

(*e, e'p*) reaction and the transition to the eikonal regime

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(Received 31 October 1995)

The reliability of the models describing the final-state interactions in (*e, e'p*) scattering at high proton energies is an important issue in view of the experiments planned at CEBAF. One of the most popular approaches adopted, the Glauber method, involves the linearization of the wave equation for the ejected proton traveling through the residual nucleus. We have studied the consequences of such an assumption for the case of the ¹²C(*e, e'p*)¹¹B* reaction at high proton momenta by comparing the results with the predictions obtained when the second-order differential equation for the proton wave is solved exactly for each partial wave. We find that the two methods give well-correlated angular distributions for momenta in the range 1–4 GeV/*c*, i.e., for kinematics relevant to the transition to the eikonal regime.

PACS number(s): 25.30.Dh, 11.80.Fv, 24.10.Eq, 24.10.Jv

In the experiments planned at CEBAF concerning (*e, e'p*) scattering [1], where the proton momentum can be larger than 1 GeV/*c*, a central issue is the reliability of models describing the final-state interactions (FSI) between the outgoing proton and the residual nucleus. For example, at moderate missing momenta *p_m* (*p_m*=*p*−*q*, where *p* is the outgoing proton momentum and *q* is the momentum transferred to the target by the electron) an accuracy within 10% is required to unambiguously identify exotic effects like color transparency [2], if any.

The most widely used approach to the problem of FSI at high energy is the Glauber approximation [3], since there is a long well-established tradition in the analysis of proton-nucleus elastic scattering [4,5]. In particular, in that context it has been shown that the validity of such an approach can arise from a nontrivial cancellation among the leading corrections to the lowest-order theory [6]. However, the generalization to the (*e, e'p*) scattering is not straightforward because the initial proton state and, in general, the kinematics are quite different.

In the impulse approximation, the basic ingredient of completely exclusive (*e, e'p*) reactions is the scattering amplitude [7,8]

$$J_{\alpha}^{\mu}(\mathbf{q}) = \int d\mathbf{r} d\sigma e^{i\mathbf{q}\cdot\mathbf{r}} \chi^{(-)*}(\mathbf{r}, \sigma) \hat{J}^{\mu}(\mathbf{q}, \mathbf{r}, \sigma) \Psi_{\alpha}(\mathbf{r}, \sigma), \quad (1)$$

where $\chi^{(-)}$ and Ψ_{α} describe the scattering- and bound-state wave functions of the nucleon knocked out from a hole with quantum numbers α , respectively. Usually, the current operator \hat{J}^{μ} is approximated by a nonrelativistic expansion in powers of the inverse nucleon mass, thus introducing uncertainties which become more important with increasing energy [9,10]. However, our interest is not focused on the comparison with experimental data, but on the analysis of the scattering wave $\chi^{(-)}$. Therefore, we have considered the simplified picture where we retain just the longitudinal component of \hat{J}^{μ} in the leading order $o(1)$ of the nonrelativistic approximation and we neglect the nucleon form factor. The cross section becomes, therefore, proportional to

$$\left| \int d\mathbf{r} d\sigma e^{i\mathbf{q}\cdot\mathbf{r}} \chi^{(-)*}(\mathbf{r}, \sigma) \Psi_{\alpha}(\mathbf{r}, \sigma) \right|^2 \equiv S_{\alpha}^D(\mathbf{q}), \quad (2)$$

which is traditionally identified as the “distorted” spectral density S_{α}^D [11] at the missing energy corresponding to the knockout hole α .

In the framework of the distorted-wave impulse approximation (DWIA) [7,8] the scattering wave function $\chi^{(-)}$ is a solution of the Schrödinger equation

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \chi = E_{c.m.} \chi, \quad (3)$$

where *m* is the reduced mass of the proton in interaction with the residual nucleus, $E_{c.m.}$ is its kinetic energy in the c.m. system, and *V* contains a local equivalent energy-dependent optical potential effectively describing the residual interaction.

