Nonperturbative treatment of gluons and pseudoscalar mesons in baryon spectroscopy

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We study baryon spectroscopy including the effects of pseudoscalar meson exchange and one gluon exchange potentials between quarks, using nonperturbative, hyperspherical method calculations. We find that a model that includes only gluon exchange cannot simultaneously describe the Roper and P-wave excitation energies. Using only pseudoscalar meson exchange partially cures this problem, but at the cost of using a relatively large pion quark coupling constant. However, one gets a similar agreement with data in a model with both effects by using a quark-meson coupling constant compatible with the measured pion-nucleon coupling constant, and a value of $\alpha_s \approx 0.35$. [S0556-2813(96)50205-0]

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Interest in studying baryon spectroscopy has been revitalized by the recent work of Glozman and Riska [1–5]. These authors point out the persistent difficulty in obtaining a simultaneous description of the masses of the P-wave baryon resonances and the Roper-nucleon mass difference. In particular they argue [2] that "the spectra of the nucleons, Δ resonances and the strange hyperons are well described by the constituent quark model, if in addition to the harmonic confinement potential the quarks are assumed to interact by exchange of the $SU(3)_F$ octet of pseudoscalar mesons." Furthermore, Ref. [5] states that gluon exchange has no relation with the spectrum of baryons.

The ideas of Glozman and Riska are especially interesting because of the good descriptions of the spectra obtained in Refs. [1–5], and because of the contradictory long-standing belief [6–9] that one-gluon exchange is a basic element of quantum chromodynamics (QCD) and the success of that interaction in baryon spectroscopy. Despite the lore, some authors had noted the difficulty in obtaining a simultaneous description of the Roper and P-wave resonances [10,11].

The purpose of this paper is to include both effects in calculating the baryon spectra using a nonperturbative technique, and to show that both kinds of effects are required for a reasonable description of the data. Including the effects of pion clouds is known to lead to a good description of nucleon properties, as well as meson-nucleon and electron-nucleon scatterings [12,13]. We note that several previous workers [14–17] have shown that including both pion exchange and gluon exchange effects leads to an improved description of the data. Those calculations typically use a shell-model diagonalization procedure to determine the eigenstates, with a truncation of states of greater than $2\hbar\omega$ excitation energy. Robson [18] and Glozman and Riska used a technique in which the differences between baryon masses is given by matrix elements of the meson exchange potential. However, nonperturbative calculations are required to handle the one-gluon exchange interaction [11,19,20]. It is therefore natural to expect that if one used only pseudoscalar meson exchange to generate all of the mass splitting, a nonperturbative treatment would be necessary. Thus a nonperturbative, all-orders treatment is needed to assess whether or not either of those two elements can be ignored. We employ the hyperspherical methods of Fabre de la Ripelle *et al.* [21] to compute the energies of the baryons.

We use a constituent quark model Hamiltonian that includes the effects of one-gluon exchange (OGE) and the exchange of pseudoscalar mesons mandated by broken chiral symmetry, V_χ, in addition to the kinetic energy and confinement terms. Thus

$$H = T + V_{\rm con} + V_{\rm OGE} + V_\chi, \tag{1}$$

where the kinetic energy T takes the nonrelativistic form

$$T = \sum_i -\frac{\nabla_i^2}{2m}, \tag{2}$$

with the u or d quark mass taken as 336 MeV to represent the nonperturbative effects that influence the properties of a single confined quark. We limit ourselves to light quarks in this first calculation, but note that the success in handling strange baryons is an important part of the work of Glozman and Riska.

Here we assume that the confining interaction $V_{\rm con}$ takes on a linear (V_L) form so that

$$V_L = \sum_{i<j} A_L |\vec{r}_i - \vec{r}_j|. \tag{3}$$

The parameter A_L is to be determined phenomenologically. The one-gluon exchange interaction between different quarks is given by the expression

$$V_{\rm OGE} = \sum_{i<j} \left(-\frac{2}{3}\frac{\alpha_s}{r_{ij}} + \frac{2}{3}\frac{\pi\alpha_s}{m^2}\frac{1}{4\pi}\frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} + \alpha_s \frac{4}{9}\frac{\pi}{m^2}\frac{1}{4\pi}\frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} \vec{\sigma}_i \cdot \vec{\sigma}_j \right), \tag{4}$$

where $r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$, $r_0 = 0.238$ fm, and α_s is a parameter to be determined phenomenologically. The value of r_0 is that of Refs. [16,17] who use $1/r_0 = 4.2$ fm^{-1}. The replacement of the usual δ function form by a Yukawa of range r_0 is intended to include the effects of the finite-sized nature of the constituent quarks. We can use values of r_0 between 0.2 and 0.3 fm without affecting the conclusions of the present work. Note that $1/r_0 \approx 4\pi f_\pi$, the chiral symmetry scale.

