Mass and width of the d' resonance in nuclei

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We calculated the mass and width of the d' resonance inside nuclei within a nucleon-Δ model by including the self-energy of the Δ in the $N\Delta$ propagator. We found that in the nuclear medium the width of the d' is increased by 1 order of magnitude while its mass changes only by a few MeV. This broadening of the width of the d' resonance embedded in nuclei is consistent with the experimental observations so that the d' can be understood as a $N\Delta$ resonance. Thus, given the freedom between either isospin 0 or isospin 2 for the d', our results give weight to the isospin-2 assignment. [S0556-2813(96)02209-1]

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I. INTRODUCTION

The possible existence of a resonance with quantum numbers $J^P=0^-$ and isospin even has been inferred from the differential cross section of the double charge exchange (DCX) reaction [1] in nuclei ranging from ^{14}C to ^{48}Ca. The authors of [1] chose the isospin-0 assignment for the resonance showing up in the DCX reaction based on the predictions of a QCD string model [2]. It was soon pointed out [3] that if the d' resonance decays into a pion and two nucleons, then it should be a solution of the equations describing the dynamics of the πNN system. That this is not the case for isospin 0 was already known before [4], since the dominant interaction of the πNN system in the isospin-0 sector is the pole part of the πN P_{11} channel. This interaction is forbidden by the Pauli principle in the case of the $J^P=0^-$ channel and as a consequence the 0^- channel is very weak (and repulsive [4]) so that no resonance can arise there. The situation of the isospin-2 sector is quite different. In this case, the dominant interaction is the πN P_{33} channel (the Δ resonance), and all existing conventional calculations [4–8] find that the 0^- channel is very attractive so that a resonance could exist in that channel. This led the authors of [3] to propose the isospin-2 assignment of the d'. Other calculations based on completely different dynamical models like the Skyrme model [9] or six-quark bags [10] have arrived at similar conclusions.

The known experimental facts of the d' resonance are the following. From the analysis of the DCX data performed in [1], its free width and its mass and width in the nuclear medium were extracted. They found for these quantities $\Gamma_{\rm free}=0.51$ MeV, $M=2065$ MeV, and $\Gamma_{\rm medium}=5$ MeV (they actually found it to be between 3 and 7 MeV and took the average value) for nuclei ranging from ^{14}C to ^{48}Ca. No value was given for the free mass of the d'. They also observed that the mass of the d' in the medium is almost independent of the nucleus. We should stress that these values were not obtained from a given theoretical model of the d' but were simply extracted from the experimental data.

In order to see if the main features of the d' can be predicted by a theory involving pions, nucleons, and Δ's, we have launched a study of the isospin-2 sector using as basic framework the nucleon-Δ interaction derived from the chiral quark cluster model [11–13]. In [5] we found that the 0^- channel is the most attractive channel of the isospin-2 sector. However, since in that calculation we neglected the nonlocal terms of the $N\Delta$ interaction as well as the contribution of the $\Delta\Delta$ channels, we found that the existing attraction was not enough to give rise to a resonance. In order to simulate the lacking attraction we varied the mass of the sigma meson that is responsible for the intermediate range attraction, until a resonance of mass 2064.4 MeV was produced. We found that the free width of the resonance is 0.6 MeV, in agreement with the value extracted in [1]. The problem of the lack of attraction of our model has been partially solved in [14] by including the nonlocal terms of the $N\Delta$ interaction. This improved model is able to produce a resonance although with a mass 80 MeV higher than the experimentally observed one. In order to get the mass of the resonance down to its correct value it probably will be necessary to include the contribution of the $\Delta\Delta$ channels. As is well known [15], the inclusion of coupled channels with a threshold above the energy in consideration leads always to additional attraction. The main result obtained in these works was that the width of the d' is strongly correlated with its mass such that if the mass of the d' approaches the πNN threshold then the width drops very fast to very small values.

Our aim in this paper is to try to understand the effects of the nuclear medium in the properties of the d' resonance. In Sec. II we will briefly review our formalism and its extension to the case when the d' is inside a nucleus. Section III will be devoted to the concept of the self-energy of the Δ due to the presence of a nuclear medium. We present our results in Sec. IV, and we conclude with their discussion in Sec. V.

