Structure of nonlocalities in meson exchange NN interactions and their role in the NN and $3N$ system

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The structure of nonlocalities implied by the meson exchange dynamics in the nucleon-nucleon interaction is investigated. It is shown that they have a considerable impact on NN as well as $3N$ observables. It is argued that, for a precise determination of their effects, a sufficiently complete calculation, consistent in the two- and three-body sector, is required.

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Probably the main challenge in present-day strong interaction few-body physics is to reach a combined and quantitative understanding of the two- and three-nucleon systems. Recently, Stadler *et al.* [1] combined meson exchange two-nucleon force models like the Paris [2] and Nijmegen [3] potentials, which provide a good description of NN data below pion production, with the full Tucson-Melbourne three-nucleon force [4,5] and obtained, in a conventional three-body Faddeev calculation, triton binding energies close to the empirical value of 8.48 MeV. However the problem is far from being solved, and many questions remain. First of all, the underlying dynamics is so far not treated consistently; e.g., the pion-nucleon form factor in the three-nucleon force is soft whereas it is hard in the two-nucleon forces employed.

Moreover the microscopic dynamics underlying the force models implies specific nonlocalities, which are however essentially suppressed in Refs. [2–5]. Their inclusion should modify the triton binding energy results [6]. Indeed various versions of the Bonn NN potential, which due to their momentum space representation can keep all prescribed nonlocalities, lead to considerably more binding than obtained with the essentially local potentials of Refs. [2,3]. This is especially true for the so-called Bonn-B potential [7], a one boson exchange (OBE) potential based on a three-dimensional reduction of the Bethe-Salpeter equation [8] proposed by Blankenbecler and Sugar (BbS [9]). This interaction, with a deuteron D-state probabilities of 4.99%, yields in a conventional Faddeev calculation a triton binding of 8.14 MeV [10]. The increase in binding energy is somewhat smaller with an alternative OBE potential (OBEPF [11]) based on a folded diagram expansion scheme [12]. With a D-state probability of 5.66% it yields 7.83 MeV binding, in a five-channel Faddeev calculation, see Ref. [11]. (Note that on the OBE level both calculational schemes provide instantaneous interactions whereas time-ordered perturbation theory originally used in the full Bonn potential [13] leads to energy dependent interactions. A corresponding OBE version (OBEPT [13]) yields, despite the very low D-state probability of 4.27%, only 6.73 MeV triton binding [14]. The reason is that due to its energy dependence the potential gets suppressed for the energies relevant in the triton.)

Part of the difference between the triton binding (about 0.3 MeV) provided by Bonn-B on one hand and OBEPF on the other hand arises from differences in the NN phase shift fits (with Bonn-B being superior in the partial waves relevant for the triton). At least 0.1 MeV remains however, which can be traced to the different nonlocal structure of the corresponding potentials.

In this Rapid Communication we want to discuss the structure of the nonlocalities present in momentum space potentials and to investigate their role for the resulting triton binding energy. We will do this mainly for the Bonn-B potential, which, as said before, is based on the Blankenbecler-Sugar (BbS) [9] reduction scheme. Therefore, the starting point is the four-dimensional Bethe-Salpeter equation (BS) [8]

$$T = K + \int d^4k \; KGT. \tag{1}$$

with an (energy-dependent) kernel consisting of all irreducible Feynman diagrams. In order to simplify this equation various reduction schemes introduce modified propagators g which lead to a three-dimensional equation. Surely, such a procedure modifies the scattering amplitude, which however can be restored by adding suitable correction terms to the original BS-kernel K, i.e., we have

$$T = V + VgT, \quad V = K + K(G-g)K + \cdots. \tag{2}$$

In the case of BbS reduction, energy components of (relative) four-momenta are restricted to be zero (whereas they are integrated over in the original BS equation). This reduction leads to the three-dimensional equation

$$T(\vec{k}',\vec{k};E) = V(\vec{k}',\vec{k};E) + \int d^3k'' \; V(\vec{k}',\vec{k}'';E) \times \frac{M}{E_{k''}} \frac{1}{E^2 - E_{k''}^2} T(\vec{k}'',\vec{k};E), \tag{3}$$

which after a suitable transformation of amplitudes

$$\tilde{V}(\vec{k}',\vec{k};E) \equiv \sqrt{\frac{M}{E_{k'}}} V(\vec{k}',\vec{k};E) \sqrt{\frac{M}{E_k}} \tag{4}$$

and the same for T acquires the usual Schrödinger form, with $E_k = \sqrt{M^2 + \vec{k}^2}$ and M the nucleon mass.

