

Ground-state energies and widths of ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei

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We extract energies and widths of the ground states of ${}^5\text{He}$ and ${}^5\text{Li}$ from recent single-level R -matrix fits to the spectra of the ${}^3\text{H}(d, \gamma){}^5\text{He}$ and ${}^3\text{He}(d, \gamma){}^5\text{Li}$ reactions. The widths obtained differ significantly from the formal R -matrix values but they are close to those measured as full widths at half maxima of the spectra in various experiments. The energies are somewhat lower than those given by usual estimates of the peak positions. The extracted values are close to the S -matrix poles calculated previously from the multiterm analyses of the N - ${}^4\text{He}$ elastic scattering data. [S0556-2813(96)02208-X]

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The ground-state properties of ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei have been studied in several experiments, see Refs. [1–3] for a review. In the most recent work [3] values of the $A=5$ widths were reported which are significantly larger than the previous ones. In the present work we reconsider the experimental data of Ref. [3]. We extract the widths and the positions of the $A=5$ levels from these data.

The present work has been done in connection with some applications of an astrophysical interest [4–6]. In addition, one may note that at present there exist difficulties in obtaining p -wave N - ${}^4\text{He}$ phase shifts at low energy from a realistic noncentral NN interaction (see, e.g., [7]) which hinder a microscopic description of light multicluster nuclear systems (see, e.g., [8]). The phase shifts are governed by the ${}^5\text{He}$ and ${}^5\text{Li}$ ground-state resonances and it would be convenient to analyze the problem directly in terms of their energies and widths.

In Ref. [3] a single-term R -matrix fit to experimental γ -ray spectra of that work was undertaken to determine the widths, while in most of the previous work the widths were estimated by measuring the full widths at half maximum (FWHM) of the spectra (see Table I in Ref. [3]). The ground-state energies were mainly estimated from the positions of the peaks (see [2]). In Ref. [9] the ${}^5\text{He}$ energy and width were extracted from an R -matrix fit to the whole set of the ${}^4\text{He}(n, n){}^4\text{He}$ cross section and polarization data in the elastic region. The p -wave R -matrix elements were approximated by resonant terms plus linear functions in energy representing contributions from distant R -matrix levels. The position of the pole of the expression obtained was found giving the energy and width of ${}^5\text{He}$. In Ref. [10] a similar type of analysis was carried out for ${}^4\text{He}(n, n){}^4\text{He}$ and ${}^4\text{He}(p, p){}^4\text{He}$ scattering using another form of the S -matrix parametrization. These multiterm analyses are considerably influenced by potential scattering interference. It is of interest to study the pole structure of the S matrix for other types of processes involving $A=5$ nuclei.

In Ref. [3] the ${}^3\text{H}(d, \gamma){}^5\text{He}$ and ${}^3\text{He}(d, \gamma){}^5\text{Li}$ spectra have been measured at $E_d=8.6$ MeV and $\theta_{\text{lab}}=90^\circ$. The spectra were fitted by the expression¹

$$\frac{\Gamma_R(E_{N\alpha})}{[E_{N\alpha} - E_{\text{res}} - \Delta(E_{N\alpha})]^2 + [\Gamma_R(E_{N\alpha})/2]^2} \quad (1)$$

times a slowly varying function in the energy $E_{N\alpha}$. Here $E_{N\alpha}$ is the relative motion energy of the $(N+\alpha)$ subsystem, and $\Delta(E_{\text{res}})=0$. The energy dependencies of the width $\Gamma_R(E)$ and of the shift function $\Delta(E)$ corresponded to the single-level R -matrix expression, that is, $\Gamma_R(E)=2\gamma^2 P(E)$ and $\Delta(E)=-\gamma^2[S(E)-S(E_{\text{res}})]$. Here γ^2 is the reduced width, while the penetrability P and the shift function S for the $\ell=1$ orbital momentum are defined as usual [11]. The overall normalizations of the spectra and the reduced widths were fitted to the experimental data while for the E_{res} energies the values from Ref. [1] were used. As a result the following values of the widths $\Gamma_R(E_{\text{res}})$ were obtained, which we shall denote Γ_R :

$$\begin{aligned} \Gamma_R({}^5\text{He}) &= (1.36 \pm 0.19) \text{ MeV}, \\ \Gamma_R({}^5\text{Li}) &= (2.44 \pm 0.21) \text{ MeV}. \end{aligned} \quad (2)$$

