

## Selection rule for fragmentations of doorway $s$ -hole states in light nuclei

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A selection rule for fragmentations of doorway  $s$ -hole states in light nuclei is given. In the two-body fragmentation process of the doorway  $s$ -hole states in light nuclei, smaller fragments than the  $\alpha$  particle ( $p$ ,  $n$ ,  $d$ ,  $t$ , and  ${}^3\text{He}$ ) are allowed, while the fragments such as the  $\alpha$  particle and the larger particles are forbidden, in spite of the fact that the  $Q$  value for the  $\alpha$  fragment is similar to or even larger than those for the  $p$ ,  $n$ , and  $d$  fragments. The selection rule comes from the spatial symmetry of the doorway  $s$ -hole states. The spectroscopic factors and partial widths for the two-body fragmentations of the  $s$ -hole state of  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$  are calculated to show the selection rule. The calculated escape widths for the  ${}^{11}\text{B}(s\text{-hole})$  and  ${}^{15}\text{N}(s\text{-hole})$  doorway states are in good correspondence with the experimental widths. This may suggest that the following scenario is realized; in light nuclei the fragmentation process of the  $s$ -hole state in the doorway stage is superior or competitive to that in the compound nuclear stage.

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### I. INTRODUCTION

The nuclear deep hole states have been systematically investigated so far with the use of quasifree knockout reactions such as  $(p,2p)$  [1–3] and  $(e,e'p)$  [4,5] and neutron pickup reactions such as  $(p,d)$  [6], etc., on light to heavy nuclear targets in order to clarify single-particle motion in nuclei. The hole spectroscopic information on binding energies and widths for various proton-hole states is deduced from the quasifree knockout data, and is summarized in the famous figure by Jacob and Maris [3]. The systematic observation of various hole binding energies gives direct evidence of the existence of inner orbital structure which is fundamental to the shell model. On the other hand, the  $n(K^-, \pi^-)\Lambda$  strangeness-exchange reaction is another tool to study the nuclear deep hole states. Since the momentum transfer of the  $(K^-, \pi^-)$  reaction is very small ( $q \approx 50$  MeV/c), the substitutional states with a neutron  $s$ -hole and  $\Lambda$ -particle  $s$ -state configuration [ $s_N^{-1}s_\Lambda$ ] as well as a [ $p_N^{-1}p_\Lambda$ ] configuration are strongly populated. With the use of the  $(K^-, \pi^-)$  reaction, Bertini *et al.* investigated the neutron-hole states in light nuclei [7].

A deep hole state is observed as a bump with large width in the excitation spectra of quasifree proton knockout reactions. For example, the proton  $s$ -hole states for  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$ , which are the typical deep hole states in  $p$ -shell nuclei, appear at  $E_x \approx 20$  MeV with  $\Gamma \approx 9$  MeV and  $E_x \approx 30$  MeV with  $\Gamma \approx 14$  MeV, respectively [1–3]. The energy and width give important information on the wave function of a deep hole state. In particular, the partial fragmentation widths which relate to how a deep hole state is fragmented should provide extremely interesting information on the fragmentation mechanism, namely, whether the deep hole state is fragmented statistically or nonstatistically. However, only the total width of the deep hole state has been known experimentally. On the other hand, theoretical studies of the partial fragmentation widths of deep hole state have also scarcely been performed so far. The continuum shell model [8] and

Green function method [9,10] have been applied to describe the experimental excitation spectra for quasifree knockout reactions such as  $(p,2p)$ . However, only the nucleon fragmentation of the final deep hole state was taken into account in their calculations.

Recently, two of the present authors (T.Y. and K.I.) [11] have demonstrated a close relation between hypernuclei and deep hole states. In the strangeness  $S = -1$  world, the strangeness-1 exchange reactions such as  $(K^-, \pi^-)$  and  $(\pi^+, K^+)$  populate strongly hypernuclear states with nucleon-hole hyperon-particle [ $N^{-1}Y$ ] configurations ( $Y = \Lambda$  and  $\Sigma$  particles); for example, hypernuclear states with the configuration of a nuclear-core deep hole ( $s$  hole in light nuclei) coupled with a  $\Lambda$  particle [7]. The core-excited hypernuclear states [ $N^{-1}Y$ ] are fragmented to some stable and/or quasis-table hyperfragments by emitting nucleons and so on, and finally the hyperfragments decay weakly. Then, in the fragmentation process of the core-excited hypernuclear states, the fragmentation mode of the nuclear-core deep hole states plays an important role [11]. On the other hand, in the  $S = -2$  world, a strangeness-2 exchange reaction such as  $(K^-, K^+)$  can produce  $\Xi^-$  hypernuclei as well as  $\Xi^-$  atomic states. The characteristic of  $\Xi^-$  systems is that a  $\Xi^-$  particle in a  $\Xi^-$  system interacts with a proton in a nucleus to be converted to two  $\Lambda$  particles ( $\Xi^- + p \rightarrow \Lambda + \Lambda + 28$  MeV) by a strong interaction. Then, a highly excited hypernuclear state with the nuclear-core proton hole coupled with two  $\Lambda$  particles is produced as an intermediate state, and is then fragmented to a double- $\Lambda$  hypernucleus ( ${}^{A_1+2}_{\Lambda\Lambda}Z_1 + A_2Z_2$ ) or twin- $\Lambda$  hypernucleus ( ${}^{A_1+1}_{\Lambda}Z_1 + {}^{A_2+1}_{\Lambda}Z_2$ ), etc. Since the  $Q$  value of the elementary process  $\Xi^- + p \rightarrow \Lambda + \Lambda$  is nearly equal to the  $s$ -state proton separation energy in light nuclei, the fragmentation of a nuclear-core  $s$ -hole state should play an important role in the fragmentation of  $S = -2$  systems [11]. The above-mentioned two examples show us that study of the fragmentation mechanism of deep hole states, in particular  $s$ -hole states in light nuclei, is needed to develop hypernuclear physics, because the fragmentation dynamics of

TABLE I. Ratios of nuclear radius ( $R$ ) to mean free path ( $\lambda$ ) of  $s$ -hole state.  $k$  and  $\Gamma$  ( $s$  hole) denote the wave number and width of the  $s$ -hole state. For  ${}^6\text{Li}$ ,  ${}^7\text{Li}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ , and  ${}^{28}\text{Si}$ , we use the experimental values of  $\Gamma$  ( $s$  hole) [1–3]. For other nuclei, the theoretical values of  $\Gamma$  ( $s$  hole) are used [12,13]. See the text.

	${}^6\text{Li}$	${}^7\text{Li}$	${}^{12}\text{C}$	${}^{16}\text{O}$	${}^{28}\text{Si}$	${}^{40}\text{Ca}$	${}^{90}\text{Zr}$	${}^{208}\text{Pb}$
$k$ ( $\text{fm}^{-1}$ )	0.91	0.89	0.81	0.78	0.70	0.67	0.58	0.47
$\Gamma$ ( $s$ hole) (MeV)	1.2	6.0	9	14	16	(20)	(24)	(27)
$\lambda$ (fm)	31	6.1	3.7	2.3	1.8	1.4	1.0	0.72
$R$ (fm)	2.56	2.40	2.46	2.73	3.08	3.48	4.25	5.50
$R/\lambda$	0.083	0.39	0.66	1.2	1.7	2.5	4.3	7.6

deep hole states plays an important role in the production of the hyperfragments.