Equation (3) can be solved for each partial wave of $\chi^{(-)}$ up to a maximum angular momentum $L_{max}(p)$, where a convergence criterion for the partial-wave expansion is satisfied. The boundary condition is such that each incoming partial wave coincides asymptotically with the corresponding component of the plane wave associated with the proton momentum *p*. Typically, this method (from now on method I) has been applied to (*e, e'p*) scattering for proton momenta below 0.5 GeV/*c* and $L_{max} < 50$ [8]. In the kinematics explored in this work a maximum $L_{max} = 120$ has been used.

At higher energies the Glauber method [3] suggests an alternative way (from now on method II) of solving Eq. (3) by linearizing it along the propagation axis \hat{z} :

$$\mathbf{r} \equiv z \frac{\mathbf{p}}{p} + \mathbf{b}, \quad (4)$$

$$\nabla^2 \approx \frac{\partial^2}{\partial z^2}, \quad (5)$$

$$\left(\frac{\partial^2}{\partial z^2} + p^2 \right) = \left(\frac{\partial}{\partial z} + ip \right) \left(\frac{\partial}{\partial z} - ip \right) \approx 2ip \left(\frac{\partial}{\partial z} - ip \right), \quad (6)$$

where \mathbf{b} describes the degrees of freedom transverse to the motion of the struck particle with momentum \mathbf{p} . With this approximation Eq. (3) becomes

$$\left(\frac{\partial}{\partial z} - ip\right)\chi = \frac{1}{2ip}V\chi. \quad (7)$$

The boundary condition is of incoming unitary flux of plane waves.

Both methods I and II solve the Schrödinger equation for the nucleon scattering wave. Relativistic effects are correctly taken into account only in a proper calculation of the kinematics. In the case of the application of the Glauber approach to unpolarized proton-nucleus scattering, this approximation does not seem to produce relevant consequences [4–6]. Nevertheless, because of the previously mentioned differences, the generalization to the $(e, e'p)$ should be tested [9,12,13].

The eikonal approximation is supposed to reproduce the exact solution of Eq. (3) for $p_m \ll q$, and in general its reliability increases with the ejectile energy [3], ideally in the limit where $\chi^{(-)}$ is expanded on an infinite number of partial waves. On the other hand, method I can be considered reliable only for nucleon energies such that the condition $L_{\max} \gg R_{\text{target}}$ is satisfied.

In a previous work [14] we have analyzed the spectral density S_α^D of Eq. (2) for the $^{12}\text{C}(e, e'p)^{11}\text{B}_{s1/2}$ reaction in parallel kinematics (\mathbf{p} along \mathbf{q}). In the range $1 < p < 2$ GeV/c and for $L_{\max} = 120$ we found a good correlation between the predictions of methods I and II, the (small) discrepancies being ascribed to the impossibility of taking into account in Eq. (7) the interference between the incoming and the reflected flux. The consequent overestimation of S_α^D with respect to method I was found to be related to kinematics and proportional to the absorption strength of the optical potential.

In the present work we have extended the calculations up to $p, q = 4$ GeV/c by improving the numerical precision of the FORTRAN code. We have considered the $^{12}\text{C}(e, e'p)^{11}\text{B}^*$ reaction in the so-called perpendicular kinematics, where p and q are kept constant and the angle between their directions, γ , is allowed to vary. The bound state Ψ_α in Eq. (2) is a solution of the Woods-Saxon potential of Comfort and Karp [15] with the quantum numbers α of the s wave. For the sake of simplicity, the contribution to V coming from the Coulomb potential has been neglected to avoid numerical problems related to the high angular momenta required. Therefore, in proper terms the results presented here refer to the $(e, e'n)$ reaction. V is an optical potential of the simple form

$$V(r) = (U + iW) \frac{1}{1 + e^{r-R/a}} \equiv (U + iW)\rho(r), \quad (8)$$

with $R = 1.2 \times A^{1/3}$ fm and $a = 0.5$ fm. The nuclear density $\rho(r)$ defined in Eq. (8) is normalized such that $\rho(0) = 1$.

