Quasibound states of η-nucleus systems

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The position and movement of poles of the amplitude for elastic η-meson scattering off the light nuclei ^{2}H, ^{3}H, ^{3}He, and ^{4}He are studied. It is found that, within the existing uncertainties for the elementary ηN interaction, all these nuclei can support a quasibound state. The values of the η-nucleus scattering lengths corresponding to the critical ηN interaction that produces a quasibound state are given. [S0556-2813(96)50305-5]

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Since meson factories cannot produce η-meson beams, these particles are available for experimental investigations only as products of certain nuclear reactions where they appear as final-state particles. Therefore, final-state interaction effects are the only source of information about the η-meson interaction with nucleons. In this connection, η-nucleus systems can play an important role in investigating the ηN dynamics, especially if they can form quasibound states. In this case, the final-state η mesons can be trapped for a relatively long time, and thus the properties of the ηN interaction can be studied.

Estimations, obtained in the framework of the optical model approach [1,2], put a lower bound on the atomic number A for which an η-nucleus bound state could exist, namely $A \geqslant 12$. In Ref. [3], the formation of η-nucleus states has been investigated, using the standard Green's function method of many-body problems. There it was found that an η^{16}O bound state should be possible. Experimentally the cross sections of pion collisions with lithium, carbon, oxygen, and aluminum, however, gave no evidence for the existence of η bound states with these nuclei [4].

A new theoretical analysis of the problem [5] predicted a binding of the η meson to ^{12}C and heavier nuclei, however, with rather large widths. The formation of an η^4He bound state was studied in a more recent work by Wycech *et al.* [6], using a modified multiple scattering theory. These authors obtained a comparatively large negative value for the real part of the η-nucleus scattering length, which was interpreted as an indication that an η-nucleus bound state could exist. We note that previous results of ours, concering the η scattering lengths with ligh nulcei [7–9], showed that the η-^{4}He scattering length can have an even larger (negative) real part than that of Ref. [6].

In Ref. [10], a preliminary investigation on the possibility of η-meson binding in the d, t, ^{3}He, and ^{4}He systems was made within the framework of the finite-rank approximation (FRA) of the nuclear Hamiltonian [11,12]. The FRA approach treats the motion of the projectile (η meson) and of the nucleons inside the nucleus separately. As a result the internal dynamics of the nucleus enters the theory only via the nuclear wave function. In [10], these wave functions were approximated by simple Gaussian forms, which reproduce the nuclear sizes only. In the present work, we perform calculations with more realistic nuclear wave functions, obtained via the so-called integro-differential equation approach (IDEA) [13–17]. We study, in particular, the position and movement of poles of the elastic amplitude of η-meson scattering off the light nuclei ^{2}H, ^{3}H, ^{3}He, and ^{4}He.

The approximate few-body equations in the FRA approach enable us to calculate the η-nucleus T matrix

$$T(\vec{k}',\vec{k};z)=\langle \vec{k}',\psi_0|T(z)|\vec{k},\psi_0\rangle, \tag{1}$$

at any complex energy. That is, we can locate the poles of the T matrix in the complex momentum plane $p=\sqrt{2\mu z}$. Here, $\vec{k}$ is the η-nucleus momentum, z the total energy of the system, μ the η-nucleus reduced mass, and ψ_0 the nuclear ground-state wave function.

For the low energies and the light nuclei with only one bound state, being considered, it appears justified to approximate the target Hamiltonian H_A by its discrete spectrum

$$H_A \approx \mathcal{E}_0|\psi_0\rangle\langle\psi_0|. \tag{2}$$

Here $|\psi_0\rangle$ stands for the ^{2}H, ^{3}H, ^{3}He, ^{4}He bound states, respectively, and $\mathcal{E}_0$ for the corresponding binding energies.

As a result, we obtain [8] for the T matrix the following equation:

$$T(z)=\sum_{i=1}^{A} T_i^0(z) + \sum_{i=1}^{A} T_i^0(z)|\psi_0\rangle \frac{\mathcal{E}_0}{(z-H_0)(z-H_0-\mathcal{E}_0)}\langle\psi_0|T(z), \tag{3}$$

where H_0 is the η-nucleus kinetic energy operator and A the number of nucleons. The $T_i^0(z)$ are Faddeev-type components of an auxiliary T operator, which obey the system of coupled equations

$$T_i^0(z)=t_i(z)+t_i(z)\frac{1}{(z-H_0)}\sum_{j\neq i} T_j^0(z). \tag{4}$$

Here t_i describes the scattering of the η meson off a nucleon at point $\vec{r}_i$, where $\vec{r}_i$ is the vector from the nuclear center of mass, which can be expressed in terms of the relative Jacobi vectors $\{\vec{r}\}$ of the nucleons. In mixed representation, the operator t_i is given by

$$t_i(\vec{k}',\vec{k};\vec{r};z)=t_{\eta N}(\vec{k}',\vec{k};z)\exp[i(\vec{k}-\vec{k}')\cdot\vec{r}_i],$$

with $t_{\eta N}(\vec{k}',\vec{k};z)$ being the off-shell ηN amplitude.

