Bose-Einstein correlations for three-dimensionally expanding, cylindrically symmetric, finite systems

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There are *two types of scales* present simultaneously in the spacelike as well as in the timelike directions in a model class describing a cylindrically symmetric, finite, three-dimensionally expanding boson source. One type of scale is related to the finite lifetime or geometrical size of the system, and the other type is governed by the rate of change of the local momentum distribution in the considered temporal or spatial direction. The parameters of the Bose-Einstein correlation function may obey an M_t scaling, as observed in S+Pb and Pb+Pb reactions at CERN SPS. This M_t scaling may imply that the Bose-Einstein correlation functions view only a small part of a large and expanding system. The full sizes of the expanding system at the last interaction are shown to be measurable with the help of the invariant momentum distribution of the emitted particles. A vanishing duration parameter can also be generated, with a specific M_t dependence, in the considered model class. [S0556-2813(96)01208-3]

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I. INTRODUCTION

The method of intensity interferometry has recently become a widely used tool for determining the space-time picture of high-energy heavy-ion collisions. Originally, the method was invented [1] to measure angular diameters of distant stars. The objects under study were approximately static and the length scales astronomical. In principle, the same method is applied to measure space-time characteristics of high-energy heavy-ion collisions, where the objects are expanding systems, with lifetimes of a few fm/c (10^{-23} sec) and length scales of a few fm (10^{-15} m).

In the case of high-energy heavy-ion collisions, intensity interferometry is pursued to infer the equation of state and identify the possible formation of a transient quark-gluon plasma state from a determination of the freeze-out hypersurface, as scanned by the Bose-Einstein correlation function (BECF); see, e.g., the contributions of the NA35, NA44, and WA80 Collaborations in Refs. [2,3]. For an introduction and review on Bose-Einstein correlations, see Refs. [4,5]. Nontrivial effects arising from correlations among space-time and momentum-space variables were studied in Ref. [6].

The recent ^{32}S+^{197}Pb reactions at 200 A GeV laboratory bombarding energy resulted in a nonexpected, symmetrical BECF-s [3] if measured in the the longitudinally comoving system (LCMS) of the boson pairs [7]. The longitudinal radius parameter was shown to measure a length scale, $R_L \propto 1/\sqrt{m_t}$, introduced in Ref. [8] for an infinite, longitudinally expanding Bjorken tube. The *side* radius parameter was thought to measure the geometrical radius and the *out* component to be sensitive to the duration of the particle freeze-out times [7,9]. *All radius component parameters turned out to be equal within the experimental errors*. Although this might be just a coincidence, in this work we show that *such a scaling behavior*, valid in a certain m_t interval, *may also be a natural consequence of a cylindrically symmetric three-dimensional hydrodynamic expansion*.[1] In this case the local temperature, the gradients of the temperature distribution, and the flow gradients generate ''thermal'' length scales in all these spacelike directions. Changes in the local temperature during the particle emission induce a temporal scale, the thermal duration. Recently, it became clear that the parameters of the BECF-s measure the *lengths of homogeneity* [8,10–12], which in turn were shown to be expressible in terms of the geometrical and the thermal lengths [13,9,12].

We shall derive here general relationships among the functional forms of the BECF-*s* as given in the laboratory (lab) frame, the LCMS frame, and the longitudinal saddle point system (LSPS) in which the functional form of the BECF-*s* turns out to be the simplest one.

A new class of analytically solvable models is introduced thereafter, describing a three-dimensionally expanding, cylindrically symmetric system for which the geometrical sizes and the duration of the particle emission are finite. In this class of the models, there are two length scales present in all directions, including the temporal one. The BECF is found to be dominated by the shorter, while the momentum distribution by the longer of these scales. The interplay between the finite ''geometrical scales'' of the boson-emitting source and the finite ''thermal scales'' shall be considered in detail.

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[1]Many earlier works discussed the m_t dependences of the radii parameters, but they did not consider the possible equality of and simultaneous m_t scaling for the transverse radius parameters. We recommend the paper by U. A. Wiedemann, P. Scotto, and U. Heinz, Phys. Rev. C **53**, 918 (1996), for an up-to-date list of references on this specific topic.

II. FORMALISM

Both the momentum spectra and the BECF-s are prescribed in the applied Wigner-function formalism [14,5]. In this formalism the BECF is calculated from the two-body Wigner function assuming chaotic particle emission. In the final expression the time derivative of the (nonrelativistic) Wigner function is approximated [7,14] by a classical emission function $S(x;p)$, which is the probability that a boson is produced at a given $x=(t,\mathbf{r})=(t,r_x,r_y,r_z)$ point in space-time with four-momentum $p=(E,\mathbf{p})=(E,p_x,p_y,p_z)$. The emission function has been related to the covariant Wigner transform of the density matrix of pion sources in Refs. [14,5] and most recently in Ref. [15], where the relation of Wigner-function formalism to the covariant current formalism [16] has also been clarified. The (off-shell) two-particle Wigner functions shall be approximated by the off-shell continuation of the on-shell Wigner-functions [14,13,9]. The particle is on the mass shell, $m^2=E^2-\mathbf{p}^2$. Please note the difference between x, indicating a four-vector in space-time, and the subscript sized x, which indexes a direction in coordinate space.

A useful auxiliary function is the Fourier-transformed emission function

$$\widetilde{S}(\Delta k;K)=\int d^4x\ S(x;K)\exp(i\Delta k\cdot x), \tag{1}$$

where

$$\Delta k=p_1-p_2,\quad K=\frac{p_1+p_2}{2}, \tag{2}$$

and $\Delta k\cdot x$ stands for the inner product of the four-vectors. Then the one-particle inclusive invariant momentum distribution (IMD) of the emitted particles, $N_1(\mathbf{p})$ is given by

$$N_1(\mathbf{p})=\widetilde{S}(\Delta k=0;\ K=p)=\frac{E}{\sigma_{\rm tot}}\frac{d\sigma}{d\mathbf{p}}, \tag{3}$$

where $\sigma_{\rm tot}$ is the total inelastic cross section. This IMD is normalized to the mean multiplicity $\langle n\rangle$ as

$$\int\ \frac{d\mathbf{p}}{E}N_1(\mathbf{p})=\langle n\rangle. \tag{4}$$

In the present paper effects arising from the final-state Coulomb and Yukawa interactions shall be neglected. The two-particle BECF can be calculated from the emission function with the help of the well-established approximation

$$C(\Delta k;K)=\frac{\langle n\rangle^2}{\langle n(n-1)\rangle}\frac{N_2(\mathbf{p}_1,\mathbf{p}_2)}{N_1(\mathbf{p}_1)N_1(\mathbf{p}_2)}\simeq 1+\frac{|\widetilde{S}(\Delta k;K)|^2}{|\widetilde{S}(0;K)|^2}, \tag{5}$$

utilized also in Ref. [13]; see Ref. [14] for further details. The corrections to this expression are known to be as small as 4–5% [12]. Note that among the eight components of Δk and K only six are independent due to the two constraints $p_1^2=p_2^2=m^2$. These constraints can be formulated alternatively as $\Delta k\cdot K=0$ and $K^2=m^2-\Delta k^2/4$. Thus the two-particle BECF depends on the off-shell emission function, which we approximate by the off-shell continuation of the on-shell emission functions.

A similar but not identical approximation used by several authors is to replace $\widetilde{S}(\Delta k;K)$ by $\widetilde{S}(\Delta k;K')$ where the off-shell K is changed to an on shell K'. The latter mean momentum is defined to be on shell as $K'^0=m^2-\mathbf{K}'^2$ where $\mathbf{K}'=\mathbf{K}=(\mathbf{p}_1+\mathbf{p}_2)/2$. The differences between these two approximation schemes are of $O(\Delta k^2/m^2)$. The above two approximation schemes coincide in the $\Delta k^2\to 0$ limit where the Bose-Einstein correlations are maximal. Since we shall make use of the $\Delta k\cdot K=0$ constraint which is exact only if the K four-vector is off shell, we shall approximate the off-shell emission function in Eq. (5) with the off-shell continuation of the on-shell emission function.

III. GENERAL CONSIDERATIONS

We model the emission function in terms of the longitudinally boost-invariant variables. The (longitudinal) proper time is $\tau=\sqrt{t^2-r_z^2}$, the space-time rapidity is $\eta=0.5\ln[(t+r_z)/(t-r_z)]$, the transverse mass is $m_t=\sqrt{E^2-p_z^2}$ and the momentum-space rapidity reads as $y=0.5\ln[(E+p_z)/(E-p_z)]$. In the transverse direction, the transverse radius $r_t=\sqrt{r_x^2+r_y^2}$ is introduced. We have

$$t=\tau\cosh(\eta),\quad r_z=\tau\sinh(\eta). \tag{6}$$

For systems undergoing a boost-invariant longitudinal expansion, the emission function may be a function of boost-invariant variables only. These are τ, r_x, r_y, p_x, p_y, and $\eta-y$. However, for finite systems the exact longitudinal boost invariance cannot be achieved and the emission function becomes a function of $\eta-y_0$ too, where y_0 stands for the midrapidity. Approximate boost invariance is recovered in the midrapidity region only, where terms proportional to $\eta-y_0$ can be neglected. Thus for finite systems undergoing a boost-invariant longitudinal expansion the emission function can be given in terms of these variables as

