a previous experiment by employing a coincidence setup [3], and it was found to be sufficiently good to warrant using the linear approximation.

III. THEORY

A. Inclusion of multiple pre-equilibrium emission in the multistep direct calculations

The FKK theory [5] of multistep direct emission has frequently been described. A summary of the formalism can be found in Ref. [2] and a more detailed description in Ref. [15]. In previous calculations for incident projectile energies up to 200 MeV a good overall description of the angular distributions has been found, but the theoretical cross sections at the lowest emission energies have been found to be systematically too low. For such high excitations of the residual nucleus it has been suggested that the discrepancy could be due to multiparticle emission, in this case two-proton emission, which is not included in the FKK calculations (Ref. [2]). The fact that this discrepancy appears to become more prominent at higher incident energy seems to be consistent with such an interpretation.

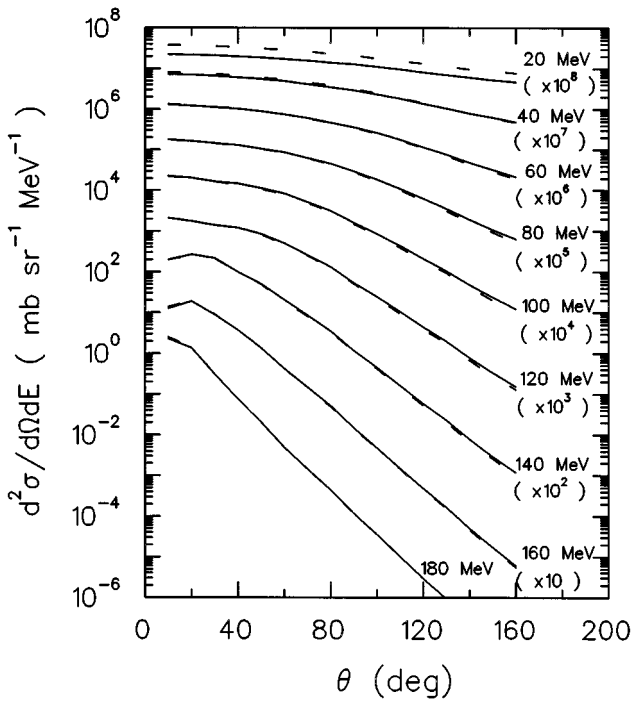

FIG. 1. Calculated (p,p') and (p,n) MSD angular distributions for a ^{181}Ta target at 200 MeV incident energy. The dashed line corresponds to neutron emission and the solid line to proton emission. The same two-body interaction strength V_0 is used in both cases. Results are multiplied by the indicated factors for display.

The original formulation only takes into account the pre-equilibrium emission of one particle (primary pre-equilibrium emission), whereas it is possible for a second accompanying nucleon (secondary pre-equilibrium emission) to carry away some of the available energy and to leave the residual nucleus in a different state. Either one of the two emerging particles can be observed in the single detector employed in inclusive experiments. The emission of more than one particle in the pre-equilibrium energy region can originate from different mechanisms:

(1) First, one can envisage a fast direct knock-out process, where the incoming nucleon imparts enough energy for both nucleons to be emitted into the continuum. One or both of the nucleons may, of course, undergo subsequent scattering or absorption by the rest of the nucleus. This process has been studied experimentally [10] and computer codes exist [16] which model this mechanism.

(2) Second, a nucleon which has been excited in a p-h excitation process to a loosely bound excited state, may subsequently be emitted.

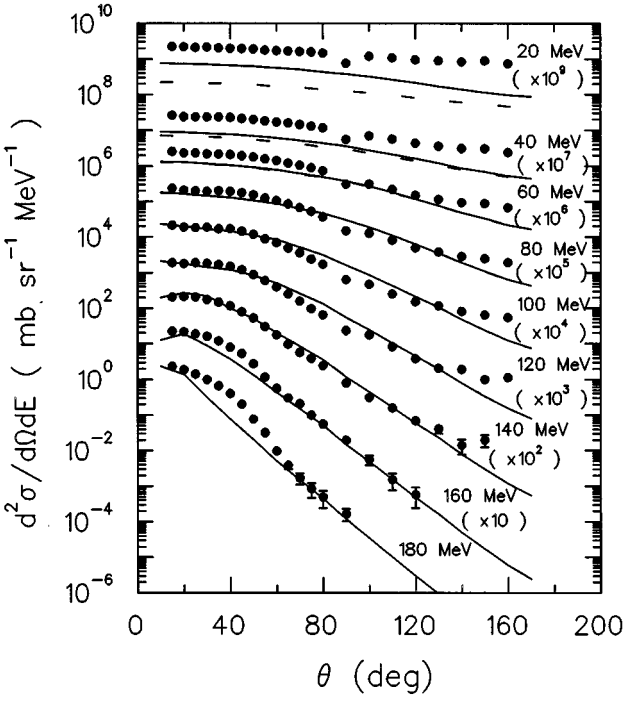

FIG. 2. Angular distributions for ^{181}Ta(p,p') at 200 MeV incident energy E_p and various emission energies $E_{p'}$. Statistical error bars are shown where these exceed the symbol size. The curves are the results of MSD calculations. The dashed line corresponds to primary emission only and the solid line to primary emission plus two-nucleon emission. Results are multiplied by the indicated factors for display and are given in the laboratory system.

