Decay out of superdeformed bands in the $A \approx 190$ mass region

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The decay out of the superdeformed (SD) bands in the 190-mass region is studied on the basis of the measured lifetimes and branching ratios of the low lying states in the SD bands of ^{192}Hg and ^{194}Pb. The decay of these bands is governed by very small admixtures of normal deformed (ND) states to the decaying SD states and by very large transition probabilities for statistical γ decay in the first potential minimum. The interaction between SD and ND states, which is responsible for the depopulation of the SD bands, can be described consistently by a simple mixing of the SD states with only the nearest neighboring ND states. Estimated values for the maximum interaction strength between ND and SD states and lower limits for the height of the barrier between first and second potential minimum are presented. [S0556-2813(96)05409-X]

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I. INTRODUCTION

Over 40 superdeformed (SD) bands have been investigated in the 190-mass region of the nuclear chart [1–3], but until very recently it had not been possible to link any of these bands to the rest of the level scheme. The search for linking transitions between the lowest SD states and the normal deformed states has been performed with great experimental effort. Only very recently was a breakthrough achieved in this search by the observation [4] of linking transitions between the yrast SD band in ^{194}Hg and normal deformed states. This observation has for the first time determined the excitation energy of a SD band in the 190-mass region. Before this exciting discovery some discrete transitions were found in coincidence with the yrast SD bands in ^{192}Hg [5] and ^{194}Pb [6] but could not be used to link the SD bands. In another experiment [7] a spectrum in coincidence with the SD band in ^{192}Hg was obtained, containing a continuous bump of transitions with an $E1$ multipolarity pattern. From this experiment and feeding studies of the SD band in ^{192}Hg [8,9] estimates for the excitation energy of the yrast SD band in this nucleus were made. The $10\hbar$ SD state in ^{192}Hg is assumed to be placed at about 4.3 MeV above the yrast line. This estimate of the excitation energy seems well supported by the experimental excitation energy of the yrast SD band in ^{194}Hg, which was found to be 4.2 MeV at spin $10\hbar$ [4].

The sudden disappearance of the SD bands at low spin together with the unobserved decay path has raised many questions concerning the mechanism involved in the decay-out process. Natural questions are the following. (1) Is the second potential minimum at the point of the decay out still separated from the first minimum by a sizable barrier? (2) Does the second minimum survive down to the SD bandhead? (3) Is a population of the SD states near the bandhead possible?

In this article we want to report how lifetimes and branching ratios of the low lying superdeformed states give essential information on the decay process. It will be shown that the decay out of the SD bands in the 190-mass region can be consistently described by a simple mixing model including the SD state and its nearest normal deformed (ND) neighbors in energy. The mixing can be associated with the height of the potential barrier that separates the first and second potential minima. Estimated values for mixing amplitudes will be given on the basis of assumptions for the excitation energies of the SD bands in ^{192}Hg and ^{194}Pb, which have so far not been firmly established. At these high excitation energies the decay out of the SD band is governed by very small mixing amplitudes between ND and SD states and is mainly due to statistical $E1$ transitions.

II. REVIEW OF THE PRESENT DATA

Lifetimes τ of SD states with sizable branchings to normal deformed states have been measured in ^{192}Hg [10–12] and ^{194}Pb [13]. The intraband SD transition quadrupole moments Q_t were calculated by using the relation $\lambda_{\text{intra}} = 1.22(1+\alpha) Q_t^2 E_\gamma^5 \langle IK20|(I-2)K\rangle^2$. Here the partial decay probability (λ_{intra}) for the intraband γ transition from a SD state with spin I and projection K (I,K) to the next lower SD state $(I-2,K)$ is given in ps^{-1} (see below), the transition energy (E_γ) in MeV, and the transition quadrupole moment (Q_t) in e b. α is the total conversion coefficient for the intraband transition. For the yrast SD bands of even-even nuclei a value of $K=0$ is expected and has been applied in the determination of Q_t. The Q_t values deduced from the lifetimes were found to be the same within the experimental errors as those measured via the Doppler-shift attenuation method (DSAM) technique in the high spin part of these bands [12,14,15]. This fact shows that the superdeformed configuration is still dominant for those band members that are involved in the first step of the depopulation of the SD band.

