Thermodynamical properties and Coulomb instabilities in hot nuclear systems with the Gogny interaction

Yi-Jun Zhang
Department of Physics, Fudan University, Shanghai 200433, People's Republic of China

Ru-Keng Su
China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, People's Republic of China and Department of Physics, Fudan University, Shanghai 200433, People's Republic of China

Hongqiu Song
China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, People's Republic of China and Institute of Nuclear Research, Academia Sinica, P.O. Box, 800204, Shanghai 200180, People's Republic of China

Fu-Min Lin
Department of Physics, Shantou University, Guangdong 515063, People's Republic of China
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The thermodynamical properties and Coulomb instability in hot nuclear system with Gogny interactions are studied by using the finite-temperature real-time Green's function method. The isotherms, critical temperature, limiting temperature, saturation density, and isothermal incompressibility for various temperatures and different asymmetric parameters are calculated. To illustrate the importance of the finite-range part and density-dependence part of the interaction, we have compared our results with that given by the Skyrme interaction and Brink-Boeker interaction. [S0556-2813(96)01008-4]

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I. INTRODUCTION

Since intermediate-energy heavy-ion collisions and high-energy proton-induced reactions indicated the possibility of the occurrence of a liquid-gas phase transition in nuclear matter, the study of the properties of nuclear matter at finite temperature has attracted increasing interest. Experimentally, many intermediate-energy heavy-ion collisions have been performed [1] to investigate the unknown features of the highly excited or hot nucleus formed in collision [2,3]. Theoretically, much effort has been devoted to studying the equation of state (EOS) for nuclear matter and to discussing the critical phenomena [4–19]. The calculated critical temperature T_C of nuclear matter for various kinds of nucleon-nucleon (NN) interactions is about 8–20 MeV.

In addition to the critical temperature, several other temperatures have been defined to discuss the properties of nuclear matter. For example, the temperature usually extracted from the moving-source fitting to the inclusive particle kinetic-energy spectrum is called the slope temperature. Employing this idea, Suraud *et al.* [1] discussed the Ar+U system at 27 MeV/nucleon and found that the resulting temperature for the fusion nucleons is about 5 MeV which is consistent with the temperature $T=4.5$–6.0 MeV obtained by measuring the relative populations of the excited states in heavy-ion reactions [3]. Another important temperature, $T_{\rm lim}$, below which the nucleus can exist in equilibrium with the surrounding vapor, and above which the nucleus is unstable and will fragment, is called the limiting temperature. It is a maximal temperature at which a hot nucleus can sustain itself before reaching mechanical instability. This is the so-called Coulomb instability. Based on a two-phase coexistence model, the limiting temperature $T_{\rm lim}$ for a hot nucleus can be obtained by solving the coexistence equations. It is found that the limiting temperature $T_{\rm lim}$ is lower than the critical temperature of the liquid-gas phase transition in nuclear matter. Therefore, the limiting temperature is more important than the critical temperature for discussing the phenomena observed in heavy-ion collisions.

The Coulomb instability of hot nuclei has been studied by us and by many other authors by using different kinds of effective NN interactions, for example, Skyrme interactions [7–9], the Brink-Boeker (BB) force [12], a quantum hadrodynamics (QHD) model [10], a relativistic mean-field theory with derivative scalar coupling [11], and a Brueckner-Hartree-Fock approach with relativistic and three-body force corrections [13]. It is found that the values of $T_{\rm lim}$ are very sensitive to the NN interactions employed for calculations. If one can extract the limiting temperature from experiment, the comparison of $T_{\rm lim}$ with theoretical predictions from different models may be used as a test for the correction of various effective interactions and models.