From the above it is immediately clear that the nonlocality structure of the original Bethe-Salpeter kernel is modified

by such reductions; in addition, the modifications depend on the kind of reduction chosen. Certainly, in order to remove such ambiguities, systematic corrections have to be done by going beyond the OBE approximation, as indicated in Eqs. (1) and (2).

In the following we would like to divide the nonlocalities into *standard* ones which are essentially scheme independent, and those called *nonstandard* which depend on the reduction scheme. In the first group we have, e.g., the M/E factors, which always arise in order to guarantee relativistic unitarity. They disappear when we do a nonrelativistic approximation neglecting three-momenta versus the nucleon mass. More important is the nonlocal structure of the πNN vertex. Choosing the pseudoscalar coupling in the Hamiltonian we obtain for the nucleon matrix element

$$\bar{u}(\vec{k}')\gamma^5 u(\vec{k})$$

$$= \sqrt{\frac{(E_{k'}+M)(E_k+M)}{4M^2}}\vec{\sigma}\cdot\left(\frac{\vec{k}'}{E_{k'}+M}-\frac{\vec{k}}{E_k+M}\right), \quad (5)$$

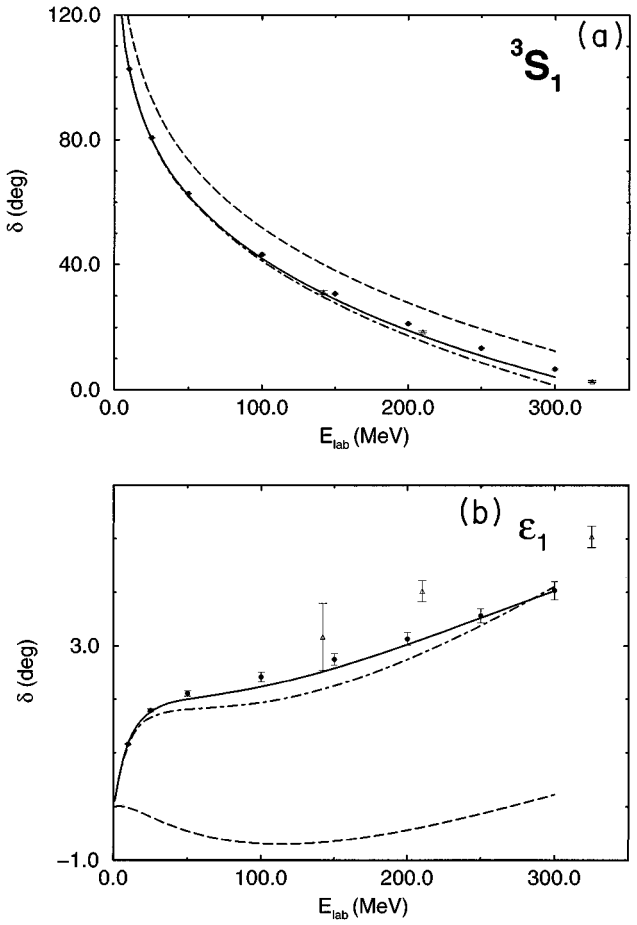

FIG. 1. (a) $^3S_1$ NN phase shift and (b) ϵ_1 mixing parameter. The solid curve denotes the predictions of the Bonn-B potential. For the dashed curve, the local approximation (5) has been applied at the πNN vertex, together with the suppression of all M/E factors. With the same approximation a readjustment of parameters leads to the dash-dotted curves. The error bars are from the empirical analysis of Nijmegen [16] and Bugg and Bryan [17].

which only after an on-shell approximation $E_{k'}=E_k$ boils down to the well-known local expression $(1/2M)\vec{\sigma}\cdot(\vec{k}'-\vec{k})$. Similar standard nonlocalities due to the structure of the Dirac spinors occur at the other $\rho NN, \omega NN, \ldots,$ vertices.