Here and below the input parameters are [3] the channel radius $R=3.6$ fm, $E_{\text{res}}=0.89$ MeV, $\gamma^2=(3.32 \pm 0.46)$ MeV in the ${}^5\text{He}$ case, and $R=3.6$ fm, $E_{\text{res}}=1.97$ MeV, $\gamma^2=(3.33 \pm 0.29)$ MeV in the ${}^5\text{Li}$ case. In Eq. (2) and below the errors correspond to those in the reduced widths.

The previous values of the width ranged from (0.55 ± 0.03) MeV to (0.85 ± 0.05) MeV in the ${}^5\text{He}$ case. In the ${}^5\text{Li}$ case the previous widths were grouped around 1.5 MeV except for one value of (2.6 ± 0.4) MeV. The latter value [12] was obtained using a method in principle similar to that of Ref. [3], while the others were obtained in several experiments by measuring FWHM's of spectra for various

¹In Eq. (1) we express the spectra in terms of the energy $E_{N\alpha}$ that coincides with E_N from Ref. [3]. Our E_{res} coincides with that of Ref. [3] in their Table II but differs from E_{res} in their Eq. (1). Our quantity $\Gamma_R(E)$ coincides with $\Gamma_{N\alpha}(E)$ from Ref. [3].

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reactions (see [1,3]) and with the help of the above-mentioned analyses [9,10] of the elastic N - ^4He scattering. The FWHM's for the spectra of Ref. [3] are rather close to the above-mentioned values.

We shall extract the poles $E = E_0 - i\Gamma/2$ of the scattering matrix from the results of Ref. [3]. These poles determine the *physical* resonance energies E_0 and widths Γ , which are different from E_{res} and $\Gamma_R = \Gamma_R(E_{\text{res}})$ entering Eq. (1). The difference proves sometimes to be quite significant; see, e.g., [13]. In particular, Γ and not Γ_R determines the lifetime of a system as is required, for example, in the applications [4–6]. One can see that $\Gamma_R(E_{\text{res}})$ cannot serve as the estimate of a width even in a narrow resonance case. The quantity $\Gamma_S(E) = [1 - \Delta'(E_{\text{res}})]^{-1} \Gamma_R(E)$ at $E = E_{\text{res}}$ could have served as such an estimate [11]. Here Δ' is a derivative over E taken at the point E_{res} . (The quantity $\Gamma_S(E)$ enters the resonant factor of a reaction when written in the form $\Gamma_S(E)/\{(E - E_{\text{res}})^2 + [\Gamma_S(E)/2]^2\}$.) In our case the widths are broad and the above-mentioned estimate is not very accurate, however.

In case of the parametrization of Eq. (1) the complex energy E of the resonance is the solution to the equation

$$[E - E_{\text{res}} - \Delta(E)]^2 + [\Gamma_R(E)/2]^2 = 0, \quad (3)$$

at the condition $k = (2\mu E)^{1/2}/\hbar = k_1 - ik_2$, $k_1 > 0$, $k_2 > 0$. In the ^5He case this equation can be represented in the form

$$(x - x_{\text{res}})^2 [\gamma^{-2} \bar{E}(x+1) + (x_{\text{res}} + 1)^{-1}]^2 + x^3 = 0,$$

where we introduced the notation $\bar{E} = \hbar^2/(2\mu R^2)$, μ being the reduced mass, and $x = E/\bar{E}$, $x_{\text{res}} = E_{\text{res}}/\bar{E}$. We obtained²

$$E_0(^5\text{He}) = (0.735 \pm 0.02) \text{ MeV},$$

$$\Gamma(^5\text{He}) = (0.57 \pm 0.02) \text{ MeV}. \quad (4)$$

Note that the value of E_0 obtained is slightly shifted downwards with respect to the position of the peak of the spectrum which is at 0.81 MeV. This is due to the asymmetric form of the spectrum.