In this paper we will discuss the same topic but a different effective NN interaction, namely, the Gogny interaction [14,17–19]. The reason for choosing the Gogny interaction is as follows. It is well known that the Skyrme interaction is a simple and useful potential for describing the theomodynamical properties of nuclear matter. But it is not a good effective interaction for simulating the mediate and long range parts of the NN interaction in a nuclear medium because the range of force in the Skyrme interaction is zero. Even though the Skyrme interaction is density dependent, because of its zero range, it favors a compact and denser

configuration, which results in a high critical temperature [8,11]. To investigate the finite-range effects, Das, Nayak, and Satpathy employed the BB interaction to study the equation of state, the Coulomb instability, and the thermodynamical properties of hot nuclear systems [12]. They found that the limiting temperature is about 5 MeV for $A=60$ (A is the nucleon number in a nucleus). This value is much lower than that of the Skyrme interaction [9]. Although the BB interaction is finite ranged, it has an obvious disadvantage, namely, density independence. The density-dependence part, as was pointed out by many authors [13,14], is very important if one desires to establish a complete self-consistent theory of the pairing and reproduce the mechanism of saturation based on the Brueckner theory. The density dependence is easy to understand if one notices that the range of summation of the G matrix depends on the Fermi energy, which is itself a function of the density, and that the projection operator of model space can be expressed as a nonlocal density [19]. The Gogny interaction is a good candidate which can overcome the disadvantages of the Skyrme interaction as well as the BB interaction because it is a density-dependence and finite-range interaction. It is of interest to study the Coulomb instability in hot nuclei with Gogny interactions and to compare our results with that of the Skyrme interaction and BB interaction for illustrating the effectiveness of density dependence as well as finite range. This is the objective of this paper.

The organization of this paper is as follows. In Sec. II we will show the formalism of our calculation. In Sec. III we will give our results including isotherms, saturation density at finite temperature, fixed asymmetric parameter, isothermal incompressibility, critical temperature of liquid-gas phase transition, Coulomb instability, and limiting temperature of the nuclear system with Gogny interactions. We will compare our results with those given by the Skyrme interaction and BB interaction to illustrating the effect of the density dependence, as well as the effect of the finite range of the interactions.

II. FORMALISM

Following Refs. [7–10], we consider the hot nucleus as a uniformly charged drop of nuclear liquid at a given temperature T and with a sharp edge in both thermal mechanical and chemical equilibrium with the surrounding vapor. Then we obtain a set of two-phase coexistence equations by requiring equality of temperature T, pressure p, neutron chemical potential μ_n, and proton chemical potential μ_p of the liquid and vapor phases, respectively:

$$p_L(T,\rho_L,\alpha_L)+p_L^{\mathrm{Coul}}(\rho_L)+p_L^{\mathrm{Surf}}(T,\rho_L)=p_V(T,\rho_V,\alpha_V), \tag{1}$$

$$\mu_L^n(T,\rho_L,\alpha_L)=\mu_V^n(T,\rho_V,\alpha_V), \tag{2}$$

$$\mu_L^p(T,\rho_L,\alpha_L)+\mu_L^{\mathrm{Coul}}=\mu_V^p(T,\rho_V,\alpha_V), \tag{3}$$

where L and V as subscripts stand for liquid and vapor, respectively. The Coulomb and surface effects are included in the liquid phase.

In our calculation, the finite-temperature real-time Green's function method [9,11,20] is used to derive the EOS for bulk asymmetry nuclear matter. Since the details of this method can be found in Refs. [11,9,20], we present here only the main steps. In this approach, we calculate the relevant physical quantities in a zero-order approximation first. The chemical potential μ_{n0} for neutrons, μ_{p0} for protons, and the single-nucleon spectra E_i can be obtained by solving the following set of equations self-consistently:

$$E_i=t_i-\mu_{i0}+\sum_j \langle ij|V(r)|ij-ji\rangle n_j, \tag{4}$$

$$\rho_q=2\sum_k n_q(k) \quad (q=n,p), \tag{5}$$

$$n_i=\frac{1}{1+\exp(E_i/k_BT)}. \tag{6}$$

In Eqs. (4) and (6), the subscript i stands for $\{q,k\}$, with $q=n$, p, and k the nucleon momentum, t_i is the kinetic energy of the single nucleon, $V(r)$ the nucleon-nucleon interaction, and k_B the Boltzmann constant. The neutron density ρ_n and proton density ρ_p in Eq. (5) are given by the following relations:

$$\rho_n=\frac{1+\alpha}{2}\rho, \quad \rho_p=\frac{1-\alpha}{2}\rho, \tag{7}$$

where α is an asymmetry parameter and ρ the total nucleon density. The first-order thermodynamical potential Ω_1 is given by

$$\Omega_1=\Omega_0+\Omega_1^{\mathrm{HF}}, \tag{8}$$

where

$$\Omega_0=-k_BT\sum_i \ln[1+\exp(-E_i/k_BT)] \tag{9}$$

TABLE I. Parameters of the Gogny D1 effective interaction.