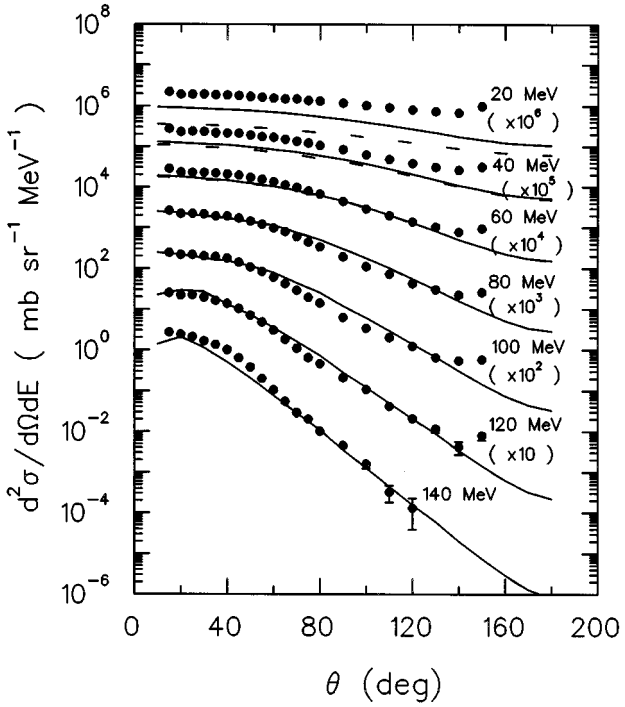

FIG. 3. Angular distributions for ^{181}Ta at 175 MeV incident energy. See also caption for Fig. 2.

(3) Finally, nucleons may be emitted from the equilibrated compound nucleus.

None of these mechanisms are included in the standard FKK theory. In this multistep direct reaction model, energy is dissipated only by the excitation of one or more particle-hole excitations. The third mechanism is expected to play only a minor role since the lowest outgoing energy measured is 20 MeV. The first mechanism was discussed briefly in Ref. [2], where it was found to be important at the higher outgoing energies. It is expected to be one of the possible reasons for the underprediction in our calculated cross sections at the higher outgoing energies. We have not included calculations using this mechanism in the present paper, since it only appears to be noticeable at very low excitation energies. Furthermore, a proper inclusion of the process in the theory requires a coherent treatment, which was not developed in the present work.

The second mechanism has been the subject of a detailed investigation by Chadwick *et al.* [8]. They have described an approximate way to take emissions of this type into account. Their procedure has been followed in the calculations in this paper. The basic expression used for the double differential cross section for two-nucleon emission is

$$\left(\frac{d^2\sigma}{dEd\Omega}\right)^{(Nj)}_{\text{mult}} = \left(\frac{d\sigma}{dE}\right)^{(Nj)}_{\text{mult}} \times \left(\frac{d^2\sigma}{dEd\Omega} \Big/ \frac{d\sigma}{dE}\right)^{(Nj)}_{\text{prim}},$$

where E=emission energy of second pre-equilibrium particle, N=pre-equilibrium stage ($p=h=N$ before emission), and j=label of type of multiple pre-equilibrium particle.

In the above expression,

$$\left(\frac{d\sigma}{dE}\right)^{(Nj)}_{\text{mult}} = \sum_{i=\pi,\nu} \int_{U=E+B}^{U_{\max}} \left(\frac{d\sigma}{dU}\right)^{(N,i)} P_j(E) T_j(E) dU$$

where i=label of type of primary-emitted particle (π= proton, ν= neutron), $(d\sigma/dU)^{(Ni)}$=differential cross section for primary pre-equilibrium emission of a nucleon of type i from stage N as a function of residual nucleus energy, obtained from angle integration, B=separation energy, U=energy of particle-hole state after primary emission, $P_j(E)$=probability of finding a particle j at energy $(E+B)$ inside a p-h exciton configuration of energy U, and $T_j(E)$=transmission coefficient representing the probability of the continuum particle escaping with energy E. Further details can be found in Ref. [8].

The basic input required for the multiple emission program consists of the primary double differential cross sections calculated by some multistep direct (MSD) code. In our case the cross sections were calculated using the Milan code [4], and hence the latter code was adapted to provide an input suitable for use with the multiple emission program of Chadwick *et al.* [8]. It is also necessary to provide double differential cross sections for both neutron and proton emission since primary emission of a neutron or proton may be accompanied by further proton emission. Because very little (p,n) data is available at the higher incident energies, one is faced with the problem of normalizing the (p,n) angular distributions correctly, i.e., choosing the appropriate V_0 value. Some (p,n) data in the energy region of interest is available for an incident energy of 160 MeV, employed in an experiment by Scobel *et al.* [17] on a ^{90}Zr target nucleus. Using (p,p') data on the same nucleus at 160 MeV obtained