In order to get a feeling for the validity of this statement we want to investigate the decay out of the SD bands in more detail. The measured total transition probability $\lambda = 1/\tau$ is

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TABLE I. Review of the experimental information on the $12\hbar$ and $10\hbar$ SD states in ^{192}Hg and ^{194}Pb, respectively. γ-ray energies E_γ [6,16] and intraband transition intensities $N_{\rm intra}$ [6,16] are given. The lifetime for ^{194}Pb was taken from Ref. [13] and the value given for ^{192}Hg is the weighted average of the values published in Refs. [10–12] (see details in text). The deduced transition quadrupole moments from the recoil distance Doppler-shift (RDDS) experiments $Q_{t,\rm RDDS}$ are given as well as the mean quadrupole moments $Q_{t,\rm DSAM}$ obtained for the upper part of the two SD bands [12,14,15].

Nucl.	I [$\hbar$]	E_γ [keV]	$N_{\rm intra}$	τ [ps]	$\lambda_{\rm intra}$ [ps^{-1}]	$\lambda_{\rm out}$ [ps^{-1}]	$Q_{t,\rm RDDS}$ [e b]	$Q_{t,\rm DSAM}$ [e b]
^{192}Hg	12	257.8	0.88 (5)	4.9 (0.7)	0.18 (3)	0.024 (11)	18.3 ($^{+1.5}_{1.6}$)	18.6 (1.1)
^{194}Pb	10	214.1	0.85 (6)	8.6 (2.2)	0.10 (3)	0.017 (8)	20.4 ($^{+7.6}_{1.9}$)	20.6 (1.3)

sensitive to the mechanism that leads to the depopulation of the SD band. It is composed of the intraband transition probability $\lambda_{\rm intra}$ and the probability $\lambda_{\rm out}$ for a transition that leaves the SD band:

$$\lambda=\lambda_{\rm intra}+\lambda_{\rm out}. \tag{1}$$

For a certain state $\lambda_{\rm out}$ and $\lambda_{\rm intra}$ can be determined from the lifetime τ and the transition intensities $N_{\rm intra}$ and $N_{\rm out}$ ($N_{\rm out}+N_{\rm intra}=1$) by

$$\lambda_{\rm out}=\frac{1}{\tau}(1-N_{\rm intra}) \tag{2}$$

and

$$\lambda_{\rm intra}=\frac{1}{\tau}N_{\rm intra}. \tag{3}$$

Here it was used that the ratio of the partial decay probabilities is equal to the ratio of the intensities:

$$\frac{\lambda_{\rm out}}{\lambda_{\rm intra}}=\frac{1-N_{\rm intra}}{N_{\rm intra}}. \tag{4}$$

In Table I the measured lifetimes τ and intraband intensities $N_{\rm intra}$ are reviewed for those SD states in ^{192}Hg and ^{194}Pb, which are involved in the first step of the decay out of the yrast SD band of these nuclei. The adopted lifetimes were taken from Ref. [13] in the case of ^{194}Pb and for the $12\hbar$ SD state in ^{192}Hg the weighted average of the values published in Refs. [10] ($4.5\pm^{1.4}_{0.8}$ ps), [11] (4.4 ± 1.5 ps), and [12] ($5.3\pm^{1.0}_{0.8}$ ps) was used. The intensities for the intraband transitions $N_{\rm intra}$ were taken from Ref. [16] for ^{192}Hg and Ref. [6] for ^{194}Pb. They are given in Table I as well as the calculated partial decay probabilities $\lambda_{\rm out}$ and $\lambda_{\rm intra}$. The adopted spin values are those reported in Ref. [17]. The obtained $\lambda_{\rm out}$ values give qualitative information on the details of the depopulation of the SD bands.

III. DECAY STRENGTH IN THE FIRST MINIMUM

In order to understand theoretically the decay out of the SD bands, more information about the decay of the normal states at high excitation energy (4–6 MeV) in the first (normal deformed) potential minimum is necessary. Their structure, the level density, and their decay properties will have strong influence on the decay properties of the SD states in the second potential minimum.