i	μ_i (fm)	W_i (MeV)	B_i (MeV)	H_i (MeV)	M_i (MeV)
1	0.7	−402.40	−100.00	−496.20	−23.56
2	1.2	−21.30	−11.77	−37.27	−68.81

$t_0=1350$ (MeV fm^4), $\alpha=1/3$
$W_{LS}=115$ (MeV fm^5), $\chi=1$

TABLE II. Comparison of the contribution for the pressure of the system from the finite-range part (FR) and the density-dependent part (DD) in the Gogny effective interaction at $T=10$ MeV and $\alpha=0$ with the IE method.

ρ (fm^{-3})	$p_{\rm DD}$ (MeV fm^{-3})	$p_{\rm FR}$ (MeV fm^{-3})
0.04	0.37	−0.86
0.08	1.86	−3.37
0.12	4.79	−7.43
0.16	9.38	−12.96
0.20	15.79	−19.88

and

$$\Omega_1^{\rm HF}=-\frac{1}{2}\sum_{i,j}\langle ij|V(r)|ij-ji\rangle n_i n_j. \tag{10}$$

The first-order Helmholtz free energy density f_1 is then calculated by the thermodynamical relation

$$f_1=\Omega_1/V_0+\sum_q \mu_{q0}\rho_q, \tag{11}$$

where V_0 is the volume of the system, and the first-order chemical potential μ_{q1} is calculated by using the following relation:

$$\mu_{q1}=\left[\frac{\partial f_1}{\partial \rho_q}\right]_T=\mu_{q0}+\mu_{qr}\quad (q=n,p), \tag{12}$$

where the rearrangement chemical potential μ_{qr} [19] which comes from the density dependence of the interaction is

$$\mu_{qr}=\frac{1}{2V_0}\sum_{i,j}\left\langle ij\left|\frac{\partial V}{\partial \rho_q}\right|ij,ji\right\rangle n_i n_j. \tag{13}$$

Finally the first-order pressure p_1 is given by

$$p_1=\sum_q \rho_q\mu_{q1}-f_1. \tag{14}$$

Now we substitute the explicit expression of the effective interaction into above equations. The Gogny D1 (GD1) effective interaction is expressed as

$$\begin{aligned}V_{NN}(\vec{r}_1-\vec{r}_2)=&\sum_{i=1,2}(W_i+B_iP_\sigma-H_iP_\tau-M_iP_\sigma P_\tau)\\&\times\exp(-|\vec{r}_1-\vec{r}_2|^2/\mu_i^2)+t_0(1+\chi_0P_\sigma)\rho^d\\&\times\delta(\vec{r}_1-\vec{r}_2)+iW_{\rm LS}(\vec{\sigma}_1+\vec{\sigma}_2)\vec{\nabla}\delta(\vec{r}_1-\vec{r}_2)\vec{\nabla},\end{aligned} \tag{15}$$

where ρ is the nucleon matter density, P_σ and P_τ the exchange operators for spin and isospin, respectively, and the parameters W_i, B_i, H_i, H_i, μ_i, t_0, χ_0, d, and $W_{\rm LS}$ are listed in Table I. Starting from the GD1 interaction, we obtain the single-nucleon spectra straightforwardly:

$$\begin{aligned}E_q(k)=&\frac{\hbar^2k^2}{2m_q}-\mu_{q0}+t_0(1+\chi_0/2)\rho^d(\rho-\rho_q)\\&+\sum_{i=1,2}(\sqrt{\pi}\mu_i)^3[(W_i+B_i/2)\rho-(H_i+M_i/2)\rho_q]\\&+\sum_{i=1,2}(\sqrt{\pi}\mu_i)^3\int\frac{d\vec{k}'}{(2\pi)^3}\exp(-\mu_i^2|\vec{k}-\vec{k}'|^2/4)\\&\times\{(H_i+2M_i)[n_p(k')+n_n(k')]\\&-(W_i+2B_i)n_q(k')\},\end{aligned} \tag{16}$$