We ignore the spin-orbit and tensor terms because our first calculation is intended to be a broad comparison of the nonperturbative effects of gluon and meson exchange. Isgur and Karl [24] found that including the tensor hyperfine forces with relative strengths predicted by the one-gluon exchange interaction is necessary to produce the splitting between the $J^\pi = 1/2^-$ and $J^\pi = 3/2^-$ nucleonic states as well as to understand their separate wave functions and consequent decay properties. Therefore we do not expect our calculations to reproduce those features. The issue of the spin-orbit interaction between quarks is a complicated one. There are many different contributions: Galilei invariant and noninvariant terms arising from one-gluon exchange see, e.g. [25], a Thomas precession term arising from the confining interaction [7], effects of exchange of scalar mesons, and the instanton induced interaction [26]. The above cited authors show that some of the various terms tend to cancel when evaluating the baryon spectra. A detailed study of the influence of the various contributions to the spin-orbit force is beyond the scope of the present work.

The effects of pseudoscalar meson octet exchange are described by the interaction [1–5]

$$V_\chi = \sum_{i<j} \alpha_{q\pi} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{3} \frac{\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F}{4m^2} \left(\mu^2 \frac{e^{-\mu r_{ij}}}{r_{ij}} - \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} \right), \quad (5)$$

where r_0 is again taken to be 0.238 fm [16]. We shall allow the strength of the meson exchange potential, $\alpha_{q\pi}$, to vary away from the expected [2] value of 0.67. This is in the spirit of the work of Refs. [1–5] who fit a very few matrix elements of V_χ to a few mass differences and predict the remainder of the spectrum. The values of the flavor SU(3) matrix elements are taken from Eq. (5.1) of Ref. [2]. We neglect the tensor force generated by the exchange of pseudoscalar mesons, as do Glozman and Riska. Similarly, retardation effects and the influence of the baryonic mass differences are neglected.

Next we turn to a brief description of the hyperspherical method, which has been in use for some time [21,27]. The idea is that the Schrödinger equation for three particles can be simplified by expressing the usual Jacobi coordinates $\vec{\xi}_1 = \vec{r}_1 - \vec{r}_2$ and $\vec{\xi}_2 \equiv 1/\sqrt{3}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$ using the hyperspherical coordinates defined by a radial distance $r = \sqrt{\xi_1^2 + \xi_2^2}$, polar angles $\omega_i = (\theta_i, \phi_i)$ of $\vec{\xi}_i$, and the additional angle ϕ defined as $\tan\phi = \xi_2/\xi_1$. The hyperspherical harmonics consist of a complete set of angular functions on the five-dimensional hypersphere. Hence the wave function and potential can be expressed in terms of linear combinations of these functions. Furthermore, Ref. [28] has shown how to construct linear combinations of these functions that form irreducible representations of the permutation group of three particles in the S state. This enables one to construct wave functions that are consistent with the Pauli exclusion principle. In particular, the requirement of constructing color-singlet states is met by treating the wave function as a product of the standard SU(6) spin-flavor wave functions, by symmetric spatial wave functions, and by the antisymmetric color wave function.

This approach means that we shall ignore the effects of mixed symmetry states. This is a reasonable starting point, since here we focus on the difference between baryon masses. The result of including the effects of the the mixed-symmetric $SU(3)^2 8$ state is to cause only a 40 MeV downshift in the nucleon mass and a 20 MeV downshift in the Roper mass in calculations using only one-gluon exchange [21]. Moreover, such a state would be an admixture to the Roper resonance as well. Our calculations indicate that the Roper mass is shifted down by a similar amount. However, including mixed-symmetry states is important for treating the charge radius of the neutron [22] and we shall do so elsewhere.

The basis of hyperspherical harmonics has a large degeneracy, which can be handled by using the optimal subset [29] which is constructed as linear combinations of potential harmonics, i.e., those states generated by allowing the potential $V_{\mathrm{con}} + V_{\mathrm{OGE}} + V_\chi$ to act on the hyperspherical harmonics of minimal order allowed by the Pauli exclusion principle. (See Ref. [27] for a detailed discussion of the general formalism.) The convergence properties of the expansion and the accuracy of using a single optimal state have been studied by several authors [30,31] with the result that the overlap between the approximate and exact eigenfunctions is generally greater than 99.5%.