In a previous paper [11] the present authors investigated the characteristic of the fragmentation mode for the  ${}^{11}\text{B}(s\text{-hole})$  [ ${}^{11}\text{C}(s\text{-hole})$ ] state by evaluating the spectroscopic factors for the  ${}^8\text{Be}+t$  and  ${}^7\text{Li}+\alpha$  ( ${}^8\text{Be}+{}^3\text{He}$  and  ${}^7\text{Be}+\alpha$ ) channels. They found a notable selection rule for fragmentation of doorway  $s$ -hole states in light nuclei, namely, in the doorway  $s$ -hole states of  $p$ -shell nuclei, the  $\alpha$ -cluster fragmentation is suppressed strongly, while the  $t$ -( ${}^3\text{He}$ -) cluster fragmentation is allowed, in spite of the fact that the  $Q$  value for the  $\alpha$ -fragmentation channel is larger than that for the  $t$ -( ${}^3\text{He}$ -)fragmentation channel. It is well known that the  $\text{SU}(3)(\lambda\mu)$ -state classification is very good to describe the structure of light nuclei [12,13], since in light nuclei the single-particle potential is given approximately as the harmonic oscillator potential and the effect of the spin-orbit potential is very small. Our selection rule is based on the spatial symmetry  $\text{SU}(3)(\lambda\mu)$  of the doorway  $s$ -hole states. The application of our selection rule to the fragmentation of the core-excited hypernuclei gave good correspondence with the experimental data [11].

The purpose of this paper is to study the spectroscopic factors and partial fragmentation widths for the two-body fragmentation channels of the doorway  $s$ -hole states of  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$ , namely, for the  $p$ -,  $n$ -,  $d$ -,  $t$ -,  ${}^3\text{He}$ -,  $\alpha$ -,  ${}^5\text{He}$ -, and  ${}^6\text{Li}$ -fragmentation channels, etc., and to show the characteristic fragmentation mode of the doorway  $s$ -hole states. The doorway  $s$ -hole states of  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$  are given in terms of the  $\text{SU}(3)(\lambda\mu)$  representation. As a result, we found that our notable selection rule [11] persists in the two-body fragmentation process of the doorway  $s$ -hole states in light nuclei, namely, that the  $p$ -,  $n$ -,  $d$ -,  $t$ -, and  ${}^3\text{He}$  fragmentations are allowed, while the  $\alpha$ -cluster and the larger-cluster fragmentations are greatly suppressed, in spite of the fact that the  $Q$  value for the  $\alpha$ -fragmentation channel is similar to or even larger than those for the  $p$ -,  $n$ -, and  $d$ -fragmentation channels. These results are in contrast to the results by the statistical model, in which a fragmentation channel with a larger  $Q$  value has a larger fragmentation width.

The wave function of a nuclear  $s$ -hole state is given generally as  $\Psi_{s\text{-hole}} = \Psi_{\text{doorway}} + \Psi_{\text{compound}}$ , where the doorway  $s$ -hole state ( $\Psi_{\text{doorway}}$ ) is orthogonal to the compound state ( $\Psi_{\text{compound}}$ ). The width of the deep hole state consists of two components, the escape width ( $\Gamma^{\uparrow}$ ) and the spreading width ( $\Gamma^{\downarrow}$ ), which corresponds, respectively, to the particle (cluster) fragmentation widths of the doorway  $s$ -hole state and the compound state produced by coupling of the doorway  $s$ -hole state with higher configurational states such as  $2h\text{-}1p$ ,  $3h\text{-}$

$2p$ , and so on. In medium-heavy and heavy nuclei, the doorway  $s$  hole produced by a quasifree knockout reaction propagating in a nuclear medium transits immediately to a compound state by strong coupling with higher configurational states, because the deep hole state appears in a highly excited energy region and the level density is very high in such an energy region. Thus the main component of the width of a deep hole state in medium-heavy and heavy nuclei is the spreading width and therefore the deep hole state is believed to be fragmented statistically. This is the standard scenario for the fragmentation of a deep hole state.

This scenario, however, may not be always realized in the case of deep hole states in light nuclei, especially in  $s$ -hole states in light nuclei such as  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$ . It is instructive to estimate the ratio of the nuclear radius ( $R$ ) to the mean free path of an  $s$ -hole state ( $\lambda$ ) for light to heavy nuclei, because the ratio ( $R/\lambda$ ) presents an index of how much of the component of the doorway  $s$ -hole state survives without exciting many-particle–many-hole states when the doorway  $s$  hole propagates in the nuclear medium. The mean free path ( $\lambda$ ) of an  $s$  hole is estimated with the use of the simple formula  $\lambda = \hbar^2 k / 2MW$ , where  $k$  is the wave number of the  $s$  hole given by  $k = \sqrt{2MT}/\hbar$  ( $M$  is the nucleon mass and  $T$  the kinetic energy of the  $s$  hole) and  $W$  is the imaginary part of the hole-nucleus optical potential which is given by  $W = 2\Gamma$  ( $\Gamma$  is the hole width). The simple formula  $\langle T \rangle = 3/4 \hbar \omega$  with  $\hbar \omega = 41/A^{1/3}$  MeV for light nuclei [ $\hbar \omega = 53/(A^{1/3} + 1.41)$  MeV for medium-heavy and heavy nuclei] is used for the  $s$ -hole kinetic energy [14]. Concerning the  $s$ -hole widths, we employ the experimental values obtained by ( $p,2p$ ) experiments [1–3]. Since there are no experimental  $s$ -hole widths in the case of medium-heavy and heavy nuclei, the theoretical values are used [14,15].

The calculated values of  $R/\lambda$  for various nuclei are shown in Table I. We see that  $R/\lambda$  is about 2.5–7.6 in the case of  ${}^{40}\text{Ca}$ – ${}^{208}\text{Pb}$ . This fact means that the doorway  $s$ -hole state produced by a quasifree knockout reaction excites many-particle–many-hole states (compound states) and then the main component of the  $s$ -hole state is a compound state which is fragmented statistically ( $\|\Psi_{\text{compound}}\|^2 \gg \|\Psi_{\text{doorway}}\|^2$ ). This is the standard scenario for the fragmentation of an  $s$ -hole state as mentioned above. On the other hand, as shown in Table I, the value of  $R/\lambda$  in light  $p$ -shell nuclei is about 1 ( ${}^{12}\text{C}$  and  ${}^{16}\text{O}$ ) or less than 1 ( ${}^6\text{Li}$  and  ${}^7\text{Li}$ ). This fact shows that in the case of  ${}^{12}\text{C}$  and  ${}^{16}\text{O}$  the fragmentation process of an  $s$ -hole state in the doorway stage is competitive with that in the compound nuclear stage ( $\|\Psi_{\text{doorway}}\|^2 \gg \|\Psi_{\text{compound}}\|^2$ ), while in the case of  ${}^6\text{Li}$  and  ${}^7\text{Li}$  the fragmentation occurs mainly in the doorway stage

( $\|\Psi_{\text{doorway}}\|^2 \gg \|\Psi_{\text{compound}}\|^2$ ). Therefore, the following fragmentation scenario for  $s$ -hole states in light nuclei such as  $^{11}\text{B}$  and  $^{15}\text{N}$  might be realized: A considerable component of the  $s$ -hole state is fragmented directly at the stage of the doorway  $s$ -hole state and the remainder transits to a compound state by coupling with higher configurational states which are fragmented statistically.

Section II is devoted to giving a description of the doorway  $s$ -hole states of  $^{11}\text{B}$  and  $^{15}\text{N}$  with use of the SU(3) group theory [12,13], and to providing the formulation of the spectroscopic factors and partial fragmentation widths for the two-body fragmentation of the doorway  $s$ -hole states of  $^{11}\text{B}$  and  $^{15}\text{N}$ . In Sec. III, we give the results for the calculated spectroscopic factors and partial fragmentation widths, and discuss the selection rule for fragmentation of doorway  $s$ -hole states in light nuclei, together with a comparison of the calculated escape widths with the experimental widths. An application of the selection rule to hypernuclei is also given briefly. Finally, we give a conclusion in Sec. IV.

