Correlations in the in-medium nucleon-nucleon cross section

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The influence of the ground state correlations on the in-medium nucleon-nucleon cross section is investigated in the framework of an extended Brueckner-Hartree-Fock theory of nuclear matter. The effect of the correlations is to overwhelm the suppression of the in-medium *NN* cross section already established in previous approximations. Moreover the resulting cross section exceeds largely, particularly for neutron-proton scattering, the free-space values in the low energy range (up to 200–250 MeV) for nuclear medium densities up to two times the saturation density.

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Transport models are a powerful approach for reproducing the collective observables in heavy-ion collisions (HIC) and to probe the equation of state (EOS) of nuclear matter. The EOS appears in the kinetic equation through the mean field felt by a nucleon freely traveling between two successive scatterings as well as through the in-medium cross section of the two nucleon scattering. It is widely recognized that both quantities must be calculated in the framework of a fully microscopic approach to the many-body problem if any reliable conclusion on the EOS of nuclear matter is to be drawn from dynamical simulation codes based on the transport model [1]. The Brueckner theory provides a satisfactory starting point for reaching this objective for the twofold reason that, on one side, it is able to handle the realistic nucleon-nucleon interaction by a renormalization procedure suitable to prevent any hard-core divergence and, on the other side, it can calculate self-consistently and simultaneously the effective interaction (G matrix) giving rise to the in-medium cross section and the nuclear mean field. There exist in the literature calculations of the in-medium *NN* cross section using the nonrelativistic Brueckner theory [2,3] as well as its relativistic version, the Dirac-Brueckner theory [4]. Although the controversy on the saturation properties of nuclear matter has not yet been solved, nevertheless both theories seem to provide comparable medium effects on the nucleon-nucleon scattering. This is an indication that the G matrix is not much affected by the peculiarities of the nuclear mean field. On the contrary, the mean field and the other quantities related to it, such as the effective mass, strongly depend upon the order at which the mean field is calculated in the hole-line expansion. It is well known [5,6], in fact, that the second order diagrams in the hole-line expansion give a contribution to the mean field which is essential for a comparison with the phenomenological optical potential. This term, sometimes called *correlation potential*, reduces the attractive Brueckner-Hartree-Fock (BHF) potential due to the excitations of the ground state correlated particles. The influence of the correlation potential has already been investigated in dynamical simulations of HIC, based on the Boltzmann-Nordheim-Vlasov (BNV) equation especially, which concerns the prediction on the low-energy transverse flows and the balance energy as well [1]. The latter quantity seems to be particularly sensitive to the momentum dependence of the mean field which is crucially modified by the introduction of the correlation term. In the present paper we want to investigate the influence of the correlation potential on both the G matrix and the effective mass. Whereas we do not expect a significant effect on the G matrix for the aforementioned argument, strong modifications have to be expected on the effective mass [6], which is of primary importance for calculating the in-medium cross section.

The BHF approximation extended to the two-hole-line mean field in order to include ground state correlations is described elsewhere [5,6]. Here we simply recall that in this case the self-consistent procedure to determine simultaneously G matrix and mean field makes use of the expansion of the mass operator truncated at the second order in the hole-line expansion. The first order term $U_1(k)$ is the usual BHF potential that is the generalization of the HF potential consisting in replacing the bare nucleon-nucleon interaction with the G matrix. Its expression is

$$U_1(k)=\frac{1}{2}\sum_{k'\leqslant k_F}\langle kk'|G|kk'\rangle_A . \tag{1}$$

The second order term $U_2(k)$ is the so-called correlation contribution which takes into account the modifications of the nuclear matter ground state due to two-particle–two-hole excitations. It is given by

$$U_2(k)=\frac{1}{2}\sum_{k'\leqslant k_F}\sum_{k_1,k_2\geqslant k_F}\frac{|\langle kk'|G|k_1k_2\rangle_A|^2}{E_k+E_{k'}-E_{k_1}-E_{k_2}+i\epsilon}. \tag{2}$$

The presence of this term deeply modifies the properties of the mean field but it does not change the nucleon binding energy calculated in the BHF limit of approximation [8]. In spite of that, it has been shown [6] that the too attractive one-hole-line term (BHF) is largely compensated by the strong repulsive two-hole-line term in the hole-line expansion for the binding energy. Within the new approximation, we can investigate how the ground state (g.s.) correlations influence the effective mass and the G matrix that determine the in-medium *NN* cross section.