At the nucleon energies here considered, the parameters U, W can only be guessed. According to the Glauber model the imaginary part should scale as $W \sim p/10$ MeV, while U/W should equal the ratio between the real and the imagi-

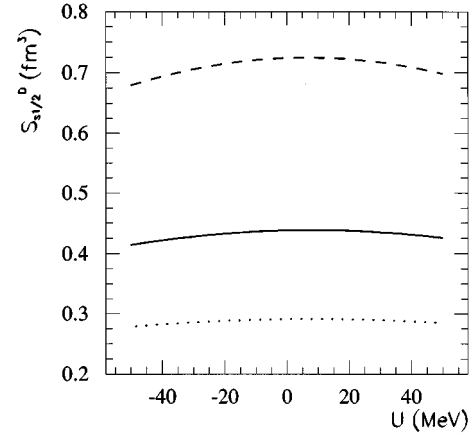


FIG. 1. The distorted spectral density $S_{s1/2}^D$ as a function of the depth U of the real part of the optical potential for the $^{12}\text{C}(e, e'p)^{11}\text{B}_{s1/2}$ reaction at the outgoing proton momentum $p = 1.4$ GeV/c and at the missing momentum $p_m = 0$. The dashed line corresponds to the depth $W = 0$ of the imaginary potential, the solid line to $W = 50$ MeV, the dotted line to $W = 100$ MeV.

nary parts of the average proton-nucleon forward-scattering amplitude, which is expected to be small in the considered kinematics [16].

In Fig. 1 $S_{s1/2}^D$ is calculated by method I as a function of U for $p_m = 0$ and $p = q = 1.4$ GeV/c, where the elementary cross section for the rescattering of the ejectile is predominantly inelastic. The three curves correspond to $W = 0, 50$, and 100 MeV, for U ranging continuously from -50 to $+50$ MeV. The $(U=0, W=0)$ point corresponds to the plane-wave impulse approximation (PWIA) result, where any rescattering between the ejectile and the residual nucleus is neglected. The middle curve ($W = 50$ MeV) shows an average 40% damping with respect to the PWIA result in agreement with the observation of the NE18 experiment in the context of a semi-inclusive $(e, e'p)$ reaction at small p_m [17]. It is evident that the sensitivity to both the sign and the magnitude of the real part of the potential is very small but for huge values of $U \gg W$, which are forbidden by the mainly absorptive character of the proton-nucleon amplitude at these kinematics. Therefore, in this work we will use $U = 0$.

Equation (7) produces by definition results that only depend on V through the ratio V/p . If V is a linear function of p (as suggested by the Glauber model), a constant absorption is produced at any value of momentum transfer, provided that $p = q$ and $p_m \approx 0$ (i.e., small γ). In Fig. 2 we show that the same property holds, with a good approximation, also for method I. The distorted spectral density is given as a function of $p = q$ for $p_m = 0$ and the choice $U = 0, W = p/50/1400$ MeV, which produces at $p = q = 1.4$ GeV/c the 40% absorption observed in the NE18 experiment. The flat curve shows that this damping remains constant down to very low values of $p = q$ ($p = q \approx 0.4$ GeV/c). However, the choice of an absorption-dominated potential is reasonable only in the kinematical region where the elementary proton-nucleon scattering amplitude is dominated by inelastic processes, i.e., for $p \geq 1$ GeV/c [16]. Only above this threshold, which is anyway relevant to the kinematics explored at CEBAF, our considerations can be applied (for applications of the

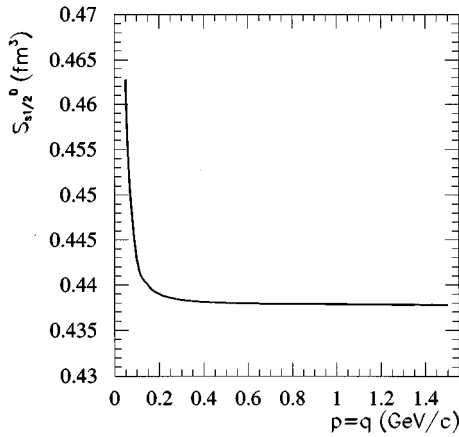


FIG. 2. The distorted spectral density $S_{s1/2}^D$ as a function of $p=q$ for the $^{12}\text{C}(e, e'p)^{11}\text{B}_{s1/2}$ reaction at $p_m=0$ and with the optical potential depths $U=0$, $W=p50/1400$ MeV.

Glauber approach to $(e, e'p)$ reactions at lower energies see Refs. [18,19]). Since the NE18 observation of a roughly q -independent 40% damping at small p_m is reproduced by both methods I and II with the optical potential of Eq. (8) and with $U=0$, $W=p50/1400$ MeV, we adopt this choice in the following also for the calculation of the angular distributions.