Thus, to calculate the T matrix (1) for any fixed value of the complex parameter $z=p^2/2\mu$, we have to proceed in three steps. First, the coupled integral equations

$$T_i^0(\vec{k}',\vec{k};\vec{r};z)=t_i(\vec{k}',\vec{k};\vec{r};z)+\int\frac{d^3k''}{(2\pi)^3}\frac{t_i(\vec{k}',\vec{k}'';\vec{r};z)}{z-\dfrac{k''^2}{2\mu}}\sum_{j\neq i}T_j^0(\vec{k}'',\vec{k};\vec{r};z) \tag{5}$$

are to be solved for a number of points $\vec{r}$ in configuration space, sufficient to perform in a second step the integration

$$\langle\vec{k}',\psi_0|\sum_{i=1}^{A}T_i^0(z)|\vec{k},\psi_0\rangle=\int d^{3(A-1)}r|\psi_0(\vec{r})|^2\sum_{i=1}^{A}T_i^0(\vec{k}',\vec{k};\vec{r};z). \tag{6}$$

Having determined these matrix elements, it remains, as a final step, to solve the integral equation

$$T(\vec{k}',\vec{k};z)=\langle\vec{k}',\psi_0|\sum_{i=1}^{A}T_i^0(z)|\vec{k},\psi_0\rangle+\mathcal{E}_0\int\frac{d^3k''}{(2\pi)^3}\frac{\langle\vec{k}',\psi_0|\Sigma_{i=1}^{A}T_i^0(z)|\vec{k}'',\psi_0\rangle}{\left(z-\dfrac{k''^2}{2\mu}\right)\left(z-\mathcal{E}_0-\dfrac{k''^2}{2\mu}\right)}\times T(\vec{k}'',\vec{k};z). \tag{7}$$

Note that after partial-wave decomposition both Eqs. (5) and (7) become one-dimensional. As an input information, we need the ground-state wave functions ψ_0 of the nuclei involved and the two-body T-matrix $t_{\eta N}$.

The $\psi_0(\vec{r})$ for $A=3$ and $A=4$ were obtained by means of the IDEA [13]. In this method the A-body bound state wave function is expanded in Faddeev-type components,

$$\Psi(\vec{r})=\sum_{i<j\leqslant A}\psi_{ij}(\vec{r}), \tag{8}$$

given as solutions of

$$(T-E)\psi_{ij}(\vec{r})=-V(r_{ij})\sum_{k<l\leqslant A}\psi_{kl}(\vec{r}), \tag{9}$$

where $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$. The IDEA is then introduced using the ansatz

$$\psi_{ij}(\vec{r})=H_{[L_m]}(\vec{r})P(\zeta_{ij},\rho)/\rho^{(D-1)/2}, \tag{10}$$

with $\rho=[2/A\Sigma r_{ij}^2]^{1/2}$ being the hyper-radius, $D=3(A-1)$, and $H_{[L_m]}(\vec{r})$ the harmonic polynomial of minimal degree $[L_m]$ [18]. For $[L_m]=0$ the IDEA reads

$$\left[T+\frac{A(A-1)}{2}V_0(\rho)-E\right]\frac{P(\zeta_{ij},\rho)}{\rho^{(D-1)/2}}=-[V(r_{ij})-V_0(\rho)]\sum_{k<l\leqslant A}\frac{P(\zeta_{kl},\rho)}{\rho^{(D-1)/2}}, \tag{11}$$

TABLE I. Positions of poles $p_0=\sqrt{2\mu E_0}$ of the η-nucleus amplitudes with $g=g'=1$ for the three values of the range parameter α.