The *NN* interaction adopted in the present calculations is the separable version of the Paris interaction, which has been extensively used in nuclear matter calculations [7]. It includes the channels $^3S_0$, $^3S_1$-$^3D_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^1D_2$,

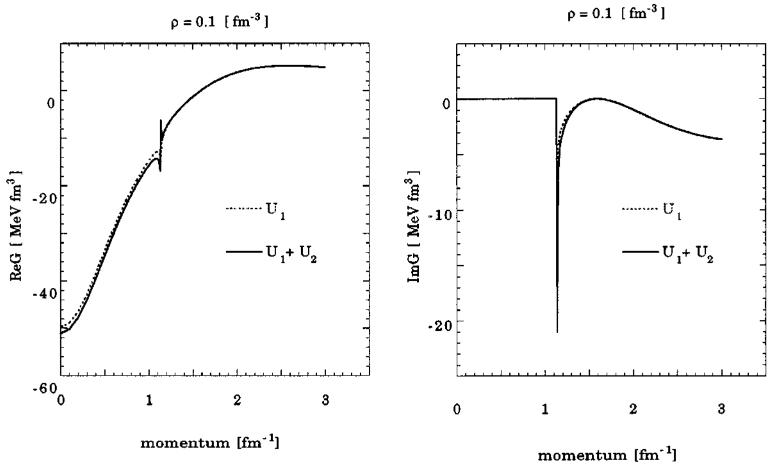

FIG. 1. Real and imaginary parts of the $^1S_0$ component of the G matrix with (full lines) and without (dashed lines) the correlation contribution U_2 to the mean field for nuclear density $\rho=0.1$ fm^{-3}. Notice that the imaginary part vanishes below the Fermi momentum.

$^3D_2$ and $^3P_2$-$^3F_2$. The missing partial waves ($L>2$) do not affect significantly the cross section in the energy domain we are interested in ($E_LAB<400$ MeV). The G matrix is self-consistently calculated along with the mean field by iteratively solving the Bethe-Goldstone equation. The continuous choice is adopted for the mean field. We call this kind of calculation a U_2 calculation while a U_1 calculation corresponds to the BHF approximation. Let us first look at the on-energy-shell G matrix. Its diagonal matrix elements at zero total momentum are the only ones required in our approximation for evaluating the NN cross section (for details see [6]). From the Bethe-Goldstone equation

$$G(q)=V_{NN}(q)+\sum_{q'} V_{NN}(q')\frac{Q(q')}{\omega_q-\omega_{q'}}G(q'), \qquad (3)$$

one can easily realize that the large cancellation occurring in the energy denominator leads to a relatively weak dependence of the G matrix on the mean field. Figure 1 displays the diagonal elements of the $^1S_0$ component of the G matrix as a function of the momentum for the baryonic density $\rho=0.1$ fm^{-3} both in the BHF and extended BHF approximations. This component is dominant in the low-energy scattering between identical particles, i.e., in neutron-neutron or proton-proton scattering. The effect of including U_2 is almost negligible both in the real and imaginary parts of the G matrix. We observe that the imaginary part of the G matrix vanishes below k_F. For rather low nucleonic densities (like the one of Fig. 1) the real part exhibits a spike at $k=k_F$ which is reminiscent of the singularity due to the pairing state of two particles in the channel $^1S_0$. The occurrence of this pairing singularity and its effect on HIC has been discussed in [3,9]. For higher values of the density the similarity between the two G matrices is much closer and the pairing singularity tends to disappear (the threshold for $^1S_0$ pairing has been calculated to be about the saturation density [10]). In Fig. 2 the same comparison is made for the $^3S_1$ component of the G matrix of the coupled channels $^3S_1$-$^3D_1$, which is the most relevant for the low-energy scattering between nonidentical particles, i.e., neutron-proton scattering. In this case, the difference between the two calculations (U_1 and U_1+U_2) amounts to 20%, at most, for the real part of the G matrix, but it is again negligible for the imaginary part. For this component the singularity is much more pronounced because of a larger pairing in the coupled channels $^3S_1$-$^3D_1$ [11].