II. FORMALISM

Since there is considerable theoretical evidence that a resonance with isospin 2 exists in the $J^P=0^-$ channel [4,6–10], we have calculated the mass and width of the d' resonance in [5,14] by solving the Lippmann-Schwinger equation for the $N\Delta$ system,

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$$T_{ij}(\sqrt{S};\vec{q}\,',\vec{q}_0)=V_{ij}(\vec{q}\,',\vec{q}_0)+\sum_k\int d\vec{q}V_{ik}(\vec{q}\,',\vec{q})\times G_0(\sqrt{S},q)T_{kj}(\sqrt{S};\vec{q},\vec{q}_0), \quad (1)$$

where $V_{ik}(\vec{q}\,',\vec{q})$ are the (in general nonlocal) $N\Delta$ interactions predicted by the chiral-quark-cluster model [11–13]. The propagator of the $N\Delta$ intermediate state is [5]

$$G_\Delta(\sqrt{S},q)=\frac{2M_\Delta}{s-M_\Delta^2+iM_\Delta\Gamma_\Delta(q)}, \quad (2)$$

where $\sqrt{S}$ is the invariant mass of the system, q is the magnitude of the $N\Delta$ relative momentum and s is the invariant mass squared of the πN subsystem given by

$$s=S+M_N^2-2\sqrt{S(M_N^2+q^2)}. \quad (3)$$

The width $\Gamma_\Delta(q)$ was taken as

$$\Gamma_\Delta(q)=\frac{2}{3}\,0.35p_0^3\frac{\sqrt{M_N^2+q^2}}{m_\pi^2\sqrt{s}}, \quad (4)$$

where

$$p_0=\left(\frac{[s-(M_N+m_\pi)^2][s-(M_N-m_\pi)^2]}{4s}\right)^{1/2}, \quad (5)$$

is the magnitude of the pion-nucleon relative momentum.

From the solution of these equations in the case when they give rise to a resonance, we obtained the properties of the d' (mass and width) in free space. As already mentioned, we found in [5] that the $J^P=0^-$ channel is the most attractive one in the isospin-2 sector. In our model this attraction originates from several facts. First of all, the quark Pauli blocking giving rise to a very strong repulsive barrier at short distances is not present in this channel [12,13]. Second, the dominant terms of the interaction at intermediate range, i.e., the sigma exchange and the tensor part of the one-pion exchange are both attractive. Finally, the contribution from the central part of the one-pion exchange and the gluons is rather weak, the last one being repulsive but restricted to very small distances. Notice that the short-range behavior has almost no effect in this channel since the nucleon and the Δ are in a relative P-wave state.

We will now extend our formalism in order to study how the properties of the d' (mass and width) are modified when it is embedded in a nuclear medium. The d' resonance behaves very differently inside a nucleus than in free space. In particular, in free space the d' cannot decay into two nucleons, while inside a nucleus such process is possible as show in Fig. 1 for a typical diagram. The main physical concept necessary in order to describe theoretically the influence of the medium in a $N\Delta$ system is the self-energy of the Δ in the medium [16–19]. This quantity has been calculated for the case of infinite nuclear matter [16–18] as well as for the case of finite nuclei [19].

In the case of a single Δ propagating through a nuclear medium, the propagator of the Δ is given by

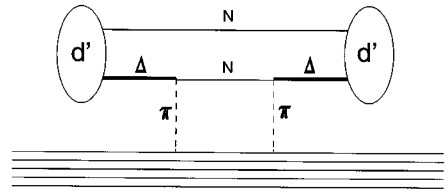

FIG. 1. Diagramatic representation of a process that gives origin to the broadening of the d' resonance in nuclei.

$$G_\Delta(E,\omega)=\frac{1}{E-M_E(\omega)+(i/2)\Gamma_E(\omega)}, \quad (6)$$

where $M_E(\omega)$ and $(1/2)\Gamma_E(\omega)$ are the effective mass and one-half of the width of the Δ in the medium,

$$M_E(\omega)=M_\Delta+\mathrm{Re}\Sigma_\Delta(\omega), \quad (7)$$

$$\tfrac{1}{2}\Gamma_E(\omega)=\tfrac{1}{2}\Gamma_\Delta-\mathrm{Im}\Sigma_\Delta(\omega), \quad (8)$$

with M_Δ and Γ_Δ being the free mass and width of the Δ, respectively, and $\mathrm{Re}\Sigma_\Delta(\omega)$ and $\mathrm{Im}\Sigma_\Delta(\omega)$ the real and imaginary parts of the self-energy of the Δ in the medium which are functions of the pion energy ω (see the next section).

