Spontaneous symmetry breaking and the dissipation of nuclear collective degrees of freedom at finite temperature

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The critical temperature for the dissipation of the collective degrees of freedom has been determined in a quantum field theoretical description of the Elliott model in which the U(3) symmetry is spontaneously broken in such a manner that rotational invariance is preserved. Results for ^{20}Ne, with a reasonable choice of parameters, seem to indicate that this critical temperature lies below the liquid-to-gas critical temperature but above the temperature of deformed-to-spherical shape transitions which have been observed in finite temperature mean field calculations. [S0556-2813(96)00108-2]

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One of the universal features of finite nuclei is a significant change in the density of states at excitation energies of 10 MeV or less [1]. At lower excitation energies the spectrum of most nuclei is sparse and dominated by a relatively small number of collective states. With increasing excitation energy, the independent particle degrees of freedom dominate and the density of states grows exponentially. As the mass number increases, the low-lying collective portion of the energy spectrum becomes more compressed and the abrupt change in the many-particle density of states occurs at lower excitation energies. Since the nuclear force is short ranged and saturates rather quickly, one would expect that such a change in the many-body level density might also occur in nuclear matter [2]. It has, therefore, been suggested that a collective-to-noncollective phase transition occurs in finite nuclei [3]. Strictly speaking, it is incorrect to speak of phase transitions in finite systems. No one can deny, however, that transitions between different regimes do take place in these systems, and that they are more or less abrupt. This issue, particularly in deformed systems, has been clouded by the fact that finite temperature mean field calculations have suggested that this phase transition is simply due to a drastic change of shape [4]. The deformed-to-spherical shape transition seen in these calculations is not seen in exact canonical calculations [5–8]. It appears to be an artifact of the finite temperature mean field approximation, and also depends on the volume of the system [9,10]. In spite of the fact that the canonical partition function above the critical temperature is dominated by the single-particle degrees of freedom, a few collective states still contribute and are extremely important in calculation of shape dependent parameters. Recent calculations of the ensemble average of the quadrupole moment squared $Q^{[2]}\cdot Q^{[2]}$ indicate that it is discontinuous in the finite temperature mean field approximation, while no discontinuity is observed in the canonical calculations [11]. In both cases this quantity does not appear to vanish at the critical temperature. It should also be noted, however, that when thermal fluctuations in the shape dependent order parameters are taken into account, by either macroscopic or microscopic procedures, a reasonable agreement with the exact canonical calculations [5] is obtained. For this reason there is some concern about attempts to confirm experimentally the existence of such shape transitions [12].

With increasing temperature, however, it is expected that these collective degrees of freedom will eventually completely dissolve, presumably below the critical temperature of the liquid-to-gas phase transition. Above this temperature, collective effects such as nuclear deformation will no longer be present. To determine this temperature in a simple model, a finite temperature extension of a quantum field theoretical description of the Elliott U(3) model [13], in which the U(3) symmetry is spontaneously broken in such a manner that rotational invariance is still preserved [14], is required. In this case six zero-energy Nambu Goldstone bosons are created, which are characterized by spin zero and 2, and the mechanism is closely related to the superconductivity model with a similar type of order or gap parameter [14]. The collective modes in such a system are these Goldstone modes. The vanishing of the gap parameters for both the $l=0$ and $l=2$ bosons at some critical temperature should signal the disappearance of the collective degrees of freedom.

The nonrelativistic model Lagrangian for the nucleon fields, which is invariant under the Elliott U(3) transformation, is given by [14]

$$L(t)=\int d^3x\,\phi^*(x)\dot{\phi}(x)-H \tag{1}$$

with

$$H=\int d^3x\,\phi^*(x)(\mathbf{x}^2-\boldsymbol{\nabla}_{\mathbf{x}}^2)\phi(x)-\frac{v_0}{2}\sum_{kq}(-)^q \times\int d^3x d^3y\,\phi^*(x)\phi^*(y)T_q^k(\mathbf{x},\boldsymbol{\nabla}_{\mathbf{x}})T_{-q}^k(\mathbf{y},\Delta_{\mathbf{y}}) \times\phi(x)\phi(y). \tag{2}$$

