Effective g_A in the pf shell

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We have calculated the Gamow-Teller matrix elements of 64 decays of nuclei in the mass range $A=41$–50. In all the cases the valence space of the full pf shell is used. Agreement with the experimental results demands the introduction of an average quenching factor $q=0.744\pm0.015$ slightly smaller but statistically compatible with the sd-shell value, thus indicating that the present number is close to the limit for large A. [S0556-2813(96)50606-0]

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The observed Gamow-Teller strength appears to be systematically smaller than what is theoretically expected on the basis of the model independent "$3(N-Z)$" sum rule. Much work has been devoted to the subject in the last fifteen years [1–4]. The heart of the problem can be summed up by defining the reduced transition probability as

$$B(\mathrm{GT})=\left(\frac{g_A}{g_V}\right)^2\langle\boldsymbol{\sigma\tau}\rangle^2,\quad \langle\boldsymbol{\sigma\tau}\rangle=\frac{\langle f||\Sigma_k\boldsymbol{\sigma}^k\boldsymbol{t}^k_{\pm}||i\rangle}{\sqrt{2J_i+1}},\quad (1)$$

and asking: Is the observed quenching due to a renormalization of the g_A coupling constant—originating in non-nucleonic effects—or is it the $\boldsymbol{\sigma\tau}$ operator that should be renormalized because of nuclear correlations?

The analysis of some pf-shell nuclei for which very precise data are available and full $0\hbar\omega$ calculations are possible, strongly suggests that most of the theoretically expected strength has been observed [5,6]. The quenching factor necessary to bring into agreement the calculated and measured values is directly related to the amplitude of the $0\hbar\omega$ model space components in the exact wave functions. This normalization factor can also be obtained from (d,p) or $(e,e'p)$ reactions and reflects the reduction in the discontinuity at the Fermi surface in a normal system. Furthermore, a similar reduction factor is found in the spin component of the $M1$ strength measured in (e,e') and (p,p') experiments [7]. As such, it is a fundamental quantity, whose evolution with mass number is of interest.

In principle there are two ways of extracting it from Gamow-Teller processes. One is to equate it to the fraction of strength seen in the resonance region in (p,n) reactions. The alternative is to calculate lifetimes for individual β decays and show that they correspond to the experimental values within a constant factor. The latter procedure is more precise, but demands high quality shell model calculations that until recently were available only up to $A=40$ [8–10].

Our aim is to extend these analyses to the lower part of the pf shell. Full $0\hbar\omega$ diagonalizations are done using the ANTOINE code [11] with the effective interaction KB3, a minimally monopole-modified version [12] of the original Kuo Brown matrix elements [13]. We refer to [14] for details of the shell-model work.

Following Ref. [15] we define quenching as follows: for β decays populating well-defined isolated states in the daughter nucleus, the square root of the ratio of the experimental measured rate to the calculated rate in a full $0\hbar\omega$ calculation is called the quenching factor. An average quenching factor q implies an average over many transitions, and may be incorporated into an effective axial vector coupling constant:

$$q=\frac{g_{A,\mathrm{eff}}}{g_A},\quad (2)$$

where g_A is the free-nucleon value of $-1.2599(25)$ [15]. Following Ref. [8] we define

$$M(\mathrm{GT})=[(2J_i+1)B(\mathrm{GT})]^{1/2},\quad (3)$$

so as to have quantities independent of the direction of the transition. Note here that our reduced matrix elements follow Racah's convention [16]. In Table I we list the $M(\mathrm{GT})$ values and compare them with the experimental results. The table contain all the transitions known experimentally. We also include the quantum numbers of the final states, the Q values, the branching ratios, and the experimental log ft values from which the $B(\mathrm{GT})$ values were obtained using

$$(f_A+f^{\epsilon})t=\frac{6146\pm6}{(f_V/f_A)B(F)+B(\mathrm{GT})}.\quad (4)$$

The value 6.146 ± 6 is obtained from the nine best-known superallowed $0^+\rightarrow0^+$ decays [15]. f_V and f_A are the Fermi and Gamow-Teller phase-space factors, respectively [17,8]. f^{ϵ} is the phase space for electron capture [18].