## II. FORMULATION

### A. Doorway $s$ -hole states in light nuclei

As is well known, the ground-state wave function of light nuclei can be described well by the SU(3)( $\lambda\mu$ ) wave function [12,13]. Since the doorway  $s$ -hole state is produced by quasifree knockout reactions such as a  $^A\text{Z}(p,2p)$ , the  $s$ -hole state produced should have the same spatial symmetry as the ground state of the target nucleus. The wave function of the  $^{12}\text{C}$  ground state has mainly the SU(3)( $\lambda\mu$ )=(04) spatial symmetry with the total quanta  $N_Q=8$ , and therefore the  $^{11}\text{B}(s\text{-hole})$  doorway state is mainly described by the SU(3)( $\lambda\mu$ )=(04) wave function with  $N_Q=8$ . In the same manner, the wave function of the  $^{16}\text{O}$  ground state has mainly the SU(3)( $\lambda\mu$ )=(00) spatial symmetry with  $N_Q=12$ , and therefore the  $^{15}\text{N}(s\text{-hole})$  doorway state is mainly described by the SU(3)( $\lambda\mu$ )=(00) wave function with  $N_Q=12$ .

In this paper, the wave function of the doorway  $s$ -hole state, SU(3)( $\lambda\mu$ ), is described by the microscopic-cluster-model technique [16,17]. The reason why we use the microscopic-cluster-model technique to describe the  $s$ -hole state is as follows. (1) The microscopic-cluster-model technique is easily able to construct the SU(3)[ $f$ ]( $\lambda\mu$ ) state, since the antisymmetrization operator among nucleons belonging to different clusters communicates with the SU(3) generators. (2) The microscopic-cluster-model wave function is free from center-of-mass motion. (3) The spectroscopic factors for fragmentation of the  $s$ -hole state into  $t$ ,  $\alpha$ ,  $^5\text{He}$ ,  $^6\text{Li}$ , and  $^7\text{Li}$  nuclei, together with proton, neutron, and deuteron, are able to be evaluated easily without complex calculation of the fractional-parentage coefficients (FPC's).

The microscopic cluster model and the partition symmetry [ $f$ ] have a close connection with each other [16,17]. For example, the microscopic  $\alpha+\alpha+3N$  and  $^{12}\text{C}+3N$  models describe the SU(3)[443]( $\lambda\mu$ ) and SU(3)[4443]( $\lambda\mu$ ) states of  $^{11}\text{B}$  and  $^{15}\text{N}$ , respectively, where  $3N$  denotes a three-nucleon cluster ( $^3\text{H}$ - $^3\text{He}$ ). (Note that a  $^{12}\text{C}$  nucleus is described by the microscopic  $3\alpha$  cluster model.) When the  $3N$  cluster is broken into  $2N$  and  $N$  clusters, the microscopic  $\alpha+\alpha+2N+N$  ( $^{12}\text{C}+2N+N$ ) cluster model can describe not only the SU(3)[443]( $\lambda\mu$ ) [SU(3)[4443]( $\lambda\mu$ )] states of  $^{11}\text{B}$  [ $^{15}\text{N}$ ] but

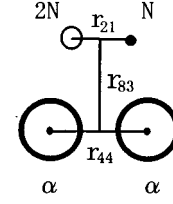


FIG. 1. Coordinate system of the microscopic  $\alpha+\alpha+2N+N$  cluster model.

also the SU(3)[4421]( $\lambda\mu$ ) [SU(3)[44421]( $\lambda\mu$ )] states of  $^{11}\text{B}$  [ $^{15}\text{N}$ ]. Thus, in the present study, we use the microscopic  $\alpha+\alpha+2N+N$  and  $^{12}\text{C}+2N+N$  cluster models for the  $^{11}\text{B}(s\text{-hole})$  and  $^{15}\text{N}(s\text{-hole})$  doorway states, respectively.

### B. $^{11}\text{B}(s\text{-hole})$ doorway state

The wave function of an SU(3)( $\lambda\mu$ )=(04) state of  $^{11}\text{B}(s\text{ hole})$  is described within the framework of the microscopic  $\alpha+\alpha+2N+N$  cluster model (see Fig. 1) with total orbital angular momentum  $L^\pi=0^+$ , total spin  $S=1/2$ , total isospin  $T=1/2$ , and total quanta  $N_Q=8$ . It should be noted that the microscopic cluster model is able to describe not only the SU(3)[443]( $\lambda\mu$ ) state but also SU(3)[4421]( $\lambda\mu$ ). The wave function of an SU(3)[ $f$ ]( $\lambda\mu$ ) state of  $^{11}\text{B}$  is given by

$$\Phi_L(^{11}\text{B}[f](\lambda\mu)) = \frac{1}{\sqrt{\nu([f](\lambda\mu))}} \sum_c a_c^{[f](\lambda\mu)L} \Phi_L(c), \quad (1)$$

$$\Phi_L(c) = \sqrt{\frac{4!4!2!}{11!}} \mathcal{A}' \{ \phi_\alpha \phi_\alpha [\phi_{2N} \phi_N]^{ST} \times [[u_{n_{44}l_{44}}(\mathbf{r}_{44}) u_{n_{21}l_{21}}(\mathbf{r}_{21})]_I u_{n_{83}l_{83}}(\mathbf{r}_{83})]_L \}, \quad (2)$$

$$c = (n_{44}l_{44}, n_{21}l_{21}, I, n_{83}l_{83}), \quad (3)$$

with  $L=I+\mathbf{l}_{83}$ ,  $I=\mathbf{l}_{44}+\mathbf{l}_{21}$ , and

$$N_Q = (2n_{44}+l_{44}) + (2n_{21}+l_{21}) + (2n_{83}+l_{83}).$$

The internal wave functions,  $\phi_\alpha$  and  $\phi_{2N}$  are given as the lowest shell model wave functions with the  $(0s)^4$  and  $(0s)^2$  configurations, respectively, and  $\phi_N$  denotes the spin-isospin function of  $N$ . The operator  $\mathcal{A}'$  antisymmetrizes nucleons belonging to different clusters. The relative wave function referring to the relative coordinate  $\mathbf{r}_{44}$  ( $\mathbf{r}_{21}$ ,  $\mathbf{r}_{83}$ ) between the  $\alpha$  and  $\alpha$  clusters [ $2N$  and  $N$ ,  $(\alpha+\alpha)$  and  $(2N+N)$ ] is expanded in terms of the harmonic oscillator wave function  $u_{n_{44}l_{44}}(u_{n_{21}l_{21}}, u_{n_{83}l_{83}})$  with the relative orbital angular momentum  $l_{44}$  ( $l_{21}$ ,  $l_{83}$ ) and node number  $n_{44}$  ( $n_{21}$ ,  $n_{83}$ ). The norm kernel matrix on the basis of  $\Phi_L(c)$  in Eq. (2) is diagonalized to obtain the expansion coefficient  $a_c^{[f](\lambda\mu)L}$  and the eigenvalue  $\nu([f](\lambda\mu))$ .

In order to see the characteristic of the fragmentation of the  $^{11}\text{B}(s\text{-hole})$  doorway state, it is instructive to calculate the spectroscopic factors and particle (cluster) fragmentation widths of all the possible two-body fragmentation channels for the  $^{11}\text{B}(s\text{-hole})$  state:  $^{10}\text{B}+n$ ,  $^{10}\text{Be}+p$ ,  $^9\text{Be}+d$ ,  $^8\text{Be}+t$ ,  $^7\text{Li}+\alpha$ , and  $^6\text{Li}+^5\text{He}$ . First we define the reduced width amplitudes for the two-body fragmentation channels. For example, the reduced width amplitudes for the  $^7\text{Li}+\alpha$ ,  $^8\text{Be}+t$ , and  $^9\text{Be}+d$  fragmentation channels are given as follows:

$$\mathcal{Y}_l^{7\text{Li}-\alpha}(r_{74}) = r_{74} \left\langle \sqrt{\frac{11!}{7!4!}} [\Phi_l(^7\text{Li}) \phi_\alpha Y_l(\mathbf{r}_{74})]_0 + \left| \Phi_0(^{11}\text{B}) \right\rangle_{r_{74}}, \quad (4)$$