Here

$$T_q^k(\mathbf{x},\nabla_{\mathbf{x}})=\sum_{\alpha\beta}\langle 1\alpha 1-\beta|kq\rangle(-1)^{1-\beta}\frac{1}{2}(a_\alpha^\dagger a_\beta+a_\beta a_\alpha^\dagger) \tag{3}$$

with

$$a_\alpha^\dagger=\frac{1}{\sqrt{2}}(x_\alpha+ip_\alpha)=\frac{1}{\sqrt{2}}(x_\alpha+\boldsymbol{\nabla}_\alpha),$$

$$a_\alpha=\frac{1}{\sqrt{2}}(x_\alpha-ip_\alpha)=\frac{1}{\sqrt{2}}(x_\alpha-\nabla_\alpha),$$

from which one easily obtains

$$T_0^0=\frac{1}{2\sqrt{3}}(\mathbf{x}^2-\nabla_{\mathbf{x}}^2),$$

$$T_q^1=\frac{i}{\sqrt{2}}(\mathbf{x}\times\boldsymbol{\nabla}_{\mathbf{x}})_q,$$

$$T_q^2=\frac{1}{6}\sqrt{\frac{4\pi}{5}}[x^2Y_q^2(\theta_x\phi_x)+\nabla^2Y_q^2(\theta_\nabla\phi_\nabla)]. \tag{4}$$

The nine operators in (4) are the complete set of generators of the U(3) group with the three operators T_q^1 forming its rotational subgroup R(3). The model Hamiltonian in (2) is often used to study the rotational spectra in nuclei [13]. For simplicity the nucleon fields $\phi(x)$ and $\phi^*(x)$ in (1) and (2) are assumed to be scalar fields,

$$[\phi(x),\phi^*(x')]_{t=t'}=\delta^3(x-x'), \tag{5}$$

since we are interested in the effects of spontaneous symmetry breaking and the role it plays in the disappearance of the collective rotational degrees of freedom in a deformed system. These fields may be expanded in terms of a complete set of three-dimensional harmonic oscillator functions,

$$\phi(x)=\sum_{nlm}R_{nlm}(x)b_{lnm}, \tag{6}$$

where

$$[b_{nlm},b_{n'l'm'}^\dagger]=\delta_{nn'}\delta_{ll'}\delta_{mm'}. \tag{7}$$

Furthermore, in order to keep the calculations tractable we shall restrict the space of the bosons to that of the *sd* shell. Admittedly the model space is small, but for a nucleus like ^{20}Ne with a ground state rotational band which is well described in the Elliott model and whose highest-lying member is well described in this model space, we hope to obtain a qualitative understanding of the disappearance of the collective degrees of freedom at finite temperature and to demonstrate that their disappearance does not coincide with the previously mentioned deformed-to-spherical shape transitions seen in finite temperature mean field calculations. However, because of the aforementioned approximations the present results must be regarded in a more qualitative manner.

In Ref. [14] it has been noted that the order parameter for the Elliott model is the same as that in the theory of superconductivity and that the corresponding vacuum of the nuclear system should have a BCS-like structure. Considering the Bogoliubov transformation for bosons [15],

$$b_{nlm}=u_{nl}\beta_{nlm}-v_{nl}(-)^m\beta_{nl-m}^\dagger,$$

$$b_{nlm}^\dagger=u_{nl}\beta_{nlm}^\dagger-v_{nl}(-)^m\beta_{nl-m},$$

$$u_{nl}^2-v_{nl}^2=1,$$

one chooses the transformation parameters to eliminate quasiparticle-quasiparticle and quasihole-quasihole terms in the Hamiltonian. This yields

$$u_{nl}v_{nl}=\frac{1}{2}\frac{\Delta_{nl}}{E_{nl}}, \tag{8}$$

$$u_{nl}^2+v_{nl}^2=\frac{\xi_{nl}}{E_{nl}}, \tag{9}$$

where ξ_{nl} are the Hartree self-consistent single-particle energies given by

$$\begin{aligned}\xi_{nl}&=2\epsilon_{nl}-\frac{2v_0}{(2l+1)^2}v_{nl}^2\sum_k(-)^k(nl||T^{[k]}||nl)^2\\&=2\epsilon_{nl}-\frac{2v_0}{2l+1}v_{nl}^2\bigg[\frac{1}{3}\epsilon_{nl}^2-\frac{1}{2}l(l+1)\\&\quad+\frac{(nl||T^{[2]}||nl)^2}{2l+1}\bigg],\end{aligned} \tag{10}$$

where ϵ_{nl} are the unperturbed harmonic oscillator single-particle energies, Δ_{nl} denote the BCS gaps, and

$$E_{nl}=\sqrt{\xi_{nl}^2-\Delta_{nl}^2} \tag{11}$$

are the quasiparticle energies. The BCS gaps satisfy the gap equations

$$\Delta_{nl}=\frac{v_0}{2l+1}\sum_k(-)^k\sum_{n'l'}(n'l'||T^{[k]}||nl)^2\frac{\Delta_{n'l'}}{2E_{n'l'}}. \tag{12}$$