$$S(x;K)d^4x=S_*(\tau,\eta,r_x,r_y)d\tau\ \tau_0\ d\eta\ dr_x\ dr_y. \tag{7}$$

Here we introduced the constant τ_0 in front of $d\eta$ due to dimensional reasons and included the Jacobian from the d^4x to the $d\tau\, d\eta\, dr_x\, dr_y$ variables into the emission function $S_*(\tau,\eta,r_x,r_y)$. The subscript $*$ indicates that the functional form of the emission function is changed with the change of the variables. Further, dependences on the mean momentum K as well as on the mid rapidity y_0 are also indicated with the subscript $*$. The effective, momentum-dependent parameters of the emission function $S_*(\tau,\eta,r_x,r_y)$ shall also be indexed with $*$ in the forthcoming. The subscript s stands for the point where the emission function is maximal [we assume that $S(x;K)$ has only one maximum for any values of K]. We do not assume at this point whether the function $-\ln S(x;K)$ is expandable into a (multivariate) Taylor series [12] around its unique minimum at the saddle point x_s or not; merely, we assume that the Fourier-transformed $\widetilde{S}(\Delta k;K)$ exists. See the Appendix for a clarifying example. We suppose, however, that the Fourier-transformed $\widetilde{S}(\Delta k;K)$ can be evaluated in terms of the τ and η variables in the small Δk region relevant for the analysis of the BECF-s. This is possible if the region around $x_s(K)$, where the Fourier integrals pick up the dominant contribution from, is sufficiently small so that within this region the τ and η dependences of t and r_z can be linearized as

$$t \simeq \tau \cosh[\eta_s] + (\eta - \eta_s)\tau_s \sinh[\eta_s], \tag{8}$$

$$r_z \simeq \tau \sinh[\eta_s] + (\eta - \eta_s)\tau_s \cosh[\eta_s], \tag{9}$$

with negligible second-order corrections. This condition is fulfilled if the characteristic sizes $\Delta\tau_*$ and $\Delta\eta$ of the considered region around $x_s(K)$ satisfied $\Delta\tau_*^2 \ll \tau_s^2$ and $\Delta\eta_*^2 \ll 1$.

The principal directions for the decomposition of the relative momentum at a given value of the mean four-momentum K are given as follows [17]: The *out* direction is parallel to the component of $\mathbf{K}$, which is perpendicular to the beam, indexed with ''out'', the *longitudinal* or *long* direction is parallel to the beam axis r_z (this component of the relative momentum is indexed with L), and the remaining direction orthogonal to both *longitudinal* and *out* is called the *side* direction, indexed with ''side''. Thus the mean and the relative momenta are decomposed as $K=(K_0,K_{\rm out},0,K_L)$ and $\Delta k=(Q_0,Q_{\rm out},Q_{\rm side},Q_L)$.

Since the particles are on mass shell, we have

$$0 = K\cdot\Delta k = K_0 Q_0 - K_{\rm out}Q_{\rm out} - K_L Q_L. \tag{10}$$

Thus the energy difference Q_0 can be expressed as

$$Q_0 = \beta_{\rm out}Q_{\rm out} + \beta_L Q_L, \tag{11}$$

where we have introduced the longitudinal and the outward component of the velocity of the pair, $\beta_L = K_L/K_0$ and $\beta_{\rm out}=K_{\rm out}/K_0$, respectively. These relations become further simplified in the LCMS, the longitudinally comoving system, introduced first in Ref. [7]. The LCMS is the frame where $K_L=0$ and thus $\beta_L=0$. We also have $\beta_{\rm out}=\beta_t$ where t stands for transverse, e.g., $r_t=\sqrt{r_x^2+r_y^2}$ and $m_t=\sqrt{m^2+p_x^2+p_y^2}$. Note that the relation $\beta_{\rm out}=\beta_t$ is independent of the longitudinal boosts, but both sides of this equation transform like $1/K_0$.

Let us express the Fourier integrals in terms of the τ and η variables in the laboratory reference frame (lab), utilizing Eqs. (8) and (9). The results in the LCMS can be obtained from the more complicated results in the lab frame by the substitution $\beta_L=0$ and $\beta_{\rm out}=\beta_t$. To simplify the notation, let us rewrite

$$\begin{aligned}\Delta k\cdot x &= Q_0 t - Q_{\rm out}r_x - Q_{\rm side}r_y - Q_L r_z\\ &\simeq Q_\tau\tau - Q_{\rm out}r_x - Q_{\rm side}r_y - Q_\eta\tau_s(\eta-\eta_s),\end{aligned} \tag{12}$$

utilizing the linearized equations (8) and (9). We have introduced the coefficients of the τ and the $\tau_s(\eta-\eta_s)$ as new variables given by

$$\begin{aligned}Q_\tau &= Q_0\cosh[\eta_s] - Q_L\sinh[\eta_s]\\ &= (\beta_t Q_{\rm out}+\beta_L Q_L)\cosh[\eta_s] - Q_L\sinh[\eta_s],\end{aligned} \tag{13}$$

$$\begin{aligned}Q_\eta &= Q_L\cosh[\eta_s] - Q_0\sinh[\eta_s]\\ &= Q_L\cosh[\eta_s] - (\beta_t Q_{\rm out}+\beta_L Q_L)\sinh[\eta_s].\end{aligned} \tag{14}$$

From these relations it follows that

$$\begin{aligned}C(\Delta k;K) &\simeq 1 + \frac{|\tilde{S}(\Delta k;K)|^2}{|\tilde{S}(0;K)|^2}\\ &\simeq 1 + \frac{|\tilde{S}_*(Q_\tau,Q_\eta,Q_{\rm out},Q_{\rm side})|^2}{|\tilde{S}_*(0,0,0,0)|^2}.\end{aligned} \tag{15}$$

Note that this expression contains a four-dimensional Fourier-transformed function, and among the four variables Q_τ, Q_η, $Q_{\rm out}$, and $Q_{\rm side}$ only three are independent due to Eq. (11). Note also that at this point the BECF may have a non-Gaussian structure, and its dependence on its variables does not factorize. The main limitation of the last approximation in Eq. (15) is that it is valid only for systems with small lengths of homogeneity, $\Delta\tau_*^2 \ll \tau_s^2$ and $\Delta\eta_*^2 \ll 1$. As we shall see in the forthcoming, this gives a lower limit in m_t for the applicability of the simple analytic results for a certain class of emission functions.

IV. CORE-HALO MODEL

If the system under consideration consists of a *core* characterized by a hydrodynamic expansion and small regions of homogeneity and a surrounding *halo* of long-lived resonances, then the above general expression can be further evaluated if the halo is characterized by sufficiently large regions of homogeneity. Indeed, the long-lived resonances may decay in a large volume proportional to their lifetime, and the decay products are emitted with a given momentum distribution from the whole volume of the decay.

The key point is the following: Let us consider an ensemble of long-lived resonances with similar momentum, emitted from a given small volume of the core. The momentum distribution of the *decay products* of these resonances will be similar to each other, independently of the approximate position of the decay. Now the approximate position of the decay is randomly distributed along the line of the resonance propagation with the weight $P(t)\propto\exp(-m_{\rm res}\Gamma_{\rm res}/E_{\rm res})$. Thus the decay products will be emitted with the same momentum distribution from a volume which is elongated along the line of resonance propagation, given by $V_{\rm decay}\simeq A_0|p_{\rm res}|/(m_{\rm res}\Gamma_{\rm res})$, where A_0 is the initial transverse size of the surface through which the resonances are emitted with a momentum $p_{\rm res}$ approximately at the time of the decay of the core, τ_s.

Thus the halo of long-lived resonances is characterized by large regions of homogeneity. (In the case of the pionic halo, the dominant long-lived resonances are ω, η, η', and K^0, all with lifetimes $1/\Gamma_{\rm res}$ greater than 20 fm/c.) If the emission function is a sum of the emission function of the core and the halo,

$$S(x;K) = S_{*,c}(\tau,\eta,r_x,r_y) + S_h(x;K), \tag{16}$$

and the Fourier-transformed emission function of the halo is sufficiently narrow to vanish at the finite resolution $Q_{\rm min}$ of the relative momentum Δk in a given experiment, then one can show [18] that

$$N_1(\mathbf{p}) = N_{1,c}(\mathbf{p}) + N_{1,h}(\mathbf{p}), \tag{17}$$

$$C(\Delta k;K) = 1 + \lambda_* \frac{|\tilde{S}_{*,c}(Q_\tau, Q_\eta, Q_{\rm out}, Q_{\rm side})|^2}{|\tilde{S}_{*,c}(0,0,0,0)|^2}, \quad (18)$$

where $N_{1,i}(\mathbf{p})$ indicates the number of particles emitted from the halo or from the core for $i=h,c$ and the effective intercept parameter

$$\lambda_* = \lambda_*(K \simeq p; Q_{\rm min}) = \left[\frac{N_{1,c}(\mathbf{p})}{N_1(\mathbf{p})}\right]^2 \quad (19)$$

is the square of the ratio of the number of particles emitted from the core to the number of all the emitted particles with a given momentum $\mathbf{p}$. This effective intercept parameter arises due to the finite relative momentum resolution, which is typically $Q_{\rm min}=10$ MeV in current heavy-ion experiments [2,3], and the comparably large region of homogeneity characterizing the halo part of the system. See Ref. [18] and references therein for a more complete account on the origin of this parameter λ_*.

We would like to raise a warning flag here: The volume, which the decay products of the long-lived resonances of a given momentum are emitted from, is large only if the decaying resonances have $|\mathbf{p}|/(m_{\rm res}\Gamma_{\rm res}) \gg 1$ fm/c. This in turn implies that the above simple picture may need further corrections for very low pt pions at rapidity $y=0$.

There is a gap in the lifetime distribution of abundant hadronic resonances: $1/\Gamma_\rho \simeq 1.3$ fm/c, $1/\Gamma_{N*} \simeq 0.56$ fm/c, $1/\Gamma_\Delta \simeq 1.6$ fm/c, and $1/\Gamma_{K*} \simeq 3.9$ fm/c, which lifetimes are of the same order of magnitude as the timescales for rescattering at the time of the last hadronic interactions. These lifetimes are also all a factor of 5–10 shorter than the lifetime of the ω meson, which is the long-lived resonance with shortest lifetime.[2] Thus the decay product of the short-lived resonances will mainly contribute to the core, which is resolvable by BEC measurements, while the decay products of long-lived hadronic resonances will mainly belong to the halo, redefined alternatively as the part of the emission function which is not resolvable in a given Bose-Einstein measurement.