Thus, in order to study the effects of the nuclear medium in the properties of the d' we solved the Lippmann-Schwinger equation (1) including the self-energy of the Δ in the $N\Delta$ propagator (2), i.e., we made in that equation the replacements

$$M_\Delta\rightarrow M_\Delta+\mathrm{Re}\Sigma_\Delta(\omega), \quad (9)$$

$$\tfrac{1}{2}\Gamma_\Delta(q)\rightarrow\tfrac{1}{2}\Gamma_\Delta(q)-\mathrm{Im}\Sigma_\Delta(\omega), \quad (10)$$

with

$$\omega=\sqrt{p_0^2+m_\pi^2}, \quad (11)$$

and p_0 given by Eq. (5).

III. THE SELF-ENERGY OF THE Δ

The self-energy of the Δ has been calculated in infinite nuclear matter [16–18] and more recently also for the case of finite nuclei [19]. The most important quantity, with respect to our particular problem, is the imaginary part of the self-energy $\mathrm{Im}\Sigma_\Delta(\omega)$. This quantity has been calculated for finite nuclei by Hjorth-Jensen, Müther, and Polls [19] within the framework of perturbative many-body theory using a basis of single-particle states appropriate for both bound hole states and for particle states in the continuum.

We show in Fig. 2 the values of $|\mathrm{Im}\Sigma_\Delta(\omega)|$ for $0\leqslant\omega\leqslant200$ MeV for the three nuclei ^{16}O, ^{40}Ca, and ^{100}Sn obtained in [19] together with the result of Nieves *et al.* [18] for infinite nuclear matter. For comparison, we show in the same figure $\Gamma_\Delta/2$, one-half of the free width of the Δ given by Eq. (4). The most striking feature that is observed in Fig. 2 is that while Γ_Δ starts at $\omega\approx140$ MeV (the mass of the pion), the imaginary part of the self-energies of infinite nuclear matter and finite nuclei starts already at $\omega=0$. This

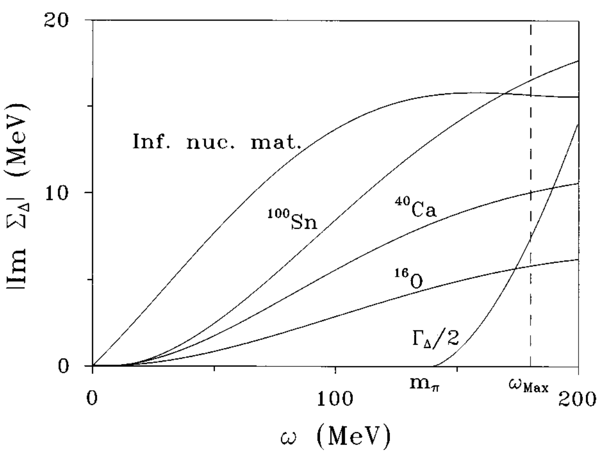

FIG. 2. Imaginary part of the self-energies of the Δ for the three nuclei ^{16}O, ^{40}Ca and ^{100}Sn [19] as well as for infinite nuclear matter [18]. Also shown in the figure is the free width of the Δ given by Eq. (4) divided by 2.

behavior simply reflects the fact that the pion can be scattered by a nucleon only if $\omega \geqslant m_\pi$ (the physical region for scattering), while the process of absorption of a pion by a nucleus can occur if the pion is at rest ($\omega = m_\pi$) or even if the pion energy is smaller than its rest mass, as long as $\omega \geqslant 0$. We have also drawn in Fig. 2 the maximum value of ω that enters in Eqs. (1)–(5) and (9)–(11):

$$\omega_{\max} = \frac{(M_{d'} - M_N)^2 - M_N^2 + m_\pi^2}{2(M_{d'} - M_N)}. \tag{12}$$

This $\omega_{\max}$ is obtained by taking $q = 0$ and $\sqrt{S} = M_{d'}$ in Eqs. (2)–(5) and (11). It is easy to see from these equations that as q increases ω decreases, such that it goes from $\omega_{\max}$ down to $-\infty$. Also, if the mass of the d' resonance decreases the value of $\omega_{\max}$ moves to the left in Fig. 2. In particular, if the mass of the resonance approaches the πNN threshold ($M_{d'} \to 2M_N + m_\pi$) then $\omega_{\max} \to m_\pi$.

