Hanbury-Brown–Twiss analysis of anisotropic transverse flow

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The effects of anisotropic transverse collective flow on the Hanbury-Brown–Twiss (HBT) correlation function is studied. There exist three different physics contributions related to flow which affect the correlation function: anisotropic source shape, anisotropic space-momentum correlations in pion emission, and the effects related to the HBT measurement of the size of a moving source in different reference frames. Resolution of these contributions experimentally can lead to a detailed understanding of both collective flow in nucleus-nucleus collisions and the HBT technique itself. A method is presented which permits the derivation of model independent relations between the radius of a source measured in a frame in which it is moving and in its rest frame.

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I. INTRODUCTION

The discovery of anisotropic transverse flow in nucleus-nucleus collisions at BNL AGS energies [1] implies that some previous results should be reevaluated taking into account the effects of flow. One set of results in this category is the measurements of collision volumes using the Hanbury-Brown–Twiss (HBT) technique. In this paper we discuss how HBT results are affected by flow. In addition we show that the HBT study of nucleus-nucleus collisions can provide valuable information on the understanding of flow itself.

By anisotropic transverse flow we generally mean directed flow. We restrict ourselves to the consideration of symmetric collisions of identical nuclei. In this analysis we use the following geometrical definitions. In the transverse plane we define as ''in$^+$'' the direction of transverse flow, ''in$^-$'' is defined as the opposite direction, and ''perp'' is the direction perpendicular to the ''in'' direction. We use the coordinate system where the x axis coincides with ''in$^+$'' and the z axis coincides with the beam direction. The reaction plane is then the plane defined by the ''in'' direction and the beam (the x-z plane). In this paper we consider only pion correlations (unless stated explicitly otherwise), and we call a pion source simply a source.

We discuss the problem essentially on a qualitative level. Our goal is to find the physics which affects the HBT measurements, not to generate a complete set of formulas to describe the general case. In the discussion we keep in mind a heuristic picture of pion production and try to understand how different features of the production affect the HBT correlation function. In this very simple picture pions are produced from two sources different in origin. ''Direct'' pions are mostly produced in deep inelastic nucleon-nucleon collisions in the zone where the nuclei overlap. This source can be relatively small and is located close to the center of the line joining the centers of the colliding nuclei. The second source of pions in our picture is resonance (mostly delta) decays. This source size should be close to the freezeout nucleon radius. In this naive picture it is clear that, for example, the ''geometry'' of the source of pions of different rapidities (or different p_t, or different orientation of pion momenta with respect to the reaction plane, etc.) can be very different. The effective source responsible for the emission of pions with a given rapidity can have nonzero longitudinal and transverse velocity. Note that we use the picture only to illustrate the sensitivity of the HBT function to different features of the source; one should remember that the real collision picture is much more complex.

In general, the difference in the effective sources for pions in different rapidity (p_t, etc.) regions can be treated mathematically as a correlation between the momentum and space position of pion emission. (Following many others we consider pion emission semiclassically, and do not discuss the space-time quantum mechanical uncertainty.) For example, if low p_t pions are produced mostly in the totally expanded stage of the collision, the extracted source size could be larger than that evaluated by using high p_t pions, emitted from the hot stage. In Sec. III we discuss how these correlations appear in the expression for the correlation function.

II. GEOMETRICAL SHAPE OF PION SOURCE

We start with the question of whether it is possible to observe different source sizes looking at the emitting object from different directions. The answer is definitely ''yes,'' for several reasons. One of them is simply the anisotropic source shape; due to transverse directed flow the effective source could be extended in the reaction plane. Directly produced pions are emitted mostly from the ''center'' (of the transverse plane), while pions from Δ (which undergo the collective motion to a greater extent) decays could have an ''off-center'' origin. Thus the source size appears to be different if measured from ''in'' or ''perp'' directions.