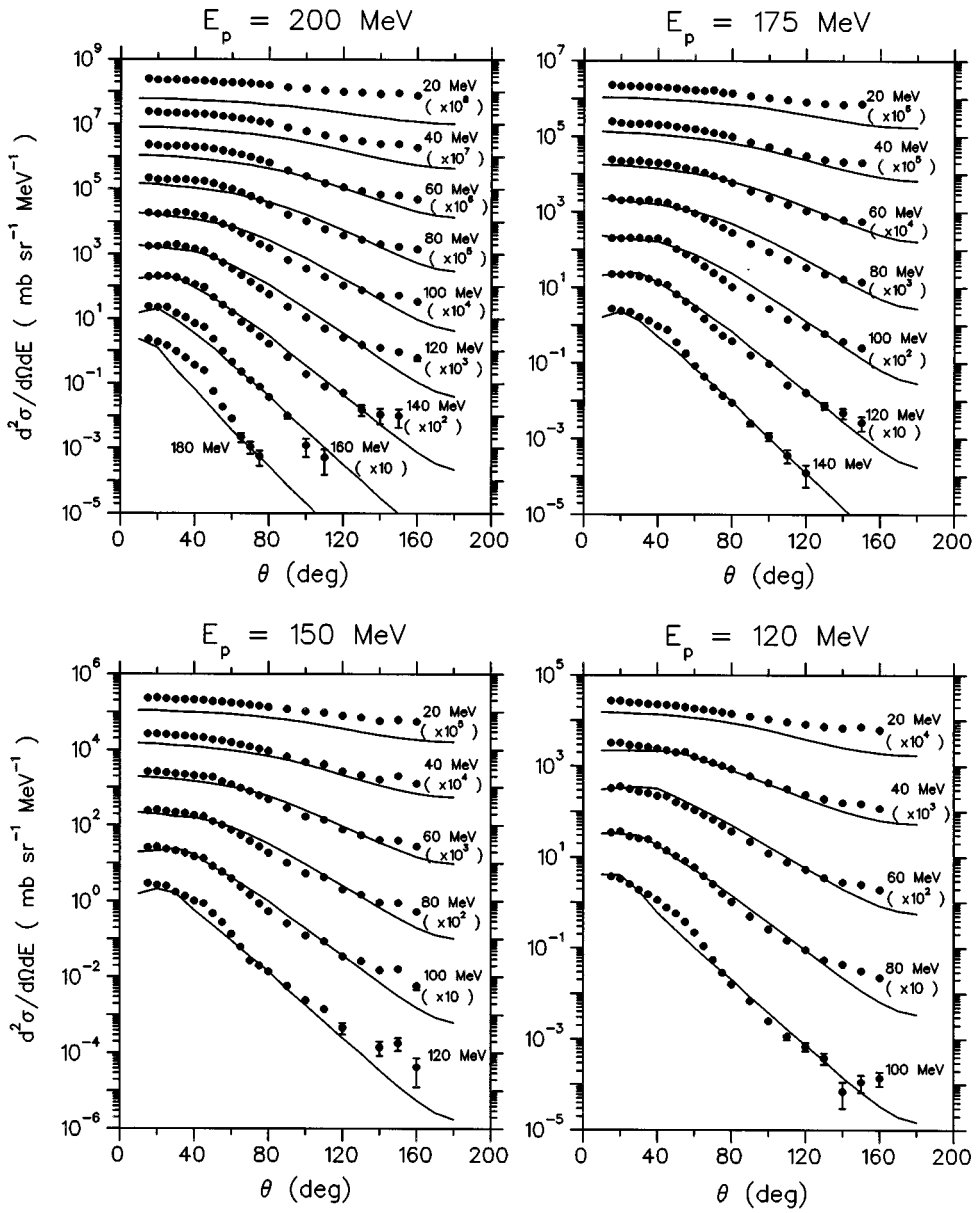

FIG. 4. Experimental angular distributions and MSD calculations for ^{115}In(p,p') at various incident energies E_p and emission energies $E_{p'}$. Two-nucleon emission has been included in the calculations. See also caption for Fig. 2.

in an earlier experiment [1], the ratio of the V_0 values for (p,p') and (p,n) can be obtained. We obtain a value of $V_0(p,p')/V_0(p,n)=1.1$ (ignoring the possibility of two-nucleon emission) which can be compared to typical values of about 1.3 calculated by Chadwick *et al.* [8] for the same reaction. This difference in the ratio gives some indication of the uncertainty associated with values extracted from the comparison of the theoretical results with the experimental data. In this work we use a ratio of 1.1 for all our targets and incident energies to determine the V_0 values to be used for the primary (p,n) cross sections once the V_0 value for (p,p') has been determined in a fit to our data.

The V_0 value for the primary (p,p') process is determined in this work by requiring a good fit between theory and experiment for some intermediate emission energy, e.g., 100 MeV for a 200 MeV incident proton. It is evident from previous work [2,3], as well as from the present data, that for the highest emission energy (typically about 20 MeV less than the incident energy), the shape of the angular distribution is not well reproduced by the theory. Hence, it is not possible to normalize the theory at such a high emission energy. At an emission energy of 100 MeV the calculated correction for two-nucleon emission is almost negligible, as will be shown below, and hence, the V_0 value can be determined from a consideration of the primary emission of protons only.

Some consideration has to be given to the matter of the violation of unitarity, raised by Chadwick *et al.* [8]. It was pointed out that if the V_0 value was determined from a calculation which did not include two-nucleon emission, an overestimated V_0 value would result from a fit to the data, and the corresponding calculated cross sections would exceed the reaction cross section. It should be emphasized, however, that since we normalize our calculations at an