	p_0 (fm^{-1})	E_0 (MeV)	α (fm^{-1})
	$-0.90259+i0.35870$	$31.456-i29.691$	2.357
ηd	$-0.84594+i0.32195$	$28.061-i24.976$	3.316
	$-0.82460+i0.30423$	$26.935-i23.006$	7.617
	$-0.56045+i0.23859$	$10.906-i11.341$	2.357
ηt	$-0.55511+i0.26826$	$10.015-i12.630$	3.316
	$-0.51725+i0.27896$	$8.0456-i12.238$	7.617
	$-0.54692+i0.24478$	$10.143-i11.354$	2.357
η^3He	$-0.50815+i0.30402$	$7.0305-i13.102$	3.316
	$-0.48310+i0.33948$	$5.0099-i13.909$	7.617
	$-0.16504+i0.27876$	$-2.0540-i3.7447$	2.357
η^4He	$-0.20215+i0.38726$	$-4.4403-i6.3718$	3.316
	$-0.25931+i0.45846$	$-5.8175-i9.6766$	7.617

where $V_0(\rho)$ is the so-called hypercentral potential [18]. Projecting Eq. (11) onto the r_{ij} space provides us, for spin dependent nucleon-nucleon potentials, with two coupled integro-differential equations for the symmetric S and mixed symmetric S' components of the function $P^n(\zeta_{ij},\rho)$, $n=S,S'$. More details and explicit equations are found in Refs. [13,15].

For the nuclear ground states, we use the fully symmetric S-wave components obtained with the semirealistic Malfliet-Tjon I–III (MT I–III) nucleon-nucleon potential [19]. The corresponding two, three, and four-body binding energies are 2.272 MeV, 8.936 MeV, and 30.947 MeV, while the root mean square (rms) radii are 1.976 fm, 1.685 fm and 1.431 fm, respectively. The omission of Coulomb effects and of the mixed-symmetry components makes ^{3}H and ^{3}He indistinguishable. In order to compensate partly for this omission, we use in Eq. (7) the experimental values for masses and binding energies of the nuclei [20].

At low energies the ηN interaction is dominated by the $N^*(1535)$ S_{11} resonance. For the ηN amplitude we, therefore, choose the separable form

$$t_{\eta N}(k',k;z)=\frac{\lambda}{(k'^2+\alpha^2)(z-E_0+i\Gamma/2)(k^2+\alpha^2)}, \quad (12)$$

with $E_0=1535$ MeV$-(m_N+m_\eta)$ and $\Gamma=150$ MeV [21]. To find the range parameter α, we use the results of Refs. [22,23]. There the same $\eta N\rightarrow N^*$ vertex function $(k^2+\alpha^2)^{-1}$ was employed, and α was determined via a two-channel fit to the $\pi N\rightarrow\pi N$ and $\pi N\rightarrow\eta N$ experimental data.

Three different values for the range parameter α are available in the literature, namely, $\alpha=2.357$ fm^{-1} [22], $\alpha=3.316$ fm^{-1} [23], and $\alpha=7.617$ fm^{-1} [22]. Since there is no criterion for singling out one of them, we use all three in our calculations. The remaining parameter λ is chosen to provide the correct zero-energy on-shell limit, i.e., to reproduce the ηN scattering length $a_{\eta N}$,

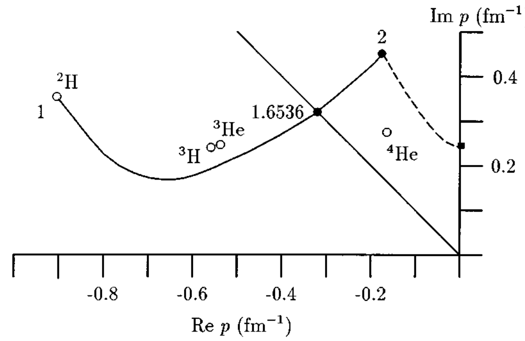

FIG. 1. The η-nucleus elastic scattering amplitude pole positions in the complex p plane. The open circles correspond to $g=1$. The solid curve is the ηd-amplitude pole trajectory when g increases from $g=1$ to $g=2$. The dashed curve shows the trajectory of the ηd pole with $g=2$ and with g' varied until the ηN interaction becomes real.

$$t_{\eta N}(0,0,0)=-\frac{2\pi}{\mu_{\eta N}}a_{\eta N}. \quad (13)$$

Different analyses provided values for the real part Re $a_{\eta N}$ in the range 0.27–0.98 fm and for the imaginary part Im$a_{\eta N}$ in the range 0.19–0.37 fm [24]. To examine at which value of $a_{\eta N}$ within the above ranges an η-nucleus bound state exists, we parametrize the scattering length as follows:

$$a_{\eta N}=(g0.55+ig'0.30)\ \text{fm}, \quad (14)$$

where g and g' are adjustable parameters. The value of $a_{\eta N}$ for $g=g'=1$ was used by Wilkin [25].