The last feature that is worth noticing in Fig. 2 is that for finite nuclei $|\text{Im}\Sigma_\Delta|$ increases with the size of the nucleus, i.e., it is largest for ^{100}Sn and smallest for ^{16}O.

The real part of the self-energy $\text{Re}\Sigma_\Delta(\omega)$ has been found to be constant and equal to $-53\rho/\rho_0$ MeV for $\omega < 190$ MeV by Nieves *et al.* [18] in the case of infinite nuclear matter. This value of $\text{Re}\Sigma_\Delta \approx -50$ MeV for infinite nuclear matter has been known already for quite a long time [16]. Since in our calculation of the d' we have always $\omega < 190$ MeV, and the calculations of [19] correspond to $\rho/\rho_0 = 0.75$, we used $\text{Re}\Sigma_\Delta(\omega) = -40$ MeV.

IV. RESULTS

Before presenting our results it is worthwhile to discuss the general properties of resonances predicted by our $N\Delta$ model. Of particular importance is the behavior of the width of a resonance when its mass is near the πNN threshold as it is the case of the d'. Let us consider first the case when the d' is in free space, i.e., it appears as a solution of Eqs. (1)–(5). Since the mass of the d' (2065 MeV) is below the $N\Delta$ threshold (2172 MeV), then if the Δ were a stable particle the d' would be simply a bound state of the $N\Delta$ system. Thus, if we make $\Gamma_\Delta(q) = 0$ in Eq. (2), then Eq. (1) has a bound-state solution, i.e., a pole, at $\sqrt{S} \approx M_{d'}$. If we now put back the width of the Δ given by Eqs. (4) and (5) the pole moves from the real axis into the complex plane, i.e., to the position $\sqrt{S} = M_{d'} - (i/2)\Gamma_{d'}$. Thus, the d' acquires its width as a direct consequence of the width of the Δ.

We have shown in Fig. 2 the width of the Δ as a function of ω. Also, in the discussion of the previous section we showed that ω varies from $\omega_{\max}$ down to $-\infty$, so that only that part of Γ_Δ lying between $\omega = m_\pi$ and $\omega = \omega_{\max}$ contributes in the integral equation (1). However, as shown by Eq. (12) $\omega_{\max} \to m_\pi$ when $M_{d'} \to 2M_N + m_\pi$, which means that as $M_{d'}$ gets closer to the πNN threshold the contribution of Γ_Δ to Eq. (1) becomes less and less important and consequently the width of the d' tends to zero in the limit $M_{d'} \to 2M_N + m_\pi$ (notice that $M_{d'}$ is less than 50 MeV above the πNN threshold). In addition, for ω near $\omega_{\max}$ (where Γ_Δ is large) $q \to 0$ so that here Γ_Δ is suppressed by a factor q^3, q^2 coming from the volume element in Eq. (1) and q coming from the interaction (which is a P wave). That explains why the width of the d' is so small in free space ($\sim$ 0.5 MeV).

If we now consider the case when the d' is embedded in a nuclear medium we have to make the replacements $M_\Delta \to M_\Delta + \text{Re}\Sigma_\Delta$ and $(1/2)\Gamma_\Delta \to (1/2)\Gamma_\Delta - \text{Im}\Sigma_\Delta$ in Eqs. (1)–(5). Therefore, by looking at Fig. 2 one expects that the effect of including the self-energy of the Δ will be an increase in the width of the d' by about 1 order of magnitude, since the area under the curves for $|\text{Im}\Sigma_\Delta|$ is roughly 1 order of magnitude larger than for $(1/2)\Gamma_\Delta$ for ω between 0 and $\omega_{\max}$.

We show in Table I the results for the mass and width of the d' resonance when it is in free space, inside ^{16}O, ^{40}Ca, or ^{100}Sn, and inside infinite nuclear matter using our standard model of [5]. As one can see, the effect on the mass of the d' is an increase of 3–4 MeV. As expected from the previous discussion, the most important effects are seen on the width, which changes by more than 1 order of magnitude. Also, as expected from Fig. 2, the broadening increases with the size of the nucleus, being maximum for the case of infinite nuclear matter. If we neglect the real part of the self-energy, then the mass of the d' returns to the free mass except by a few tenths of MeV, while there is almost no change in any of the widths of the d' given in Table I.