V. CLASSES OF SIMPLE CORE FUNCTIONS

If the emission function of the core can be factorized,

$$S_{*,c}(\tau,\eta,r_x,r_y) = H_*(\tau) G_*(\eta) I_*(r_x,r_y), \quad (20)$$

where $H_*(\tau)$ stands for the effective emission function in proper time, $G_*(\eta)$ stands for the effective emission function in space-time rapidity, and $I_*(r_x,r_y)$ stands for the effective emission function in the transverse directions, then the expression for the BECF can be further simplified as

$$C(\Delta k;K) = 1 + \lambda_* \frac{|\tilde{H}_*(Q_\tau)|^2 |\tilde{G}_*(Q_\eta)|^2 |\tilde{I}_*(Q_{\rm out}, Q_{\rm side})|^2}{|\tilde{H}_*(0)|^2 |\tilde{G}_*(0)|^2 |\tilde{I}_*(0,0)|^2}. \quad (21)$$

[2]Note that with a resolution of typically $Q_{\rm min}=10$ MeV, the resonance ω will be considered as long-lived resonance [18].

If the $I_*(r_x,r_y)$ function is symmetric for rotations in the (r_x,r_y) plane around its maximum point $r_{x,s}$, then one may introduce $Q_t = \sqrt{Q_{\rm side}^2 + Q_{\rm out}^2}$ to find

$$C(\Delta k;K) = 1 + \lambda_* \frac{|\tilde{H}_*(Q_\tau)|^2 |\tilde{G}_*(Q_\eta)|^2 |\tilde{I}_*(Q_t)|^2}{|\tilde{H}_*(0)|^2 |\tilde{G}_*(0)|^2 |\tilde{I}_*(0)|^2}. \quad (22)$$

Such factorization around the saddle point happens, e.g., for the new class of analytically solvable models if certain conditions are satisfied, as discussed in the subsequent part. From the above expression it is clear that for this type of model the dependence of the BECF on the components of the relative momentum can be diagonalized with an appropriate choice of the three independent components of the relative momentum. Note that the assumed existence of the Fourier-transformed distribution functions is a weaker condition than the assumption of the analytic form of the Fourier-transformed function; see the Appendix for an example. Another example was given, e.g., in Ref. [19] for a $H(\tau)$ distribution for which $\tilde{H}(Q_\tau)$ is not analytic function at $Q_\tau=0$ and $|H(Q_\tau)|^2$ does not start with a quadratic term. In mathematical statistics it is well known that the Fourier-transformed stable distributions are not analytic at $Q=0$ [20]. On the other hand, there are many physically interesting Gaussian models which correspond to the multivariate second-order Taylor expansion of the above general results, i.e., the analytic form of the corresponding Fourier-transformed function. The out-longitudinal cross term [12] has been recently discovered also in this context. To study the properties of the BECF, let us apply a Gaussian approximation to the effective distribution functions as

$$H_*(\tau) \propto \exp[-(\tau-\tau_s)^2/(2\Delta\tau_*^2)], \quad (23)$$

$$G_*(\eta) \propto \exp[-(\eta-\eta_s)^2/(2\Delta\eta_*^2)], \quad (24)$$

$$I_*(r_x,r_y) \propto \exp\{-[(r_x-r_{x,s})^2 + (r_y-r_{y,s})^2]/(2R_*^2)\}. \quad (25)$$

Apart from the momentum-dependent parameters $\Delta\tau_*$, $\Delta\eta_*$, and R_*, the mean emission point may also be momentum dependent in the above expression, $\tau_s=\tau_s(K)$, $\eta_s=\eta_s(K)$, $r_{x,s}=r_{x,s}(K)$ and $r_{y,s}=r_{y,s}(K)$. For the sake of simplicity we do not specify the normalization constants in Eq. (25) since they cancel from the BECF, which is given by

$$C(\Delta k;K) = 1 + \lambda_* \exp(-Q_\tau^2\Delta\tau_*^2 - Q_\eta^2\tau_s^2\Delta\eta_*^2 - Q_t^2 R_*^2). \quad (26)$$

This is a diagonal form of the BECF-*s* for which the factorization property, Eq. (20), and the Gaussian approximation for the core, Eqs. (23)–(25), are simultaneously satisfied. In the present form of the BECF, there are no cross terms among the chosen variables. Now, let us rewrite this form using the standard HBT coordinate system [17] to find

$$C(\Delta k;K) = 1 + \lambda_* \exp(-R_{\rm side}^2 Q_{\rm side}^2 - R_{\rm out}^2 Q_{\rm out}^2 - R_L^2 Q_L^2 - 2R_{{\rm out},L}^2 Q_{\rm out} Q_L), \quad (27)$$

$$R_{\rm side}^2 = R_*^2, \quad (28)$$

$$R_{\rm out}^2 = R_*^2 + \delta R_{\rm out}^2, \tag{29}$$

$$\delta R_{\rm out}^2 = \beta_t^2 (\cosh^2[\eta_s]\Delta\tau_*^2 + \sinh^2[\eta_s]\tau_s^2\Delta\eta_*^2), \tag{30}$$

$$R_L^2 = (\beta_L \sinh[\eta_s] - \cosh[\eta_s])^2 \tau_s^2 \Delta\eta_*^2 + (\beta_L \cosh[\eta_s] - \sinh[\eta_s])^2 \Delta\tau_*^2, \tag{31}$$

$$R_{{\rm out},L}^2 = \beta_t \cosh[\eta_s](\beta_L \cosh[\eta_s] - \sinh[\eta_s])\Delta\tau_*^2 + \beta_t \sinh[\eta_s](\beta_L \sinh[\eta_s] - \cosh[\eta_s])\tau_s^2\Delta\eta_*^2. \tag{32}$$

This result is nonperturbative in terms of the variable η_s and is valid in any frame. The main limitations of this result are the assumed Gaussian model class [Eqs. (23)–(25)] and the assumed smallness of the emission region around $x_s(K)$ so that t and r_z dependences could be linearized in terms of τ and η.

The above equations simplify a lot in the LCMS system, where $\beta_L = 0$:

$$\delta R_{\rm out}^2 = \beta_t^2 (\cosh^2[\eta_s]\Delta\tau_*^2 + \sinh^2[\eta_s]\tau_s^2\Delta\eta_*^2), \tag{33}$$

$$R_L^2 = \cosh^2[\eta_s]\tau_s^2\Delta\eta_*^2 + \sinh^2[\eta_s]\Delta\tau_*^2, \tag{34}$$

$$R_{{\rm out},L}^2 = -\beta_t \sinh[\eta_s]\cosh[\eta_s](\Delta\tau_*^2 + \tau_s^2\Delta\eta_*^2). \tag{35}$$

The lifetime information $\Delta\tau_*^2$ and the invariant measure of the longitudinal size along the τ_s=const hyperbola, $\tau_s^2\Delta\eta_*^2$, appear in a mixed form in the $R_{\rm out}^2$, R_L^2, and the $R_{{\rm out},L}^2$ source parameters even in the LCMS frame. The amount of these mixings is controlled by the value of $\eta_s^{\rm LCMS}$. This relationship clarifies the physical significance of the $\eta_s^{\rm LCMS}$, the space-time rapidity of the maximum of the emission function in the LCMS frame: $\eta_s^{\rm LCMS}$ is the *cross-term generating hyperbolic mixing angle* for cylindrically symmetric, finite systems undergoing longitudinal expansion and satisfying the factorization property, Eq. (20). If $\eta_s^{\rm LCMS} = 0$, no mixing of temporal and longitudinal components appear in the LCMS. In some limited sense one may call η_s the cross-term generating hyperbolic mixing angle in any frame, because if $\eta_s = 0$ in a certain frame, then cross terms can be diagonalized away as follows.

Let us define the LSPS, the longitudinal saddle-point system, to be the frame where $\eta_s = 0$. Since η_s is a function of K in a fixed frame, $\eta_s = \eta_s(K)$, the LSPS frame may depend on K (e.g., on transverse mass of the pair). In the LSPS frame the out-long cross term and the mixing of the temporal and timelike information can be diagonalized. We have, in the LSPS,

$$\delta R_{\rm out}^2 = \beta_t^2 \Delta\tau_*^2, \tag{36}$$

$$R_L^2 = \tau_s^2\Delta\eta_*^2 + \beta_L^2\Delta\tau_*^2, \tag{37}$$

$$R_{{\rm out},L}^2 = \beta_t\beta_L\Delta\tau_*^2, \tag{38}$$

as follows from Eqs. (30)–(32). Introducing the new variables $Q_0 = \beta_t Q_{\rm out} + \beta_L Q_L$ and $Q_t = \sqrt{Q_{\rm out}^2 + Q_{\rm side}^2}$, we obtain for the correlation function

$$C(\Delta k;K) = 1 + \lambda_* \exp(-\Delta\tau_*^2 Q_0^2 - \tau_s^2\Delta\eta_*^2 Q_L^2 - R_*^2 Q_t^2). \tag{39}$$

From this relationship we also see that $Q_0({\rm LSPS}) = Q_\tau$ and $Q_L({\rm LSPS}) = Q_\eta$; cf. Eq. (26).