For the electromagnetic transitions of the ND states at the excitation energy of the SD states statistical transitions as well as collective $E2$ and $M1$ transitions should be considered. We calculated the statistical $E1$ transition probability using the Fermi-gas model for the nuclear level density. The Fermi-gas model parametrization from Ref. [18] was adopted where the level density $\rho_n=1/D_n$ at an excitation energy E_x for states with spin I and parity Π is expressed as

$$\rho_n(E_x,I,\Pi)=\frac{2I+1}{8\sqrt{6\pi}\Im^{3/2}(E_x-E_{\rm yr})}\left(\frac{E_x-E_{\rm yr}}{a}\right)^{-3/4}\times\exp\{2\sqrt{a(E_x-E_{\rm yr})}\}. \tag{5}$$

Here $E_{\rm yr}$ is the energy of the yrast state at spin I and $\Im$ is the moment of inertia obtained by fitting the high spin part of the yrast line with a second order expansion of the rotational energy in terms of $I(I+1)$. Values of $\Im=60\hbar^2$ MeV^{-1} and $\Im=50\,\hbar^2$ MeV^{-1} were used for ^{192}Hg and ^{194}Pb, respectively. Level density parameters of $a=22.3$ MeV^{-1} and 22.4 MeV^{-1} were used for ^{192}Hg and ^{194}Pb, calculated from the parametrization of the nuclear level density in Ref. [19]. To account for the pairing correlations at spin I a spin-dependent pairing gap Δ_I was introduced by which the excitation energy E_x was renormalized to $U=E_x-2\Delta_I$ ($\Delta_0=12/\sqrt{A}$ [20]). For the calculations we have used a 15–20% reduced pairing gap around spin $10\hbar$. In the following the discussion will be restricted to the renormalized energy U. The $E1$ decay probability is then given by

$$\lambda_n^{E1}=\xi_{E1}\int_0^{U-E_{\rm yr}}\frac{\rho(U-E_\gamma)}{\rho(U)}f_{\rm GDR}(E_\gamma)E_\gamma^3dE_\gamma. \tag{6}$$

Here ξ_{E1} is the $E1$ strength fitted to neutron resonance data from Ref. [21]. The neutron separation energy for ^{192}Hg and ^{194}Pb was estimated to $S_n=9.5$ MeV from an extrapolation of calculated separation energies of light lead and mercury isotopes in Ref. [22]. The average γ strength in this mass region is about $\langle\Gamma_\gamma\rangle=0.1$ eV [21]. The giant dipole resonance (GDR) function

$$f_{\rm GDR}(E_\gamma)=\frac{\Gamma_R E_\gamma}{(E_R-E_\gamma)^2+\Gamma_R E_\gamma} \tag{7}$$

was used with parameters of $E_R=13.6$ MeV and $\Gamma_R=4.4$ MeV estimated from the values for the natural lead and mercury isotopes in Ref. [23].

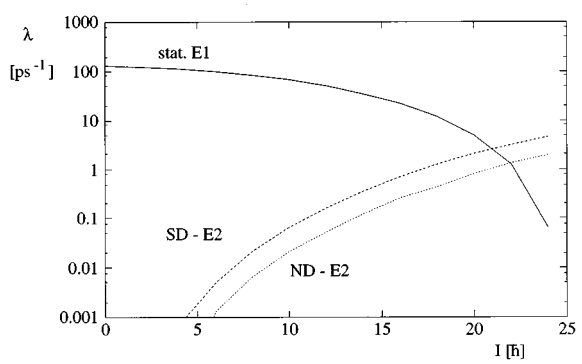

FIG. 1. Spin dependence of the statistical $E1$-transition probability at an excitation energy of $U=5.5$ MeV for ^{192}Hg. Transition probabilities for collective $E2$ transitions in a ND rotational band with $\Im=60\hbar^2$ MeV^{-1} and a SD rotational band with $\Im=90\hbar^2$ MeV^{-1} are shown for comparison.

Figure 1 shows the spin dependence of the transition probability λ_n^{E1} for statistical $E1$ transitions in ^{192}Hg at an excitation energy of $U=5.5$ MeV. Also transition probabilities are shown for collective $E2$ transitions in rotational bands with moments of inertia of $\Im=60\,\hbar^2$ MeV^{-1} (ND) and $\Im=90\,\hbar^2$ MeV^{-1} (SD) and quadrupole moments of $Q_{\rm ND}=5\,e$ b and $Q_{\rm SD}=20\,e$ b, respectively. Around spin $12\hbar$ the $E1$ transition probability is about 300 times larger than the strongest collective $E2$ transition. Assuming that collective $M1$ transition probabilities are of the order of the $E2$ transition probabilities, the statistical $E1$ transitions clearly dominate. The estimates above were previously used to discuss the decay of the SD band in ^{192}Hg [8,10,24,25].

One should keep in mind that $E2$ and $M1$ transitions might also contribute to the statistical decay of the ND states under certain conditions. However, we have restricted our discussion to $E1$ transitions, which seems to be supported by the observed $E1$ character of the statistical spectrum following the decay of the SD band in ^{192}Hg [7].