$$\mathcal{Y}_l^{8\text{Be}-t}(r_{83}) = r_{83} \left\langle \sqrt{\frac{11!}{8!3!}} [\Phi_l(^8\text{Be}) \phi_t Y_l(\mathbf{r}_{83})]_0 + \left| \Phi_0(^{11}\text{B}) \right\rangle_{r_{83}}, \quad (5)$$

$$\mathcal{Y}_l^{9\text{Be}-d}(r_{92}) = r_{92} \left\langle \sqrt{\frac{11!}{9!2!}} [\Phi_l(^9\text{Be}) \phi_d Y_l(\mathbf{r}_{92})]_0 + \left| \Phi_0(^{11}\text{B}) \right\rangle_{r_{92}}, \quad (6)$$

where the integration is made for all coordinates other than the relative radial coordinate between the two clusters. The internal wave functions of the deuteron and triton are given as the lowest shell model wave functions with  $(0s)^2$  and  $(0s)^3$ , respectively. The reduced width amplitudes for the other fragmentation channels are given as the same as those in Eqs. (4)–(6). Concerning the internal wave functions for  $^{10}\text{B}$ ,  $^{10}\text{Be}$ ,  $^9\text{Be}$ ,  $^8\text{Be}$ ,  $^7\text{Li}$ ,  $^6\text{Li}$ , and  $^5\text{He}$ , we use the  $\text{SU}(3)(\lambda\mu) = (22)$ ,  $(22)$ ,  $(31)$ ,  $(40)$ ,  $(30)$ ,  $(20)$ , and  $(10)$  wave functions, respectively, which are obtained by the microscopic-cluster-model technique. For example, the  $^7\text{Be}(\lambda\mu) = (30)$ ,  $^8\text{Be}(\lambda\mu) = (40)$ , and  $^9\text{Be}(\lambda\mu) = (31)$  wave functions are given, respectively, within the frame of the microscopic  $\alpha+t$ ,  $\alpha+\alpha$ , and  $\alpha+\alpha+n$  cluster models,

$$\Phi_l(^7\text{Li}) = \frac{1}{\sqrt{\nu(30)}} \sqrt{\frac{4!3!}{7!}} \mathcal{A}'\{\phi_\alpha \phi_t u_{nl}(\mathbf{r}_{43})\}_{2n+l=3}, \quad (7)$$

$$\Phi_l(^8\text{Be}) = \frac{1}{\sqrt{\nu(40)}} \sqrt{\frac{4!4!}{8!}} \mathcal{A}'\{\phi_\alpha \phi_\alpha u_{nl}(\mathbf{r}_{44})\}_{2n+l=4}, \quad (8)$$

$$\Phi_l(^9\text{Be}) = \frac{1}{\sqrt{\nu(31)}} \sum_{n_{44}l_{44}, n_{81}l_{81}} a_{n_{44}l_{44}, n_{81}l_{81}}^l \Phi_l(n_{44}l_{44}, n_{81}l_{81}), \quad (9)$$

$$\begin{aligned} \Phi_l(n_{44}l_{44}, n_{81}l_{81}) \\ = \sqrt{\frac{4!4!}{9!}} \mathcal{A}'\{\phi_\alpha \phi_\alpha \phi_n [u_{n_{44}l_{44}}(\mathbf{r}_{44}) u_{n_{81}l_{81}}(\mathbf{r}_{81})]_l\}, \end{aligned} \quad (10)$$

with  $\mathbf{l} = \mathbf{l}_{44} + \mathbf{l}_{81}$  and  $(2n_{44} + l_{44}) + (2n_{81} + l_{81}) = 5$ . The expansion coefficient  $a_{n_{44}l_{44}, n_{81}l_{81}}^l$  and eigenvalue  $\nu(31)$  in Eq. (9) are obtained by diagonalization of the norm kernel matrix on the basis of  $\Phi_l(n_{44}l_{44}, n_{81}l_{81})$  given in Eq. (10). The other notations are self-explanatory. Then the spectroscopic factors for the  $^7\text{Li}+\alpha$ ,  $^8\text{Be}+t$ , and  $^9\text{Be}+d$  channels are defined as the integrated values for the square of each reduced width amplitude,

$$S^2[^7\text{Li}(l) + \alpha] = \int_0^\infty dr_{74} [\mathcal{Y}_l^{7\text{Li}-\alpha}(r_{74})]^2, \quad (11)$$

$$S^2[^8\text{Li}(l) + t] = \int_0^\infty dr_{83} [\mathcal{Y}_l^{8\text{Be}-t}(r_{83})]^2, \quad (12)$$

$$S^2[^9\text{Be}(l) + d] = \int_0^\infty dr_{92} [\mathcal{Y}_l^{9\text{Be}-d}(r_{92})]^2. \quad (13)$$

The spectroscopic factors for the other fragmentation channels are the same as those in Eqs. (11)–(13).

The partial fragmentation width for the  $^{11}\text{B}(s\text{-hole})$  doorway state to the  $^{10}\text{B}+n$ ,  $^{10}\text{Be}+p$ ,  $^9\text{Be}+d$ ,  $^8\text{Be}+t$ ,  $^7\text{Li}+\alpha$ , and  $^6\text{Li}+^5\text{He}$  channels is calculated by the separation energy method [18] and is defined as

$$\Gamma_{cl} = 2P_l(k_c a_c) \gamma_l^2(a_c), \quad (14)$$

$$P_l(k_c a_c) = \frac{k_c a_c}{F_l^2(k_c a_c) + G_l^2(k_c a_c)}, \quad (15)$$

$$\gamma_l^2(a_c) = \frac{3\hbar^2}{2\mu a_c^2} \frac{a_c^3}{3} \mathcal{Y}_l^2(a_c), \quad (16)$$

where  $c$  denotes a fragmentation channel and  $P_l$  and  $\gamma_l^2$  are the penetration factor and reduced width, respectively.  $F_l$  and  $G_l$  denote the regular and irregular Coulomb wave functions, respectively. The wave number  $k_c$  is given by  $k_c = \sqrt{2\mu_c Q_c / \hbar^2}$ , where  $Q_c$  and  $\mu_c$  present the  $Q$  value and reduced mass for the fragmentation channel  $c$ , and  $a_c$  denotes the channel radius. The total sum of the partial fragmentation widths  $\Gamma_{cl}$  corresponds to the escape width ( $\Gamma^\dagger$ ), and is given by

$$\Gamma^\dagger = \sum_{cl} \Gamma_{cl}. \quad (17)$$

### C. $^{15}\text{N}(s\text{-hole})$ doorway state

The wave function of an  $\text{SU}(3)(\lambda\mu) = (00)$  state of  $^{15}\text{N}(s\text{ hole})$  is described within the framework of the microscopic  $^{12}\text{C} + 2N + N$  cluster model with total orbital angular momentum  $L^\pi = 0^+$ , total spin  $S = 1/2$ , total isospin  $T = 1/2$ , and total quanta  $N_Q = 12$ . It should be noted that the microscopic cluster model is able to describe not only the  $\text{SU}(3)[4443](\lambda\mu)$  state but also  $\text{SU}(3)[44421](\lambda\mu)$ . The wave function of an  $\text{SU}(3)[f](\lambda\mu)$  state of  $^{15}\text{N}$  is given by