At variance with the G matrix, the effective mass, which is given by

$$\frac{m}{m*}=1+\frac{m}{\hbar^2k_F}\frac{dU}{dk_F}, \qquad (4)$$

is strongly affected by the ground state correlations. As can be seen in Fig. 3, these correlations add a repulsive component to the mean field with the effect of reducing the depth of the mean field. In addition, the typical wiggle around the Fermi momentum is much more pronounced. The global ef-

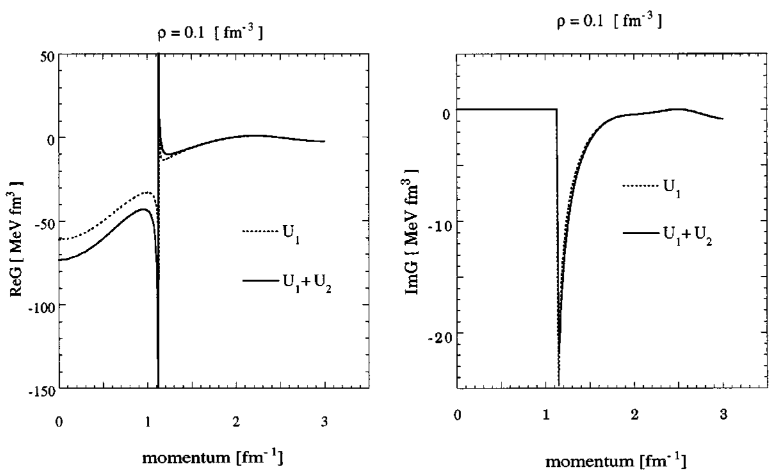

FIG. 2. Real and imaginary parts of the $^3S_1$ component of the $^3S_1$-$^3D_1$ channel of the G matrix with (full lines) and without (dashed lines) the correlation contribution U_2 to the mean field for nuclear density $\rho=0.1$ fm^{-3}.

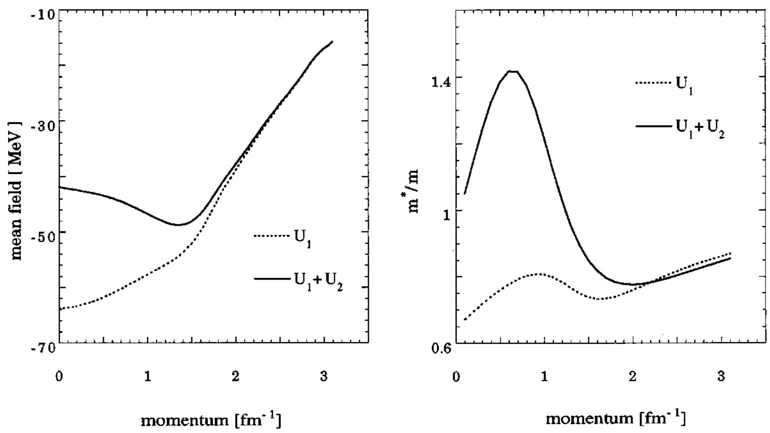

FIG. 3. Nucleon mean field and effective mass with (full lines) and without (dashed lines) the correlation contribution U_2 to the mean field for nuclear density $\rho = 0.1\ \mathrm{fm}^{-3}$.

fect is to enhance the effective mass to values even higher than the bare mass in a momentum range typical of low-energy *NN* collisions. The drastic change of m/m^* going from the first to the second order in the hole-line expansion raises the problem of the possible convergence of this quantity, which deserves further investigation as such. The effective mass enters the expression of the cross section through the initial and final level density. Thus the cross section is very sensitive to the effective mass, whereas we saw that the G matrix is not so much affected by the inclusion of U_2. In conclusion we may predict the *NN* cross section to be very sensitive to the g.s. correlations.

The G matrix is the scattering amplitude between two particles colliding in the nuclear medium. The medium affects the scattering cross section through both the Pauli blocking, which prevents the two particles from scattering into occupied states, and the mean field due to the surrounding nucleons, which they feel during their free motion . The density of states is related to the self-energy in a given approximation and eventually depends upon the effective mass m^*. In this paper, the calculations of the G matrix are restricted only to total momentum $\vec{P} = 0$ so that the laboratory frame coincides with the center of mass frame. The pairing singularity, occurring in the G matrix at $P = 0$, has been washed out for the reason that even a small amount of total momentum is enough to make the gap function quite negligible. In heavy-ion collisions, to which the in-medium cross section is mainly addressed, the colliding nucleons have mostly a finite total momentum. In the c.m. frame, the elastic differential cross section for an unpolarized neutron-proton system is given by