We use RQMD (relativistic quantum molecular dynamics) 1.08 [2] generated events to evaluate the approximate magnitude of the effect. We study production point distributions of pions with rapidity (in the laboratory frame) around $y_{\mathrm{lab}}\approx 3$ which is close to beam rapidity in Au+Au collisions for a projectile energy of 11.4 GeV/nucleon. We select pions which are emitted in the transverse plane in the $+x$, $-x$,

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TABLE I. The first and second moments and correlation coefficients of spatial and temporal distribution of charged pions in Au+Au collisions as seen from different directions. Pions are required to lie in the rapidity interval $2.7<y_{\rm lab}<3.2$ and have transverse momentum $0.14<p_t<0.25$ GeV. The impact parameter $3.0<b<6.0$ fm. Units for the first moments are fm and for the second moments fm^2.

	$+x$	$-x$	$+y$	$-y$		$+x$	$-x$	$+y$	$-y$
$\langle x\rangle$	4.9	-0.5	1.6	1.7	$\langle xy\rangle-\langle x\rangle\langle y\rangle$	0.2	0.0	0.1	-0.7
$\langle y\rangle$	-0.2	-0.2	3.0	-3.0	$\langle xz\rangle-\langle x\rangle\langle z\rangle$	9.3	1.6	7.3	9.1
$\langle z\rangle$	11.2	9.2	9.9	9.3	$\langle xt\rangle-\langle x\rangle\langle t\rangle$	12.2	-7.1	5.3	9.1
$\langle t\rangle$	19.7	16.7	18.2	17.3	$\langle yz\rangle-\langle y\rangle\langle z\rangle$	-2.4	-2.1	4.7	-2.5
$\langle x^2\rangle-\langle x\rangle^2$	12.8	17.4	17.6	15.8	$\langle yt\rangle-\langle y\rangle\langle t\rangle$	-1.6	-3.0	9.3	-8.6
$\langle y^2\rangle-\langle y\rangle^2$	19.1	16.6	16.9	17.1	$\langle zt\rangle-\langle z\rangle\langle t\rangle$	72.8	68.4	74.7	72.7
$\langle z^2\rangle-\langle z\rangle^2$	69.4	61.1	68.7	66.2					
$\langle t^2\rangle-\langle t\rangle^2$	92.3	92.2	98.0	99.1					

and y directions, and analyze their distributions in x, y, z, and t. Pions are considered as being emitted in $+x$ direction if their transverse momentum components satisfy the condition $0<|p_y|/p_x<0.5$. All calculations are performed in the center of mass (Au+Au) frame. To suppress fluctuations related to very rare cases of decays of very long-lived resonances, we consider only pions produced within the first 50 fm/c after the collision. The results are summarized in Table I.

From the first moments, one can see that the center of the observed pion source is shifted from the center line in the direction of flow. The variances of the distributions can be considered as squares of the effective source sizes in different directions. These are the sizes which one would measure if the pions were to carry the information on their production points (this is not the case in reality). Due to the collision symmetry with respect to the reaction plane some coefficients in Table I (such as $\langle y\rangle$ for the $+x$ and $-x$ cases) are expected to be zero and others (such as $\langle x\rangle$ for $+y$ and $-y$ cases) are expected to be equal. The values actually obtained in the calculation can be used to estimate roughly the uncertainty in the results.

The effective source velocities for each of the cases of $+x$, $-x$, $+y$, and $-y$ directions of pion emission can be estimated from Table I using the formula

$$v_i=\frac{\langle r_i t\rangle-\langle r_i\rangle\langle t\rangle}{\langle t^2\rangle-\langle t\rangle^2}, \tag{1}$$