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TABLE I. Experimental and theoretical $M(\mathrm{GT})$ matrix elements. The experimental data have been taken from [20]. $I_\beta+I_\epsilon$ are the branching ratios. All other quantities are explained in the text.

Process	$2J_n^\pi,2T_n^\pi$	Q (MeV)	$I_\beta+I_\epsilon$ (%)	log ft	M(GT) Expt.	M(GT) Theor.	W
^{41}Sc(β^+)^{41}Ca	7^-,1	6.496	99.963(3)	3.461(7)	2.999	4.083	6.172
^{42}Sc*(β^+)^{42}Ca	12^+,2	3.851	100	4.17(2)	2.497	3.389	11.127
^{42}Ti(β^+)^{42}Sc	2^+,0	6.392	55(14)	3.17(12)	2.038	2.736	3.086
^{43}Sc(β^+)^{43}Ca	7^-,3	2.221	77.5(7)	5.03(2)	0.677	0.764	6.172
	5^-,3	1.848	22.5(7)	4.97(3)	0.726	0.878	
^{44}Sc(β^+)^{44}Ca	4_1^+,4	2.497	98.95(4)	5.30(2)	0.392	0.741	6.901
	4_2^+,4	0.998	1.04(4)	5.15(3)	0.466	0.205	
	4_3^+,4	0.353	0.010(2)	6.27(8)	0.128	0.295	
^{44}Sc*(β^+)^{44}Ca	12^+,4	0.640	1.20(7)	5.88(3)	0.324	0.276	11.127
^{45}Ca(β^-)^{45}Sc	7^-,3	0.258	99.9981	5.983(1)	0.226	0.079	13.802
^{45}Ti(β^+)^{45}Sc	7^-,3	2.066	99.685(17)	4.591(2)	1.123	1.551	6.172
	5^-,3	1.342	0.154(12)	6.24(4)	0.168	0.280	
	7^-,3	0.654	0.090(10)	5.81(5)	0.276	0.397	
	9^-,3	0.400	0.054(5)	5.60(4)	0.351	0.712	
^{45}V(β^+)^{45}Ti	7^-,1	7.133	95.7(15)	3.64(2)	1.801	2.208	6.172
	5^-,1	7.093	4.3(15)	5.0(2)	0.701	0.428	
^{46}Sc(β^-)^{46}Ti	8^+,2	0.357	99.9964(7)	6.200(3)	0.187	0.277	13.093
^{47}Ca(β^-)^{47}Sc	7^-,5	1.992	19(10)	8.5(3)	0.012	0.262	16.331
	5^-,5	0.695	81(10)	6.04(6)	0.212	0.235	
^{47}Sc(β^-)^{47}Ti	5^-,3	0.600	31.6(6)	6.10(1)	0.198	0.235	13.802
	7^-,3	0.441	68.4(6)	5.28(1)	0.508	0.611	
^{47}V(β^+)^{47}Ti	5_1^-,3	2.928	99.552(15)	4.901(5)	0.555	0.896	4.365
	3_1^-,3	1.378	0.049(6)	6.08(6)	0.143	0.107	
	1_1^-,3	1.1337	0.285(10)	5.10(2)	0.442	0.563	
	3_2^-,3	0.765	0.071(3)	5.36(2)	0.327	0.514	
	5_2^-,3	0.761	0.0091(7)	6.25(4)	0.118	0.278	
	5_3^-,3	0.402	0.0172(9)	5.41(3)	0.309	0.202	
	3_3^-,3	0.379	0.0067(5)	5.77(4)	0.204	0.204	
	1_2^-,3	0.134	0.0021(6)	5.18(9)	0.403	0.780	
^{47}Cr(β^+)^{47}V	3^-,1	7.451	96.1(13)	3.70(2)	0.942	1.186	4.365
	5^-,1	7.363	3.9(13)	5.1(2)	0.442	0.646	
^{48}Sc(β^-)^{48}Ti	12_1^+,4	0.661	90.0(3)	5.532(13)	0.484	0.780	22.256
	12_2^+,4	0.485	9.85(9)	6.010(17)	0.279	0.331	
^{48}V(β^+)^{48}Ti	8_1^+,4	1.719	89.0(9)	6.175(7)	0.192	0.345	9.259
	6^+,4	0.791	3.33(7)	6.565(10)	0.123	0.090	
	8_2^+,4	0.775	7.76(9)	6.180(6)	0.191	0.181	
^{48}Cr(EC)^{48}V	2^+,2	1.233	100	4.294(7)	0.559	0.709	5.346
^{48}Mn(β^+)^{48}Cr	8_1^+,0	11.670	6.5(25)	5.4(2)	0.469	0.527	9.259
	8_2^+,0	9.101	10.1(24)	4.6(1)	1.178	2.179	
	8_3^+,0	8.876	4.0(9)	5.0(1)	0.743	0.172	
	8_4^+,0	8.497	8.0(7)	4.58(5)	1.206	1.361	
	10_1^+,0	8.235	3.2(4)	4.90(6)	0.834	0.651	
^{49}Ca(β^-)^{49}Sc	3^-,7	2.178	91.5(7)	5.075(4)	0.455	1.007	13.093
	5_1^-,7	1.190	7.0(7)	5.12(5)	0.432	0.209	
	1^-,7	0.769	0.66(7)	5.42(5)	0.306	0.757	
	5_2^-,7	0.524	0.21(6)	5.3(2)	0.351	0.591	
^{49}Sc(β^-)^{49}Ti	7^-,5	1.994	99.94(1)	5.71(1)	0.309	0.469	16.331
	9^-,5	0.371	0.010(3)	6.9(2)	0.079	0.072	
	5^-,5	0.232	0.05(1)	5.6(1)	0.351	0.389	
^{49}V(EC)^{49}Ti	7^-,5	0.602	100	6.2(1)	0.176	0.130	10.691