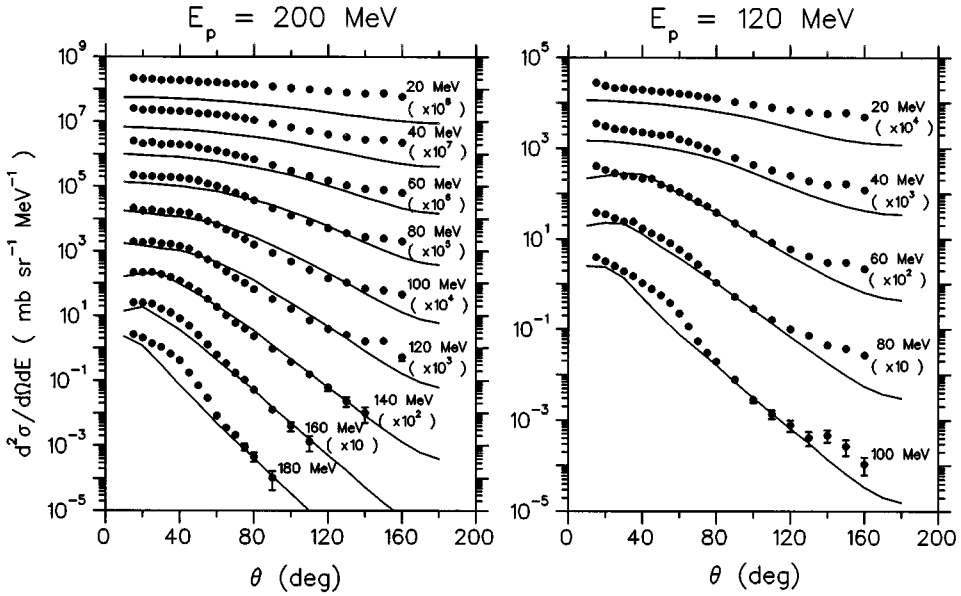

FIG. 5. Experimental angular distributions and MSD calculations for ^{141}Pr(p,p'). See also caption for Fig. 2.

emission energy where the multinucleon contribution can be neglected, the V_0 value determined is generally independent of whether multinucleon emission has been included or not. This also holds for past practice. Because the calculated cross sections are fairly sensitive to the V_0 value adopted, as a result of the V_0^{2n} dependence of the nth step contribution, care has to be taken that the unitarity is not violated. As will be shown in Sec. IV A for the targets considered, the total cross sections obtained are not in conflict with the unitarity requirement.

In practice, it was found that the (p,n) and (p,p') double-differential cross sections are very similar in shape and magnitude for the same two-body interaction strength V_0. In Fig. 1 the calculated (p,n) and (p,p') angular distributions for a ^{181}Ta target at 200 MeV incident energy for primary emission only are compared. When using the primary (p,p') cross sections with a V_0 value scaled down by the factor 1/1.1 as described above to approximate the (p,n) contribution, the multinucleon emission contribution is practically the same as when using the (p,n) cross sections explicitly. This could simplify the calculations as the (p,n) cross sections do not have to be calculated, although in this work they were calculated explicitly.

Another approximation inherent in the method of Chadwick *et al.* [8] involves an estimate of the transmission coefficients for the second proton or neutron emitted. This is particularly important at the lower emission energies because of the restricting effect of the Coulomb barrier on protons. These transmission coefficients have been calculated from a Gamow factor given in Ref. [18]. The angular distribution of the second emitted nucleon is assumed to be the same as the angular distribution of a primary process causing this emission. This approximation is reasonable since the secondary emission is fairly isotropic in the energy range where it makes the largest contribution.

Since the effect of multinucleon emission in the process considered by Chadwick *et al.* [8] is expected to be more important for the highest incident energy, consider first the angular distribution for ^{181}Ta for an incident proton energy of 200 MeV. In Fig. 2 the experimental and theoretical angular distributions are compared for the cases of (i) no multiple nucleon emission and (ii) two-nucleon emission included in the calculations. It is evident that at the highest excitation energies of the residual nucleus the multiple emission leads to a significant improvement in the agreement between experiment and theory. For lower excitation energies the effect of multiple emission is very small. In Fig. 3 a similar comparison is shown for ^{181}Ta at an incident energy of 175 MeV.

In the calculations of Figs. 2 and 3, no energy dependence of the effective interaction in the multistep parts of the calculations has been considered. As will be shown in Sec. IV B, the remaining discrepancy at the lowest outgoing energy would be much smaller if we used an energy dependence.

Discrepancies between theoretical and experimental cross sections are also prominent at high emission energies, and may be linked with multiple emission as well, but in this case with the direct two-particle knock-out mechanism referred to as the first type earlier in this section. This interpretation is suggested by the calculations of Ref. [2].

B. Calculational details

A level density parameter a inversely proportional to the mass number A of the target nucleus has been assumed, as in Ref. [2], viz., $a=A/8.5$ MeV^{-1}. In the calculations a value must also be chosen for the spin cutoff parameter σ in the spin distribution formula. We have used a value of $\sqrt{0.24nA^{2/3}}$, as suggested by [19]. Here, n is the number of particles plus holes excited in each successive step of the reaction cascade, i.e., $n=2$.

A Yukawa potential of range 1 fm was used for the two-body effective interaction, of which the strength V_0 is adjusted to best reproduce the data. The calculated cross sections were normalized by choosing values of V_0 to give the best overall agreement with the experimental angular distribution at an emission energy corresponding to half the incident energy.