$$\sigma(\theta) = \frac{m^{*2}}{4\pi^2\hbar^4} \sum_{SS_zS_z'} |G^S_{S_zS_z'}(\theta)|^2, \tag{5}$$

where θ is the scattering angle and the E dependence is understood. G^S is obtained summing up all the on-shell $G^{SJT}_{L'L}$ partial wave components corresponding to the same spin S (for more details see Ref. [2]). In the case of collision between identical particles, i.e., neutron-neutron or proton-proton, the antisymmetrization requirement amounts to a linear combination of the $G(\theta)$ and $G(\pi-\theta)$ as follows:

$$\sigma(\theta) = \frac{m^{*2}}{16\pi^2\hbar^4} \sum_{SS_zS_z'} |G^S_{S_zS_z'}(\theta) + (-1)^S G^S_{S_zS_z'}(\pi-\theta)|^2, \tag{6}$$

where only the $T=1$ partial waves are included. Equation (4) can be analytically integrated over to give the total cross section for neutron-proton scattering:

$$\sigma_{\mathrm{tot}}(E) = \frac{m^{*2}}{4\pi^2\hbar^4} \sum_{SJT} \sum_{L'L} \frac{(2J+1)}{4\pi} |G^{JST}_{L'L}|^2. \tag{7}$$

An analogous expression is obtained for the total cross section of scattering between identical particles by integrating Eq. (5).

The cross section depends on the energy of the two colliding nucleons, or equivalently, on the energy E of one nucleon in the frame where the other one is at rest, and on the density of the surrounding medium that has been supposed to be completely equilibrated. In our calculations, the energy range extends up to $E = 300$ MeV, where the medium effects turn out to be the most sizable. Only two values of the

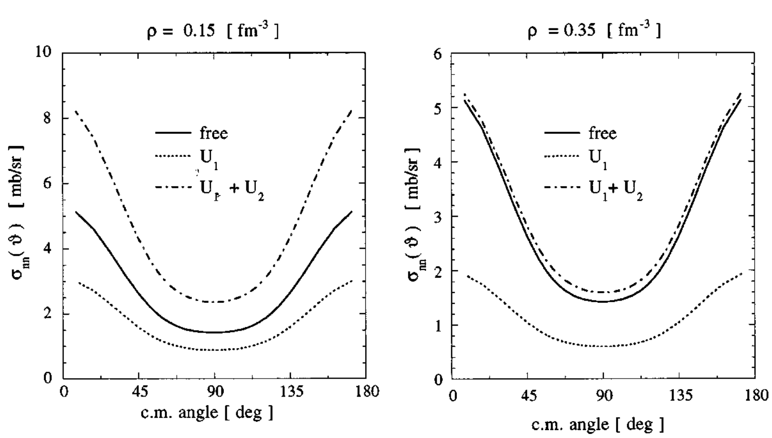

FIG. 4. Differential neutron-neutron cross section σ_{nn} with (dot-dashed line) and without (dotted line) the correlation contribution U_2 to the mean field for scattering energy $E = 100$ MeV. The free space cross section (solid line) is shown for comparison.

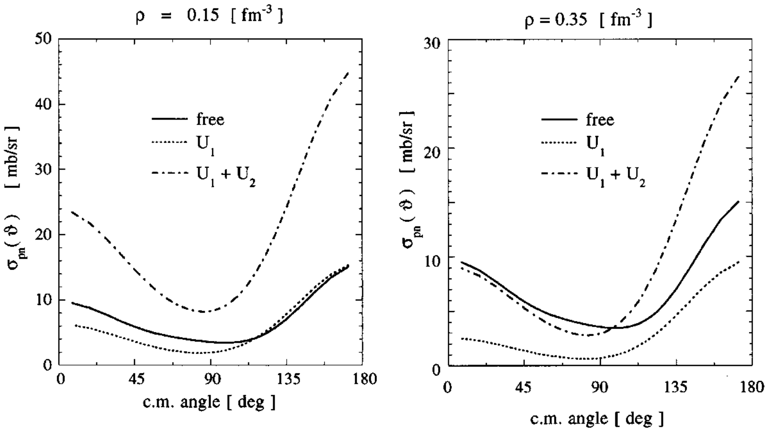

FIG. 5. Differential proton-neutron c.m. cross section σ_{pn} with (dot-dashed line) and without (dotted line) the correlation contribution U_2 to the mean field for scattering energy $E=100$ MeV. The free space cross section (solid line) is shown for comparison.