In the pure Glauber theory W is not a free parameter and would result in a larger value. As previously mentioned, the increase of W leads to a larger discrepancy between the results of methods I and II, but the overall agreement is not too much spoiled [14]. However, the W -value suggested by the Glauber model produces a sensibly larger absorption than observed in the NE18 experiment (for more details on this topic see Refs. [18,20–22]).

In Figs. 3 and 4 the distorted spectral density $S_{s1/2}^D$ is shown as a function of γ for $q=p=1.4$ and 4 GeV/c, respectively. Because of the large range of angles considered, several diffraction minima are explored while the size of the distribution falls down by many orders of magnitude. The dotted line represents the result with no final-state interac-

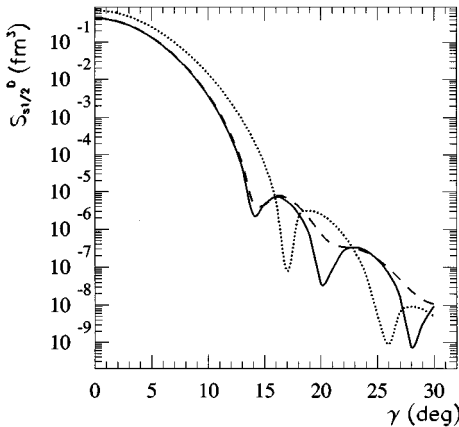


FIG. 3. The distorted spectral density $S_{s1/2}^D$ as a function of the angle γ between p and q for the $^{12}\text{C}(e, e'p)^{11}\text{B}_{s1/2}$ reaction with $p=q=1.4$ GeV/c. The dotted line shows the PWIA result. The solid and dashed lines are the results of methods I and II, respectively (see text).

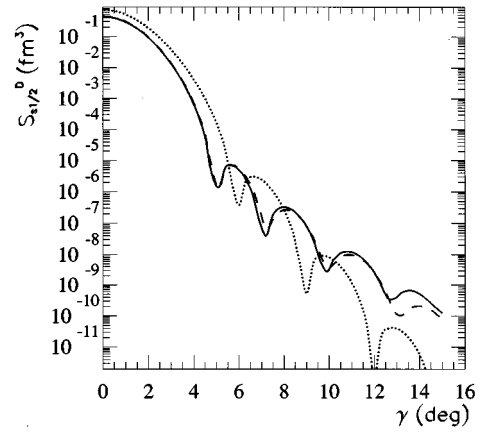


FIG. 4. The same as in Fig. 3, but for $p=q=4$ GeV/c.

tions (PWIA), which is of course identical in both methods. The solid and the dashed lines are the results of method I and II, respectively. The two angular distributions are rather well correlated in all the kinematics here explored, except in the diffraction minima for $p=q=1.4$ GeV/c. We have checked that these discrepancies are smoothed with increasing p, q until they almost disappear at $p=q=4$ GeV/c, as it is clear in Fig. 4. In any case, through all the kinematics considered the oscillatory patterns are very close to each other across a remarkably large range of variation in size.

It must be noticed that the rich diffractive pattern of the angular distributions is partially due to the nontrivial structure of the PWIA contribution, which itself contains many local minima. We have already shown in a previous work [14] that if the Woods-Saxon bound state Ψ_α is substituted by a pure harmonic oscillator, so to produce an exponentially decreasing angular distribution in PWIA, the results of both methods I and II still show an oscillatory pattern at large angles due to FSI. Thus, the natural interpretation is that the diffractive minima, which are reminiscent of the angular distribution for elastic proton-nucleus scattering [4], derive from the fact that the ejected proton is testing coherently the residual nucleus. This is peculiar of a completely exclusive reaction, where the residual nucleus does not fragment. Energy-integrated distributions (i.e., for a semi-inclusive $(e, e'p)$ reaction [20,23]) are by definition less sensitive to the structure of the recoiling $(A-1)$ system, thus leading to very different angular shapes.