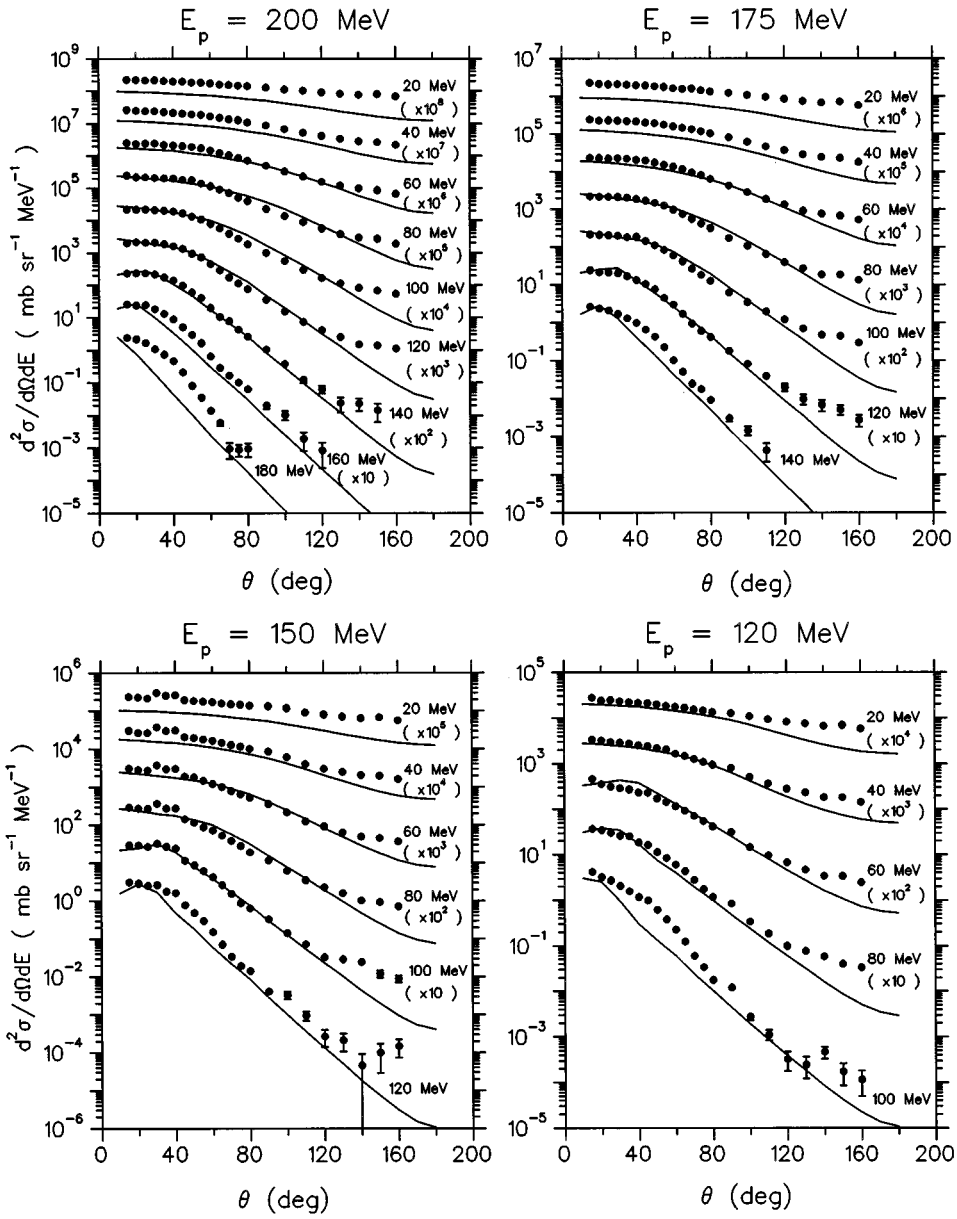

FIG. 6. Experimental angular distributions and MSD calculations for ^{167}Er(p,p'). See also caption for Fig. 2.

There is strong evidence from previous studies that the effective nucleon-nucleon interaction is energy dependent [20,1–3,21], which means that a different V_0 should be used for successive stages in the multistep calculations. The effect of this dependence of V_0 is considered in detail in Sec. IV B. However, to simplify the study of the influence of multinucleon emission, and because the E dependence of V_0 is not known *a priori*, the energy dependence of V_0 has been ignored in all our other calculations.

The number of partial waves $L_{\max}$ used in the DWBA calculations varied between 30 (for incident energy $E_p = 120$ MeV) and 70 (for incident energy $E_p = 200$ MeV), and the number of steps in the cascade employed in the calculations varied from five for the lower to eight for the higher incident energies. For some previous calculations up to 200 MeV, angular momentum transfer values up to $L = 8$ have been used. The effect of including values up to $L = 12$ has been investigated, and it was found that there is a minimal difference in the angular distributions compared to those calculated with $L = 8$. Hence, the latter limit was generally used.

A significant improvement over previous calculations with the Milan code was to include more than one p-h transition for a particular energy bin and angular momentum transfer in the multistep part of the code. This restriction appeared to lead to some spurious results [6] in some cases where the angular distributions showed a marked sensitivity to the particular set of configurations chosen (pairs of Nilsson single-particle states for a p-h transition). In particular, the V_0 values extracted will then not be reliable, which will, in turn, affect conclusions regarding the energy and mass dependence of the strength of the effective interaction. In addition, a larger number of configurations were chosen for the first step compared to previous calculations, viz., typically about 12 configurations, and the same number was also used for the multistep part.