To be specific, we display the specific nucleon and Δ wave functions. The nucleon wave function is given by

$$\psi^N = \frac{1}{\sqrt{2}} (\chi^\rho \eta^\rho + \chi^\lambda \eta^\lambda) u_N(r) r^{-5/2}, \quad (6)$$

where $\chi^\rho, (\eta^\rho)$ are the mixed antisymmetric spin (flavor) wave functions and $\chi^\lambda, (\eta^\lambda)$, are the mixed-symmetric spin (flavor) wave functions. The Δ wave function is given by

$$\psi^\Delta = \chi^{3/2} \eta^{3/2} u_\Delta(r) r^{-5/2}. \quad (7)$$

The radial wave functions u_N and u_Δ are obtained by solving the differential equation

$$\left[\frac{\hbar^2}{m} \left(-\frac{d^2}{dr^2} + \frac{15/4}{r^2} \right) + V_{N,\Delta}(r) - E \right] u_{N,\Delta}(r) = 0, \quad (8)$$

where the potentials $V_{N,\Delta}(r)$ are obtained by reexpressing the interactions above in terms of a quark-quark interaction V_{qq} such that

$$V_{qq}(r_{ij}) = V^0(r_{ij}) + V^S(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j + V^\chi(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F. \quad (9)$$

The term V^0 includes both the confining and spin-independent part of the quark-quark interaction. Then the potential $V(r)$ of Eq. (8) is given by

$$V_N(r) = \frac{48}{\pi} \int_0^1 [V^0(ru) - V^S(ru) + C_N V^\chi(ru)] \sqrt{1-u^2} u^2 du, \quad (10)$$

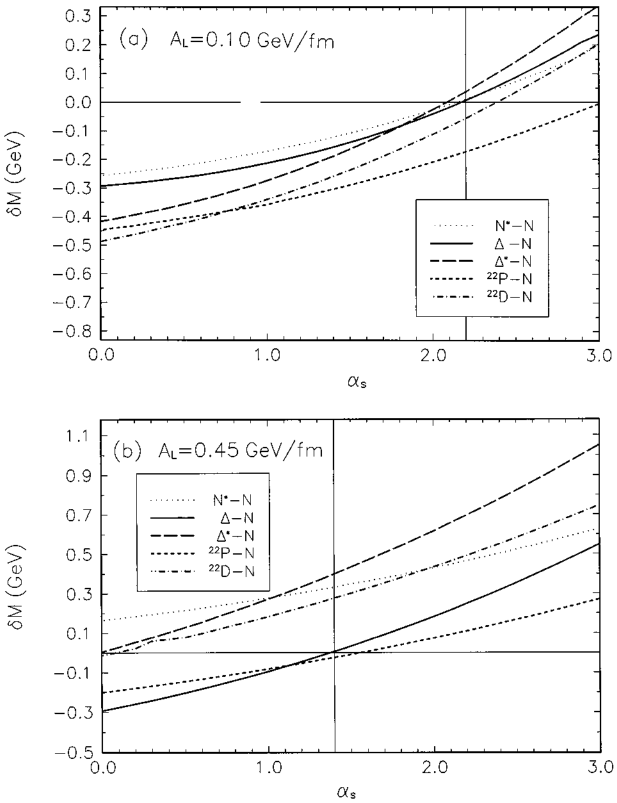

FIG. 1. Baryon mass splitting versus α_s, with $V^\chi=0$. Differences between the computed and measured values of the mass splitting Δ (in GeV) are shown. (a) $A=0.10$ GeV/fm; (b) $A=0.45$ GeV/fm.

$$V_\Delta(r)=\frac{48}{\pi}\int_0^1[V^0(ru)+V^S(ru)+C_\Delta V^\chi(ru)]\sqrt{1-u^2}u^2du,$$

where $C_N=14/3$ and $C_\Delta=4/3$ are obtained by taking the matrix elements of the flavor-spin matrix $\vec{\sigma}_i\cdot\vec{\sigma}_j\vec{\lambda}_i^F\cdot\vec{\lambda}_j^F$ in the appropriate wave functions. The differential equations are solved using the renormalized Numerov method formulated by Johnson [32].