Let us study an expansion in terms of $\epsilon = |Y - y_0|/\Delta\eta$, where Y is the rapidity belonging to K the mean momentum of the pair and $\Delta\eta$ is the geometrical size of the expanding system in the space-time rapidity variable, satisfying $\Delta\eta > \Delta\eta_*$. It is obvious that in the lab frame $\eta_s^{\rm lab} = Y + O(\epsilon)$, since in the $\epsilon \to 0$ limit we recover boost invariance and the particle emission must be centered around the only scale: the rapidity of the pair. Similarly, we see that $\eta_s^{\rm LCMS} = 0 + O(\epsilon)$. It follows that the cross term and the crossing of temporal and longitudinal information in the lab frame comprise a leading-order effect,

$$\delta R_{\rm out}^2 = \beta_t^2 (\cosh^2[Y]\Delta\tau_*^2 + \sinh^2[Y]\tau_s^2\Delta\eta_*^2) + O(\epsilon), \tag{40}$$

$$R_L^2 = \frac{\tau_s^2\Delta\eta_*^2}{\cosh^2[Y]} + O(\epsilon), \tag{41}$$

$$R_{{\rm out},L}^2 = -\beta_t \frac{\sinh[Y]}{\cosh[Y]} \tau_s^2\Delta\eta_*^2 + O(\epsilon). \tag{42}$$

On the other hand, the mixing of the temporal and longitudinal information is only next-to leading order in the LCMS according to Eq. (35), i.e., $R_{{\rm out},L}^2({\rm LCMS}) = 0 + O(\epsilon)$. However, if the $|Y - y_0| \ll \Delta\eta$ condition is not satisfied, the out-long cross term might be large even in the LCMS, as has been demonstrated numerically in Ref. [21].

The cross-term generating mixing angle η_s vanishes exactly in the LSPS frame, becomes a small parameter in the LCMS if $|Y - y_0|/\Delta\eta \ll 1$, and becomes leading order in any frame significantly different from the LSPS or LCMS. Thus we confirm the recent finding [22] that the out-longitudinal cross term can be diagonalized away if one finds the (transverse mass dependent) longitudinal rest frame of the source.

Note that cylindrical symmetry around the center of the particle emission as assumed by Eqs. (23)–(25) is a stronger requirement than the cylindrical symmetry of the emission function around the beam axis. This latter symmetry implies only that both the requirements $S(t, r_x, r_y, r_z; K_0, K_{\rm out}, K_L) = S(t, -r_x, r_y, r_z; K_0, -K_{\rm out}, K_L)$ and $S(t, r_x, r_y, r_z; K_0, K_{\rm out}, K_L) = S(t, r_x, -r_y, r_z; K_0, K_{\rm out}, K_L)$ should be simultaneously fulfilled. Thus cylindrical symmetry around the beam axis is compatible with a different Gaussian radius in the side and out directions,

$$I_*(r_x, r_y) \propto \exp\left(-\frac{[r_x - r_{x,s}(K)]^2}{2R_{*,x}^2} - \frac{r_y^2}{2R_{*,y}^2}\right), \tag{43}$$

with $R_{*,x} \neq R_{*,y}$. Cylindrical symmetry around the beam axis implies only that $r_{x,s}(K_0, K_{\rm out}, K_L) = -r_{x,s}(K_0, -K_{\rm out}, K_L)$ and $r_{y,s}(K) = 0$. In the low transverse momentum limit, when $K_{\rm out} = 0$, the relations $r_{x,s}(K_{\rm out} = 0) = 0$ and $R_{*,x}(K_{\rm out} = 0) = R_{*,y}(K_{\rm out} = 0)$ also follow from cylindrical symmetry around the beam axis. If $R_{*,x} \neq R_{*,y}$ at a given nonvanishing value of the mean transverse momentum $K_{\rm out}$, the generalized version of Eqs. (26) and (39) for the BECF reads as

$$C(\Delta k;K)=1+\lambda_* \exp(-\Delta\tau_*^2 Q_\tau^2-\Delta\eta_*^2\tau_s^2 Q_\eta^2-R_{*,x}^2 Q_{\rm out}^2 -R_{*,y}^2 Q_{\rm side}^2). \tag{44}$$

In such a case, Eqs. (28) and (29) are also modified as

$$R_{\rm side}^2=R_{*,y}^2, \tag{45}$$

$$R_{\rm out}^2=R_{*,x}^2+\delta R_{\rm out}^2. \tag{46}$$

This implies that the difference between the out and side radius parameters is *not restricted* by cylindrical symmetry around the beam axis to positive values only, since $R_{\rm out}^2 - R_{\rm side}^2=\delta R_{\rm out}^2+R_{*,x}^2-R_{*,y}^2$ which can also be negative if $R_{*,y}$ is sufficiently large [23]. However, cylindrical symmetry does imply that $R_{\rm out}=R_{\rm side}$ in the $K_{\rm out}\to 0$ limit.

Up to this point, we have reviewed the properties of BECF-*s* without reference to any particular model, for some more and more limited classes of simple emission functions. We have obtained certain model-independent relations [cf. Eqs. (15), (18), and (22)] which are valid for some non-Gaussian as well as Gaussian source functions. We have studied the relations between source parameters with a method which is nonperturbative in terms of η_s, but perturbative in terms of $\Delta\eta_*^2$ and $\Delta\tau_*^2/\tau_s^2$.

Let us study the properties of an analytically solvable model class in the subsequent parts.

VI. NEW CLASS OF ANALYTICALLY SOLVABLE MODELS

For central heavy-ion collisions at high energies the beam or r_z axis becomes a symmetry axis. Since the initial state of the reaction is axially symmetric and the equations of motion do not break this pattern, the final state must be axially symmetric too. However, in order to generate the thermal length scales in the transverse directions, the flow field must be either three dimensional or the temperature distribution must have significant gradients in the transverse directions. Furthermore, the local temperature may either increase during the duration of the particle emission because of the reheating of the system caused by the hadronization [24] and/or intensive rescattering processes or decrease because of the expansion and the emission of the most energetic particles from the interaction region. An example for such a time-dependent temperature was given, e.g., by the solid line of Fig. 1. in Ref. [25].

We study the following model emission function for high-energy heavy-ion reactions:

$$S(x;K)\,d^4x=\frac{g}{(2\pi)^3}\,m_t\cosh(\eta-y)\exp\left(-\frac{K\cdot u(x)}{T(x)}+\frac{\mu(x)}{T(x)}\right)H(\tau)\,d\tau\,\tau_0\,d\eta\,dr_x\,dr_y. \tag{47}$$

Here g is the degeneracy factor, the prefactor $m_t\cosh(\eta-y)$ corresponds to the flux of the particles through a τ=const hypersurface according to the Cooper-Frye formula [26] and the four-velocity $u(x)$ is

$$u(x)=\left(\cosh[\eta]\left(1+b^2\frac{r_x^2+r_y^2}{\tau_0^2}\right)^{(1/2)},\ b\frac{r_x}{\tau_0},\ b\frac{r_y}{\tau_0},\ \sinh[\eta]\left(1+b^2\frac{r_x^2+r_y^2}{\tau_0^2}\right)^{(1/2)}\right)$$
$$\simeq\left(\cosh[\eta]\left(1+b^2\frac{r_x^2+r_y^2}{2\tau_0^2}\right),\ b\frac{r_x}{\tau_0},\ b\frac{r_y}{\tau_0},\ \sinh[\eta]\left(1+b^2\frac{r_x^2+r_y^2}{2\tau_0^2}\right)\right), \tag{48}$$

which describes a scaling longitudinal flow field merged with a linear transverse flow profile. The transverse flow is assumed to be nonrelativistic in the region where there is a significant contribution to particle production. The local temperature distribution $T(x)$ at the last interaction points is assumed to have the form

$$\frac{1}{T(x)}=\frac{1}{T_0}\left(1+a^2\frac{r_x^2+r_y^2}{2\tau_0^2}\right)\left(1+d^2\frac{(\tau-\tau_0)^2}{2\tau_0^2}\right), \tag{49}$$

and the local rest density distribution is controlled by the chemical potential $\mu(x)$ for which we have the ansatz

$$\frac{\mu(x)}{T(x)}=\frac{\mu_0}{T_0}-\frac{r_x^2+r_y^2}{2R_G^2}-\frac{(\eta-y_0)^2}{2\Delta\eta^2}. \tag{50}$$

The parameters R_G and $\Delta\eta$ control the density distribution with finite geometrical sizes. The proper-time distribution of the last interaction points is assumed to have the following simple form:

$$H(\tau)=\frac{1}{(2\pi\Delta\tau^2)^{(1/2)}}\exp[-(\tau-\tau_0)^2/(2\Delta\tau^2)]. \tag{51}$$

The parameter $\Delta\tau$ stands for the width of the freeze-out hypersurface distribution; i.e., the emission is from a layer of hypersurfaces which tends to an infinitely narrow hypersurface in the $\Delta\tau\to 0$ limit.

The emission function, specified by Eqs. (47)–(51), is not invariant to boosts, neither in the longitudinal nor in the transverse directions. Although the flow profile, Eq. (48), was assumed to be invariant under longitudinal boosts, the finite longitudinal size which enters the model through the chemical potential in Eq. (50) breaks the longitudinal boost invariance of the emission function. [Note that the longitudinal boost invariance of the flow profile is supported by the NA35, NA49, and NA44 measurements for $R_L(M_t)$, to the best currently available experimental precision [2,3,27]].

This emission function corresponds to a Boltzmann approximation to the local momentum distribution of a longitudinally expanding, finite system which expands into the transverse directions with a transverse flow which is nonrelativistic at the saddle point. The transverse gradients of the local temperature at the last interaction points are controlled by the parameter a. The strength of the flow is controlled by the parameter b. The parameter $c=1$ is reserved to denote the speed of light, and the parameter d controls the strength of the change of the local temperature during the course of particle emission.

Note that other shapes of the temperature profile lead to the same result if $1/T(x)$ starts with the same second-order Taylor expansion around $r_x=r_y=0$. The physical significance of the transverse temperature profile is that it concentrates the emission of the particles with high transverse mass to a region which is centered around $r_x=r_y=0$ and which narrows as the transverse mass increases. The Gaussian approximation to the inverse temperature profile is thus a technical simplification only; other decreasing temperature profiles have similar effects as follows from the above picture. Similarly, the significance of the temporal changes of the temperature is that it creates different effective emission times for particles with different transverse mass, and the Gaussian approximation is a technical simplification.