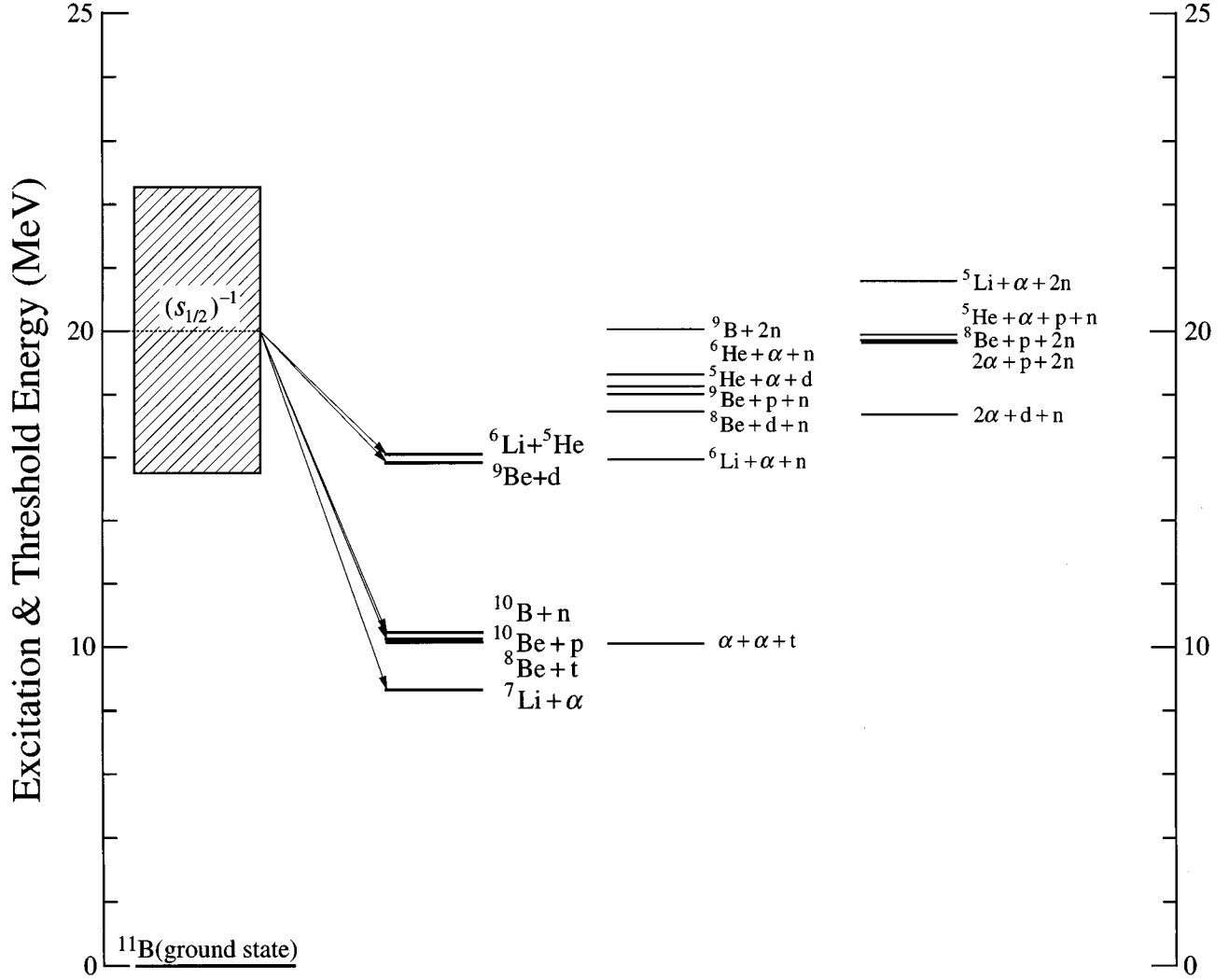


FIG. 2. Various thresholds for fragmentation channels of the  $s$ -hole state of  $^{11}\text{B}$ . The excitation energy and width of the  $^{11}\text{B}(s\text{-hole})$  state are  $E_x \approx 20$  MeV and  $\Gamma \approx 9$  MeV, respectively, and are shown in the left side.

$$\Phi_L(^{15}\text{N}[f](\lambda\mu)) = \frac{1}{\sqrt{\nu[f](\lambda\mu)}} \sum_c a_c^{[f](\lambda\mu)L} \Phi_L(c), \quad (18)$$

$$\begin{aligned} \Phi_L(c) = & \sqrt{\frac{12!2!}{15!}} \mathcal{A}' \{ [\phi_{2N} \phi_N]^{ST} \\ & \times [\phi_l(^{12}\text{C}) [u_{n_r l_r}(\mathbf{r}) u_{n_\rho l_\rho}(\rho)]_L \}, \end{aligned} \quad (19)$$

$$c = (l, n_r l_r, n_\rho l_\rho, I), \quad (20)$$

with  $\mathbf{L} = \mathbf{l} + \mathbf{I}$ ,  $I = I_r + I_\rho$ , and  $N_Q = (2n_r + l_r) + (2n_\rho + l_\rho) + 8$ . The internal wave function  $\phi_l(^{12}\text{C})$  is given by the  $\text{SU}(3)[f](\lambda\mu) = [444](04)$  wave function. The relative wave function referring to the relative coordinate  $\mathbf{r}$  ( $\rho$ ) between  $^{12}\text{C}$  and  $(2N+N)$  ( $2N$  and  $N$ ) is expanded in terms of the harmonic oscillator wave function  $u_{n_r l_r}$  ( $u_{n_\rho l_\rho}$ ) with the relative orbital angular momentum  $l_r$  ( $l_\rho$ ) and node number  $n_r$  ( $n_\rho$ ). The other notations are self-explanatory. The norm

kernel matrix on the basis of  $\Phi_L(c)$  in Eq. (19) is diagonalized to obtain the expansion coefficient  $a_c^{[f](\lambda\mu)L}$  and the eigenvalue  $\nu[f](\lambda\mu)$ .

The spectroscopic factors and partial widths for the two-body fragmentations of the  $^{15}\text{N}(s\text{-hole})$  state, namely,  $^{14}\text{C} + p$ ,  $^{14}\text{N} + n$ ,  $^{13}\text{C} + d$ ,  $^{12}\text{C} + t$ ,  $^{11}\text{B} + \alpha$ ,  $^{10}\text{B} + ^5\text{He}$ ,  $^{10}\text{Be} + ^5\text{Li}$ , and  $^9\text{Be} + ^6\text{Li}$ , are evaluated in the same manner as in the case of  $^{11}\text{B}(s\text{-hole})$ . The total sum of the partial fragmentation widths corresponds to the escape width ( $\Gamma^\dagger$ ).

### III. RESULTS AND DISCUSSION

First it is instructive to show various thresholds for fragmentation channels of the  $s$ -hole state of  $^{11}\text{B}$  ( $^{15}\text{N}$ ), which are given in Fig. 2 (Fig. 3), together with the excitation energy ( $E_x$ ) and width ( $\Gamma$ ) for the  $s$ -hole state [ $E_x \approx 20$  (30) MeV and  $\Gamma \approx 9$  (14) MeV in  $^{11}\text{B}$  ( $^{15}\text{N}$ )] [1–3]. In this paper, the excitation energies of the  $^{11}\text{B}(s\text{-hole})$  and  $^{15}\text{N}(s\text{-hole})$  states are fixed to  $E_x = 20$  and 30 MeV, respectively. In the case of  $^{11}\text{B}$ , the two-body fragmentation channel with the largest  $Q$  value measured from the excitation energy of the  $s$ -hole state