which gives, for example, $v_x^{\{+x\}}\approx 0.13$, $v_x^{\{-x\}}\approx -0.08$, $v_y^{\{+y\}}\approx 0.10$, and for all direction of pion emission $v_z\approx 0.76$. Note the difference between the magnitudes of $v_x^{\{+x\}}$ and $v_x^{\{-x\}}$ which is just the difference in transverse flow velocities due to anisotropic flow. In collisions at this energy, flow is mostly carried by nucleons. Pions are involved in flow mainly through baryon resonance decays. It is important for our analysis that the pions from Δ decays almost do not "remember" the flow, since the change in pion momenta due to flow is small. The physical reason is the same as that for the low p_t enhancement. When the resonance decays, the products share the resonance momentum in proportion to the ratio of their masses. The pion from $\Delta\rightarrow\pi p$ decay carries only about 1/7 of the momentum of the Δ. Thus we can expect for pions only about an extra 10 MeV/c of transverse momentum in the direction of flow, if one takes reasonable values for baryon flow [6]. Analysis of experimental data [7] yields an average value of $v_t\approx 0.3$; for the longitudinal component it gives $v_{\rm long}\approx 0.5$. These values could be different from our estimate due to the particular region of pion phase space considered.

The HBT analysis in principle permits to investigate the source size in many dimensions, studying the dependence of the correlation function on different components of the relative pion pair momentum. Usually the so-called "long-side-out" coordinate system is used for such an analysis. We remind the reader that the "long-side-out" coordinate system is defined in the following way (see Fig. 1). "Long" is the direction of the beam, "out" is the direction of the pion pair transverse momentum, "side" is the direction in the transverse plane perpendicular to the "out" direction. In the case of anisotropic flow the picture becomes more complicated; one must also take into account the direction of flow. Consequently, we denote, for example, by "in$^+$-out" the "out" direction for the case when the pion pair momentum points in the direction of flow; and "in$^-$-side" source size would

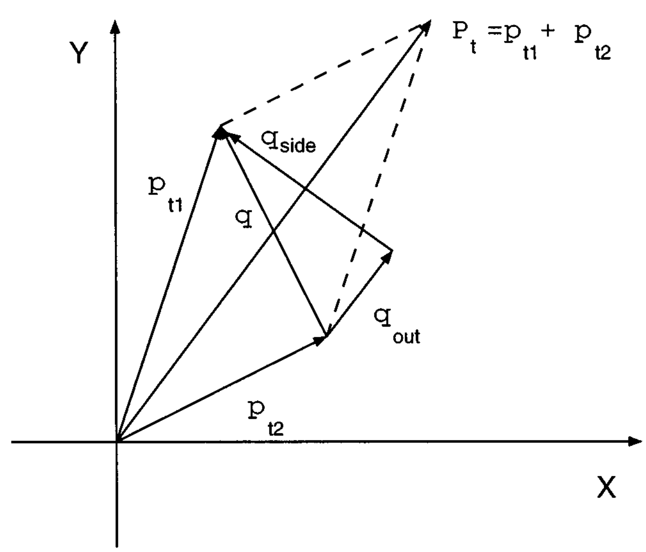

FIG. 1. The definition of transverse momenta in the "long-side-out" coordinate system.

mean the mean size of the source in the ''side'' direction as it appears for pions emitted in the direction opposite to flow.

If one considers something like ''in-out'' and ''in-side'' source sizes, many interesting possibilities appear. One can see from Table I that, for example, the $+x$ and $-x$ effective sizes are quite different. One of the reasons for this is shadowing. Direct pions are produced most often in the region where colliding nuclei overlap. Let us look at the collision from the flow (''in$^+$'') direction, where one expects the spectators and most of the nucleons of the projectile. Certainly these nucleons distort the real image of the pion source. From this direction one can see mostly the pions from the nucleon fireball and, consequently, one measures this fireball size. For an observer from the opposite (in the transverse plane) ''in$^-$'' direction the source image is not distorted by shadowing so the picture could be quite different. From this direction one sees simultaneously the hot pion fireball and, spatially separated from it, the nucleon fireball. The ''out'' size is the effective transverse size of the source in the direction of the transverse momentum of the pion pair; ''in$^+$-out'' and ''in$^-$-out'' sizes should be determined by the sizes of the two sources and their separation. This could explain the difference in ''out'' sizes from Table I seen from ''in$^+$'' ($\sqrt{12.8}$ fm) and ''in$^-$'' directions ($\sqrt{17.4}$ fm). ''In$^+$-side'' and ''in$^-$-side'' sizes are the dimensions of the source perpendicular to the direction of the pion momentum as seen from ''in$^+$'' and ''in$^-$'' directions. The difference in ''in$^+$-side'' and ''in$^-$-side'' sizes ($\sqrt{19.1}$ fm and $\sqrt{16.6}$ fm, respectively) is mostly due to the difference in sizes of nucleon and pion fireballs, not to their separation. Another reason for the difference could be the shadowing of the pion source by nucleons in ''in$^+$'' case, which effectively results in a larger observed source size.