For the case of $a=b=d=0$, we recover the case of longitudinally expanding finite systems as presented in Ref. [9]. The finite geometrical and temporal length scales are represented by the transverse geometrical size R_G, the geometrical width of the space-time rapidity distribution $\Delta\eta$, and the mean duration of the particle emission $\Delta\tau$. Effects arising from the finite longitudinal size were calculated analytically first in Ref. [28] in certain limited regions of the phase space. We assume here that the finite geometrical and temporal scales as well as the transverse radius and proper-time dependence of the inverse of the local temperature can be represented by the mean and the variance of the respective variables; i.e., we apply a Gaussian approximation, corresponding to the forms listed above, in order to get analytically tractable results. We have first proposed the $a=0$, $b=1$, and $d=0$ version of the present model, and elaborated also the $a=b=d=0$ model [9] corresponding to longitudinally expanding finite systems with a constant freeze-out temperature and no transverse flow. Soon after the parameter b was introduced [12] and it has been realized that the maximum of the emission function for a given mean momentum K has to be close to the beam axis, fulfilling $r_{x,z}\ll\tau_0$, in order to get a transverse mass scaling law for the parameters of the Bose-Einstein correlation functions in certain limiting cases [29]. In this region around the beam axis, however, the transverse flow is nonrelativistic [12] even for the case $b=1$ if this region is sufficiently small. Sinyukov and collaborators classified the various cases of ultrarelativistic transverse flows [11,30], and introduced a parameter which controls the transverse temperature profile, corresponding to the $a\neq b=0$ case. We have studied [29] the model class $a\neq 0$, $b\neq 0$, and $d=0$, which we extend here to the $d\neq 0$ case too.

The integrals of the emission function are evaluated using the saddle-point method [8,10,12]. The saddle point coincides with the maximum of the emission function, parametrized by $(\tau_s, \eta_s, r_{x,s}, r_{y,s})$. These coordinate values solve simultaneously the equations

$$\frac{\partial S}{\partial \tau}=\frac{\partial S}{\partial \eta}=\frac{\partial S}{\partial r_x}=\frac{\partial S}{\partial r_y}=0. \tag{52}$$

These saddle-point equations are solved in the LCMS, the longitudinally comoving system, for $\eta_s^{\rm LCMS}\ll 1$ and $r_{x,s}\ll\tau_0$. The approximations are self-consistent if $|Y-y_0|\ll 1+\Delta\eta^2 m_t/T_0-\Delta\eta^2$ and $\beta_t\ll\tau_s^2\Delta\eta_*^2/(bR_*^2)$, which for the considered model can be simplified as $\beta_t=p_t/m_t\ll(a^2+b^2)/b/\max(1,a,b)$. The transverse flow is nonrelativistic at the saddle point if $\beta_t\ll(a^2+b^2)/b^2/\max(1,a,b)$. We assume that $\Delta\tau<\tau_0$ so that the Fourier integrals involving $H(\tau)$ in the $0\leq\tau<\infty$ domain can be extended to the $-\infty<\tau<\infty$ domain. The radius parameters are evaluated here to the leading order in $r_{x,s}/\tau_0$. Thus terms of $O(r_{x,s}/\tau_0)$ are neglected; however, we keep all the higher-order correction terms arising from the nonvanishing value of η_s in the LCMS. We calculate both the radius parameters and the invariant momentum distribution in Gaussian saddle-point approximation. We shall discuss the limitations of the saddle-point method after presenting these results on the BECF and IMD.

For the model of Eq. (47) the saddle-point approximation for the integrals leads to an effective emission function which can be factorized similarly to Eq. (20). Thus the radius parameters of the model are expressible in terms of the homogeneity lengths $\Delta\eta_*$, R_*, and $\Delta\tau_*$ and the position of the saddle point η_s, i.e., the cross-term generating hyperbolic mixing angle. The saddle point in the LCMS is given by $\tau_s=\tau_0$, $\eta_s^{\rm LCMS}=(y_0-Y)/[1+\Delta\eta^2(1/\Delta\eta_T^2-1)]$, $r_{x,s}=\beta_t bR_*^2/(\tau_0\Delta\eta_T^2)$, and $r_{y,s}=0$. Note that the space-time rapidity of the saddle point $\eta_s^{\rm LCMS}$ depends on the boost-invariant difference y_0-Y which can be evaluated in any frame. The radius parameters or *lengths of homogeneity* [8,12] are given in the LCMS by Eqs. (27)–(29) and (33)–(35), and we obtain

$$\frac{1}{R_*^2}=\frac{1}{R_G^2}+\frac{1}{R_T^2}\cosh[\eta_s^{\rm LCMS}], \tag{53}$$

$$\frac{1}{\Delta\eta_*^2}=\frac{1}{\Delta\eta^2}+\frac{1}{\Delta\eta_T^2}\cosh[\eta_s^{\rm LCMS}]-\frac{1}{\cosh^2[\eta_s^{\rm LCMS}]}, \tag{54}$$

$$\frac{1}{\Delta\tau_*^2}=\frac{1}{\Delta\tau^2}+\frac{1}{\Delta\tau_T^2}\cosh[\eta_s^{\rm LCMS}]. \tag{55}$$

where the *thermal length scales* are given by

$$R_T^2=\frac{\tau_0^2}{a^2+b^2}\frac{T_0}{M_t}, \tag{56}$$

$$\Delta \eta_T^2 = \frac{T_0}{M_t}, \tag{57}$$

$$\Delta \tau_T^2 = \frac{\tau_0^2}{d^2} \frac{T_0}{M_t}. \tag{58}$$

Here $M_t = \sqrt{K_0^2 - K_L^2}$ is the transverse mass belonging to the mean momentum K. In the region of the Bose-Einstein enhancement, where the relative momentum of the pair is small, M_t satisfies $M_t = \frac{1}{2}(m_{t,1} + m_{t,2})$ $[1 + O(y_1 - y_2) + O((m_{t,1} - m_{t,2})/(m_{t,1} + m_{t,2}))]$. Note the distinction between the subscripts for the transverse direction, indicated by t and the use subscripts for the ''thermal'' scales indicated by T. It is timely to emphasize at this point that the parameters of the Bose-Einstein correlation function coincide with the (rapidity and transverse mass dependent) *lengths of homogeneity* [8] in the source, which physically can be identified with that region in coordinate space where particles with a given momentum are emitted from. The above relations indicate that these lengths of homogeneity for simple thermal models can be basically obtained from two type of scales in the framework of the saddle-point method. These scales have different momentum dependence and are referred to as ''thermal'' and ''geometrical'' scales.

In contrast to the homogeneity lengths, which can be defined even without thermalization, the ''thermal scales'' cannot be introduced without at least approximate local thermalization. Thus the thermal scales originate from the factor $\exp[-p \cdot u(x)/T(x)]$. They measure that region in space-time where thermal smearing can compensate the change of the local momentum distribution which in turn is induced by either the gradients of the flow field or the gradients of the temperature field. This is to be contrasted to the ''geometrical'' scales, which originate from the $\exp[\mu(x)/T(x)]$ factor which controls the density distribution. The geometrical scales can be interpreted as the regions in space-time where there is significant density to have particle emission. Obviously, for locally thermalized systems both the geometrical and thermal scales influence the regions of homogeneity and the smaller scale will be the dominant one. Since the four-momentum p is explicit in the factor $\exp[-p \cdot u(x)/T(x)]$ and enters the ''geometrical'' scales only through the momentum dependence of the saddle point, the momentum dependences for the ''thermal'' and ''geometrical'' scales shall be in general different from each other. Note also that in the above expression for $\Delta \eta_*$ a third type of scale is also present in the term $-1/\cosh^2[\eta_s]$, which stems from the $m_t \cosh[\eta - y]$ Cooper-Frye prefactor in Eq. (47). Thus this term is related to the shape of the freeze-out hypersurface distribution (which distribution tends to a single hypersurface if $\Delta \tau \to 0$).

The parameters of the BECF-s are dominated by the smaller of the geometrical and the thermal scales not only in the spatial directions, but in the temporal direction too according to Eqs. (53)–(58). These analytic expressions show that *even a complete measurement of the parameters of the BECF as a function of the mean momentum K may not be sufficient to determine uniquely the underlying phase-space distribution* [29,8,9,12,13]. We also can see that the LCMS frame approximately coincides with the LSPS frame for pairs with $|y_0 - Y| \ll 1 + \Delta \eta^2 M_t/T_0 - \Delta \eta^2$ and the terms arising from the nonvanishing values of η_s can be neglected. In this approximation, the cross-term generating hyperbolic mixing angle $\eta_s \approx 0$; thus, we find the leading-order LCMS result

$$C(\Delta k;K) = 1 + \lambda_* \exp(-R_L^2 Q_L^2 - R_{\rm side}^2 Q_{\rm side}^2 - R_{\rm out}^2 Q_{\rm out}^2), \tag{59}$$

with a vanishing out-long cross term $R_{{\rm out},L} = 0$. To leading order, the parameters of the correlation function are given by

$$R_{\rm side}^2 = R_*^2, \tag{60}$$

$$R_{\rm out}^2 = R_*^2 + \beta_t^2 \Delta \tau_*^2, \tag{61}$$

$$R_L^2 = \tau_0^2 \Delta \eta_*^2. \tag{62}$$

Observe that the difference of the side and out radius parameters is dominated by the lifetime *parameter* $\Delta \tau_*$. Thus a vanishing difference between the $R_{\rm out}^2$ and $R_{\rm side}^2$ can be generated dynamically if the duration of the particle emission is large, but the thermal duration $\Delta \tau_T$ becomes sufficiently small [cf. Eq. (55)]. This in turn can be associated with intensive changes in the local temperature distribution during the course of the particle emission.

Observe, that the BECF in an arbitrary frame can be obtained from combining Eqs. (53)–(58) with the general expressions given by Eqs. (27)–(32). In that case, the value of $\eta_s = Y + \eta_s^{\rm LCMS} = Y + (y_0 - Y)/[1 + \Delta \eta^2 (1/\Delta \eta_T^2 - 1)]$ has to be used in Eqs. (27)–(32).