Since by including the self-energy of the Δ the mass of the d' increases by 3–4 MeV, in order to account for this mass shift we have readjusted the mass of the sigma meson from 234 MeV to 231.5 MeV in our standard model. These results are shown in Table II. The mass of the d' in the medium is now in agreement with the experimentally extracted value. Notice that as observed in [1] the mass of the d' in the medium is almost independent of the nucleus in which the d' is embedded. However, the mass and the width of the d' in free space are now 2061.1 MeV and 0.47 MeV, respectively. We should point out that in [1] they extracted the mass of the d' embedded in nuclei and both the free width and the width in the medium, but not the free mass. Thus, the value 2061.1 MeV is not in contradiction with the value of the mass given in [1]. The free width is in very good

TABLE I. Mass and width of the d' resonance in free space, finite nuclear matter, and infinite nuclear matter for m_σ=234.0 MeV.

Case	$M_{d'}$ (MeV)	$\Gamma_{d'}$ (MeV)
Free space	2064.4	0.62
^{16}O	2067.8	6.18
^{40}Ca	2068.0	10.98
^{100}Sn	2068.5	16.88
Infinite nuclear matter	2068.9	23.96

TABLE II. Mass and width of the d' resonance in free space, finite nuclear matter, and infinite nuclear matter for m_σ=231.5 MeV.

Case	$M_{d'}$ (MeV)	$\Gamma_{d'}$ (MeV)
Free space	2061.1	0.47
^{16}O	2064.6	5.68
^{40}Ca	2064.8	10.30
^{100}Sn	2065.2	15.83
Infinite nuclear matter	2065.6	23.06

agreement with the value 0.51 MeV obtained in [1] while the widths in the medium for the nuclei ^{16}O and ^{40}Ca are comparable to those extracted in [1] although somewhat higher in the case of ^{40}Ca (they obtained between 3 and 7 MeV).

In order to understand the overestimation of the width of the d' in ^{40}Ca shown in Table II one should be aware of the following. The self-energies of [19] are r dependent, where r is the radial coordinate. They gave the values of Im$\Sigma_\Delta(\omega)$ shown in Fig. 2 which correspond to a ''typical'' radius of 1.5 fm. While this value of r is perhaps appropriate for light nuclei (carbon, oxygen), it is not for medium or heavy nuclei. In particular, in the case of ^{40}Ca the wave functions of the nucleons of the $1d_{3/2}$ and $1f_{7/2}$ shells, which are the relevant ones for the DCX reaction, peak at around 3 fm. Therefore, if one looks at the results shown in Fig. 2 of [19], one sees that Im$\Sigma_\Delta(\omega)$ should be roughly a factor of 2 smaller for r around 3 fm. If we repeat the calculation given in Table II for ^{40}Ca with Im$\Sigma_\Delta(\omega)$ divided by two, we get instead of 10.30 MeV, a value of 5.43 MeV for the width of the d'. This would be in good agreement with the observations of [1]. Using a similar argument the shift of the mass of the d' due to the medium (see Tables I and II) would be reduced. Since Re$\Sigma_\Delta(\omega)$ is a linear function of the density [18] and for $r\sim3$ fm the nuclear density has decreased by more than a factor 2, repeating again the calculation of Table II leads us to an estimate of the shift of the mass of the d' due to the medium, not being more than 2 MeV.

V. DISCUSSION

Thus, as we have just shown, our model is able to give a consistent description of the properties of the d' obtained in the analysis of [1] just by assuming that the d' is a $N\Delta$ resonance of isospin 2, i.e., the isospin-2 assignment of the d' is consistent with everything we know about pions, nucleons, and Δ's both when they are in free space or when they are embedded in nuclear matter. This strongly enhances the case for the isospin-2 assignment of the d'.

Finally, since in our opinion there is enough theoretical evidence indicating a resonance with isospin 2, we like to suggest a new experimental search of it. The reaction $\pi^- d \to \pi^+ X$, where $X=\pi^- nn$, is best suited for this purpose since X necessarily has isospin 2. This reaction was used in the past [20–22] to search for a possible $\pi^- nn$ bound state. In that case, they were looking mainly at the region near the πNN threshold, although data were taken also for energies above this threshold. However, in the region of about 50 MeV above the πNN threshold (where the d' is) they used a very large energy step ($\sim$15 MeV) and a resolution of about 3 MeV so that they could not possibly see the d'. In order to detect the d' a resolution better than 0.5 MeV would be needed and they should look in the mass region between 2065 MeV and at most a couple of MeV below this value.

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