nuclear medium density, $\rho=0.15$ and 0.35 fm^{-3}, have been selected, at least for the differential cross section. These are typical values reached by the density of the compound system formed in an HIC during the expanding and compressing phase, respectively. In Fig. 4 the differential cross section for scattering between two identical nucleons, i.e., *p-p* or *n-n* scattering, is displayed for the energy $E=100$ MeV. In the case of the *p-p* scattering the Coulomb repulsion has been neglected. The free-space cross section is plotted in comparison with the in-medium cross section with correlations (U_2 calculation) and without correlations (U_1 calculation). The U_1 calculation exhibits the predictable reduction due to the attractive effects of the nucleonic medium in agreement with other calculations based on different approaches (relativistic Dirac-Brueckner theory [4] and nonrelativistic Brueckner [2] and Green's function approaches [3]). The effect of correlations (U_2 calculation) is a repulsion so strong as to increase the in-medium cross section even above the free-space cross section. At higher density, there is an almost perfect compensation between the attractive and repulsive effect of the nuclear medium so that $\sigma(\theta)\approx\sigma_{\text{free}}(\theta)$. At low energy, the cross section is mainly dominated by the *S* wave but other channels are also present giving rise to a slight anisotropy. In the neutron-proton differential cross section at the same energy, showed in Fig. 5, the *S* wave ($T=1$) is still present but now the coupled channels *S-D* ($T=0$) gives the dominant contribution. The medium effect in a U_2 calculation is once again a strong enhancement of $\sigma(\theta)$ in comparison with a U_1-calculation. It is much stronger than in the previous case due to the presence of the *S-D* wave which contains the largest part of the tensor force. Comparing it with the free scattering, we observe no medium effects in the forward directions ($\theta\leqslant\pi/2$), but larger and larger deviations are found at backwards angles. Whereas $\sigma(\theta)$ in U_1 calculations is still comparable with the results of Ref. [3], the effect of the correlations is to make the anisotropy more remarkable: the forward $\sigma(\theta)$ is two times $\sigma_{\text{free}}(\theta)$ but the backward value is four times at the saturation density of the medium. At higher density only the backward $\sigma(\theta)$ stays above $\sigma_{\text{free}}(\theta)$.

Last, we discuss the total cross section, Eq. (7), which provides us with an overall picture of the density and the energy dependence of *NN* scattering in the nuclear medium . Figure 6 displays $\sigma_{\text{tot}}(E)$ for proton-proton or neutron-neutron collisions (left-hand side) and for neutron-proton collisions (right-hand side) for two densities. From a U_1 calculation the expected suppression is found in agreement with previous microscopic calculations [3,4]. It is easily interpreted as being due to the attractive action of the surrounding nucleonic medium on two colliding nucleons. Including U_2

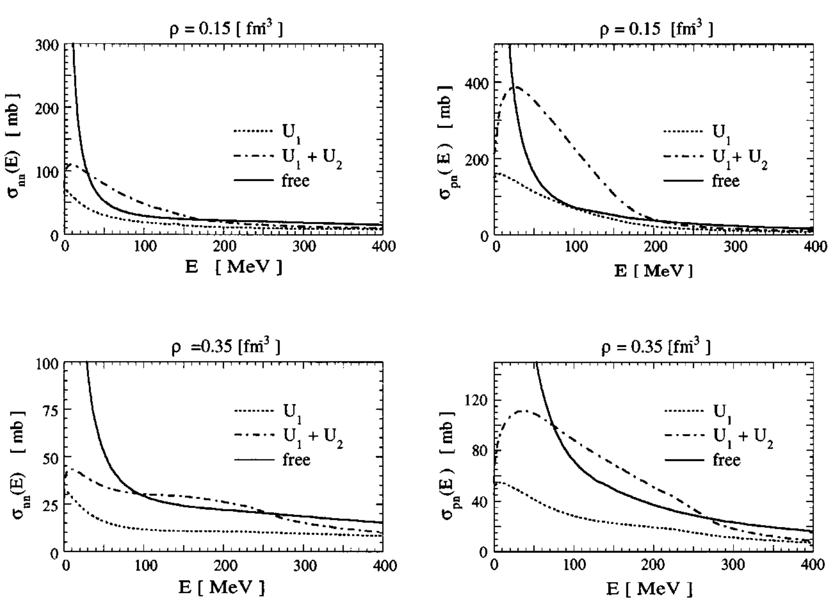

FIG. 6. Total neutron-neutron (σ_{nn}) and proton-neutron cross section (σ_{pn}) with (dot-dashed line) and without (dotted line) the correlation contribution U_2 to the mean field. The free space cross section (solid line) is shown for comparison. Please notice the different scales in the *y* axis.