If this picture is correct one could study very interesting effects, varying the pseudorapidity of the correlated pair and the relative (azimuthal) angle between the pion pair and the reaction plane. In connection with the arguments made above it is also interesting to study in detail the pion triple differential distributions calculated with respect to the reaction plane. For example, the shadowing discussed above could result in nonzero values of the third harmonic Fourier coefficient of the pion azimuthal distribution. The rapidity (pseudorapidity) dependence, if observed, could provide information on the localization of dense nucleon matter.

It is very interesting to compare the results of pion interferometry with the results of source size measurements by the two-proton correlation technique. In our (oversimplified) picture the proton source is different from the pion source. The effects discussed in the next two sections are also different for the two approaches, which makes the comparison more difficult but also more interesting.

III. HBT FUNCTION AND SPACE-MOMENTUM CORRELATIONS

All of the arguments related to the difference between the ''in$^+$'' and ''in$^-$'' sizes given above can be formulated in terms of the correlation between the pion momentum and the space-time location of its production point. The reason for a separate discussion (in the previous section) is that the space-momentum correlations widely discussed in the literature are generally related only to isotropic transverse flow (any collective movement-phenomena in multiparticle production we call flow in this paper). The subject of our study is anisotropic flow.

Below we rederive the expressions for effective source radii measured using the HBT technique, having in mind that this derivation can be helpful in understanding the flow contribution to the correlation function. Under a few simple assumptions the correlation function can be written in the ''on-shell'' form [3]:

$$C(\mathbf{q},\mathbf{P}) = 1 + \frac{\int d^4x_1\, d^4x_2\, S(x_1,\mathbf{P}/2)S(x_2,\mathbf{P}/2)e^{-iq(x_1-x2)}}{[\int d^4x\, S(x,\mathbf{P}/2)]^2}, \tag{2}$$

where $S(x,\mathbf{p})$ is the source function, $q=p_1-p_2$ is the relative momentum, and $P=p_1+p_2$ is the total momentum of the pair, $P=(E,\mathbf{P})$; p_1 and p_2 are pion four-momenta. By consideration of the exact form of the correlation function one can generate the corrections [4] to the source parameters calculated with Eq. (2).

The mathematics of the space-momentum correlation is very simple; the correlation means that the source function does not factorize:

$$S(x,\mathbf{p}) \neq S_s(x)S_m(\mathbf{p}). \tag{3}$$

A consequence of this is that the values presented in Table I depend on the pion momentum. We come to a trivial conclusion: the interferometry of pions with definite momentum is sensitive to the source which emits pions with just this momentum. The measured source sizes are not the sizes of the whole source but only of the effective region which emits the proper pions (the lengths of homogeneity [8]).