The values (4) should be somewhat corrected for the following reason. The value of 0.89 MeV taken in Ref. [3] as E_{res} was actually obtained [2] as the mean position of the resonant peak. In general, this position differs from the optimal E_{res} value or, which is the same, from the energy at which the resonant phase shift reaches 90° . We changed the value of E_{res} to shift the position of the peak to the value of 0.89 MeV. The resulting E_{res} value, $E_{\text{res}} = 0.98$ MeV, is close to those given in Refs. [1,9,14] for elastic n - ^4He scattering. It is not clear whether the small change in the E_{res} value

made is quite compatible with the γ^2 value deduced in Ref. [3] from the fit to the data. The point is that in order to allow for spectrum calibration errors the fitting procedure in Ref. [3] included shifts up or down in energy of the entire observed spectrum resulting from a convolution with a detector response function. The information on the latter function listed in the paper [3] does not suffice to perform the convolution. If, nevertheless, one adopts the shifted E_{res} value one obtains the following energy and width of ^5He :

$$E_0(^5\text{He}) = (0.80 \pm 0.02) \text{ MeV},$$

$$\Gamma(^5\text{He}) = (0.65 \pm 0.02) \text{ MeV}. \quad (5)$$

These values are in remarkable agreement with the values of $E_0 = 0.77$ MeV and $\Gamma = 0.64$ MeV, and also with the values of $E_0 = 0.78$ MeV and $\Gamma = 0.72$ MeV obtained in Refs. [9] and [10], respectively, from multiterm analyses of the elastic n - ^4He scattering. The measured FWHM's (see Table I in Ref. [3]) are also in accordance with the width value obtained.

In the ^5Li case, $\Delta(E)$ and $\Gamma_R(E)$ entering Eq. (3) include Coulomb functions with complex arguments and their derivatives. They were calculated with the help of the computer code of Ref. [15]. Equation (3) was solved with a Newton-type method. The results corresponding to Eqs. (4) and (5) are

$$E_0(^5\text{Li}) = (1.635 \pm 0.03) \text{ MeV},$$

$$\Gamma(^5\text{Li}) = (1.16 \pm 0.03) \text{ MeV}, \quad (6)$$

and

$$E_0(^5\text{Li}) = (1.72 \pm 0.03) \text{ MeV},$$

$$\Gamma(^5\text{Li}) = (1.28 \pm 0.03) \text{ MeV}, \quad (7)$$

respectively. The E_0 values are in accordance with the value of $E_0 = 1.6$ MeV obtained in Ref. [10] from the multiterm analysis of elastic p - ^4He scattering. Similarly to the ^5He case, these values are shifted downwards with respect to the peak position of 1.97 MeV. The widths obtained are lower than the value of $\Gamma = 1.45$ MeV from Ref. [10] and than the mean FWHM value [1] of ≈ 1.5 MeV, but they are close to the FWHM value of (1.24 ± 0.03) MeV for the spectrum of Ref. [3].

It may be noted that the above S -matrix pole widths cannot be "reduced" as were the R -matrix widths in Ref. [3] to obtain reduced widths γ^2 . The reduced widths of ^5He and ^5Li extracted in Ref. [3] turned out to be equal, which showed that charge symmetry is not violated. The ratio of the R -matrix widths [3] thus has a direct physical meaning, as well as the S -matrix pole results.

In conclusion, the experimental data of Ref. [3] on the $^3\text{H}(d, \gamma)^5\text{He}$ and $^3\text{He}(d, \gamma)^5\text{Li}$ reactions allowed safe extrapolations to the S -matrix poles. From these extrapolations rather accurate values of the complex energies of $A = 5$ nuclei have been determined. They are in good agreement with those obtained from the multiterm analyses of N - ^4He cross section and polarization data set in the elastic region. In spite

²A pleasant peculiarity of the result of Eq. (4) for the width is a considerable reduction of the relative errors as compared to Eq. (2). The same holds true for Eqs. (5)–(7) below and this is due to the difference in the γ^2 dependence in the equations determining the widths. Note as well that the highest (lowest) E_0 value, of course, corresponds to the lowest (highest) Γ value in Eq. (4) and in the relations below.

of the broad resonances the widths obtained proved to be close to those measured as FWHM's. However, the real parts of the poles are shifted downwards with respect to the positions of the peaks in the spectra.

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