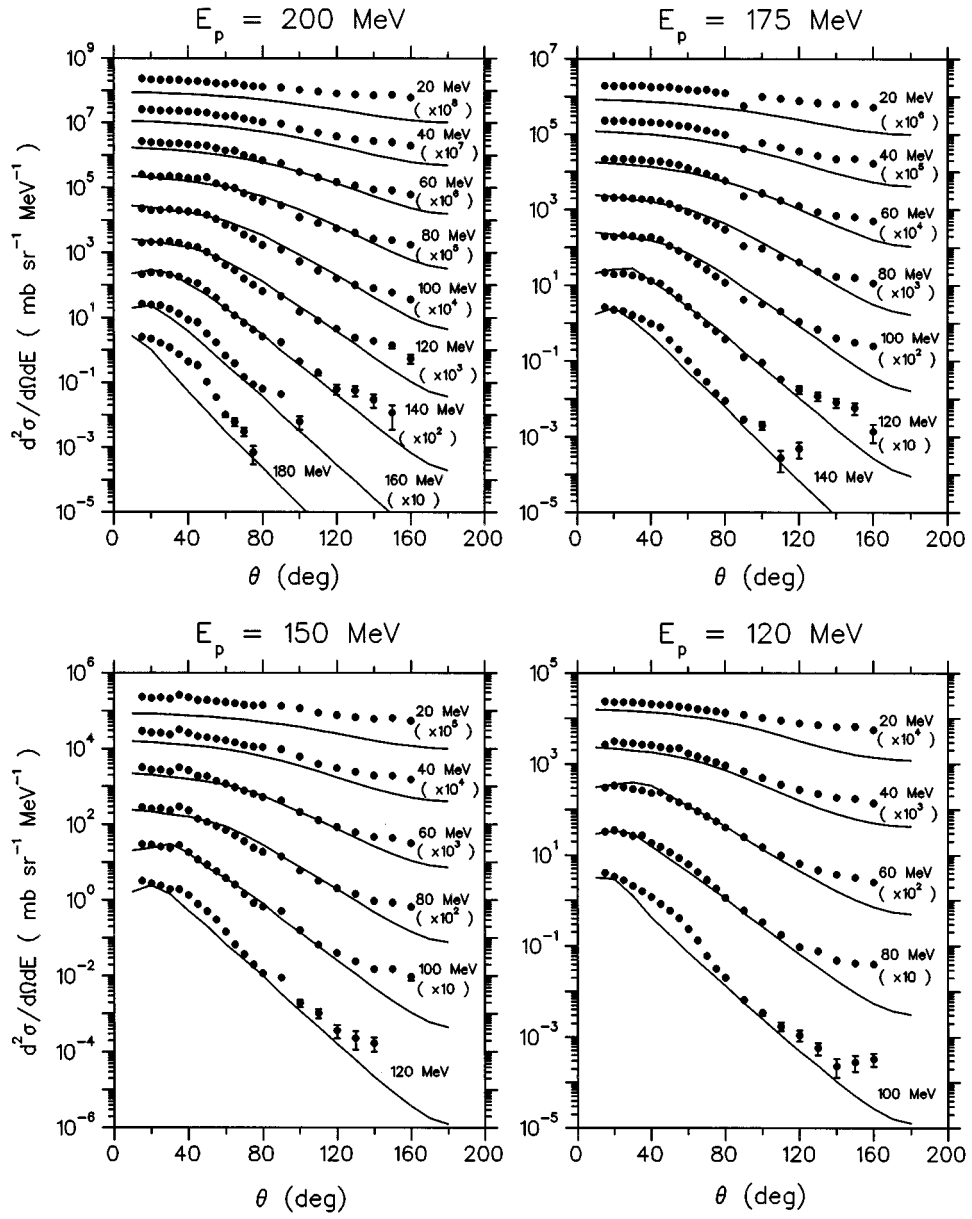

FIG. 7. Experimental angular distributions and MSD calculations for ^{173}Yb(p,p'). See also caption for Fig. 2.

IV. RESULTS AND DISCUSSION

A. Comparison of theoretical and experimental angular distributions

The angular distributions for the targets ^{115}In, ^{141}Pr, ^{167}Er, ^{173}Yb, and ^{181}Ta, at proton incident energies between 120 and 200 MeV are given in Figs. 2 to 8 for a range of energies of the emitted proton (or excitation energy of the residual nucleus $U=E_p-E_{p'}$, in the case of one-particle emission). The theoretical angular distributions, calculated as described in Sec. III B and including two-nucleon emission, are compared with the experimental data.

For ^{181}Ta at 200 MeV incident energy, the total reaction cross section given by the optical model, using the Schwandt potential [22], is 1574 mb. (Using the Madland potential [23] it is 1379 mb.) For comparison, the reaction cross section can also be calculated from a semiempirical energy-dependent formula given in Ref. [24], which gives a value of 1450 mb, reasonably consistent with the optical model values. Our total calculated cross section for (p,p') is 882 mb, with a contribution of 153 mb due to two-nucleon emission, and for (p,n) it is 619 mb, with a contribution of 250 mb from two-nucleon emission. This gives a total cross section of 1501 mb, which is less than the reaction cross section of 1574 mb. There will also be a small contribution from processes such as (p,d) and (p,α). For a ^{90}Zr target, for example, with protons incident at 160 MeV, this contribution has been estimated by Chadwick *et al.* to be 60 mb. For ^{181}Ta at 120 MeV incident energy, the reaction cross section is 1628 mb (based on the Schwandt potential), while the total calculated (p,p') cross section is 896 mb (174 mb is due to two-nucleon emission) and for (p,n) it is 560 mb (265 mb is due to two-nucleon emission), giving a total of 1456 mb. Thus the unitarity requirement is not violated. For the other two incident energies, 150 and 175 MeV, the calculated and reaction cross sections have values close to the ones illustrated here. For the other four targets this trend is generally also true, with cross sections of similar magnitude as in the examples above.