Let us now turn to the numerical results. We shall calculate the masses of the ground-state and first-radial excitation for S, P, and D waves in the nucleon and Δ channels and compare our predictions with all of these nonstrange four- and three-star baryon resonances with masses below 1800 MeV found in the particle data tables [23]. For the purpose of clarity, our procedure will be to show only five mass differences in the figures: those between the nucleon and the $\Delta(1232)$, Roper $N(1440)$, $\Delta(1600)$, ^{22}P, and ^{22}D. We use the standard spectroscopic notation $^{2I+1\ 2S+1}L$. A final comparison of our predictions with all of the lowest 13 states will be presented later.

The first model we shall consider includes the one-gluon exchange but neglects the effects of the meson exchange interaction V_χ. The differences between the computed and measured values of the mass splitting $\delta M=M^{\rm theory}-M^{\rm expt}$ are shown as a function of α_s in Fig. 1. A curve passes through the horizontal line when the computed value of the indicated mass difference is equal to the experimental value of that difference. This notation is used in each of the figures.

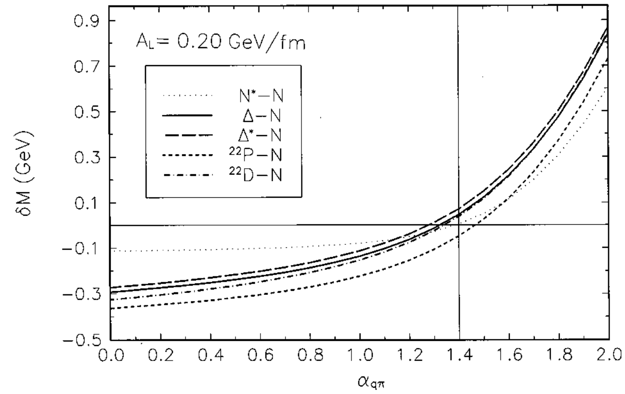

FIG. 2. Baryon splitting Glozman-Riska model, neglecting the one-gluon exchange interaction, $V_{\rm OGE}=0$. The mass differences (Δ in GeV) are shown as a function of $\alpha_{q\pi}$.

The results of Fig. 1(a) show that the model can account for all mass differences except for the splitting between the P-wave resonance and the nucleon. A large value of $\alpha_s\approx2.2$ is used to obtain the fit with $A=0.10$ GeV/fm. If one uses instead $A=0.45$ GeV/fm, one is able to account for the Δ-nucleon and P-wave nucleon splitting but not the Roper mass, as shown in Fig. 1(b). This agreement is obtained also for a large value of $\alpha_s\approx1.4$ that roughly corresponds to the original theory of Refs. [6,7], which works reasonably well except for the Roper mass.

One may also study the converse situation of keeping pseudoscalar meson exchange and ignoring the gluonic exchange, which represents a nonperturbative treatment of the Glozman-Riska theory. The results, shown in Fig. 2, indicate that this version of the nonrelativistic quark model is very successful if one allows the freedom to vary the value of $\alpha_{q\pi}$ away from the expected value of 0.67 anticipated in Ref. [2]. Using a factor of two increase so that $\alpha_{q\pi}\approx1.4$ improves immensely the agreement with experiment. No such agreement can be obtained if one insists on using the value 0.67.

The third model we consider is the most general, in which both the color magnetic and pseudoscalar meson exchange terms are included. Both of these terms contribute to the N-Δ splitting [12], so that including both effects can be reasonably expected to lead to smaller values of α_s and $\alpha_{q\pi}$ than used in Figs. 1 and 2. The results for this general model are shown in Fig. 3. One obtains a good description of the data, with the energy of the ^{22}P state as the expected single exception. Furthermore, the value of α_s is about 0.35 instead of about 2 required if this is the sole physics responsible for the Δ-nucleon mass splitting. A small value is preferred because this interaction is derived using perturbation theory. Still another nice feature is that the value of $\alpha_{q\pi}\approx1$, which is close to the value expected [33] from the measured pion nucleon coupling constant, $g_{\pi N}$. The relation between the pion-quark coupling constant, g, and $g_{\pi N}$ is $g_{\pi N}=(m_u/g_A m_N)g$ [2]. Using the experimentally measured axial coupling constant $g_A=1.26$ along with our quark mass $m_u=$ 336 MeV and $g_{\pi N}^2/4\pi=14.2$ gives $\alpha_{q\pi}=g^2/4\pi=1.15$. The use of $g_A=1.26$ accounts for known relativistic effects, which change the quark wave functions but do not modify the spectrum [8]. The use of $\alpha_{q\pi}\approx1$ to reproduce the differences between baryon masses therefore represents a