TABLE I. (*Continued*.)

Process	$2J_n^\pi,2T_n^\pi$	Q (MeV)	$I_\beta+I_\epsilon$ (%)	log ft	M(GT) Expt.	M(GT) Theor.	W
^{49}Cr(β^+)^{49}V	7^-,3	2.631	12(2)	5.6(1)	0.304	0.335	5.346
	5_1^-,3	2.540	37(2)	5.02(2)	0.593	0.817	
	3^-,3	2.478	50(2)	4.81(2)	0.755	1.033	
	5_2^-,3	1.116	0.081(9)	5.80(4)	0.242	0.312	
	3_2^-,3	0.969	0.028(6)	6.15(8)	0.161	0.182	
	5_3^-,3	0.396	0.0011(2)	6.75(7)	0.081	0.264	
	3_3^-,3	0.322	1.9(7) 10^{-4}	7.3(2)	0.043	0.195	
^{49}Mn(β^+)^{49}Cr	5^-,1	7.715	93.6(26)	3.67(3)	1.364	1.704	5.346
	7^-,1	7.443	6.4(26)	4.8(2)	0.764	0.623	
^{50}Ca(β^-)^{50}Sc	2^+,8	3.118	99.0(13)	4.14(2)	0.667	0.956	6.901
^{50}Sc(β^-)^{50}Ti	8^+,6	4.213	8.4(18)	6.7(1)	0.116	0.208	20.471
	12^+,8	3.689	88.4(15)	5.39(1)	0.525	0.572	
	8^+,8	2.741	0.58(4)	7.01(4)	0.081	0.125	
	10^+,8	2.007	1.58(5)	5.99(2)	0.263	0.358	