To give an idea of how the correlation function depends on the values discussed in the previous section and presented in Table I, we derive below the expressions for the source radii. We consider the correlation function with a fixed value of $\mathbf{P}$. In this case the correlation function depends only on three variables. From the fact that the pions are on the mass shell it follows that

$$q_0 = \mathbf{qP}/E = \mathbf{qV}, \tag{4}$$

where $\mathbf{V}$ is the velocity of the pion pair. Then, defining $r=x_1-x_2=(t,\mathbf{r})$ the correlation function can be rewritten in the form

$$C(\mathbf{q},\mathbf{P}) = 1 + \frac{\int d^4x_1\, d^4x_2\, S(x_1,\mathbf{P}/2)S(x_2,\mathbf{P}/2)e^{-i\mathbf{q}(\mathbf{V}t-\mathbf{r})}}{[\int d^4x\, S(x,\mathbf{P}/2)]^2}. \tag{5}$$

The effective ''mean square source size'' can be defined [5] through the second derivative of the correlation function with respect to the component of $\mathbf{q}$ in the corresponding direction. There are diagonal

$$R_i^2 = -1/2(\partial^2 C(\mathbf{q})/\partial^2 q_i)|_{\mathbf{q}=0} = \langle (V_i t - r_i)^2\rangle/2, \tag{6}$$

and cross terms [4],

$$R_{ij}^2 = -1/2(\partial^2 C(\mathbf{q})/\partial q_i \partial q_j)|_{\mathbf{q}=0} = \langle (V_i t - r_i)(V_j t - r_j)\rangle/2, \tag{7}$$

TABLE II. Matrix of source radii (in fm^2) found using the HBT correlation function measured from the different directions with respect to flow in the center of mass system of the colliding nuclei. The input parameters are taken from Table I.

	R_x^2	R_y^2	R_z^2	R_{xy}^2	R_{xz}^2	R_{yz}^2
$x+$	16.9	19.1	12.6	0.8	-1.5	-1.0
$x-$	25.3	16.6	11.8	-1.1	3.3	1.2
$y+$	17.6	24.0	12.9	-1.9	2.7	0.6
$y-$	15.8	24.8	14.7	2.8	1.2	-0.2

where we have introduced the notation

$$\langle f\rangle=\frac{\int d^4x_1\, d^4x_2\, fS(x_1,\mathbf{P}/2)S(x_2,\mathbf{P}/2)}{[\int d^4x\, S(x,\mathbf{P}/2)]^2}. \tag{8}$$

The interpretation of the expressions for the radii is straightforward: R^2 is the mean square of the distance between pions at the moment the second pion being produced (see also the discussion of this question in [9]).

In the ''long-side-out'' coordinate system

$$\mathbf{q}(\mathbf{V}t-\mathbf{r})=-q_{\rm side}r_{\rm side}-q_{\rm out}(V_{\rm out}t-r_{\rm out}) -q_{\rm long}(V_{\rm long}t-r_{\rm long}), \tag{9}$$

and it follows that

$$2R^2_{\rm side}=\langle r^2_{\rm side}\rangle, \tag{10}$$

$$2R^2_{\rm out}=\langle(r_{\rm out}-V_{\rm out}t)^2\rangle=\langle r^2_{\rm out}\rangle-2\langle r_{\rm out}V_{\rm out}t\rangle+\langle(V_{\rm out}t)^2\rangle, \tag{11}$$

$$2R^2_{\rm long}=\langle(r_{\rm long}-V_{\rm long}t)^2\rangle =\langle r^2_{\rm long}\rangle-2\langle r_{\rm long}V_{\rm long}t\rangle+\langle(V_{\rm long}t)^2\rangle, \tag{12}$$

$$2R^2_{\rm out,long}=\langle(r_{\rm long}-V_{\rm long}t)(r_{\rm out}-V_{\rm out}t)\rangle. \tag{13}$$

In the absence of flow

$$\langle r_{\rm side}\rangle=0, \tag{14}$$

due to azimuthal symmetry of the collision [4]. For the same reason $R^2_{\rm side,long}=R^2_{\rm side,out}=0$. For the case of nonzero flow these results are valid only for ''in'' sizes due to the symmetry of the collision with respect to the reaction plane. For the case of ''perp'' sizes all of the terms are in the general case nonzero.