where [6,19]

$$d\vec{k}=\begin{cases}k^2dk & \text{for an infinite system,}\\ \left(k^2-\dfrac{3\pi k}{4R}+\dfrac{3\pi}{8R^2}\right)dk & \text{for a finite system,}\end{cases} \tag{17}$$

with R the radius of the system, $m_q=m$ the mass of the nucleon, and

$$\rho_q=\frac{1}{\pi}\int_0^\infty n_q(k)dk,$$

$$n_q(k)=\frac{1}{1+\exp[E_q(k)/k_BT]}. \tag{18}$$

Solving Eqs. (16)–(18) with given temperature T, total nucleon density ρ, and asymmetry parameter α, we can obtain the zero-order chemical potential μ_{q0} and single-particle spectra $E_q(k)$. The rearrangement chemical potential given by Eq. (13) becomes [19]

$$\mu_{qr}=\frac{1}{8}t_0d\rho^d[3-(1+2\chi_0)\alpha^2],\quad q=n,p, \tag{19}$$

$$\mu_{q1}=\mu_{q0}+\mu_{qr}. \tag{20}$$

The pressure can be obtained from Eqs. (11) and (14). We find

$$\begin{aligned}p_1=&\frac{k_BT}{\pi^2}\int_0^\infty dk\ln\left[\frac{1}{n_n(k)n_p(k)}\right]+\frac{t_0\rho^d}{2}\left[\rho^2-\frac{\rho_n^2+\rho_p^2}{2}+\left(\frac{\rho^2}{2}-\rho_n^2-\rho_p^2\right)\right]\chi_0+\frac{1}{8}t_0d\rho^{d+2}[3-(1+2\chi_0)\alpha^2]\\&+\sum_{i=1,2}\frac{(\sqrt{\pi}\mu_i)^3}{2}\left[\left(W_i+\frac{B_i}{2}\right)\rho^2-\left(H_i+\frac{M_i}{2}\right)(\rho_n^2+\rho_p^2)\right]+\sum_{i=1,2}(\sqrt{\pi}\mu_i)^3\int\int\frac{d\vec{k}}{(2\pi)^3}\frac{d\vec{k}'}{(2\pi)^3}\exp(-\mu_i^2|\vec{k}-\vec{k}'|^2/4)\\&\times\{(H_i+2M_i)[n_{n,p}(k)n_{n,p}(k')]+(W_i+2B_i)[n_n(k,k')+n_p(k,k')]\},\end{aligned} \tag{21}$$

where

$$n_{n,p}(k)=n_n(k)+n_p(k), \quad n_n(k,k')=n_n(k)n_n(k').$$

The above equations can be solved by two different treatments: (1) Expanding the right-hand sides of the above equations into a series of momentum and carrying out the calculation up to order $O(k^n)$. These are the so-called degeneracy-correction expansions by Jaqaman [8] and adopted by Jaqaman [8] and Das *et al.* [12] for their calculations. We refer to this treatment as the momentum expansion approximation (MEA). (2) Solving the above coupling integral equations exactly and self-consistently. Hereafter we refer to this treatment as IE. In order to study the approximation of the MEA, we calculate thermodynamical properties up to $O(k^4)$ and compare the result with that given by solving the integral equation (IE) exactly.

Before the calculation, we must take into account the Coulomb interaction and surface effect because in the above discussion the Coulomb interaction is switched off and the surface effect is not considered. For simplicity, we use an average Coulomb potential per proton in a uniformly charged sphere:

$$V_p^{\mathrm{Coul}}(\rho)=\frac{6Ze^2}{5R}, \tag{22}$$

where Z is the number of protons in the liquid droplet and R the radius of the liquid droplet [9]. The exchange term of the Coulomb interaction has been neglected. When the Coulomb interaction is switched on, the single-particle spectra given by Eq. (16) for protons and the chemical potential of protons all have an additional term: V_p^{Coul} and $\mu_p^{\mathrm{Coul}}=V_p^{\mathrm{Coul}}$, respectively. The contribution of the pressure is expressed as

$$p_L^{\mathrm{Coul}}(\rho)=\frac{Z^2e^2}{5AR}\rho, \tag{23}$$

where $A=N+Z$ is the number of nucleon and N the number of neutrons in the liquid droplet.