Note also that in our results higher-order terms arising from the nonvanishing value of η_s in the LCMS are summed up, while in Refs. [12] the first subleading corrections were found.

VII. INVARIANT MOMENTUM DISTRIBUTIONS

The IMD plays a *complementary role* to the measured Bose-Einstein correlation function [9,13,29]. Thus a *simultaneous analysis* of the Bose-Einstein correlation functions and the IMD may reveal information both on the temperature and flow profiles and on the geometrical sizes.

For the considered model, Eq. (47), the invariant momentum distribution can be calculated in such a manner that the Cooper-Frye prefactor $m_t \cosh(\eta - y)$ is kept exactly and the saddle-point approximation is applied to the remaining Boltzmann and proper-time factors, $\exp[-p \cdot u/T(x) + \mu(x)/T(x)]H(\tau)$. This calculation yields

$$N_{1,c}(\mathbf{p}) = \frac{g}{(2\pi)^3} (2\pi \overline{\Delta \eta_*^2} \tau_0^2)^{1/2} (2\pi \overline{R_*^2}) \frac{\overline{\Delta \tau_*}}{\Delta \tau} \times m_t \cosh[\overline{\eta_s}] \exp(+\overline{\Delta \eta_*^2}/2) \times \exp[-p \cdot u(\overline{x_s})/T(\overline{x_s}) + \mu(\overline{x_s})/T(\overline{x_s})]. \tag{63}$$

The quantities $\overline{\Delta \eta_*^2}$ and $\overline{\eta_s}$ are defined as

$$\frac{1}{\overline{\Delta \eta_*^2}} = \frac{1}{\Delta \eta^2} + \frac{1}{\Delta \eta_T^2} \cosh[\overline{\eta_s}], \quad \overline{\eta_s} = \frac{(y_0 - y)}{1 + \Delta \eta^2/\Delta \eta_T^2}, \tag{64}$$

and the modified saddle point is located in the LCMS at $\overline{\tau_s}=\tau_s=\tau_0$, $\overline{\eta_s}$, $\overline{r}_{x,s}=\beta_t b\overline{R}_*^2/(\tau_0\Delta\eta_T^2)$ and $\overline{r}_{y,s}=0$. The modified radius and lifetime parameters can be obtained by evaluating the R_* and $\Delta\tau_*$ parameters at the space-time rapidity coordinate of the modified saddle point, $\overline{R}_* = R_*(\eta_s^{\rm LCMS}\to\overline{\eta_s})$ and $\overline{\Delta\tau_*}=\Delta\tau_*(\eta_s^{\rm LCMS}\to\overline{\eta_s})$. Thus the modified quantities (indicated by overline) differ from the unmodified parameters of the saddle-point approximation by the contributions of the Cooper-Frye prefactor. This happens because Eq. (63) is obtained by applying the saddle-point method for Eq. (3) with the model emission function, Eq. (47), in such a way that the Cooper-Frye prefactor $m_t\cosh[\eta-y]$ is kept exactly and the remaining factors are approximated with the saddle-point technique in the LCMS [29], described in detail in the previous section. Note that $\overline{R}_*\simeq R_*$ and $\overline{\Delta\tau_*}\simeq\Delta\tau_*$ in the midrapidity region, where $\eta_s^{\rm LCMS}$, $\overline{\eta_s}\ll 1$.

In Eq. (63) the exact shape of the four-velocity field can be used, which is given in the first line of Eq. (48). When evaluating $\mu(\overline{x}_s)/T(\overline{x}_s)$ in any frame, the invariant difference $\overline{\eta}_s - y_0^{\rm LCMS}=\overline{\eta}_s+y-y_0$ should be used.

The momentum distribution as given in Eq. (63) can be rewritten into a more explicit shape, which is more suitable for analytic study. This can be done if one neglects terms of $O(\overline{r}_{x,s}{}^3/\tau_0^3)$ in the exponent, which is the same order of accuracy which has been utilized for the solution of the saddle-point equations. Further, a term in the exponent $(m_t/T_0)\cosh(\overline{\eta_s})$ is approximated by its second-order Taylor expansion $(m_t/T_0)(1+0.5\overline{\eta_s}^2)$. These approximations yield

$$\exp\left(-\frac{p\cdot u(\overline{x_s})}{T(\overline{x_s})}+\frac{\mu(\overline{x_s})}{T(\overline{x_s})}\right)$$
$$\simeq\exp\left(\frac{\mu_0}{T_0}\right)\left(-\frac{(y-y_0)^2}{2(\Delta\eta^2+\Delta\eta_T^2)}\right)$$
$$\times\exp\left[-\frac{m_t}{T_0}\left(1-f\,\frac{\beta_t^2}{2}\,\frac{T_G}{T_0+T_G\,\cosh[\overline{\eta_s}]}\right)\right],$$

where the geometrical contribution to the effective temperature is indicated by $T_G(m_t)=T_0R_G^2/R_T^2(m_t) = (a^2+b^2)m_tR_G^2/\tau_0^2$ and the fraction f is defined as $f=b^2/(a^2+b^2)$, satisfying $0\leqslant f\leqslant 1$.

The Boltzmann factor is further simplified in the midrapidity region $|y-y_0|\ll 1+(m_t/T_0)\Delta\eta^2$, where $\cosh(\overline{\eta_s})\simeq 1$. The resulting expression for the IMD is given by

$$N_{1,c}(\mathbf{p})=\frac{g}{(2\pi)^3}\,(2\pi\overline{\Delta\eta_*^2}\tau_0^2)^{1/2}(2\pi R_*^2)\,\frac{\Delta\tau_*}{\Delta\tau}\,m_t\cosh[\overline{\eta_s}]\exp(+\overline{\Delta\eta_*^2}/2)\;\;\exp(\mu_0/T_0)$$
$$\times\exp\left(-\frac{(y-y_0)^2}{2(\Delta\eta^2+\Delta\eta_T^2)}\right)\exp\left[-\frac{m_t}{T_0}\left(1-f\,\frac{\beta_t^2}{2}\right)\right]\exp\left(-f\,\frac{m_t\beta_t^2}{2(T_0+T_G)}\right), \tag{65}$$

which is describing a momentum distribution peaked at midrapidity, corresponding to the finiteness of the considered source. A detailed analytic study of the invariant momentum distribution of Eq. (65) can be performed as follows.

For the considered model, the rapidity width $\Delta y(m_t)$ of the invariant momentum distribution at a given m_t shall be dominated by the *longer* of the thermal and geometrical length scales. If the condition $a\simeq 0$ is fulfilled, i.e., $f\approx 1$, the longer of the thermal and geometrical scales shall also dominate T_*, the effective temperature (slope parameter) at a midrapidity y_0. The following relations hold:

$$\Delta y^2(m_t)=\Delta\eta^2+\Delta\eta_T^2(m_t),$$
$$\frac{1}{T_*}=\frac{f}{T_0+T_G(m_t=m)}+\frac{1-f}{T_0}. \tag{66}$$

That is why the IMD measurements can be considered to be complementary to the BECF data.

In the special limiting case when gradients of the temperature are negligible, $a=0$ and $f=1$, we have $T_*=T_0+mb^2R_G^2/\tau_0^2$. If the flow velocity at the geometrical radius $\langle u_t\rangle\equiv bR_G/\tau_0$ is independent of the particle type, we obtain a relation

$$T_*=T_0+m\langle u_t\rangle^2. \tag{67}$$

A similar relation for the mass dependence of the mean (transverse) kinetic energy was introduced in Ref. [31] for longitudinally expanding systems with nonrelativistic transverse flows. This simple relation (67) for the effective temperature can be considered as a special case of the more general Eq. (66).

The measured IMD can be obtained from the IMD of the core as given above and from the measured $\lambda_*(\mathbf{p})$ parameters of the BECF as

$$N_1(\mathbf{p})=\frac{1}{\sqrt{\lambda_*(\mathbf{p})}}\,N_{1,c}(\mathbf{p}); \tag{68}$$

see, e.g., Ref. [18] for further details.

The invariant momentum distribution described by Eq. (65) features two types of low transverse momentum enhancement as compared to a static thermal source with a slope parameter T_*. One may introduce the *volume factor* or $V_*(y,m_t)$ which yields the momentum-dependent size of the region, where the particles with a given momentum are emitted from

$$V_*(y,m_t)=(2\pi\overline{\Delta\eta_*^2}\tau_0^2)^{1/2}(2\pi R_*^2)\,\frac{\Delta\tau_*}{\Delta\tau}. \tag{69}$$

The *rapidity-independent low-p_t enhancement* is a consequence of the transverse mass dependence of this effective volume, which may depend on m_t for certain limiting cases in the following way:

$$V_*(y,m_t)\propto\left(\frac{T_0}{m_t}\right)^{k/2}, \qquad (70)$$

where $k=0$ for a static fireball ($a=b=d=0$ and $\Delta\eta_T^2 \gg \Delta\eta^2$). The case $k=1$ is satisfied for $a=b=d=0$ and $\Delta\eta^2 \gg \Delta\eta_T^2$, which describes long, one-dimensionally expanding finite systems [9]. The case $k=2$ corresponds to $a=b=0\neq d$ and $\Delta\eta^2\gg\Delta\eta_T^2$, describing longitudinally expanding systems with cooling. The case $k=3$ corresponds to $a\neq 0$, $b\neq 0=d$, i.e., three-dimensionally expanding, cylindrically symmetric, finite systems possibly with a transverse temperature profile [29], and the $k=4$ case corresponds to the same but with a $d\neq 0$ parameter, describing the temporal changes in the local temperature during the particle emission process appended with the condition $\Delta\tau_T\ll\Delta\tau$. Thus the inclusion of this effective m_t-dependent volume factor into the data analysis not only would undoubtedly increase the precision of the measurements of the slope parameters, but in turn it also could shed light on the dynamics of the particle emission from such complex systems.