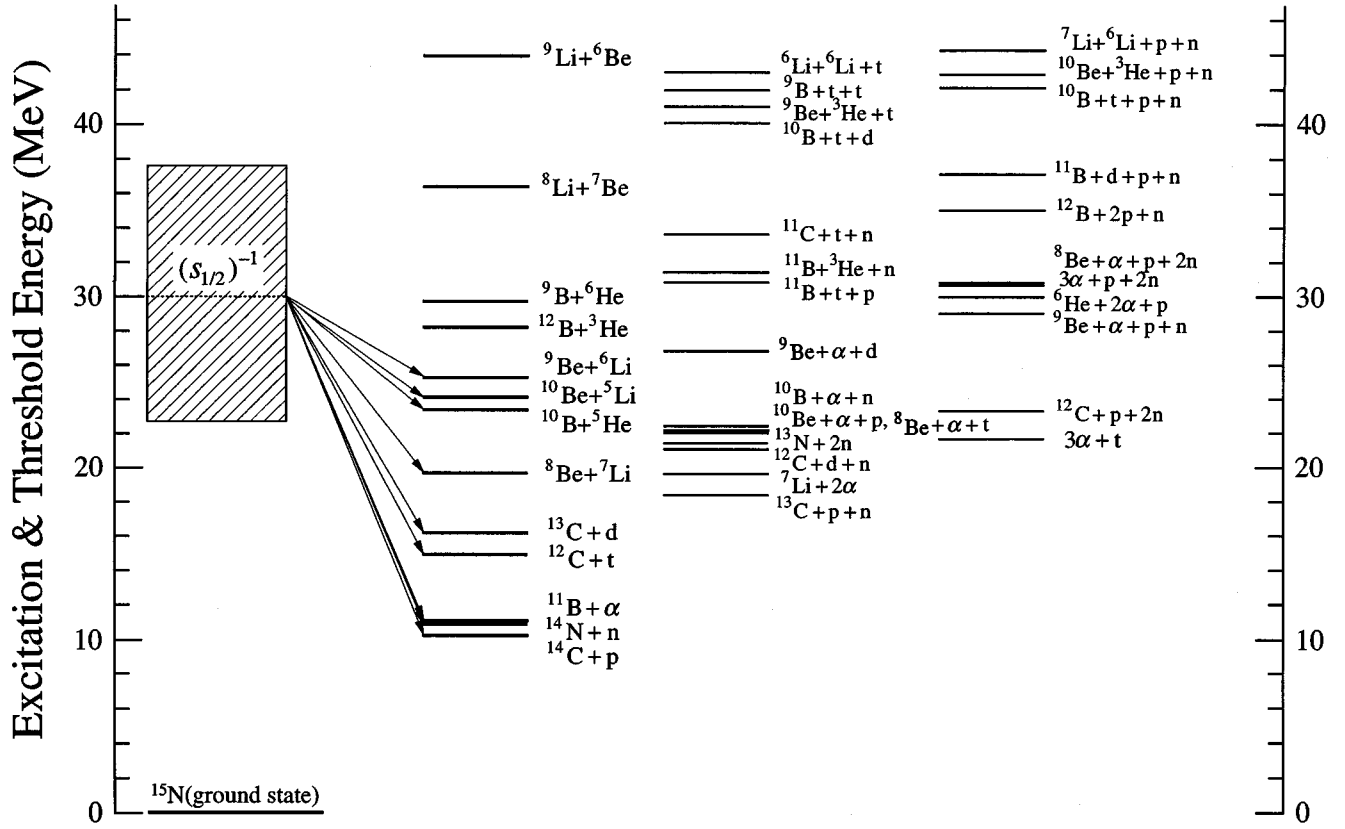


FIG. 3. Various thresholds for fragmentation channels of the  $s$ -hole state of  $^{15}\text{N}$ . The excitation energy and width of the  $^{15}\text{N}(s\text{-hole})$  state are  $E_x \approx 30$  MeV and  $\Gamma \approx 14$  MeV, respectively, and are shown in the left side.

is the  $\alpha$ -cluster fragmentation channel ( $^7\text{Li} + \alpha$ ) with  $Q = 11.0$  MeV, and that with the second largest  $Q$  value is the  $^8\text{Be} + t$  channel ( $Q = 8.5$  MeV) which is almost degenerate in energy with  $^{10}\text{Be} + p$  (8.5 MeV) and  $^{10}\text{B} + n$  (8.2 MeV). These four channels should play an important role in the fragmentation of the  $^{11}\text{B}(s\text{-hole})$  state. For the three-body fragmentation channels, only the  $\alpha + \alpha + t$  channel ( $Q = 8.6$  MeV) appears around the  $^{10}\text{Be} + p$ ,  $^{10}\text{B} + n$ , and  $^8\text{Be} + t$  channels. The three-body fragmentation channel with the second largest  $Q$  value is the  $^6\text{Li} + \alpha + n$  channel ( $Q = 4.0$  MeV) and the four-body fragmentation channel with the largest  $Q$  value is the  $2\alpha + d + n$  channel (2.6 MeV). On the other hand, in the case of  $^{15}\text{N}$ , the two-body fragmentation channel with the largest  $Q$  value is the proton fragmentation channel ( $^{14}\text{C} + p$ ) with  $Q = 19.8$  MeV, which is almost degenerate in energy with  $^{14}\text{N} + n$  ( $Q = 19.2$  MeV) and  $^{11}\text{B} + \alpha$  (19.1 MeV), and that with the second largest  $Q$  value is the  $^{12}\text{C} + t$  channel (15.2 MeV). It should be noted that the  $Q$  values of these four channels are larger than those in  $^{11}\text{B}$ . These four channels should play an important part in the fragmentation of the  $^{15}\text{N}(s\text{-hole})$  state. The three-body fragmentation channel with the largest (second largest)  $Q$  value is the  $^{13}\text{C} + p + n$  ( $^7\text{Li} + \alpha + \alpha$ ) channel with  $Q = 12.0$  (10.6) MeV, which appears around the  $^8\text{Be} + ^7\text{Li}$  channel (7.6 MeV). This is in contrast to the case of  $^{11}\text{B}$ , in which the three-body channel with the largest  $Q$  value appears around the  $^7\text{Li} + \alpha$ ,  $^8\text{Be} + t$ ,  $^{10}\text{Be} + p$ , and  $^{10}\text{B} + n$  channels.

In this section, we give the calculated spectroscopic factors and partial fragmentation widths for the two-body frag-

mentation channels of the doorway  $s$ -hole states of  $^{11}\text{B}$  and  $^{15}\text{N}$ , and discuss the escape width ( $\Gamma^\dagger$ ) and the selection rule for fragmentation of doorway  $s$ -hole states in light nuclei. An application of the selection rule to hypernuclei is discussed briefly.

#### A. Fragmentation of $^{11}\text{B}(s\text{-hole})$ doorway state

Since the  $^{11}\text{B}(s\text{-hole})$  state has total quanta  $N_Q = 8$ , the norm kernel on the basis given in Eq. (2) within the framework of the microscopic  $\alpha + \alpha + 2N + N$  cluster model is diagonalized in the condition of  $N_Q = 8$ . The resultant eigenstates are classified into six  $\text{SU}(3)[443](\lambda\mu)$  and nine  $\text{SU}(3)[4421](\lambda\mu)$  labels as shown in Table II, together with the results obtained within the framework of the microscopic  $\alpha + \alpha + t$  cluster model. According to Elliott's rule [12,13] the possible  $L$  values (orbital angular momenta) for a given  $(\lambda\mu)$

TABLE II.  $\text{SU}(3)[443](\lambda\mu)$  and  $[4421](\lambda\mu)$  states with quanta  $N_Q = 8$  obtained by the microscopic  $\alpha + \alpha + 2N + N$  cluster model technique, together with  $\text{SU}(3)(\lambda\mu)$  states with quanta  $N_Q = 8$  obtained by the microscopic  $\alpha + \alpha + t$  cluster model technique.

$\alpha + \alpha + 2N + N$		$\alpha + \alpha + t$
$\text{SU}(3)[443](\lambda\mu)$	$\text{SU}(3)[4421](\lambda\mu)$	$\text{SU}(3)(\lambda\mu)$
(04) (23) (42)	(04) (23) (42)	(04) (23) (42)
(31) (21) (20)	$(31)^2 (21)^2 (20)$	
	(50)	

TABLE III. Calculated spectroscopic factors ( $S^2$ ) and partial fragmentation widths ( $\Gamma$ ) of  $^{11}\text{B}(\lambda\mu)_L = (04)_{0^+}$  states.  $T$  denotes the isospin of  $^{10}\text{B}$  or  $^{10}\text{Be}$  and  $a$  expresses the channel radius (see the text).