Nonstandard nonlocalities arise in potential terms which depend on the energy components of four-momenta. The best example is the meson propagator, which in the original Feynman amplitude can be written as

$$\frac{1}{(k_0'-k_0)^2-(\vec{k}'-\vec{k})^2-m^2}, \quad (6)$$

with energy components completely unrelated to corresponding three-momenta. (m is the meson mass.) The BbS reduction yields the well-known local expression $-1/[(\vec{k}'-\vec{k})^2+m^2]$, whereas in another reduction scheme advocated by Gross [15] a nonlocal correction term appears, i.e., $1/[(E_{k'}-E_k)^2-(\vec{k}'-\vec{k})^2-m^2]$. Another important example is the ρNN tensor coupling involving $\sigma^{\mu\nu}(k'-k)_\nu$, which after a BbS reduction goes into $\sigma^{\mu i}(\vec{k}'-\vec{k})_i$ whereas the Gross scheme implies an additional nonlocal term $\sigma^{\mu 0}(E_{k'}-E_k)$.

These examples show that there are numerous nonlocalities with prescribed structure which can (and should) be kept in momentum space representations. Clearly, the aforementioned nonrelativistic and on-shell approximations are *a priori* not justified even for extremely low-energy NN scattering since the three-momenta are integrated over in the scattering equation. The question then arises how important are such nonlocalities for the evaluation of observables.

We will first look at the actual role of these nonlocalities in NN scattering. Figure 1 shows the $^3S_1$ phase shift and ϵ_1 mixing parameter, which have strong impact on the triton binding. Obviously the removal of the nonlocal structure at the πNN vertex together with the neglection of M/E factors

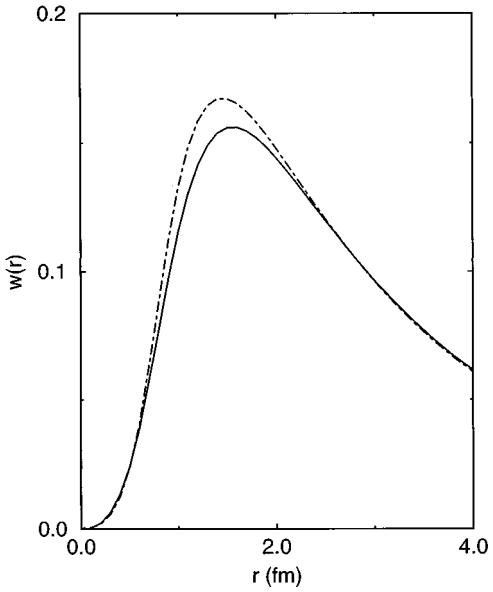

FIG. 2. Deuteron D-wave. The solid curve denotes the result of the Bonn-B potential. The dash-dot curve corresponds to those of Fig. 1.

TABLE I. Deuteron and low-energy scattering parameters predicted by Bonn-B [7] and various local approximations of it. The numbers in square brackets, after the ^{3}H binding energies, are estimated corrections due to the nonequivalence of the phases of the models (cf. text).

	Bonn-B	No M/E factor	Static π	Expt.
Deuteron:				
ϵ_d (MeV)	2.2246	2.2246	2.2246	2.224 575
P_d (%)	4.99	5.20	5.40	-
Q_d (fm^2)	0.278	0.279	0.278	0.2859±0.0003
A_S (fm$^{-1/2}$)	0.8860	0.8859	0.8868	0.8846±0.0016
η	0.0264	0.0264	0.0260	0.0256±0.0004
Neutron-proton low-energy scattering (scattering length a, effective range r):				
$^1S_0$: a_s (fm)	−23.75	−23.75	−23.75	−23.758±0.010
r_s (fm)	2.71	2.70	2.68	2.75±0.05
$^3S_1$: a_t (fm)	5.424	5.423	5.428	5.424±0.004
r_t (fm)	1.76	1.76	1.77	1.759±0.005
^{3}H binding energy (MeV) (five-channel calculation):				
	−8.16	−8.04	−7.81[+0.1]	−8.48