Note that the m_t-dependent effective volume factor, Eq. (70), enhances the production of direct pions at low p_t, as compared to that of heavy resonances with mass M_h at low p_t, by a factor of $f_e=V_*(m_\pi)/V_*(M_h)=(M_h/M_\pi)^{(k/2)}$, where $k=0$, 1, 2, 3, or 4 is related to the dimension of the expansion. We argue in Sec. IX that the $k=3$ or $k=4$ case is supported by NA44 data. This yields an enhancement factor of $f_e=(770/140)^{(k/2)}=12.90$ or 30.25, respectively, for the production of direct pions from the core at low p_t as compared to ρ mesons. See also Fig. 2. of Ref. [18] for a more detailed account on this topic.

The *rapidity-dependent low-p_t enhancement*, which is a generic property of the longitudinally expanding finite systems [32], reveals itself in the rapidity dependence of the effective temperature, defined as the slope of the exponential factors in the IMD in the low-pt limit at a given value of the rapidity. The leading order [32] result is

$$T_{\rm eff}(y)=\frac{T_*}{1+a(y-y_0)^2} \quad \text{with} \quad a=\frac{T_0T_*}{2m^2}\left(\Delta\eta^2+\frac{T_0}{m}\right)^{-2}. \qquad (71)$$

Please note that this analysis of the low transverse mass region of the IMD relies on the applicability of the saddle-point method in the low transverse momentum region too. Thus it may be valid for kaons or heavier particles (as well as locally very cold pionic systems). However, in case of pions, the self-consistency of the applied formulas and their region of validity has to be very carefully checked. This region of the applicability of the saddle-point technique for the considered finite systems is discussed in detail in the next section. Note that the low transverse momentum region is populated by a number of resonance decays. For the long-lived resonances, thus, a nontrivial $1/\sqrt{\lambda_*(p)}$ factor may appear and contribute to both the rapidity-dependent and the rapidity-independent low-p_t enhancement. Although this factor is measurable from the shape analysis of the BECF, care is required to study the contribution of the decay products of short-lived resonances to the momentum distribution of pions. Kaons or other heavier particles thus provide a cleaner test for these analytic results as compared to pions [33].

The *high-p_t enhancement or decrease* refers to the change of the effective temperature at midrapidity with increasing m_t. The large transverse mass limit T_∞ shall be in general different from the effective temperature at low pt given by T_* since

$$T_\infty=\frac{2T_0}{2-f}, \quad \frac{T_\infty}{T_*}=\frac{2}{2-f}\left(1-f\,\frac{T_G(m)}{T_0+T_G(m)}\right). \qquad (72)$$

Utilizing $T_G/T_0=R_G^2/R_T^2$, the high-p_t enhancement or decrease turns out to be controlled by the ratio of the thermal radius $R_T(m_t=m)$ to the geometrical radius R_G. One obtains $T_\infty>T_*$ if $R_T^2(m)>R_G^2$ and similarly $T_\infty<T_*$ if $R_T^2(m)<R_G^2$. Since for large colliding nuclei R_G is expected to increase, a possible high-pt decrease in these reactions may become a geometrical effect, a consequence of the large size.

VIII. LIMITATIONS

The simple analytic formulas presented in the previous sections are obtained in a saddle-point approximation for the evaluation of the space-time integrals. This approximation is known to converge to the exact result in the limit the integrated function develops a sufficiently narrow peak; i.e., both $\Delta\eta_*^2\ll 1$ and $\Delta\tau_*^2/\tau_s^2\ll 1$ are required. This in turn gives a lower limit in m_t for the applicability of the formulas for the class of models presented in the previous section. (However, the saddle-point method may give precise results even when the integrand does not develop a narrow peak. For the emission function of Ref. [13] the saddle-point method gives an exact result, because that emission function can be rewritten in Gaussian form.)

From the requirement $\Delta\eta_*^2\ll 1$, we have

$$m_t\cosh[\eta_s]/T_0\gg 1+1/\cosh^2[\eta_s]-1/\Delta\eta^2. \qquad (73)$$

From the requirement $\Delta\tau_*^2/\tau_s^2\ll 1$, one gets

$$m_t\cosh[\eta_s]/T_0\gg(1-\tau_s^2/\Delta\tau^2)/d^2. \qquad (74)$$

These are the conditions governing the validity of the calculation of the parameters of the BECF as presented in Sec. VI. Compared to the condition (73), the condition of validity of the calculation of the invariant momentum distribution is less stringent, since one needs to satisfy only $\overline{\Delta}\eta_*^2\ll 1$, which yields

$$m_t\cosh[\overline{\eta_s}]/T_0\gg 1-1/\Delta\eta^2. \qquad (75)$$

For the case of the NA44 measurement, one may *estimate* the region of reliability for the analytic formulas presented in the previous parts using inequalities (73) and (74). Since the data indicate $R_{\rm side}\simeq R_{\rm out}$ within errors, the inequality $\Delta\tau_*^2/\tau_s^2\ll 1$ and its consequence, Eq. (74), seem to be well justified. In the inequality (73) the finite longitudinal size plays an important role. For infinite systems, $\Delta\eta=\infty$, the calculations of BECF parameters are reliable for $m_t \gg 2T_0$ (since $\eta_s = 0$ for infinite systems in the LCMS), while the calculations for the IMD are reliable to $m_t \gg T_0$. In the midrapidity region where NA44 data were taken, one has $\eta_s \simeq 0$ and for *finite* systems one finds $m_t \gg T_0(2-1/\Delta\eta^2)$. Note that this estimated lower limit in m_t is extremely sensitive to the precise value of $\Delta\eta$ in the region $\Delta\eta \simeq 1/\sqrt{2} \approx 0.7$. Thus for finite systems the region of applicability of our results extends to lower values of m_t than for infinite systems which were recently studied in great detail in Ref. [23]. The inequality (73) can be used in basically two ways: Either one assumes a value for T_0 and then obtains the lower limit in m_t for the applicability of the saddle-point method, or one assumes that the saddle-point method is applicable to a certain value of m_t (e.g., in case it gives a good description of data), and then one obtains an upper limit for the corresponding T_0 parameter, the central temperature at the mean time of last interactions.

An upper transverse momentum limit is obtained for the validity of the calculations in Secs. VI and VII from the requirement $\tau_{x,s}/\tau_0 < 1$ or $r_{x,s}^2/\tau_0^2 \ll 1$. This condition and the requirement $br_{x,s}/\tau_0 < 1$ are fulfilled simultaneously if

$$\beta_t \ll \frac{a^2+b^2}{b\ \max(1,b)\max(1,a,b)}. \tag{76}$$

If $a^2+b^2 \simeq 1$, this condition simplifies to $\beta_t \ll 1/b$.

When comparing to data, detailed numerical studies may be necessary [23] to check the precision of the saddle-point integration. In a subsequent paper we plan to present these studies together with a detailed comparison of the model to the available NA44 data.

IX. LIMITING CASES

Observe that both the thermal and the geometrical length scales enter both the parameters of the Bose-Einstein correlation function and those of the invariant momentum distribution. We limit the discussion in this part to the midrapidity region $\eta_s^{\rm LCMS} \simeq 0$ and we assume $\Delta\eta_*^2 \ll 1$; i.e., we neglect the $1/\cosh^2[\eta_s]$ term in Eq. (54). Various limiting cases can be obtained as combinations of basically the relative size of the thermal and the geometrical scales in the transverse, longitudinal, and temporal directions. These in turn are the following.

(i) If $R_T(M_t) \gg R_G$ in a certain M_t interval, we have also $T_0 \gg T_G(m_t)$ at the same transverse mass scale. In this region, the side radius parameter shall be determined by the geometrical size $R_{\rm side} = R_* \simeq R_G$; hence, it shall be transverse mass independent.

The m_t distribution at midrapidity shall be proportional to $\exp(-m_t/T_0)$.

(ii) If $\Delta\eta_T \gg \Delta\eta$, we have $R_L \simeq \tau_0 \Delta\eta$ and the rapidity width of the IMD shall be dominated by the thermal scale, $\Delta y^2(m_t) \simeq \Delta\eta_T^2 = T_0/m_t$.

(iii) If $\Delta\tau_T \gg \Delta\tau$, the temporal duration shall be measured by $R_{\rm out}^2 - R_{\rm side}^2 \simeq \beta_t^2 \Delta\tau^2$. The invariant momentum distribution shall be influenced only through the $\Delta\tau_*/\Delta\tau \approx 1$ factor in V_*.

These cases are rather conventional limiting cases. An unconventional limit complements each as follows.

(iv) If $R_T(M_t) \ll R_G$ in a certain M_t interval, we have also $T_0 \ll T_G(m_t)$ at the same transverse mass scale. In this region, the side radius parameter shall be determined by the thermal size $R_{\rm side} = R_* \simeq R_T(M_t)$; hence, it shall be transverse mass dependent, $R_{\rm side}^2 \propto \tau_0^2 T_0/M_t$.

The m_t distribution at midrapidity shall be proportional to $\exp(-m_t/T_*)$. If $a^2 \ll b^2$, we have $T_* \simeq T_G(m)$ as follows from Eq. (66).

(v) If $\Delta\eta_T \ll \Delta\eta$, we have the leading order LCMS result $R_L \simeq \tau_0 \Delta\eta_T \simeq \tau_0 \sqrt{T_0/M_t}$, and the rapidity width of the IMD shall be dominated by the geometrical scale, $\Delta y^2 \simeq \Delta\eta^2$.

(vi) If $\Delta\tau_T \ll \Delta\tau$, the *thermal* duration shall be measured by $R_{\rm out}^2 - R_{\rm side}^2 \simeq \beta_t^2 \Delta\tau_T^2 \simeq \beta_t^2 \tau_0^2 T_0/(d^2 M_t)$. For large values of the transverse mass, the model thus shall feature a dynamically generated vanishing duration parameter, which has a specific transverse mass dependence. The invariant momentum distribution shall be influenced only through the $\Delta\tau_*/\Delta\tau \simeq 1/\sqrt{m_t}$ factor in V_*.