Now we take the surface effect into account. For the liquid droplet with a surface, we consider the surface effect on pressure as the same as Refs. [7,8]. The formula for the temperature dependence of the surface tension $\gamma(T)$ reads [21]

$$\gamma(T)=(1.14\ \ \mathrm{MeV\,fm^{-2}})\left[1+\frac{3T}{2T_C}\right]\left[1-\frac{T}{T_C}\right]^{3/2}, \tag{24}$$

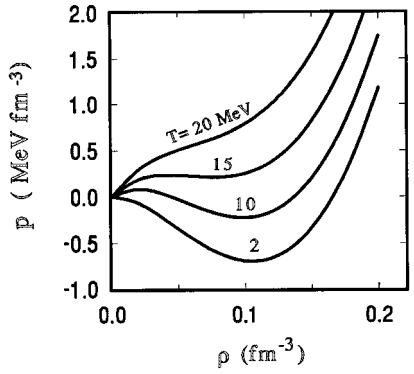

FIG. 1. Isotherms for the symmetric nuclear system.

where T_C is the critical temperature for infinite symmetric nuclear matter. The additional pressure brought out from the surface tension of the liquid droplet is

$$p_L^{\mathrm{surf}}(T,\rho)=-2\gamma(T)/R, \tag{25}$$

where the nuclear density ρ is related to the droplet radius R by the relation $A=\frac{4}{2}\pi R^3\rho$ for a given nucleon number A.

III. RESULTS AND DISCUSSIONS

Now we are in a position to calculate the relevant thermodynamical quantities by means of the Gogny interaction and the above formalism. Our results are shown in Figs. 1–8 and Tables II–IV.

The p-ρ isotherms for $\alpha=0$ symmetric nuclear system are shown in Fig. 1. The critical temperature T_C can be found from the inflection point of p-ρ isotherms, which satisfies

$$\frac{\partial p}{\partial \rho}=\frac{\partial^2 p}{\partial \rho^2}=0. \tag{26}$$

We found that $T_C=15.90$ MeV for the Gogny interaction; above T_C no liquid-gas mixed phase can exist. In order to illustrate the contributions of the finite-range part and the density-dependent part of the Gogny interaction independently, we calculate the contributions of these two parts to pressure separately and show our results in Table II. As shown in Table II, the contribution of the finite range of Gogny interactions to pressure p_{FR} is negative, but the density-dependence part p_{DD} is positive. This result can easily be understood if one notices that the finite range of interactions will soften the compact and hard configurations of

TABLE III. Critical temperature T_C from μ_n-ρ curves and μ_p-ρ curves with different asymmetry parameters for the Gogny effective interaction with the IE and the MEA methods.

α	T_C^n (MeV) (with IE)	T_C^p (MeV) (with IE)	T_C^n (MeV) (with MEA)	T_C^p (MeV) (with MEA)
0.0	15.90	15.90	14.60	14.60
0.1	14.12	17.64	13.00	16.52
0.3	10.30	21.03	8.92	20.10
0.5	5.58	24.41	4.34	23.67

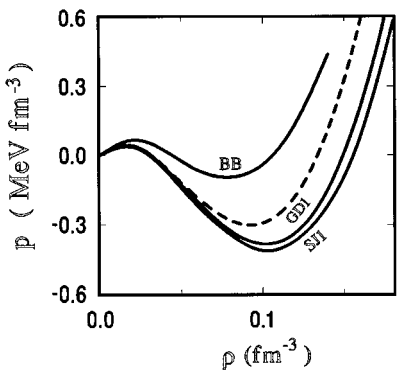

FIG. 2. Isotherms ($T=8$ MeV) for the symmetric nuclear system with the Brink-Boeker (BB) interaction, Gogny (GD1) interaction, and Skyrme (SJ1) interactions. The dashed line is calculated by the momentum expansion approximation.

zero-range interactions and then, equally, it plays an ''attractivelike'' role and reduces the pressure. But the density-dependence part, because it has the same behavior as that of the Skyrme interaction, will give us positive result. Another important result shown by Table II is that the values of p_{FR} and p_{DD} have the same order of magnitude. It means that the finite-range part and the density-dependence part of interaction have the same importance for the thermodynamical properties of nuclear systems. In this sense, the Gogny interaction has an advantage over that of the Skyrme interaction and BB interaction.