in the Bonn-B potential provides a drastic change. Of course, as Fig. 1 demonstrates, by readjusting suitable parameters a description with roughly the same quality can be again obtained within this local approximation. Thus, if the interaction is considered to be purely phenomenological, this approximation does not seem to matter at all, at least in the NN system. However, the parameters in meson exchange NN interactions (coupling constants and form-factor parameters) are physical and will be ultimately fixed from QCD. Therefore it indeed matters already at this stage whether an approximation is done which forces one to change parameters.

After a refit of the S-wave phase shifts, the ϵ_1 mixing parameters and the deuteron binding energy the effect of the above approximation can still be seen in the deuteron D-wave, see Fig. 2. Compared to the original Bonn-B wave function a sizable increase occurs which reaches out well beyond 2 fm, in agreement with the findings of Gibson *et al.* [18] who compared the original Bonn-B result with a strictly local but phase equivalent version obtained by inversion methods. The deuteron D-state probability p_D increases by about 0.4% and the triton binding decreases by almost 0.4 MeV (see "static π" in Table I). The local approximation at the πNN vertex is mainly responsible for this shift: If only M/E factors are suppressed the decrease of the triton binding (again after a suitable refit) is only 0.12 MeV, cf. case "no M/E factor" in Table I.

Further nonlocalities occur in the ρ-, σ-, and ω-exchange pieces; their combined effect on the triton binding is expected to be small since, according to the calculations by Gibson *et al.* in Ref. [18], the effect of all nonlocalities in the Bonn-B potential is to increase the triton binding by 0.30 MeV, which roughly agrees with the shift we obtained in our calculations using static π exchange and leaving out all M/E factors.

However one should realize that part of these effects are scheme dependent (i.e., are generated by nonstandard nonlo-

TABLE II. Deuteron and low-energy scattering parameters predicted by Bonn-B [7] and variants of it where the Gross prescription is applied to the ρNN tensor coupling only ($G1$) or also to the meson propagators ($G2$). The numbers in square brackets, after the ^{3}H binding energies, are estimated corrections due to the nonequivalence of the phases of the models (cf. text).

	Bonn-B	$G1$	$G2$	Expt.
Deuteron:				
ϵ_d (MeV)	2.2246	2.2245	2.2246	2.224 575
P_d (%)	4.99	5.47	5.64	-
Q_d (fm^2)	0.278	0.279	0.277	0.2859±0.0003
A_S (fm$^{-1/2}$)	0.8860	0.8856	0.8870	0.8846±0.0016
η	0.0264	0.0265	0.0262	0.0256±0.0004
Neutron-proton low-energy scattering (scattering length a, effective range r):				
$^1S_0$: a_s (fm)	−23.75	−23.75	−23.75	−23.758±0.010
r_s (fm)	2.71	2.71	2.71	2.75±0.05
$^3S_1$: a_t (fm)	5.424	5.421	5.428	5.424±0.004
r_t (fm)	1.76	1.76	1.77	1.759±0.005
^{3}H binding energy (MeV) (five-channel calculation):				
	−8.16	−8.01[−0.02]	−7.85[−0.02]	−8.48