We have shown that for the $^{12}\text{C}(e, e'p)^{11}\text{B}_{s1/2}$ reaction and for outgoing proton momenta in the range $1 < p < 4$ GeV/c (relevant to the planned experiments at CEBAF) the eikonal approximation to the scattering wave of the ejectile produces FSI effects very similar to the ones obtained when the complete second-order differential equation is solved exactly up to 120 partial waves. The angular distributions are in good agreement up to very large angles, where the absolute size can fall down by many orders of magnitude. The observed oscillatory pattern can be interpreted as a coherent diffractive scattering between the ejectile and the residual nucleus. Therefore, completely exclusive $(e, e'p)$ reactions are best suited to verify this prediction.

We would like to thank O. Benhar, S. Boffi, S. Jeschonek, N. N. Nikolaev, and S. Simula for many stimulating discussions.

- [1] A. Saha (spokesperson), CEBAF Proposal No. E-91-006; R. Milner (spokesperson), CEBAF Proposal No. E-91-007; D. F. Geesaman (spokesperson), CEBAF Proposal No. E-91-013.
- [2] L. L. Frankfurt, G. A. Miller, and M. I. Strikman, *Annu. Rev. Nucl. Part. Sci.* **45**, 501 (1994); N. N. Nikolaev, *Int. J. Mod. Phys. E* **3**, 1 (1994); A. Bianconi, S. Boffi, and D. E. Kharzeev, *Yad. Fiz.* **57**, 1732 (1994).
- [3] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. Brittain and L. G. Dunham (Interscience, New York, 1959), Vol. 1; R. J. Glauber and G. Matthiae, *Nucl. Phys.* **B21**, 135 (1970).
- [4] R. H. Bassel and C. Wilkin, *Phys. Rev.* **174**, 1179 (1968).
- [5] G. D. Alkhazov, S. I. Belostotsky, and A. A. Vorobyev, *Phys. Rep.* **42**, 89 (1978).
- [6] S. J. Wallace, *Phys. Rev. C* **12**, 179 (1975).
- [7] S. Frullani and J. Mougey, *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1984), Vol. 13.
- [8] S. Boffi, C. Giusti, and F. D. Pacati, *Phys. Rep.* **226**, 1 (1993).
- [9] A. Picklesimer, J. W. Van Orden, and S. J. Wallace, *Phys. Rev. C* **32**, 1312 (1985).
- [10] S. Gardner and J. Piekarewicz, *Phys. Rev. C* **50**, 2822 (1994).
- [11] S. Boffi, C. Giusti, and F. D. Pacati, *Nucl. Phys.* **A386**, 599 (1982).
- [12] R. D. Amado, J. Piekarewicz, D. A. Sparrow, and J. A. McNeil, *Phys. Rev. C* **28**, 1663 (1983).
- [13] E. Rost, J. R. Shepard, and D. Murdock, *Phys. Rev. Lett.* **49**, 448 (1982).
- [14] A. Bianconi and M. Radici, *Phys. Lett. B* **363**, 24 (1995).
- [15] J. Comfort and B. C. Karp, *Phys. Rev. C* **21**, 2162 (1980).
- [16] C. Lechanoine-LeLuc and F. Lehar, *Rev. Mod. Phys.* **65**, 47 (1993).
- [17] NE18 Collaboration, N. C. R. Makins *et al.*, *Phys. Rev. Lett.* **72**, 1986 (1994).
- [18] V. R. Pandharipande and S. C. Pieper, *Phys. Rev. C* **45**, 791 (1992).
- [19] M. Radici, S. Boffi, S. C. Pieper, and V. R. Pandharipande, *Phys. Rev. C* **50**, 3010 (1994).
- [20] N. N. Nikolaev, A. Szczurek, J. Speth, J. Wambach, B. G. Zakharov, and V. R. Zoller, *Phys. Rev. C* **50**, R1296 (1994); *Nucl. Phys.* **A582**, 665 (1995).
- [21] A. Kohama, K. Yazaki, and R. Seki, *Nucl. Phys.* **A536**, 716 (1992).
- [22] L. L. Frankfurt, E. J. Moniz, M. M. Sargsyan, and M. I. Strikman, *Phys. Rev. C* **51**, 3435 (1995).
- [23] A. Bianconi, S. Jeschonnek, N. N. Nikolaev, and B. G. Zakharov, *Phys. Lett. B* **338**, 123 (1994).