Some combinations of cases (i)–(vi), are especially interesting, such as the following.

(vii) If all the finite geometrical source sizes R_G, $\Delta\eta$ and $\Delta\tau$ are large compared to the corresponding thermal length scales, we have, in the LCMS,

$$\Delta\tau_*^2 \simeq \frac{\tau_0^2}{d^2}\frac{T_0}{M_t}, \tag{77}$$

$$R_L^2 \simeq \tau_0^2 \frac{T_0}{M_t}, \tag{78}$$

$$R_{\rm side}^2 \simeq \frac{\tau_0^2}{a^2+b^2}\frac{T_0}{M_t}. \tag{79}$$

Thus, if $d^2 \gg a^2+b^2 \approx 1$, the model may feature *a dynamically generated vanishing duration parameter*. In this case, the model *predicts an M_t scaling for the duration parameter* as

$$\Delta\tau_*^2 \propto \frac{1}{M_t}. \tag{80}$$

This prediction could be checked experimentally if the error bars of the measured radius parameters were decreased to such a level that the difference between the out and side radius parameters would be significant.

Alternatively, if the vanishing duration parameter of the BECF is generated due to a very fast hadronization process as discussed in Ref. [25], then one has

$$\Delta\tau_*^2 \simeq \Delta\tau^2 \propto \text{const}; \tag{81}$$

i.e., in this case the duration parameter becomes independent of the transverse mass.

If the finite source sizes are large compared to the thermal length scales and if we also have $a^2+b^2 \approx 1$, one obtains an *M_t scaling* for the parameters of the BECF,

$$R_{\rm side}^2 \simeq R_{\rm out}^2 \simeq R_L^2 \simeq \tau_0^2 \frac{T_0}{M_t}, \tag{82}$$

valid for $\beta_t \ll 1/b$.

Note that this relation is independent of the particle type and has been observed in the recent NA44 data in S+Pb reactions at CERN SPS [3]. Preliminary NA49 data for Pb+Pb at CERN SPS are also compatible with this scaling law [27]. This M_t scaling may be valid to arbitrarily large transverse masses with $\beta_t \approx 1$ if $b \ll 1$. The lower limit of the validity of this relation is given by the applicability of the saddle-point method, Eqs. (73) and (74). To generate a vanishing difference between the side and out radii and an M_t scaling for the BECF radii simultaneously, the parameters have to satisfy the inequalities (73) and (74) as well as $b \ll a^2 + b^2 \approx 1 \ll d^2$, i.e., the cooling should be the fastest process, the next dominant process within this phenomenological picture has to be the development of the transverse temperature profile, and finally the transverse flow shall be relatively weak. If the temporal changes of the temperature are not intensive enough, then a small lifetime parameter can also be obtained by a fast hadronization and simultaneous freeze-out as discussed in Ref. [25] with $\Delta\tau \approx 0$.

We would like to emphasize that there are *a number of conditions* in the model which need to be satisfied simultaneously to get the scaling behavior, which is supported by nine NA44 data points (three for kaons and six for pions [3]). One has to wait for *future data points* to learn more about the experimental status of the scaling. The model presented in this paper *may describe more complex* transverse momentum dependences of the parameters of the Bose-Einstein correlation function, too; the M_t scaling is only one of its virtues in a specific limiting case. However, it is rather difficult to get a limiting case with $R_L \approx R_{\rm side} \approx R_{\rm out} \propto 1/\sqrt{M_t}$ in analytically solvable models. Such a behavior is related to the cylindrical symmetry of the emission function.

Thus the symmetry of the BECF in the LCMS can be considered as a strong indication for a *three-dimensionally expanding, cylindrically symmetric* source, possibly with a transverse and temporal temperature profile. The LCMS frame is selected if the mean emission point or saddle point stays close to the symmetry axis even for particles with a large transverse mass, $r_{x,s}(m_t) \ll \tau_0$, and if the finite longitudinal size introduces only small difference between the LCMS and LSPS frames, i.e., $|y - y_0| \ll 1 + \Delta\eta^2(m_t/T_0 - 1)$. In this case considered in Sec. VI the emission function is cylindrically symmetric and so the BECF is symmetric in the LCMS of the pair (and *not* in the center of mass system of the pair [34]).

(viii) It is interesting to investigate the other limiting case when $R_T \gg R_G$, $\Delta\eta_T \gg \Delta\eta$ and $\Delta\tau_T \gg \Delta\tau$ by combining the limiting cases (i)–(iii). In this case one obtains

$$R_L^2 \simeq \tau_0^2 \Delta\eta^2 = R_{L,G}^2, \quad R_{\rm side}^2 \simeq R_G^2, \quad R_{\rm out}^2 \simeq R_G^2 + \beta_t^2 \Delta\tau^2, \tag{83}$$

$$\Delta y^2(m_t) \simeq \Delta\eta_T^2 = \frac{T_0}{m_t}, \quad T_* = T_0. \tag{84}$$

Thus, if the thermal length scales are larger than the geometrical sizes in all directions, the BECF measurement determines the geometrical sizes properly, and the p_t and the dn/dy distributions are determined by the temperature of the source. In this case the momentum distribution reads as

$$N_1(\mathbf{p}) \propto m_t \cosh(y - y_0) \exp\left(-\frac{m_t \cosh(y - y_0)}{T_0} \right), \tag{85}$$

which is a thermal distribution for a static source located at the midrapidity y_0.

Thus, *two length scales are present* in all the three principal directions of three-dimensionally expanding systems. The BECF radius parameters are dominated by the *shorter* of the thermal and geometrical length scales. However, the rapidity width of the $d^2N/dy/dm_t^2$ distribution, $\Delta y^2(m_t)$, is the quadratic sum of the geometrical and the thermal length scales; thus, it is dominated by the *longer* of the two. Similarly, the effective temperature is dominated by the *higher* of the two temperature scales for $f \approx 1$ according to Eq. (66). The effective temperature of the m_t distribution is decreasing in the target and projectile rapidity region in this class of analytically solvable models.

This study is a generalization of the basic ideas presented and illustrated in Ref. [13] for the case of three-dimensionally expanding, cylindrically expanding, cylindrically symmetric finite systems with a scaling longitudinal flow, weak transverse flow, and a transverse and temporal temperature profile.

X. SUMMARY

A general formulation is presented for the two-particle Bose-Einstein correlation function for cylindrically symmetric systems undergoing collective hydrodynamic expansion; cf. Eqs. (15), (18), (21), and (22). Note that these relations were shown to be valid for certain limited classes of emission functions. The resulting class of Bose-Einstein correlation functions, however, includes non-Gaussian correlation functions too. The case of Gaussian correlation functions is studied in detail and the radius parameters are expressed in the lab, LCMS, and LSPS systems, where the functional form of the correlation functions becomes more and more simplified. The cross-term generating hyperbolic mixing angle is identified with the value of the η variable of the saddle point in the considered frame.

A class of Gaussian models is introduced which in some regions of the model parameters may obey an M_t *scaling for the side, out, and longitudinal radius parameters*. A vanishing effective duration of the particle emission may be generated by the temporal changes of the local temperature during the evaporation. The model predicts an M_t *scaling also for the duration parameter* in this limiting case.

Finally, we stress that *both* the invariant momentum distribution and the Bose-Einstein correlation function may carry only partial information about the phase-space distribution of particle emission. However, their *simultaneous analysis* sheds more light on the dynamics and the geometrical source-sizes.

Note added in proof. We would like to draw attention to a recent paper by T. Csörgő, P. Lévai, and B. Lörstad (Report No. hep-ph/9603373, to be published in Acta Physica Slovaca), where the model and the analytical approximations presented above were subjected to a careful numerical study in various domains of the parameter space and the results were shown in 16 figures, completed after this manuscript had been submitted for publication. For graphical illustration of the main ideas presented in the present article, we recommend another paper: T. Csörgő and B. Lörstad, Report No. hep-ph/9511404, in *Proceedings of the XXVth International Symposium on Multiparticle Dynamics*, Stara Lesna, Slovakia, 1995 edited by L. Sandor *et al.* (World Scientific, Singapore, 1996).

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APPENDIX

In this appendix we give a simple example when the Fourier-transformed emission function exists, but the Gaussian version of the saddle-point method is not applicable. Let us consider the one-dimensional Lorentzian distribution function

$$S(r)=\frac{1}{\pi R}\,\frac{1}{(1+r^2/R^2)}. \tag{A1}$$

Here r is a real variable (in one dimension). The corresponding correlation function is

$$C(q)=1+|\tilde{S}(q)|^2, \tag{A2}$$

with

$$\tilde{S}(q)=\int_{-\infty}^{\infty}dr\; S(r)\exp(-iqr), \tag{A3}$$

which yields

$$C(q)=1+\exp(-2|q|R). \tag{A4}$$

This function is not analytic at $q=0$ because it depends on the modulus of q, and for positive values of q its Taylor expansion starts with a linear term. This is to be contrasted with the results for the saddle-point method. If the Gaussian version of the saddle-point method is applicable, then $\tilde{S}(q)$ can be expanded into a Taylor series around $q=0$ as

$$\tilde{S}(q)=1+i\langle r\rangle q-\langle r^2\rangle q^2/2+\cdots\ . \tag{A5}$$

Here the average of a function of variable r is defined as

$$\langle f(r)\rangle=\int_{-\infty}^{\infty}dr f(r)S(r), \tag{A6}$$

and the two-particle correlation function can be written as

$$C(q)\approx 2-q^2R_G^2\approx 1+\exp(-q^2R_G^2), \tag{A7}$$

with

$$R_G^2=\langle r^2\rangle-\langle r\rangle^2. \tag{A8}$$

Since for the considered function $R_G^2=\infty$, the Gaussian saddle-point method is not applicable. Still, the Fourier-transformed emission function and the BECF exist, as given by Eq. (A4).

Cases similar to this are characterized by non-Gaussian correlation functions [20]. Similar examples can be found among multivariate distributions, too.

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