Since $a_{\eta N}$ is complex, the η-nucleus Hamiltonian is non-Hermitian and its eigenvalues are generally complex. In this case, eigenvalues attributed to quasibound states are located in the second quadrant of the complex p plane [26]. The energy $E_0=p_0^2/2\mu$ corresponding to a pole at $p=p_0$.

$$E_0=\frac{1}{2\mu}[(\mathrm{Re}p_0)^2-(\mathrm{Im}p_0)^2+2i(\mathrm{Re}p_0)(\mathrm{Im}p_0)], \quad (15)$$

TABLE II. The parameter g generating the η-nucleus amplitude poles $p_0=\sqrt{2\mu E_0}$ on the diagonal for the three values of the range parameter α and $g'=1$.

	g	p_0 (fm^{-1})	E_0 (MeV)	α (fm^{-1})
	1.6536	$-0.32527+i0.32527$	$-i9.7026$	2.357
ηd	1.5605	$-0.33541+i0.33541$	$-i10.317$	3.316
	1.5260	$-0.33670+i0.33670$	$-i10.397$	7.617
	1.3624	$-0.33515+i0.33515$	$-i9.5266$	2.357
ηt	1.3055	$-0.35190+i0.35190$	$-i10.503$	3.316
	1.2436	$-0.35186+i0.35186$	$-i10.500$	7.617
	1.3306	$-0.34034+i0.34034$	$-i9.8239$	2.357
η^3He	1.2171	$-0.36267+i0.36267$	$-i11.155$	3.316
	1.1421	$-0.37631+i0.37631$	$-i12.010$	7.617
	0.86222	$-0.20641+i0.20641$	$-i3.4679$	2.357
η^4He	0.80813	$-0.26522+i0.26522$	$-i5.7255$	3.316
	0.79578	$-0.35215+i0.35215$	$-i10.094$	7.617

TABLE III. The η-nucleus scattering lengths for the parameter g of Table II, which generate the condition for binding ($\mathrm{Re}E=0$).

	$\alpha=2.357$ (fm^{-1})	$\alpha=3.316$ (fm^{-1})	$\alpha=7.617$ (fm^{-1})
ηd	$0.171+i5.99$	$-0.198+i4.57$	$-0.318+i3.52$
ηt	$-3.65+i3.49$	$-2.91+i3.02$	$-2.19+2.70$
$\eta ^3\mathrm{He}$	$-3.49+i3.67$	$-2.66+i3.31$	$-1.96+i2.86$
$\eta ^4\mathrm{He}$	$-3.43+i2.60$	$-2.81+i2.14$	$-2.30+i1.72$

has a negative real part, $\mathrm{Re}\,E_0<0$, only if p_0 is above the diagonal of this quadrant. Such a pole is related to a quasibound state, which for $\mathrm{Re}\,p_0\to 0$ goes over into a real bound state. For p_0 in the second quadrant but below the diagonal we have $\mathrm{Re}\,E_0>0$. Therefore this diagonal is critical: When crossing it from below a pole gains the physical meaning of a quasibound state.

Fixing g and g' of Eq. (14) to $g=g'=1$ and varying the complex momentum $p=\sqrt{2\mu z}$, we located the poles close to the origin $p=0$. The results obtained are given in Table I. For one choice of the range parameter, namely $\alpha=2.357$ fm^{-1}, the positions of the poles found are shown by the open circles in Fig. 1. It is seen that for the ηd, ηt, and $\eta ^3$He systems, these poles lie below the diagonal, while for the $\eta ^4$He system the pole is in the quasibound region.

Increasing g while keeping $g'=1$, the below-diagonal poles are moving towards, and finally cross, the diagonal. In the deuteron case, the corresponding trajectory is depicted in Fig. 1 by the solid curve that crosses the diagonal when $g=1.6536$.

To find the relationship of poles above the diagonal to physical bound states, we gradually removed the imaginary part of $a_{\eta N}$ by fixing g and decreasing g' in Eq. (4) to zero. The imaginary part of the Breit-Wigner factor in Eq. (12) was also decreased, using the same parameter g', so that it goes over into $(z-E_0+ig'\Gamma/2)^{-1}$. For $g'=0$ the Hamiltonian becomes Hermitian and, hence, the bound state poles in this case must be on the positive imaginary axis. The dashed curve in Fig. 1 is the trajectory of the ηd bound-state pole (with $g=2$) when g' decreases from 1 to 0. It is seen that the final position of the pole lies on the positive imaginary axis. This supports our interpretation of poles above the diagonal as quasibound states.