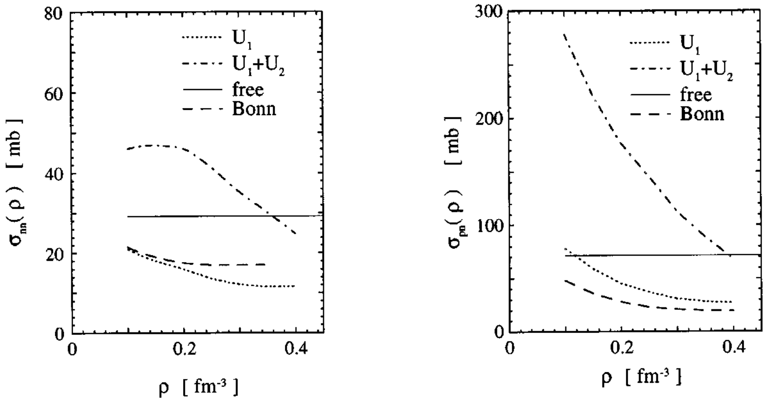

FIG. 7. Total neutron-neutron (σ_{nn}) and proton-neutron cross section (σ_{pn}) with (dot-dashed line) and without (dotted line) the correlation contribution U_2 to the mean field as a function of the nuclear density for an incoming energy of 100 MeV in the laboratory frame. The free space cross section (solid horizontal line) and the one reported in Ref. [4] are shown for comparison.

implies a strong enhancement of $\sigma_{\mathrm{tot}}(E)$ due to the reduction of attractive action of the effective interaction, i.e., the on-shell G matrix, but also to the renormalization of the nucleon mass. The strong reduction of the S-D component due to the ground state correlations makes these effects much more sizable in the cross section between nonidentical particles. As a consequence, the in-medium $\sigma_{\mathrm{tot}}(E)$ exceeds the free-space cross section in an energy domain whose extension depends on the nucleonic density. In Fig. 7, $\sigma_{\mathrm{tot}}(E)$ has been plotted versus the nuclear matter density at the fixed energy, $E=100$ MeV in the laboratory frame. The horizontal line represents the free-space cross section. The presence of g.s. correlations enhances the cross section in the domain of densities up to two times the saturation value. At higher density $\sigma_{\mathrm{tot}}(E)$ approaches the free scattering value. In fact, at high enough density, the attractive nature of the BHF potential balances the repulsive effect of the g.s. correlations and then σ_{tot} resemble the free-space cross section. In Fig. 7, the results of Ref. [4] are also reported. The deviation from our U_1 calculations is within 50% and even less at high density, but it is much larger from our U_2 calculations. It is worth noticing that the cross section obtained from a U_1 calculation nicely overlaps the Dirac-Brueckner value. This result supports the supposition that the difference from BHF calculations is not to be attributed to relativistic effects, as claimed in Ref. [4], but is due to the different content of tensor force of the Bonn potential with respect to the Paris potential.

The influence of the surrounding medium on the NN scattering cross section has been studied within the BHF theory extended to include the ground state correlations. This is in keeping with the influence of the nucleon mean field that this approach is able to calculate self-consistently along with the in-medium scattering amplitude, i.e., on-shell G matrix, at the same level of approximation. These are the two properties of the EOS to be incorporated into kinetic equations in order to describe the one-body dynamics of HIC. Skipping any discussion on the effects of the mean field given elsewhere [1], from the previous results we may predict important consequences on HIC mainly in the low-energy domain. As an effect of the strong enhancement of the in-medium cross section, especially for neutron-proton collisions, the rate of collisions would strongly increase as long as the density is not too high. This occurs in the early stage as well as in the latest steps of the scattering process. In the first case, pre-equilibrium processes and also the nuclear stopping power would be affected. In turn, the prediction on the balance energy, which results from a delicate competition between attractive and repulsive scatterings, could remarkably diverge from previous calculations. In the second case, all processes related to the freeze-out could be influenced by the presumable dilatation of the freeze-out time due to the fact the collision rate is still remarkable at low density according to the present predictions.

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