To exhibit the above statement transparently, we compare $T=8$ MeV, $\alpha=0$ isotherms of SJ1, GD1, and BB interactions in Fig. 2, where the SJ1 curve is given by our previous paper [9] and the BB curve by Ref. [12]. Since the curve of the BB interaction in Ref. [12] is calculated by the MEA, we calculate, respectively, the isotherms of the GD1 interaction by the MEA and IE approaches and show the result of the MEA by a dashed line. From Fig. 2, we get two results, first, $p_{\mathrm{BB}}>p_{\mathrm{GD1}}>p_{\mathrm{SJ1}}$ for a fixed density. Second, the method of the MEA [up to $O(k^4)$] can only be used to calculate the pressure in the low density region; in the middle and high density regions ($\rho>0.05$ fm^{-3}), the errors are considerable.

Now we discuss the asymmetric effect. The $T=8$ MeV isotherms of the GD1 interaction for various asymmetric parameters are shown in Fig. 3. The critical asymmetry parameter α_C is 0.693 (for $T=8$ MeV), above which liquid-gas coexistence is impossible. This result is similar to that of Skyrme interactions [11]. The pressure p vs α curves with fixed $T=6$ MeV, $\rho=0.1$ fm^{-3} for BB, GD1, and SKI are shown in Fig. 4. We see that the pressures given by different interactions all increase with α, but rapidly for the BB interaction and slowly for the SKI interaction. This result is of course inconsistent with our density-dependence and finite-range arguments.

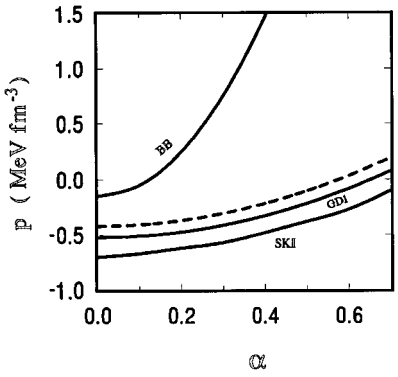

FIG. 4. Pressure vs asymmetric parameter curves for the nuclear system with the Brink-Boeker interaction (BB), Gogny interaction (GD1), and Skyrme interaction (SKI) where we fix $T=6$ MeV, $\rho=0.1$ fm^{-3}. The dashed line is calculated by the momentum expansion approximation.

For finite nuclear systems, as pointed out by Refs. [6,20], it will be more advantageous to work with the chemical potential μ rather than to work with pressure p because of the finite-size effect. The proton and neutron have different single energies and chemical potentials due to the Coulomb interaction and asymmetric effect. Because the proton and neutron are not in chemical equilibrium, although they may be in thermal equilibrium, their chemical potentials are not related to each other. Since the proton and neutron have different chemical potentials, they will also appear to have different critical temperatures T_C^p and T_C^n, respectively. As was argued by Refs. [6,9,20], we must choose the smaller of T_C^p

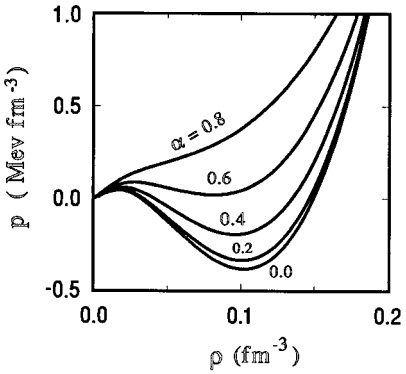

FIG. 3. $T=8$ MeV isotherms for various asymmetric parameters.