Calculating the reduced matrix elements for T^0 and T^1 yields

$$\Delta_{nl}=v_0\sum_{n'l'}\left\{\left[\frac{1}{3}\epsilon_{nl}^2-\frac{1}{2}l(l+1)\right]\delta_{nn'}\delta_{ll'}+(g_{ll'}^{nn'})^2\right\}\frac{\Delta_{n'l'}}{2E_{n'l'}}. \tag{13}$$

Note this form corrects that given in Ref. [14]. In the above equation,

$$g_{ll'}^{nn'}=\frac{1}{2l+1}(n'l'||T^{[2]}||nl), \tag{14}$$

where

$$(n'l'||T^{[2]}||nl)=\frac{2}{\sqrt{6}}\sqrt{\frac{4\pi}{5}}(n'l'||x^2Y^{[2]}(\theta_x,\phi_x)||nl)R_{l'l}^{n'n} \tag{15}$$

and

$$R_{l'l}^{n'n}=\int x^2dxR_{n'l'}(x)x^2R_{nl}(x). \tag{16}$$

In the sd shell for the Hamiltonian operator given in (2) it is easy to show that

$$(g_{00}^{11})^2=0,\quad (g_{02}^{10})^2=\frac{20}{3},$$

$$(g_{20}^{01})^2=\frac{4}{3},\quad (g_{22}^{00})^2=\frac{7}{3}. \tag{17}$$

It is straightforward to extend these equations to finite temperature by setting the expectation values of the quasiparticle occupation probabilities to the Bose factor [16,17], i.e.,

$$\langle\beta_{nlm}^{\dagger}\beta_{nlm}\rangle=f_B(E_{nl})=\frac{1}{e^{E_{nl}/T}-1}, \tag{18}$$

$$\langle\beta_{nlm}\beta_{nlm}^{\dagger}\rangle=1-f_B(E_{nl}). \tag{19}$$

The Hartree single-particle energies at temperature T are

$$\xi_{nl}=2\epsilon_{nl}-\frac{2v_0}{2l+1}\left[\frac{1}{3}\epsilon_{nl}^2-\frac{1}{2}l(l+1)+(g_{ll}^{nn})^2\right]\times\left[f_B(E_{nl})+v_{nl}^2\coth\left(\frac{E_{nl}}{2T}\right)\right], \tag{20}$$

and the gap equations at finite T are given by

$$\Delta_{nl}=v_0\sum_{n'l'}\left\{\left[\frac{1}{3}\epsilon_{nl}^2-\frac{1}{2}l(l+1)\right]\delta_{nn'}\delta_{ll'}+(g_{ll'}^{nn'})^2\right\}\times\frac{\Delta_{n'l'}}{2E_{n'l'}}\coth\left(\frac{E_{n'l'}}{2T}\right). \tag{21}$$

It is easy to see that all the gaps vanish at the same critical temperature T_c.

In the sd shell there is only one single-particle energy,

$$\epsilon_{10}=\epsilon_{02}=\epsilon,$$

which, since the zero of energy is unknown, is a free parameter. Its value together with that of the coupling strength v_0 then uniquely specify the model. One of these parameters, v_0, has been eliminated by insisting that the solution Δ_{10} of the uncoupled s-wave gap equation reproduces the empirical gap at zero temperature [19,20],

$$\Delta_0=12A^{-1/2}$$

(which is approximately 3.5 for $A=20$). The solution to the uncoupled gap equation is

$$\Delta_0^2=\epsilon^2\left\{\frac{4}{9}+\frac{4}{27}v_0\epsilon-\frac{5}{324}(v_0\epsilon)^2\right\}=\frac{144}{A}.$$

This has a real solution for $v_0\epsilon$ only if

$$\epsilon>3\sqrt{\frac{20}{A}}.$$

The quadratic has two solutions, one above and one below $v_0\epsilon=24/5=4.8$. The lower solution has been selected since physically one expects $v_0\epsilon\leqslant 4.8$. Now further requiring $v_0\epsilon>0$ it follows that

$$\epsilon<\frac{18}{\sqrt{A}}.$$

In Table I the zero temperature gaps, the value of v_0, and the critical temperature are given as a function of ϵ for ^{20}Ne. For ϵ in the range 3.2 to 3.8 MeV the critical temperature lies well above that for the low-lying deformed-to-spherical shape transitions seen in finite temperature mean

TABLE I. The coupling strength v_0, the $l=0$ and $l=2$ gaps at zero temperature, $\Delta_{10}(0)$ and $\Delta_{02}(0)$, and the critical temperature T_c, all as functions of the single-particle energy ϵ.