By varying the enhancing factor g for each of the η-nucleus systems under consideration, we found the values which generate quasibound states on the diagonal. They are given in Table II. These values correspond to an ηN attraction, which generates an η-nucleus binding with $\mathrm{Re}\,E_0=0$. Further increase of g moves the poles up and to the right, enhancing the binding and reducing the widths of the states. The value of $\mathrm{Re}\,a_{\eta N}$ that provides the critical binding lies within the range [0.27,0.98] fm used in the literature. Therefore, an η-nucleus quasibound state may exist for $A\geqslant 2$. However, as can be seen in Table I and II, the widths of such quasibound states could be small only for the $\eta ^4$He system.

In Table III we present the η-nucleus scattering lengths calculated with parameters generating the critical η-nucleus binding. From this table we see that the real part of the η-nucleus scattering length can be small despite the existence of a quasibound state. This is due to the non-Hermitian nature of the ηN interaction. Being complex, this interaction generates critical poles rather far from the origin and their influence on the scattering length (the value of the amplitude at the origin) is not very strong.

In conclusion, we have shown that the uncertainties in the ηN scattering length allow for choices of parameters in the ηN amplitude that may generate poles in the η-nucleus amplitudes considered, which can be attributed to quasibound states.

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[1] Q. Haider and L. C. Liu, Phys. Lett. B **172**, 257 (1986).
[2] L. C. Liu and Q. Haider, Phys. Rev. C **34**, 1845 (1986).
[3] G. L. Li, W. K. Cheng, and T. T. S. Kuo, Phys. Lett. B **195**, 515 (1987).
[4] R. E. Chrien *et al.*, Phys. Rev. Lett. **60**, 2595 (1988).
[5] H. C. Chiang, E. Oset, and L. C. Liu, Phys. Rev. C **44**, 738 (1991).
[6] S. Wycech, A. M. Green, and J. A. Niskanen, University of Helsinki Report No. HU–TFT–95–8, 1995.
[7] S. A. Rakityansky, S. A. Sofianos, and V. B. Belyaev, in *Symposium on Effective Interactions in Quantum Systems*, edited by S. A. Sofianos (UNISA, Pretoria, 1994).
[8] S. A. Rakityansky, S. A. Sofianos, W. Sandhas, and V. B. Belyaev, Phys. Lett. B **359**, 33 (1995).
[9] V. B. Belyaev, S. A. Rakityansky, S. A. Sofianos, W. Sandhas, and M. Braun, Few-Body Syst. Suppl. **8**, 312 (1995).
[10] S. A. Rakityansky, S. A. Sofianos, V. B. Belyaev, and W. Sandhas, in *The International Conference Mesons and Light Nuclei* (MESONS–95), Prague, 1995, Few-Body Syst. Suppl. (to be published).
[11] V. B. Belyaev and J. Wrzecionko, Sov. J. Nucl. Phys. **28**, 78 (1978).
[12] V. B. Belyaev, in *Lectures on the Theory of Few-Body Systems* (Springer-Verlag, Heidelberg, 1990).
[13] M. Fabre de la Ripelle, H. Fiedeldey, and S. A. Sofianos, Phys. Rev. C **38**, 449 (1988).
[14] M. Fabre de la Ripelle, H. Fiedeldey, and S. A. Sofianos, Few-Body Syst. **6**, 157 (1989).

[15] W. Oehm, S. A. Sofianos, H. Fiedeldey, and M. Febre de la Ripelle, Phys. Rev. C **44**, 81 (1991).

[16] W. Oehm, S. A. Sofianos, H. Fiedeldey, and M. Fabre de la Ripelle, Phys. Rev. C **43**, 25 (1991).

[17] W. Oehm, S. A. Sofianos, H. Fiedeldey, and M. Fabre de la Ripelle, Phys. Rev. C **42**, 2322 (1990).

[18] M. Fabre de la Ripelle, in *Lecture Notes in Physics*, edited by L. S. Fereira, A. C. Fonseca, and L. Streit (Springer-Verlag, New York, 1987), Vol. 273, pp. 283–323.

[19] R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969); Ann. Phys. (N.Y.) **61**, 425 (1970).

[20] A. H. Wapstra and G. Audi, Nucl. Phys. **432**, 1 (1985).

[21] Particle Data Group, Phys. Rev. D **50(3)**, 1319 (1994).

[22] R. S. Bhalerao and L. C. Liu, Phys. Rev. Lett. **54**, 865 (1985).

[23] C. Bennhold and H. Tanabe, Nucl. Phys. **A530**, 625 (1991).

[24] M. Batinic and A. Svarc, Los Alamos e-print archive: nucl-th/9503020, 1995.

[25] C. Wilkin, Phys. Rev. C **47**, R938 (1993).

[26] W. Cassing, M. Stingl, and A. Weiguny, Phys. Rev. C **26**, 22 (1982).