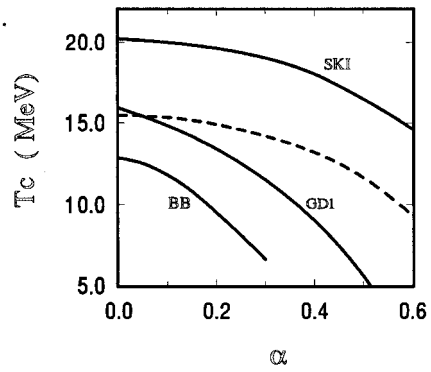

FIG. 5. The asymmetric parameter dependence of the critical temperature for the nuclear system with the Brink-Boeker interaction (BB), Gogny interaction (GD1), and Skyrme interaction (SKI). The dashed line is calculated by the momentum expansion approximation.

TABLE IV. Relevant equilibrium values at $T=T_{\lim}$ with different nucleon number A.

A	$T_{\lim}$ (MeV)	ρ_L (fm^{-3})	ρ_V (fm^{-3})	p_V (MeV fm^{-3})	μ_V^n (MeV)	μ_V^p (MeV)	α_V
10	9.32	0.166	0.017	0.070	−15.44	−15.60	0.004
50	8.19	0.161	0.014	0.052	−12.92	−13.86	0.027
109	6.77	0.161	0.011	0.034	−9.58	−11.89	0.078
150	6.11	0.161	0.010	0.027	−7.79	−11.25	0.128
208	5.42	0.160	0.009	0.022	−5.67	−10.87	0.214

and T_C^n as the correct critical temperature. The critical temperatures T_C^n given by μ_n vs ρ isotherms and T_C^p given by μ_p vs ρ isotherms with different α for the GD1 interaction are shown in Table III and Fig. 5. For comparison, we show our results calculated by the MEA and IE together in Table III. We found that T_C^n decreases but T_C^p increases as α increases. T_C^n is always less than T_C^p, and $T_C^p - T_C^n$ becomes larger when α increases. In Fig. 5, we draw $T_C = T_C^n$ vs α curves for BB, GD1, and SKI interactions simultaneously. We see that for a fixed α, the critical temperature of SKI is higher, GD1 middle, and BB lower. Besides, the discrenpancy between the MEA and IE in Fig. 5 is remarkble.

Now we turn to discuss the isothermal incompressibility K_T which is an important quantity reflecting the property of the nuclear system. K_T is defined as

$$K_T = 9\left(\frac{\partial p}{\partial \rho}\right)\Bigg|_{\rho=\rho_0}, \tag{27}$$

where $\rho_0(\alpha, T)$ is the saturation density at a particular temperature and asymmetric parameter when the free energy is minimum. In order to compare the result given by the GD1 interaction with that given by the BB interaction in Ref. [12], we show the ρ_0 vs α curves and $K_T(\alpha)/K_T(\alpha=0)$ vs α curves in Figs. 6 and 7, where the solid lines refer to $T=6$ MeV and dashed lines for $T=7$ MeV, respectively. We find from Fig. 6 that, for a fixed temperature, the saturation density ρ_0 for the GD1 interaction is larger than that of the BB interaction, no matter whether the temperature is large or small. We also find that ρ_0 decreases as α increases for both two cases, but the decrements are different. The saturation density ρ_0 decreases rapidly for the BB interaction. Similar behavior for $K_T(\alpha)/K_T(\alpha=0)$ vs α is found in Fig. 7. We see for a fixed temperature and α, the isothermal incompressibility $K_T(\alpha)$ for the GD1 interaction is larger than that of the BB interaction. The softness of K_T for the BB interaction is partly due to a missing density dependence and partly due to the finite-range enhancement. The effect of the finite-range force for the BB interaction is larger than that of the GD1 interaction.