ϵ (MeV)	v_0 (MeV)	$\Delta_{10}(0)$ (MeV)	Δ_{02} (MeV)	T_c (MeV)
3.0	1.600	6.338	2.594	2.127
3.2	0.717	3.582	1.411	3.524
3.4	0.415	3.068	1.091	4.954
3.6	0.228	2.837	0.898	7.684
3.8	0.100	2.728	0.767	15.295
4.0	0.009	2.685	0.672	145.43

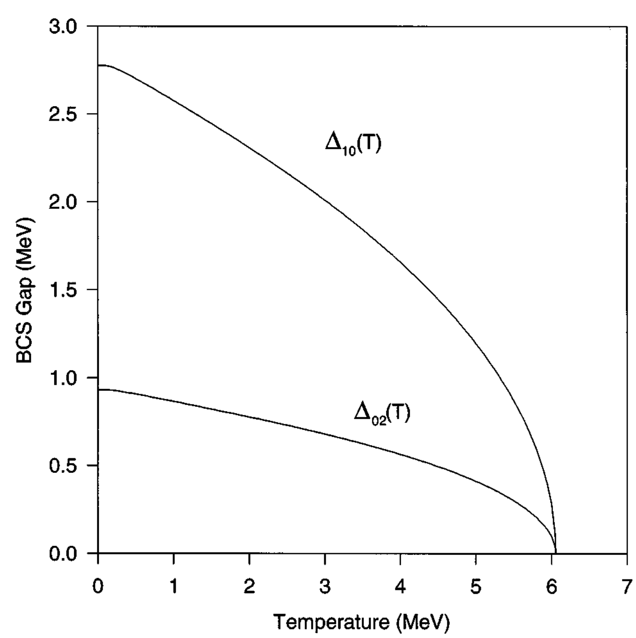

FIG. 1. The $l=0$ BCS gap $\Delta_{10}(T)$ (MeV), and the $l=2$ BCS gap $\Delta_{02}(T)$ (MeV) as a function of the temperature T (MeV) for single-particle energy $\epsilon=3.5$ MeV.

field calculations, $T_c \approx 2.1$ MeV [3,18,7], but below that for the bulk liquid-to-gas transition, $T_{\mathrm{LG}} \approx 15-20$ MeV [21,22]. (Recent experimental measurements seem to indicate that the critical temperature for the liquid-to-gas phase transition may be as low as 5 MeV in finite nuclei [23].) For the coupling strength values $v_0 = 0.1$–0.7 MeV [8] one obtains resonable values, and for the 0^+-2^+ energy level splitting

$$E_{2^+} - E_{0^+} = \Delta_{10} - \Delta_{02}.$$

(Experimentally this energy level splitting is approximately 1.6 MeV [24] and is fitted in the exact SU_3 shell model calculations with $v_0 = 0.09$ MeV [8].) Moreover, for $\epsilon = 3.2$ MeV and $v_0 = 0.717$ MeV a reasonable value for the 0^+-2^+ energy level splitting (2.1 MeV) as well as $\Delta_{10} \approx \Delta_0$ is obtained, and $T_c = 3.5$ MeV, which is above the critical temperature for the deformed-to-spherical shape transition but below that of the liquid-to-gas transition. This then indicates the total dissolution of the collective degrees of freedom prior to nuclear breakup. As ϵ approaches its upper bound the gaps and critical temperature diverge, indicating a breakdown of the weak-coupling BCS theory. Unfortunately, although the results are sensitive to the values of v_0 or equivalently the energy level splitting, there is no reliable way to fix these values precisely in such a simple model. Although the critical temperatures obtained are a bit large for the model space considered, what is important is that they are significantly higher than the critical temperatures of the previously observed deformed-to-spherical shape transitions. Realistic calculations are therefore essential to verify the existence and the value of the critical temperature for such a transition.

In Fig. 1 the $l = 0$ and $l = 2$ gaps are plotted as a function of temperature for $\epsilon = 3.5$ MeV corresponding to $v_0 = 0.312$ and $T_c = 6.058$ MeV. Note that the s-wave pairs are always more deeply bound than the d-wave pairs at any temperature, but they both become unbound at the same temperature.

In the present paper we have presented a very simple but general mechanism for the dissipation of collective degrees of freedom due to nuclear deformation by means of spontaneously breaking U(3) symmetry in such a manner that rotational invariance is preserved. The present calculations in ^{20}Ne in the Elliott model, with a realistic choice of parameters, indicate that the temperature at which these collective degrees of freedom disappear lies well above the temperature of the collective-to-noncollective transition observed in canonical and finite temperature Hartree-Fock calculations but below the liquid-to-gas critical temperature. More realistic effective interactions which contain SU(3) symmetry breaking terms, such as the tensor and spin-orbit interaction, are not expected to alter the present results in any significant manner.

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