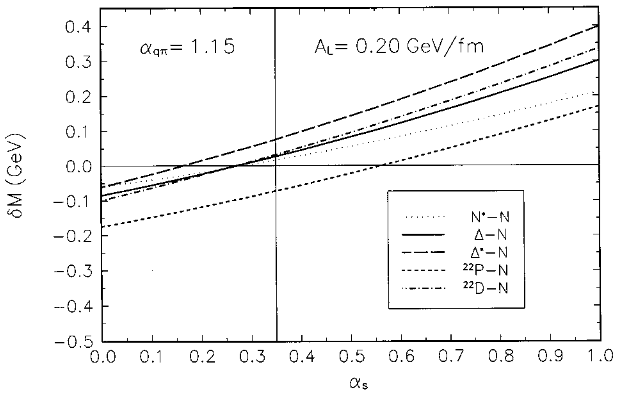

FIG. 3. Baryon splitting with the complete Hamiltonian. The mass differences (Δ in GeV) are shown as a function of α_s.

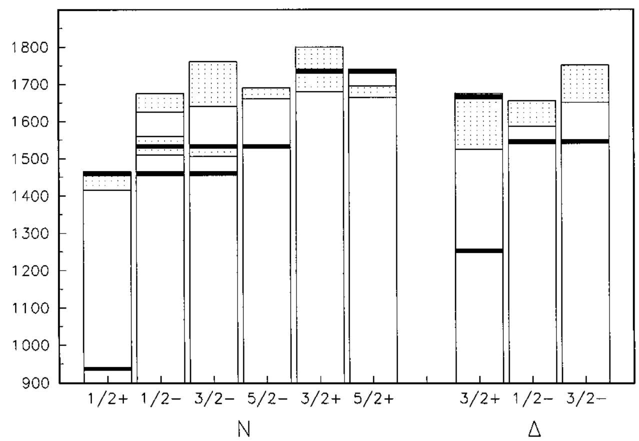

FIG. 4. S-, P- and D-wave energy levels—complete Hamiltonian with $\alpha_s=0.35$, $\alpha_{q\pi}=1.15$, and $A_L=0.20$ GeV/fm. Bars (theory), boxes (experiment).

significant improvement in the theory.

As shown in Fig. 4, the masses of the $\Delta(1232)$, $N(1440)$, $\Delta(1600)$, and the ${}^{22}D$ are very well reproduced. The masses of the P-wave states are off by at least 50 MeV. Nevertheless, this is good agreement considering that there are essentially only two free parameters in this third model. Furthermore, in addition to the interactions mentioned above, the effects of including the mixed-symmetric states lowers the mass of the nucleon by about 40 MeV [21], but is not expected to influence the masses of the P-wave states. Thus we expect that the P-wave states can ultimately be reasonably well described.

We have obtained a reasonably good description of the energies of states, so that it is worthwhile to begin discussing some of the properties of the wave functions. We note that the value of $A_L=0.20$ GeV/fm, which yields a nucleon rms radius of 0.46 fm is significantly smaller than the experimental value $\sim$0.8 fm, but much larger than obtained, $\approx$0.3 fm, in work using only one-gluon exchange such as that of Refs. [19,20]. Including the relativistic recoil correction, also invoked by Capstick and Isgur, is known to increase the computed value of the radius. Similar effects occur by including the influence of the meson cloud on the nucleon radius, and the effects of other components of the wave function. We plan to include such effects, along with tensor and spin-orbit forces and retardation effects, in future work. This would enable us to obtain a realistic treatment and to compute the decay properties of the excited states. We also plan to consider strange baryons.

This brings us to a summary of the effects of one-gluon and one-pseudoscalar meson exchange interactions. If we include only the effects of one-gluon exchange, we do not get a good description of the even and odd parity resonant states. This description is improved if we include only the effects of meson exchange. However, both gluonic and pseudoscalar meson exchange are expected from the underlying theory. Thus although we verify several of the statements of Refs. [1–5], a theory that includes both gluon and meson exchange seems somewhat more plausible. Indeed, our most general model is defined by using a value of $\alpha_{q\pi}$ equal to that provided by the pion-nucleon coupling constant and a value of $\alpha_s=0.35$ about equal to that provided by perturbation theory. With this model, nonrelativistic calculations including confinement, one-gluon and pseudoscalar meson exchange can describe the light-quark baryon spectrum reasonably well.

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