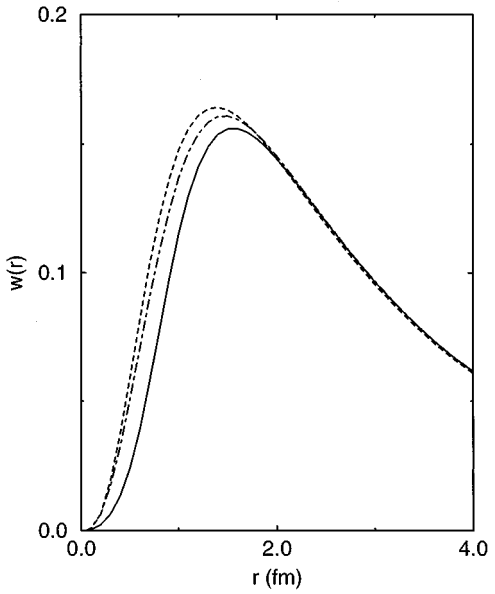

FIG. 3. Deuteron D-wave. The solid curve denotes the result of the Bonn-B potential. For the dash-dotted curve, the ρ-meson tensor vertex coupling is evaluated according to the Gross reduction scheme [15] whereas for the dashed curve the Gross prescription is applied also to the meson propagators (see text).

calities) and thus might change when systematic corrections beyond the OBE approximation [see Eq. (2)] will be applied. Indeed, if we modify in the Bonn-B potential the meson propagators as well as the ρNN tensor coupling according to the Gross reduction scheme as outlined before (a prescription which has been used in the first Bonn OBE potentials [19] 20 years ago) we obtain, again after a suitable refit to the original Bonn-B results (cf. Table II) a considerably different deuteron D-wave (see Fig. 3) which is much closer to the result obtained from the local approximation discussed before. Thus starting from this alternative but *a priori* equally justified prescription the triton binding is somewhat smaller than in the BbS reduction scheme, and the modifications introduced by the local approximations are now considerably reduced.

Since the potentials employed in this study are not exactly phase equivalent there is a word in place about the uncertainties in the predicted triton binding energies arising from this nonequivalence. For the $^1S_0$ partial wave we can rely on a paper by Gibson and Stephenson who studied the dependence of the triton binding on variations of the effective-range parameters [20]. Using the scale set by their investigations we estimate a decrease of the triton binding by about 20 (50) keV for the models with no M/E factor (static π) if phase equivalence would be fulfilled. The models presented in Table II are practically phase equivalent so that no corrections should occur for the $^1S_0$.

In the case of $^3S_1-{}^3D_1$ the effective-range parameters (a_t, r_t) of all models are in very good agreement. However, there are some deviations in the mixing parameter ϵ_1. In order to estimate their effects we generated variants of the model Bonn-B with comparable deviations. It turned out that deviations in ϵ_1 of ± 0.2 degrees at $E_{\rm lab}=300$ MeV roughly correspond to a change of ± 20 keV in the triton binding. (Note that this scale can be also deduced from the models Bonn-A, -B, and -C presented in (Table A.1 of) Ref. [7].) Since most of the models yield an ϵ_1 slightly above the one of Bonn-B we estimate an increase in the triton binding energy of at most 20 keV. An exception is the model "static π" where ϵ_1 is somewhat lower at small energies (cf. Fig. 1) which should lead to an overestimation of the triton binding by about 50 keV. The estimated total corrections due to the nonequivalence of the phases are compiled in Tables I and II. Obviously they are quite small and therefore not of any relevance for the conclusions of the paper.

In summary, nonlocalities in the NN interaction implied by the meson exchange dynamics play an important role and should not be suppressed. They appreciably change results for NN phase shifts and the deuteron binding. A comparably good description of such observables without these nonlocalities is possible, however at the expense of having to change physical meson parameters. Even after a refit, effects remain in the triton leading to a sizable increase in the binding energy, i.e., without three-body forces, the predicted value is closer to the empirical value. The actual results depend however strongly on the calculational scheme chosen since the structure of the prescribed nonlocalities is different for different schemes. In order to reduce these ambiguities the OBE approximation should be avoided by systematically including higher order contributions. This is anyhow required for a consistent evaluation of two- and important three-body forces involving the Δ isobar. It is important to use an efficient calculational scheme in order to reach sufficiently fast convergence. This is not the case for time-ordered perturbation theory applied in the full Bonn potential, which leads to energy-dependent interactions. A scheme leading to instantaneous potentials in all orders based, e.g., on a folded diagram expansion converges much faster in bound nuclear systems [12].

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