	$Q$ (MeV)	[443](04)		[4421](04)		$a$ (fm)
		$S^2$	$\Gamma$ (MeV)	$S^2$	$\Gamma$ (MeV)	
$^{10}\text{B}^{T=0}(0^+) + n$	8.2	0.033	0.21	0.179	1.14	4.5
$^{10}\text{B}^{T=0}(2_1^+) + n$	3.7	0.007	0.01	0.041	0.06	4.0
$^{10}\text{B}^{T=0}(2_2^+) + n$	2.7	0.070	0.06	0.385	0.35	4.0
$^{10}\text{B}^{T=1}(0^+) + n$	8.5	0.011	0.07	0.060	0.39	4.5
$^{10}\text{B}^{T=1}(2_1^+) + n$	5.1	0.003	0.01	0.013	0.03	4.0
$^{10}\text{B}^{T=1}(2_2^+) + n$	4.1	0.023	0.04	0.129	0.23	4.0
$^{10}\text{Be}^{T=1}(0^+) + p$	8.5	0.022	0.13	0.119	0.72	4.5
$^{10}\text{Be}^{T=1}(2_1^+) + p$	5.1	0.002	0.00	0.027	0.05	4.0
$^{10}\text{Be}^{T=1}(2_2^+) + p$	4.1	0.047	0.06	0.257	0.31	4.0
$^9\text{Be}(1^-) + d$	3.9	0.177	0.58	0.243	0.80	3.6
$^9\text{Be}(3^-) + d$	0.5	0.155	0.00	0.213	0.00	3.3
$^8\text{Be}(0^+) + t$	8.5	0.137	1.01	0.000	0.00	3.5
$^8\text{Be}(2^+) + t$	5.6	0.171	0.56	0.000	0.00	3.3
$^8\text{Be}(4^+) + t$	(-2.9)	0.173	0.00	0.000	0.00	3.0
$^7\text{Li}(1^-) + \alpha$	11.0	0.000	0.00	0.000	0.00	
$^7\text{Li}(3^-) + \alpha$	6.4	0.000	0.00	0.000	0.00	
$^6\text{Li}(0^+) + ^5\text{He}$	3.0	0.000	0.00	0.000	0.00	
$^6\text{Li}(2^+) + ^5\text{He}$	0.0	0.000	0.00	0.000	0.00	
Total sum			2.75		4.08	

are  $L = K, K+1, \dots, K+\lambda$  for  $K \neq 0$  and  $L = \lambda, \lambda-2, \dots, 1$  or  $0$  for  $K=0$  with the integer  $K$  taking the values  $K = \mu, \mu-2, \dots, 1$  or  $0$ . It should be noted that the  $[443](\lambda\mu)$  state within the framework of  $\alpha + \alpha + 2N + N$  is exactly the same as the  $(\lambda\mu)$  state within the framework of  $\alpha + \alpha + t$ , and is orthogonal with the  $[4421](\lambda\mu)$  state. Since the doorway  $s$ -hole state of  $^{11}\text{B}$  is produced by quasifree knockout reactions such as a  $^{12}\text{C}(p, 2p)$ , the  $s$ -hole state produced should have the same spatial symmetry as the ground state of  $^{12}\text{C}$ ,  $\text{SU}(3)(\lambda\mu) = (04)$ . Therefore, the two  $(\lambda\mu) = (04)$  representations in Table II, namely,  $[443](04)$  and  $[4421](04)$ , which are degenerate in energy, are mainly responsible for the  $^{11}\text{B}(s\text{-hole})$  doorway state.

The calculated spectroscopic factors are shown in Table III for the fragmentation of the  $\text{SU}(3)[f](\lambda\mu)L^\pi = [443](04)0^+$  and  $[4421](04)0^+$  eigenstates into the  $^{10}\text{Be} + p$ ,  $^{10}\text{B} + n$ ,  $^9\text{Be} + d$ ,  $^8\text{Be} + t$ ,  $^7\text{Li} + \alpha$ , and  $^6\text{Li} + ^5\text{He}$  channels together with their excited channels. They are summarized in Table IV, in which the values are the total sums for

TABLE IV. Summary of the calculated partial fragmentation widths of  $^{11}\text{B}(\lambda\mu)_L = (04)_{0^+}$  states given in Table III. See the text.

	[443](04)		[4421](04)	
	$S^2$	$\Gamma$ (MeV)	$S^2$	$\Gamma$ (MeV)
$^{10}\text{B} + n$	0.147	0.40	0.807	2.20
$^{10}\text{Be} + p$	0.071	0.19	0.403	1.08
$^9\text{Be} + d$	0.332	0.58	0.456	0.80
$^8\text{Be} + t$	0.481	1.57	0.000	0.00
$^7\text{Li} + \alpha$	0.000	0.00	0.000	0.00
$^6\text{Li} + ^5\text{He}$	0.000	0.00	0.000	0.00
Total sum		2.75		4.08

the ground-state channel and excited channels; for example,  $S^2(^8\text{Be} + t) = S^2[^8\text{Be}(0^+) + t] + S^2[^8\text{Be}(2^+) + t] + S^2[^8\text{Be}(4^+) + t]$ . A surprising result is seen, that the spectroscopic factors for the fragmentation of the two  $(\lambda\mu)L^\pi = (04)0^+$  eigenstates into the  $^7\text{Li} + \alpha$  and  $^6\text{Li} + ^5\text{He}$  channels are exactly zero, while those into the  $^{10}\text{Be} + p$ ,  $^{10}\text{B} + n$ ,  $^9\text{Be} + d$ , and  $^8\text{Be} + t$  channels are nonzero. This means that the  $^{11}\text{B}(s\text{-hole})$  doorway state can be fragmented into the less-than- $\alpha$ -cluster channels, but fragmentation into the more-than- $\alpha$ -cluster channels is forbidden. In particular, the result that the  $\alpha$ -cluster fragmentation is forbidden is striking if we take into account the fact that the  $Q$  value for the  $\alpha$ -cluster fragmentation channel is the largest among the other channels (see Fig. 2). This notable fact is called the selection rule for fragmentation of doorway  $s$ -hole states in light nuclei, and was found by two of the present authors (T.Y. and K.I.) for the first time [11].

The reason why  $\alpha$ -cluster fragmentation is forbidden in the  $^{11}\text{B}(s\text{-hole})$  doorway state is understood intuitively as shown in Fig. 4. According to the shell model, the  $^{11}\text{B}(s\text{-hole})$  state is given as the  $(0s)^3(0p)^8$  configuration [Fig. 4(a)]. In the case of the  $^8\text{Be} + t$  channel, the eight nucleons in the  $p$  shell are correlated to form the  $^8\text{Be}$  cluster [Fig. 4(b)] and then the spectroscopic factors are  $S^2 \approx 0.14-0.17$  as seen Table III. On the other hand, in the case of the  $^7\text{Li} + \alpha$  channel, the four nucleons in the  $p$  shell are correlated to form the  $\alpha$  cluster and then the remainder nucleus,  $^7\text{Li}$ , corresponds to the  $s$ -hole state, which is orthogonal to the  $^7\text{Li}$  ground-band states [see Fig. 4(c)]. Therefore  $\alpha$ -cluster fragmentation is forbidden in the fragmentation of the  $^{11}\text{B}(s\text{-hole})$  state.