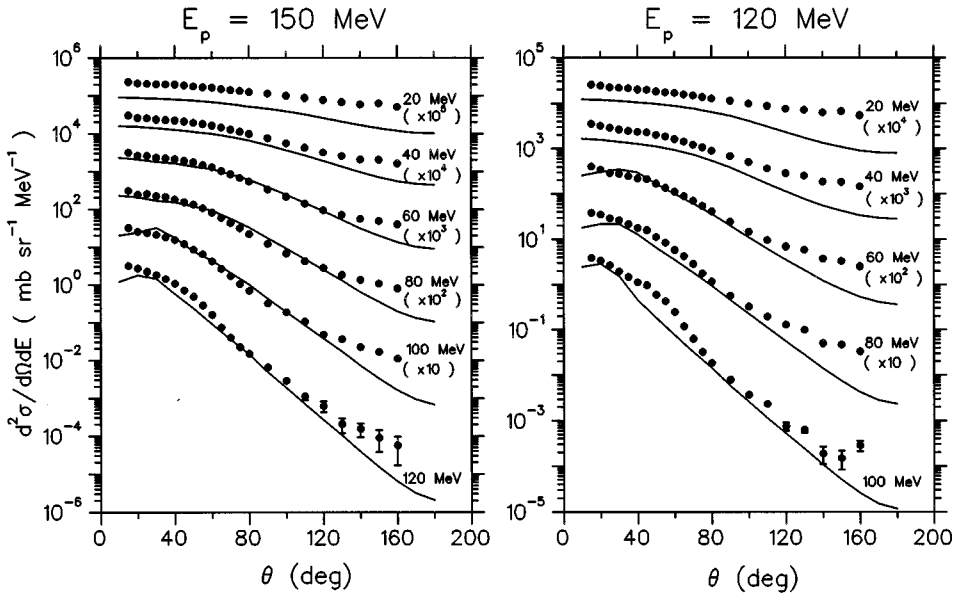

FIG. 8. Experimental angular distributions and MSD calculations for ^{181}Ta(p,p'). See also caption for Fig. 2.

Our MSD calculations reproduce the angular distributions for all the targets quite well. The contribution of multiparticle emission can be seen to be quite important for high excitations and forward angles. There is, however, still a shortfall by a factor of 2–3 in the theoretical values at high excitation. However, the use of an energy-dependent V_0 increases the cross section in this region, as will be discussed in Sec. IV B, and can possibly explain the remaining discrepancy. There may also be a small multistep compound contribution in this energy region, but it is likely to be small at an outgoing energy as high as 20 MeV. It has also been suggested [25,26] that multistep compound emission can also result from transitions from the P (direct) to the Q (compound) chain after the first step, even at high incident energies where the feeding of the Q chain from the entrance channel is negligible.

At the lower excitation energy the contribution of multinucleon emission calculated according to the method of Chadwick *et al.* [8] is negligible. Some contribution due to collective excitations may still add to the cross section at these low excitation energies, as well as the knock-out contribution already mentioned.

B. The effective two-body interaction

Our calculated values of the effective interaction strength V_0 are based on a simple Yukawa force of 1 fm range. It should be noted that a factor of 0.25 in the first step of the multistep chain, originally introduced into the Milan code to distinguish between (p,n) and (p,p') reactions [27], but subsequently also used for (p,p') reactions, has been retained for the sake of consistency in the comparison of V_0 values. Ideally, the distinction between protons and neutrons should be based on a two-component formulation of the theory.

Two aspects of the strength of the effective interaction are noteworthy, namely, the energy dependence and the dependence on the mass number of the target. The energy dependence of V_0 has been noted in several studies and, in particular, the relationship to theoretical optical model studies which imply an exponential variation [20]. Our extracted values again show a definite energy dependence, but it appears to be a slightly weaker dependence on the energy than the optical model prediction [1–3].

Figure 9 shows calculations where an energy-dependent V_0 has been used. The solid line shows a theoretical calculation with a constant V_0, the dashed line an exponential energy variation found previously [1] of

$$V_0 \propto \exp[-0.0049E]$$

and the dotted line with a linear dependence, namely,

$$V_0 = 18.2 - 0.048E.$$

The slope of the linear dependence was chosen to approximate the energy variation of V_0 as found in this study. As may be seen in Fig. 9, the specific choice of the energy dependence implies some uncertainty at low emission energies. However, it is significant that the use of an energy dependence increases the cross section at higher excitations, because it has been shown that multinucleon emission does not account completely for the discrepancy in this region. The two effects taken together, with appropriate rescaling of the effective interaction strength, can explain this discrepancy.

Previous studies have shown that the strength of the effective interaction V_0 may be mass dependent. Whereas similar values of V_0 were found for several nuclei with $89 \leqslant A \leqslant 100$ [3], a higher value was found for ^{58}Ni and a lower value for ^{197}Au [2]. The targets in the present study were chosen to investigate the nature of the variation from mass 100 to about 200. In Fig. 10 we show this dependence for the different incident energies employed. A definite general decrease in V_0 as a function of mass is observed. This confirms the result of [2] which provided tentative evidence for a possible target-mass dependence. Some of the possible explanations for such an A dependence could be:

(1) The A dependence of the level density. Reference [3] indicates that the pre-equilibrium data are not sensitive to the shell structure of the target nucleus, which can, thus, be eliminated as a source of the mass dependence. The expres-