A quick look at Table I shows that the calculated values are systematically larger than the experimental ones. In order to obtain the effective g_A, first we normalize the M(GT) to the ''expected'' total strength, W (listed in Table I) and defined by

$$W=\begin{cases} |g_A/g_V|[(2J_i+1)3|N_i-Z_i|]^{1/2} & \text{for } N_i\neq Z_i, \\ |g_A/g_V|[(2J_f+1)3|N_f-Z_f|]^{1/2} & \text{for } N_i=Z_i. \end{cases} \tag{5}$$

In Fig. 1 are plotted the experimental values versus the theoretical ones for

$$R(\mathrm{GT})=M(\mathrm{GT})/W. \tag{6}$$

The points follow nicely a straight line whose slope gives the average quenching factor, $q=0.744\pm0.015$. Most R(GT) values are much smaller than 1, reflecting the fact that the strength in the decay window is small and fragmented. As a consequence, each individual decay may be sensitive to small uncertainties in the calculations, which can be averaged out by summing the total strength for each nucleus. Therefore we introduce a new quantity:

$$T(\mathrm{GT})=\left[\sum_f R^2(\mathrm{GT},i\rightarrow f)\right]^{1/2}. \tag{7}$$

In the corresponding plot in Fig. 2 the points again follow closely the $q=0.744$ line.

The comparison with the results in other regions is suggestive of a trend:

pf shell, $q=0.744\pm0.015$ (this work);

sd shell, $q=0.77\pm0.02$ [9];

p shell, $q=0.82\pm0.015$ [10].

In the figures, both the lines for $q=0.744$ and $q=0.77$ are drawn and it is clear that there is not much to choose between them, and indeed, the average quenching factor of 0.77 has been extensively used in many pf-shell calcula-

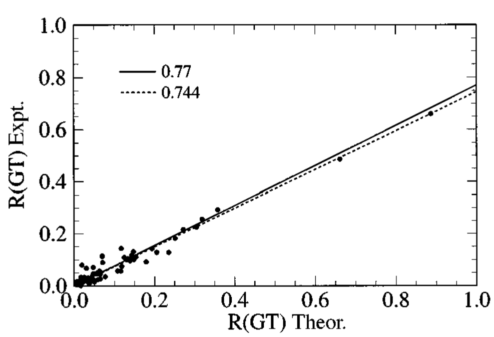

FIG. 1. Comparison of the experimental matrix elements R(GT) with the theoretical calculations based on the ''free-nucleon'' Gamow-Teller operator. Each transition is indicated by a point in the x-y plane, with the theoretical value given by the x coordinate of the point and the experimental value by the y coordinate.

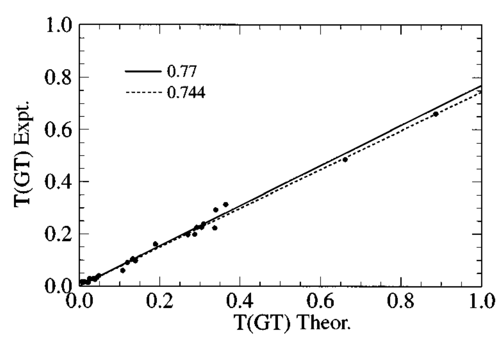

FIG. 2. Comparison of the experimental values of the sums T(GT) with the corresponding theoretical value based on the ''free-nucleon'' Gamow-Teller operator. Each sum is indicated by a point in the x-y plane, with the theoretical value given by the x coordinate of the point and the experimental value by the y coordinate.

tions, either in direct diagonalizations [14,5,6] or shell-model Monte Carlo studies [19], leading to agreement with global Gamow-Teller strengths [as measured in (n,p) and (p,n) reactions] and lifetimes.

Nevertheless, the results in the three regions point to a decrease of q with mass number, and the closeness of the sd and pf values suggests that we have reached the large-A regime. This observation is quite consistent with the numbers extracted by Osterfeld from (p,n) data in heavier nuclei (see Fig. 6 in [3]).

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