We want to stress that if the assumed high excitation energies of the SD states are correct, the decay of ND states at these energies will be dominated by statistical transitions. Therefore one should expect a highly fragmented decay consistent with the experimental findings so far.

IV. DECAY OF THE SD STATES BY MIXING WITH ND STATES

In order to study the mechanism leading to the decay out of the SD band we applied the mixing model that was first proposed by Vigezzi *et al.* [26,27]. In this model the mixing between SD and ND states implies that the real SD states at spin I have besides a dominant superdeformed part $|s_I\rangle$ also a small normal deformed admixture $|n_I\rangle$:

$$|\Psi_I\rangle=a_s(I)|s_I\rangle+a_n(I)|n_I\rangle,$$

while the amplitudes fulfill $a_s^2(I)+a_n^2(I)=1$. The measured lifetimes and intensities can be reproduced using this mixing model and they give information about the mixing amplitudes of the mixed state within the SD band.

For the decay out of the SD bands we assume that the matrix elements of the electromagnetic transition operator $\hat{T}_{\rm EM}$ between pure SD and pure ND wave functions are vanishing ($\langle s|\hat{T}_{\rm EM}|n\rangle=0$). Therefore the decay out of the band is governed only by matrix elements of the type $\langle s|\hat{T}_{\rm EM}|s\rangle$ and $\langle n|\hat{T}_{\rm EM}|n\rangle$. The results of the previous section have shown (see Fig. 6) that the decay out is dominated by the matrix elements of the statistical decay of normal deformed states. Under the assumption that statistical $E1$ transitions are dominating, the only relevant transition probability is λ_n^{E1} [see Eq. (1)]. In this case the partial decay probability $\lambda_{\rm out}$ for the decay out of the SD band is given by

$$\lambda_{\rm out}=a_n^2(I)\lambda_n^{E1}. \tag{8}$$

This important relation expresses the fact that the decay out of the SD band is forced by the interplay between the large statistical $E1$ transition probability of the highly excited normal deformed states and decreasing transition probabilities in the SD band due to the decreasing transition energies (λ_s scales with E_γ^5). This interplay enables SD states, which are clearly dominated by a SD configuration, to decay to normal deformed states. The calculation of λ_n^{E1} with respect to the excitation energy of the SD band, presented in Table II, allows an estimation of the mixing amplitudes $a_n^2(I)$. Table II shows the resulting squared normal mixing amplitudes a_n^2 for different assumptions of the excitation energy of the SD states in ^{192}Hg and ^{194}Pb. It is very remarkable that these squared mixing amplitudes a_n^2 are extremely small ($a_n^2\leqslant 0.01$).

TABLE II. Calculated average level spacing D_n, calculated $E1$ transition probability λ_n^{E1}, and deduced squared mixing amplitude a_n^2 at different excitation energies for spins $12\hbar$ and $10\hbar$ in ^{192}Hg and ^{194}Pb, respectively.

	I [$\hbar$]	Energy above yrast [MeV]	U [MeV]	D_n [eV]	λ_n^{E1} [ps^{-1}]	a_n^2
		3.0	4.1	928	4.9	0.005
		3.5	4.6	262	7.4	0.003
^{192}Hg	12	4.0	5.1	82	11.2	0.002
		4.5	5.6	27	16.8	0.001
		3.0	4.2	776	5.1	0.0033
		3.5	4.7	219	7.7	0.0022
^{194}Pb	10	4.0	5.2	68	11.7	0.0015
		4.5	5.7	23	17.5	0.0010

V. NEAREST NEIGHBOR MIXING APPROACH

The mixing amplitudes in Table II can be used for further investigation of the decay mechanism. A very interesting property in this context is the squared interaction strength v^2 between SD and ND states, which is responsible for the mixing. So far we have not discussed the mixing itself but rather its consequences for the mixing amplitudes and the level lifetimes. One might expect that due to the high level density of the ND states the mixing is rather complex and many ND states contribute to the admixture. We cannot exclude that this is true; however, there are some arguments that justify a very simple approach.

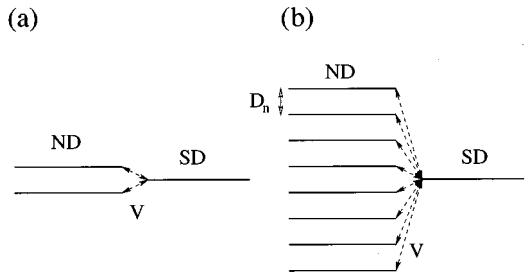

FIG. 2. Schematic drawing of the situation in which the SD level is situated in the middle between two ND states and interacts only with the two nearest neighbors (a) or with several ND states (b) with equidistant spacing D_n.