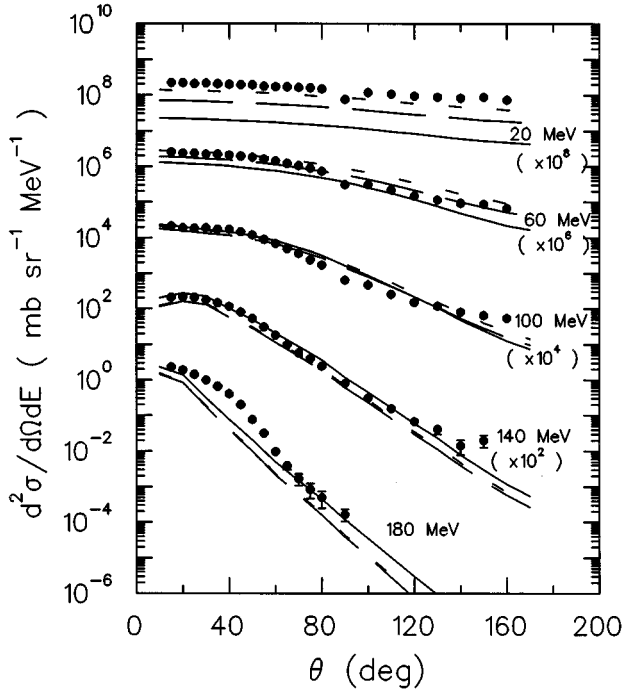

FIG. 9. Theoretical predictions for ^{181}Ta(p,p') with different functional forms of the energy dependence of the effective interaction in the multistep calculations. The solid line is a calculation with no energy dependence, the broken line an exponential energy dependence, and the dashed line a linear energy dependence. See the text for the parameters used in the different forms.

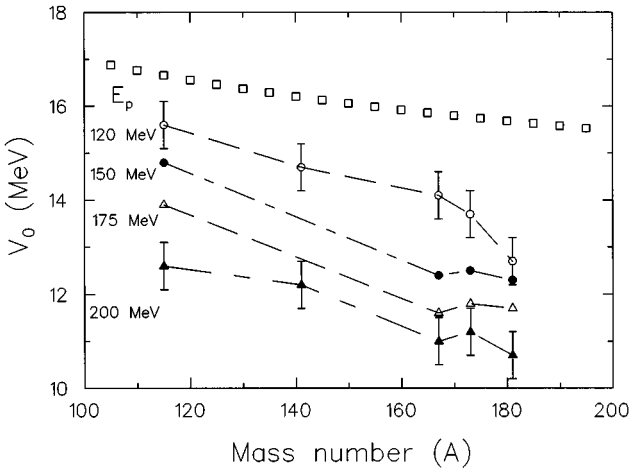

FIG. 10. The extracted V_0 values as a function of the mass of the target. Error bars reflecting the estimated theoretical uncertainty are only shown for the top and bottom data sets, and the dashed lines serve to guide the eye. The uppermost set of values have been calculated with arbitrary normalization using a semiclassical level density expression from Ref. [28].

sion for the level density, used in the present study, is based on the Fermi gas model and could be inadequate, since it has only been verified at low excitation energies. This would be reflected in an A dependence of the effective interaction. The semiclassical approximation used in Ref. [28] predicts a level density parameter which has an A dependence somewhat different from that usually employed ($a \propto A$), leading to a trend in the V_0 values which is similar to that found in the present study, as shown in Fig. 10.

(2) Shell-model two-body effective interactions in a finite model space exhibit a mass dependence [29]. Inadequacies in the description of particle and hole states according to a simplistic spherical Nilsson shell model, such as the ordering of single-particle states, may also contribute to an A dependence.

(3) The optical potentials employed in this work [22] have only a minor A-dependent part, based on the asymmetry term. Other global optical potentials have used parameters with a definite dependence on A [30]. It is conceivable that some systematic effect in the global potentials could cause an A-dependent effect in V_0.

V. SUMMARY AND CONCLUSIONS

It has been shown that the statistical multistep direct theory of Feshbach, Kerman, and Koonin [5] reproduces the (p,p') continuum angular distributions of the selected target nuclei reasonably well over a range of incident energies from 100 to 200 MeV, and a wide range of proton emission energies, for an angular range between 15° and 160°. The contribution of two-nucleon emission, calculated by the method of Chadwick *et al.* [8], leads to an improvement in the agreement between theory and experiment for forward angles at the highest excitation energies. We have also shown that for an energy-independent V_0, the calculated cross sections are not in conflict with the unitarity requirement on the reaction cross section.

This study has confirmed earlier indications that the extracted strengths of the effective interaction are mass dependent. We have also suggested that a large part of this mass dependence derives from a mass dependence of the level density. This has important consequences for the use of multistep direct theory in applications such as calculating activation cross sections, background contributions to giant resonances and Gamow-Teller strength distributions, and medical radiotherapy dosages.

The multinucleon contribution calculated does not account completely for the discrepancy between theory and experiment at low emission energies. This is partly as a result of using an energy-independent V_0 in the multistep part of the calculation. An energy-dependent V_0 increases the calculated cross section, and with appropriate rescaling of V_0, the two effects taken together can give a good reproduction of the angular distribution at high excitation energies, but different assumed energy dependencies lead to slightly different predictions. There is clearly a need to delineate this energy dependence more accurately than is known at present. Ideally, a more realistic interaction form than a simple Yukawa potential should be used, which would also make possible the prediction of polarization observables such as the analyzing power. We conclude that, although multiparticle emission is physically appealing as an important process at the incident energies employed in this study, better information regarding the exact energy dependence of the

strength of the effective interaction is required for an accurate assessment of quantitative improvements of the theoretical formulation.

There may also be some contribution from multistep compound emission [3]. At higher emission energies the multistep compound component is negligible, but there are indications that knock-out contributions which involve a multiparticle emission mechanism, not included in this paper, are important.

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