Finally we discuss the Coulomb instability of the Gogny model. For a given nucleus, starting from a properly low temperature, we solve the coexistence equations (1)–(3) for the nuclear density ρ_L of the liquid phase, the nuclear density ρ_V, and the asymmetry parameter α_V of the vapor phase. Then we increase the value of the temperature and repeat the above calculation. We find that when the temperature reaches a certain value, the co-existence equations have no solution. This value of temperature gives the limiting temperature $T_{\lim}$. We present the A dependence of $T_{\lim}$ calculated from the GD1 interaction for nuclei along the well-known β-stability line,

$$Z = 0.5A - 0.3\times 10^{-2} A^{5/3}, \tag{28}$$

in Table IV and Fig. 8. The equilibrium values of the relevant quantities at the limiting temperature for different A are shown in Table IV. We see from this table that when A increases from 10 to 208, $T_{\lim}$ decreases from 9.32 MeV to 5.42 MeV monotonously. The rate of decrement is smaller in large A regions than that in small A regions. For comparison, we have drawn in Fig. 8 the $T_{\lim}$ vs A curves calculated from the SKI, GD1, and BB interactions simultaneously. It can be

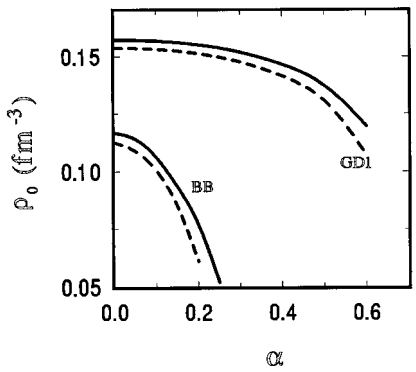

FIG. 6. The asymmetric parameter α and temperature dependence of saturation density ρ_0 for the nuclear system with the Brink-Boeker interaction (BB) and Gogny interaction (GD1). We choose the temperature as $T=6$ MeV and 7 MeV. The curves of the BB interaction are chosen from Ref. [12].

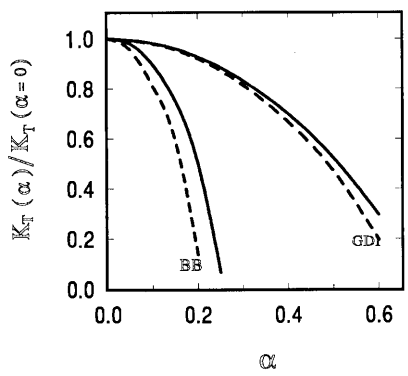

FIG. 7. Same as Fig. 6, but for isothermal incompressibility.

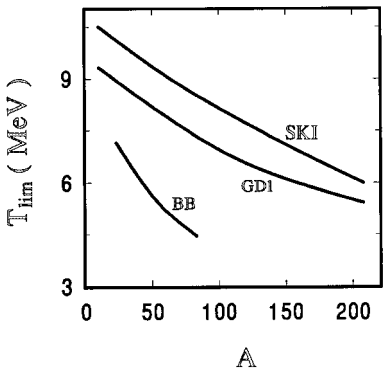

FIG. 8. Mass number A dependence of limiting temperature $T_{\rm lim}$ calculated with the Skyrme interaction (SKI), Gogny interaction (GD1), and Brink-Boeker interaction (BB).

seen that the three curves have a similar tendency but the decreasing slopes are different. The decrement of $T_{\rm lim}$ for the BB interaction is more rapid than the SKI and GD1 interactions. The basic reason for this rapid decrease comes from the missing of density dependence in the BB interaction as was discussed before. Because the decrement is too rapid, the limiting temperature, for example, 3.1 MeV for $A=150$, seems too small for comparison with experiment.

In summary, we have studied the isotherms of various temperature and asymmetric parameters and calculated the critical temperature, limiting temperature, isothermal incompressibility, and saturation density for the Gogny interaction in details. We have used two approaches, namely, the momentum expansion approximation up to $O(k^4)$ and solving integral equations exactly, to calculate thermodynamical quantities. We have found that the momentum expansion approximation method can only be used in low density regions; beyond this region, the discrepancy is remarkable. To illustrate the importance of the finite-range part and density-dependence part of the interaction, we have compared our results with those given by the Skyrme interaction and Brink-Boeker interaction, and calculated the contributions of these two parts of the GD1 interaction to pressure carefully. We have found that even though the contributions of these two parts are opposite, their importance for the thermodynamical properties of nuclear systems is almost equal. In this sense, the Gogny interaction may be a better interaction to describe the nuclear system.

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