It can be assumed that at the high excitation energy of the SD states all ND states are composed of a complex mixture of many states with different structures [28]. The resulting states all have about the same structures in common with the SD state in the second potential minimum. This means that for each ND state $|n^{(i)}\rangle$ the matrix element $v_i^2=|\langle n^{(i)}|\hat{V}|s\rangle|^2$ with the SD state $|s\rangle$ is very similar and can be approximated by a mean interaction $\langle v^2\rangle$. On average the ND states are well separated as can be seen from the average level spacings D_n in Table II, which are of the order of 30 eV. This value is large compared to the natural width of the ND states, which is of the order of some meV. These facts make us believe that the actual mixing situation is rather simple and only a few states are involved.

Let us discuss some simple situations in order to get an estimate for this average interaction $\langle v^2\rangle$. In Fig. 2(a) the SD state is placed in the middle between two ND states that are separated by the average spacing D_n. In this case the wave function of the state $|\Psi_I\rangle$ with the dominant SD component is given in first order perturbation by

$$|\Psi_I\rangle=|s_I\rangle-\frac{2\langle v\rangle_2}{D_n}(|n_I^{(1)}\rangle+|n_I^{(2)}\rangle). \tag{9}$$

Here $|n_I^{(1)}\rangle$ and $|n_I^{(2)}\rangle$ denote the next ND level above and below the SD level at spin I, respectively, and $\langle v\rangle_2$ denotes the average interaction strength for the case of two ND levels mixing into the SD wave function. The squared normal deformed mixing amplitude is then given by

$$a_n^2=\left(1-\frac{|\langle s_I|\Psi_I\rangle|^2}{|\langle\Psi_I|\Psi_I\rangle|^2}\right). \tag{10}$$

Using this expression and the wave function in Eq. (9) one can easily obtain the mean squared interaction $\langle v^2\rangle_2$ in the case of two ND states mixing weakly ($a_n^2<<1$) to the SD state:

$$\langle v^2\rangle_2\approx\tfrac{1}{8}D_n^2a_n^2. \tag{11}$$

A more complicated situation is given if we take into account the interaction with n different ND levels [see Fig. 2(b)]. It is still assumed that the ND levels have an average spacing of D_n. This situation can be described in first order perturbation theory by the following wave function $|\Psi_I\rangle$:

TABLE III. Maximum interaction strength $v_{\max}$ for the mixing of ND states with the SD states at spin $12\hbar$ and $10\hbar$ in ^{192}Hg and ^{194}Pb, respectively, at an excitation energy of 4.5 MeV above the yrast line.

Nucl.	I [$\hbar$]	E [MeV]	D_n [eV]	$v_{\max}$ [eV]	$\Gamma_{\max}$ [eV]	$W_{\min}$ [MeV]	$A_{\min}$ [$\hbar$]
^{192}Hg	12	4.5	27	0.27	0.017	1.5	7.8
^{194}Pb	10	4.5	23	0.24	0.016	1.5	7.8

$$|\Psi_I\rangle=|s_I\rangle-\sum_{i=1}^{n}|n_I^{(i)}\rangle\frac{\langle v^2\rangle_n}{x_i}. \tag{12}$$

Here $\langle v^2\rangle_n$ is the average interaction of each ND level with the SD level and x_i is the separation between the ith ND level and the SD level. The mean squared interaction $\langle v^2\rangle_n$ for this case can be calculated in a similar way as for the previous case by using Eq. (10). In the limit of $n\to\infty$ the following expression for $\langle v^2\rangle_n$ is obtained:

$$\langle v^2\rangle_n\approx\frac{1}{\pi^2}D_n^2a_n^2. \tag{13}$$

From the two scenarios above we conclude that the average interaction of the SD state with each ND level decreases with an increasing number of ND levels involved in the mixing. In order to simplify the calculations it was assumed that the SD level is placed in the middle between the nearest ND states. If this restriction is not made, one can show that the mean interaction will be always smaller than that given in Eq. (11).