The expressions for the radii can be rewritten through the moments of one-particle production point spatial distributions. For example,

$$\begin{aligned}R_x^2&=\langle(x-V_xt)^2\rangle/2\\&=\langle[(x_1-x_2)-V_x(t_1-t_2)]^2\rangle/2\\&=\langle(x_1-\langle x_1\rangle)^2\rangle-2V_x(\langle x_1t_1\rangle-\langle x_1\rangle\langle t_1\rangle)\\&\quad+V_x^2\langle(t_1-\langle t_1\rangle)^2\rangle,\end{aligned} \tag{15}$$

and therefore the radii can be estimated using Table I. The results of radii calculation are presented in Table II taking into account that in this case the mean transverse and longitudinal velocities of pions are approximately 0.38 and 0.87, respectively (in the center of mass of colliding nuclei).

Due to the relatively small values of pion pair transverse velocities the radii R_x^2 and R_y^2 are not very different from the values of $\langle x^2\rangle-\langle x\rangle^2$ and $\langle y^2\rangle-\langle y\rangle^2$ in Table II. They still resemble the features of the true source geometry. Due to the sizable longitudinal pion velocity the values of R_z^2 are very different from the values of $\langle z^2\rangle-\langle z\rangle^2$. Note also the possibility for nonzero values of cross terms in the radii matrix. The relatively large value of R_{xz}^2 in the $-x$ case could indicate spatial separation of the pion and nucleon fireballs.

IV. HBT MEASUREMENTS OF A MOVING SOURCE

The values given in Table II are the parameters of the HBT correlation function as measured in the center of mass frame of the colliding nuclei. As was shown in Sec. II, the pion source in this frame has nonzero collective longitudinal and transverse velocities. Here we study how this motion distorts the HBT measurements of the source sizes.

In some models it is possible to perform all necessary calculations in covariant form by introducing the four-velocity of the source. However it is useful to derive model independent formulas by performing Lorentz transformations between different systems. Our goal is to establish the relationship between the source size in its rest frame and the HBT correlation function measured in the frame where the source is moving. The correlation function by definition is the ratio of the two-particle invariant density and the product of the invariant one-particle densities; it is invariant under Lorentz transformations. Using this property one can write

$$C(\mathbf{q},\mathbf{P})=C(\mathbf{q}',\mathbf{P}') =1+\frac{\int d^4x_1'd^4x_2'S(x_1',\mathbf{P}'/2)S(x_2',\mathbf{P}'/2)e^{-iq'(x_1'-x_2')}}{[\int d^4x'S(x',\mathbf{P}'/2)]^2}, \tag{16}$$

where the integrals are evaluated in the source rest frame, and the prime denotes the coordinate and momentum values in this system.

Let us assume that our source moves with velocity $\mathbf{v}$. We start with the case $v^2\ll1$. In this case the momentum transformation equations are very simple:

$$q_0'=q_0-\mathbf{vq}=\mathbf{q}(\mathbf{V}-\mathbf{v}), \tag{17}$$

$$\mathbf{q}'=\mathbf{q}-\mathbf{v}q_0=\mathbf{q}-\mathbf{v}(\mathbf{qV}), \tag{18}$$

$$\mathbf{P}'=\mathbf{P}-\mathbf{v}E. \tag{19}$$

Then

$$q'(x_1'-x_2')=q'x'=[\mathbf{q}(\mathbf{V}-\mathbf{v})]t'-[\mathbf{q}-\mathbf{v}(\mathbf{qV})]\mathbf{r}'. \tag{20}$$

One evaluates the second derivatives of expression (16) with respect to q_i to obtain the radii. For example,

$$R_x^2 = -1/2(\partial^2 C(\mathbf{q})/\partial^2 q_x)|_{\mathbf{q}=0} = \langle [t'(V_x - v_x) - x' + V_x(\mathbf{r}'\mathbf{v})]^2 \rangle'/2, \tag{21}$$

$$R_{xy}^2 = \langle [t'(V_x - v_x) - x' + V_x(\mathbf{r}'\mathbf{v})] \times [t'(V_y - v_y) - y' + V_y(\mathbf{r}'\mathbf{v})] \rangle'/2. \tag{22}$$