The discussion above enables us to give a total upper limit for the mean interaction strength $\langle v^2\rangle_{\max}$, which is given by

$$\langle v^2\rangle_{\max}=\tfrac{1}{8}D_n^2a_n^2. \tag{14}$$

Table III shows the estimated maximum values for $v_{\max}=\sqrt{\langle v^2\rangle_{\max}}$ at an assumed excitation energy of 4.5 MeV above yrast for the $12\hbar$ and $10\hbar$ SD states in ^{192}Hg and ^{194}Pb, respectively. One can see from Tables II and III that the maximum average interaction $v_{\max}$ is much smaller than the average level spacing D_n. The small values of $v_{\max}$ with respect to the average level spacing make us strongly believe that only the mixing with the closest ND levels accounts for the admixture of ND wave functions to the real state $|\Psi_I\rangle$.

It is clear that the mixing amplitude depends strongly on the relative position of the SD state with respect to the nearest ND states. This position might vary strongly in different SD bands or even for different states within one SD band. Therefore a different intensity behavior or development of the ND admixture with spin in various SD bands in the 190-mass region might find its explanation in different relative positions of the SD and ND states involved.

VI. BARRIER BETWEEN THE FIRST AND SECOND MINIMA

The mixing of ND and SD states can be associated with the tunneling through the potential barrier that separates the first and second potential minima. For the purpose of con-

necting these two descriptions it is useful to introduce a width Γ with the coupling matrix element v^2 by using the Fermi golden rule (maximum values for Γ are given in Table III):

$$\Gamma = \frac{2\pi v^2}{D_n}. \tag{15}$$

This width Γ is a measure for the rate T ($T=\Gamma/\hbar$) of changing the nuclear shape from a superdeformed to a normal deformed one.

The tunneling through the potential barrier is often treated in a semiclassical approach [8,24,25,29] in which the second potential minimum is approximated by a harmonic oscillator potential with frequency ω_s and the potential barrier is described by an inverted parabola with frequency ω_b. In this standard parametrization the tunneling width Γ_{tunn} is given by

$$\Gamma_{\text{tunn}} = \frac{\hbar\omega_s}{2\pi}\exp\left(-\frac{2\pi}{\hbar\omega_b}W\right), \tag{16}$$

where W is the barrier height. In our discussion we used the estimates of $\hbar\omega_b=\hbar\omega_s=0.6$ MeV as were used in Refs. [9,8,24–27]. Since the tunneling width Γ_{tunn} also describes the transition of the nuclear shape from SD to ND, it can be identified with the width Γ of the mixing description ($\Gamma_{\text{tunn}}=\Gamma$). This leads to lower limits for the barrier height W, which are shown in Table III. Instead of the barrier heights W and $W_{\min}$ the related actions A and $A_{\min}$ for the tunneling can be used. The parameters $W_{\min}$ and $A_{\min}$ can be calculated according to the above by the relation

$$\frac{A_{\min}}{\hbar} = \frac{\pi W_{\min}}{\hbar\omega_b} = -\frac{1}{2}\ln\left(\frac{(2\pi)^2}{\hbar\omega_s}\frac{\langle v^2\rangle_{\max}}{D_n}\right). \tag{17}$$

The barrier height W depends strongly on the assumed shape of the barrier and on the chosen parameter for the barrier frequency ω_b. Therefore it is not a good measure for the barrier properties. However, the tunneling action A depends only on the oscillator frequency ω_s of the second potential minimum. Varying ω_s by a factor of 2 results only in a change of the action A by about 5% and shows that the action is mainly independent of the assumed shape of the potential. The lower limits for the tunneling action are presented in Table III. We will compare these values with results from other decay studies below.

VII. EXTRAPOLATIONS TO LOWER SPINS

The extremely small amount of mixing between SD and ND states in the first step of the decay out suggests investigating the situation for the next lower SD states. The intensities of the next lower SD transitions are known in both nuclei [6,16], but because of their low intensity, it has so far not been possible to determine the lifetimes of the SD states which decay via those transitions. From the results at spin 12 $\hbar$ and 10$\hbar$ in ^{192}Hg and ^{194}Pb, respectively, one can assume that there is no dramatic change in the structure of the next lower SD states. Namely, we assume that the transition quadrupole moments remain constant for the 214.4 keV and 170.7 keV transitions in ^{192}Hg and ^{194}Pb, respectively. Using this assumption it is easy to obtain an estimated partial probability $\lambda_{\text{out}}^{\text{est}}$ for the decay out as well as the mixing amplitudes of these states. Table IV summarizes the intensities N_{intra} of the intraband transitions, the calculated $E1$ transition probabilities λ_n^{E1}, and the estimated values for $\lambda_{\text{out}}^{\text{est}}$ and a_n^2.