The other radii can be computed in an analogous way. The prime on the bracket ($\langle\ \rangle'$) means that the mean value is evaluated using $S(x_1, \mathbf{P}'/2)$ instead of $S(x_1, \mathbf{P}/2)$. In fact the source function depends on $\mathbf{P}$ rather slowly and for an estimate of the radii one can neglect the difference for the case of small source velocity, which we consider here. Note that Eqs. (22) and (23) show explicitly the dependence of the correlation function radii parameters on source velocity $\mathbf{v}$.

The physical interpretation of Eqs. (22) and (23) is the same as for the case of the source being at rest, as can be shown by performing an approximate Lorentz transformation. The HBT correlation function measures the distance between the pions at the moment of production of the second pion, which is

$$t\mathbf{V} - \mathbf{r} = (t' + \mathbf{v}\mathbf{r}')\mathbf{V} - (\mathbf{r}' + \mathbf{v}t') = t'(\mathbf{V} - \mathbf{v}) - \mathbf{r}' + \mathbf{V}(\mathbf{r}'\mathbf{v}), \tag{23}$$

as calculated to the first order of v.

For the case of $v \ll 1$ the values of the second moments of the spatial and temporal distributions are very close to each other in both the systems (note that in the source rest frame the quantities $\langle t'\mathbf{r}' \rangle$ are zero). Taking this into account and using the values from Table I one can estimate the distortion of the correlation function due to transverse directed flow by considering Lorentz boosts only in the transverse plane. We find, for example, that about half of the difference between R_x^2 in $+x$ and $-x$ cases is due to the transverse flow, with the remaining part attributable to the difference in source geometry for the two cases.

We consider the case of arbitrary v and use the exact Lorentz transformations for one particular case when the flow velocity and pair velocity are directed along the x axis. In this case,

$$q_0' = \frac{q_0 - v q_x}{\sqrt{1-v^2}} = \frac{q_x V - v q_x}{\sqrt{1-v^2}} = q_x \frac{V - v}{\sqrt{1-v^2}}, \tag{24}$$

$$q_x' = \frac{q_x - v q_0}{\sqrt{1-v^2}} = \frac{q_x - v V q_x}{\sqrt{1-v^2}} = q_x \frac{1 - vV}{\sqrt{1-v^2}}, \tag{25}$$

where V is the pion pair velocity, and v is the flow velocity. From this formula and expressions for q_0' and q_x' one can derive that

$$R_x^2 = \left\langle \left(\frac{t'(V-v)}{\sqrt{1-v^2}} - \frac{x'(1-vV)}{\sqrt{1-v^2}} \right)^2 \right\rangle' = \frac{\langle [t'(V-v) - x'(1-vV)]^2 \rangle'}{1-v^2}. \tag{26}$$

For a simpler case of instantaneous freezeout ($t=0$) and of a Gaussian source the analogous formula was derived earlier in [10]. Formula (26) is more general; it gives the dependence of HBT radii on the source velocity independent of any model. For anisotropic transverse flow the flow velocity depends on the orientation of pion momenta with respect to the reaction plane. This results in different apparent radii for the "in$^+$," "in$^-$," and "perp" cases even if the real source geometry is the same in all cases.

V. CONCLUSION

Anisotropic transverse flow produces anisotropy both in the pion source geometry and in the space-momentum correlation. It also affects the interferometry size measurements because of nonzero effective velocity of the source. These three are the main phenomena responsible for the dependence of the HBT correlation function on the orientation of pion pair momentum with respect to the reaction plane. A detailed study of the experimental data and model predictions is necessary to disentangle all three effects, but such studies could be beneficial to our understanding of flow in nucleus-nucleus collisions and of the HBT technique itself.

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