TABLE IV. Average level spacing D_n and $E1$ transition probability λ_n^{E1} for spins 10$\hbar$ and 8$\hbar$ in ^{192}Hg and ^{194}Pb, respectively. The estimated partial decay probability for the decay out $\lambda_{\text{out}}^{\text{est}}$ and the squared mixing amplitudes a_n^2 were calculated assuming an excitation energy of 4.5 MeV above yrast for the next higher SD state and constant transition quadrupole moments Q_t (taken from Table I) for the intraband transitions in both nuclei. The intensities N_{intra} for the intraband transitions were taken from Refs. [6,16].

	I [$\hbar$]	N_{intra}	U [MeV]	D_n [eV]	λ_n^{E1} [ps^{-1}]	$\lambda_{\text{out}}^{\text{est}}$ [ps^{-1}]	a_n^2
^{192}Hg	10	0.08(2)	5.5	40	16	0.748	0.047
^{194}Pb	8	0.64 (9)	5.4	36	15	0.014	0.0009

While the situation in both nuclei seems very similar in the first step of the decay process a remarkable difference in the development of the mixing amplitudes can be observed in the second step of the decay out. The difference is already obvious from the development with spin of the branchings N_{intra} in ^{192}Hg and ^{194}Pb.

It is even more evident from the difference of the mixing amplitudes for the two lowest states in the two nuclei:

$$^{192}\text{Hg}: a_n^2(12)=0.001 \quad \text{and} \quad a_n^2(10)=0.047,$$

$$^{194}\text{Pb}: a_n^2(10)=0.001 \quad \text{and} \quad a_n^2(8)=0.0009.$$

In the picture developed above, the mixing amplitude of a SD state at a given excitation energy depends only on the interaction strength v^2 and the relative position of the SD level with respect to the nearest neighboring ND levels. The estimated mixing amplitudes for the 10$\hbar$ and 8$\hbar$ states in ^{192}Hg and ^{194}Pb, respectively, raise the question of whether the main difference in the two SD band arises from a completely different behavior of the interaction strength or, in other words, the barrier height, or if this difference is due to very different placements of the SD levels with respect to the nearest ND states. Clearly in the latter case the branching ratio N_{intra} and the mixing amplitudes a_n^2 would display a chaotic character.

VIII. DISCUSSION AND SUMMARY

In the previous sections we have shown that the decay out of SD bands in the 190-mass region can be described by a very small admixture of normal deformed states to the superdeformed ones. This admixture is estimated to be less than 1% ($a_n^2 \leqslant 0.01$) for the first step of the decay process in ^{192}Hg and ^{194}Pb, where about 15% of the intensity leaves both SD bands.

Calculations have shown that at the estimated excitation energy of about 4.5 MeV above the yrast line for the 12$\hbar$ SD state in ^{192}Hg statistical transitions of the ND states dominates the decay out. For simplicity we have assumed the

TABLE V. Lower limits for the tunneling action A for the lowest two states with observed intraband transitions in the SD bands in ^{192}Hg and ^{194}Pb. For comparison values from Khoo *et al.* [8] and Shimizu *et al.* [24] are also presented.

		$A_{\min}$ [$\hbar$]	A [$\hbar$]	
Nucl.	State	Present work	Khoo *et al.* [8]	Shimizu *et al.* [24]
^{192}Hg	$12\hbar$	7.8	8.1	4.5
	$10\hbar$	5.5	5.4	3.7[a]
^{194}Pb	$10\hbar$	7.8		4.4
	$8\hbar$	7.1		3.3[b]

[a]Calculated from slope $dA/dI=0.38$.
[b]Calculated from slope $dA/dI=0.54$.

excitation energy of the SD state at $10\hbar$ in ^{194}Pb to be 4.5 MeV even though no reliable estimate for this energy was given before. In our present work we assumed the statistical transitions to show mainly $E1$ characteristics. This is in agreement with the measured multipolarity pattern of the continuous spectrum of γ rays in coincidence with the SD band in ^{192}Hg [7]. We have given arguments for a very simple mixing of the SD state with only the nearest neighboring ND states in energy. The decay out could be consistently described within this approach and upper limits for the interaction strength between SD and ND states could be given ($v_{\max} \leqslant 0.5$ eV). In this picture it is not possible to give exact values for the interaction strength, since this depends strongly on the relative position of SD and ND states.

Using the common tunneling approach we have calculated lower limits for the height W of the barrier that separates first and second potential minima. Since W depends on the shape of the barrier, we prefer to give lower limits for the tunneling action A.

With the assumption of a continuous constant quadrupole moment in the lower part of the SD bands we could estimate mixing amplitudes a_n^2 and the related parameters ($v_{\max}^2$, $W_{\min}$, $A_{\min}$) for the $10\hbar$ and $8\hbar$ SD states in ^{192}Hg and ^{194}Pb, respectively.

We now want to discuss the resulting values for the action of the tunneling process in comparison with the results of previous studies of the decay in ^{192}Hg and ^{194}Pb.

Table V shows the lower limits of A from our studies compared to values from Khoo *et al.* [8] for ^{192}Hg and Shimizu *et al.* [24] for ^{192}Hg and ^{194}Pb. The agreement of our lower limit with the values given by Khoo *et al.* is remarkable. These values have been obtained by simulating the measured feeding distribution of the SD band in ^{192}Hg with Monte Carlo calculations and using these results to extrapolate the barrier properties to the decay out spin region. The action A was obtained by fitting the intensity of the SD intraband transitions using the approach proposed by Vigezzi *et al.* [26,27] in the weak coupling limit (for the definition of $\Gamma_{s,n}$ see [26,27]):

$$\overline{N_{\text{out}}} \approx \sqrt{\frac{\pi}{2}\frac{\Gamma}{D_n}\frac{\Gamma_n}{\Gamma_s}}. \tag{18}$$

The values by Shimizu *et al.* are much lower than the other results. However, these calculations have been performed for a much lower excitation energy of the SD band in ^{192}Hg [24]. At that excitation energy of 3.2 MeV above yrast we find a lower limit for the action of $A_{\min}=5.7\hbar$ for the $12\hbar$ state in ^{192}Hg. This value is still larger than that given by Shimizu *et al.* The interaction between SD and ND states is overestimated in these calculations and as a consequence the transition quadrupole moments would change dramatically, which is not in accordance with the experiments.

Because of the agreement with the values from Khoo *et al.*, one could assume that the situation is now well described. However, the results for ^{194}Pb do not fit into a consistent picture. In order to describe the mixing amplitudes of the $8\hbar$ SD state in ^{194}Pb following the arguments used by Khoo *et al.*, one would find the action to decrease much slower with spin compared to ^{192}Hg. However, the calculations of Shimizu *et al.* show that the slope of the action with spin is larger in ^{194}Pb ($dA/dI=0.54$) compared to ^{192}Hg ($dA/dI=0.38$). This result of the microscopic calculations is also in agreement with the approach that pairing correlations are the driving force for changing the nuclear shape and should be more important in Pb than in Hg isotopes. From these arguments one would expect a similar or larger slope of the action in ^{194}Pb compared to ^{192}Hg.

In the approach of Vigezzi *et al.* [see Eq. (18)] this obvious discrepancy between proposed development of the action [8] and the expectations from theory [24] is not explained.

At this point we want to remind the reader that the A values given in our work are only lower limits. One can explain the tremendous difference of the intensities and mixing amplitudes by a very different placement of the SD levels with respect to the nearest ND states at different stages of the decay out. In this case one would not need a very different behavior of the interaction strength v^2 or, in other words, of the barrier height W or tunneling action A.

We want to explain this in more detail. The $10\hbar$ SD state in ^{192}Hg might be rather close to a ND state, at least much closer compared to the relative position of the $12\hbar$ state. The large increase of the mixing amplitude with decreasing spin is then not due to a slight decrease in the action but mainly to the different relative position. In ^{194}Pb the situation is the opposite. Here the $10\hbar$ SD state is closer to the nearest ND state compared to the relative position of the $8\hbar$ state. So even with a larger decrease of the action with decreasing spin in this nucleus the mixing amplitudes remain rather constant in both states.

The picture of this nearest neighbor mixing could give a natural explanation of different decay points and different developments of the intraband intensities for different identical SD bands of even-even nuclei.

Concerning the question of whether the SD minimum might survive down to a SD bandhead our analysis does not provide a clear answer. Since the relative positions of SD states with respect to the nearest ND states is random, it would depend on the very special case of how the amplitudes develop. An interesting suggestion from the discussion in the present paper is the possibility that the mixing between the SD and ND states has a random character. If this is indeed the case, then we need to study much more experimental cases in order to learn about the average mixing behavior.

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