This intuitive understanding is supported by the following  $\text{SU}(3)$  algebra. According to the  $\text{SU}(3)$  group theory, the  $^7\text{Li}$

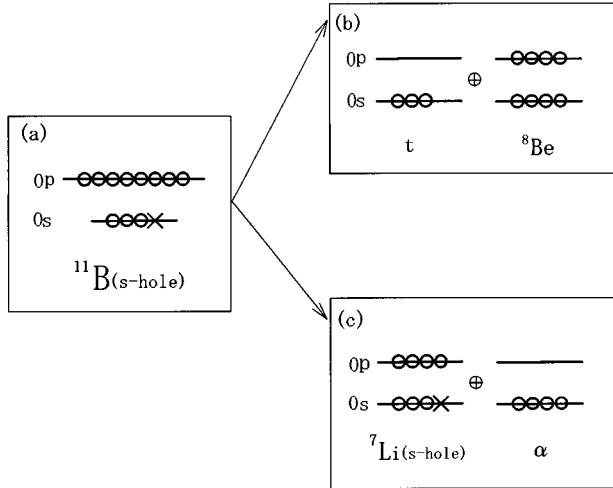


FIG. 4. Intuitive figure to understand the reason why the  $\alpha$ -cluster fragmentation is forbidden in the  $^{11}\text{B}(s\text{-hole})$  doorway state. (a) the  $(0s)^3(0p)^8$  shell model configuration for the  $^{11}\text{B}(s\text{-hole})$  state. (b) In the  $^8\text{Be}+t$  fragmentation channel, the eight nucleons in the  $(0p)$  shell of  $^{11}\text{B}(s\text{-hole})$  are correlated to form the  $^8\text{Be}$  cluster and then the spectroscopic factors are  $S^2 \approx 0.14\text{--}0.17$  (see Table III). (c) In the  $^7\text{Li}+\alpha$  fragmentation channel, the four nucleons in the  $(0p)$  shell of  $^{11}\text{B}(s\text{-hole})$  are correlated to form the  $\alpha$  cluster and then the remainder nucleus  $^7\text{Li}$  is the  $s$ -hole state, which is orthogonal to the  $^7\text{Li}$  ground-band states, and therefore  $\alpha$ -cluster fragmentation is forbidden in the  $^{11}\text{B}(s\text{-hole})$  doorway state.

and  $\alpha$  nuclei belong, respectively, to  $(\lambda_1\mu_1)=(30)$  with quanta  $N_Q=3$  and  $(\lambda_2\mu_2)=(00)$  with  $N_Q=0$ , and the relative wave function between the  $^7\text{Li}$  and  $\alpha$  clusters is described by the  $(\lambda_r\mu_r)=(50)$  state with  $N_Q=5$ . Therefore, the  $(\lambda\mu)$  representations of  $^{11}\text{B}$  constructed by the three representations  $(\lambda_1\mu_1)=(30)$ ,  $(\lambda_2\mu_2)=(00)$ , and  $(\lambda_r\mu_r)=(50)$ , are given by

$$(30) \otimes (00) \otimes (50) = (80), (61), (42), \text{ and } (23). \quad (21)$$

Consequently, the  $(\lambda\mu)=(04)$  representation of  $^{11}\text{B}$  is not able to be constructed from the  $(30)$ ,  $(00)$ , and  $(50)$  ones, so that  $\alpha$ -cluster fragmentation is forbidden. On the other hand, in the case of  $t$ -cluster fragmentation, the  $^8\text{Be}$  and  $t$  nuclei correspond, respectively, to  $(\lambda_1\mu_1)=(40)$  with  $N_Q=4$  and  $(\lambda_2\mu_2)=(00)$  with  $N_Q=0$ , and the relative wave function between the  $^8\text{Be}$  and  $t$  clusters is given by the  $(\lambda_r\mu_r)=(40)$  state with  $N_Q=4$ . Therefore the  $(\lambda\mu)$  representations of  $^{11}\text{B}$  constructed by the three representations  $(\lambda_1\mu_1)=(40)$ ,  $(\lambda_2\mu_2)=(00)$ , and  $(\lambda_r\mu_r)=(40)$  are given by

$$(40) \otimes (00) \otimes (40) = (80), (61), (42), (23), \text{ and } (04). \quad (22)$$

Thus the  $(\lambda\mu)=(04)$  representation of  $^{11}\text{B}$  can be constructed from the  $(40)$ ,  $(00)$ , and  $(40)$  ones, so that  $t$ -cluster fragmentation is allowed. The same situation is realized for the other channels,  $^{10}\text{B}+n$ ,  $^{10}\text{Be}+p$ ,  $^9\text{Be}+d$ , and  $^6\text{Li}+^5\text{He}$  (see Table V).

Here, let us return to Tables III and IV. The spectroscopic factor of the  $[4421](04)$  state for the  $t$ -cluster fragmentation channel is exactly zero, which is in contrast to the case of the

TABLE V. SU(3)-algebra analysis of whether the  $(\lambda\mu)=(04)$  representation can be constructed from the three SU(3) representations  $(\lambda_1\mu_1)$ ,  $(\lambda_2\mu_2)$ , and  $(\lambda_r\mu_r)$ .

$A^1Z_1 + A^2Z_2$	$(\lambda_1\mu_1)$	$(\lambda_2\mu_2)$	$(\lambda_r\mu_r)$	$(\lambda\mu)=(00)$
$^{10}\text{B}+n$	(22)	(00)	(20)	allowed
$^{10}\text{Be}+p$	(22)	(00)	(20)	allowed
$^9\text{Be}+d$	(31)	(00)	(30)	allowed
$^8\text{Be}+t$	(40)	(00)	(40)	allowed
$^7\text{Li}+\alpha$	(30)	(00)	(50)	forbidden
$^6\text{Li}+^5\text{He}$	(20)	(10)	(50)	forbidden

$[443](04)$  state. This is a characteristic of the  $[4421](04)$  state which is orthogonal to the  $[443](04)$  state. Since the  $[4421]$  spatial symmetry state has more freedom for the nucleon part outside the  $\alpha$  and  $\alpha$  core than does the  $[443]$  state, the values of the spectroscopic factors for the  $^{10}\text{Be}+p$  and  $^{10}\text{B}+n$  channels in the case of  $[4421](04)$  are larger than those in the case of  $[443](04)$ , while those for the  $^8\text{Be}+t$  ( $^9\text{Be}+d$ ) channel in the former are smaller than (nearly equal to) those in the latter. Thus, the  $[4421](04)$  and  $[443](04)$  eigenstates contribute mainly to the nucleon and triton fragmentations of the doorway  $s$ -hole state, respectively, and for the deuteron fragmentation both the  $[4421](04)$  and  $[443](04)$  eigenstates contribute nearly equally.

The calculated partial fragmentation widths of the  $^{11}\text{B}(s\text{-hole})$  doorway state are also given in Tables III and IV, in which the channel radius  $a$  [see Eq. (14)] is taken at some point outside the final peak of the reduced width amplitude  $\mathcal{Y}_l(r)$  defined in Eqs. (4)–(6) [18]. Reflecting the fact that the spectroscopic factor for the  $^7\text{Li}+\alpha$  channel is zero, the  $\alpha$ -cluster fragmentation width becomes exactly zero, in spite of the fact that the  $Q$  value ( $Q=11.0$  MeV) of the  $^7\text{Li}+\alpha$  channel is largest among the other channels [cf.  $Q(^{10}\text{Be}+p)=8.5$  MeV,  $Q(^{10}\text{B}+n)=8.2$  MeV,  $Q(^9\text{Be}+d)=3.9$  MeV, and  $Q(^8\text{Be}+t)=8.5$  MeV]. One sees also zero partial width for the  $^6\text{Li}+^5\text{He}$  channel ( $Q=4.4$  MeV). This result is in great contrast to that of the statistical model, because the model predicts a larger fragmentation width for a fragmentation channel with a larger  $Q$  value.

The total sums of the partial widths for  $[443](04)$  and  $[4421](04)$  are  $\Gamma([443])=2.75$  MeV and  $\Gamma([4421])=4.08$  MeV, respectively, as shown in Tables III and IV. Reflecting the characteristic of the spectroscopic factors, about 60% (90%) of  $\Gamma([443])$  [ $\Gamma([4421])$ ] comes from the triton (nucleon) channel. Since both the  $[443](04)$  and  $[4421](04)$  eigenstates contribute to the doorway  $s$ -hole state, the escape width ( $\Gamma^\dagger$ ) of the  $s$ -hole state of  $^{11}\text{B}$  corresponds to the sum of  $\Gamma([443])$  and  $\Gamma([4421])$  and thus  $\Gamma^\dagger \approx 7$  MeV, which is nearly equal to the experimental result ( $\approx 9$  MeV). [Note that the excitation energy of  $^{11}\text{B}(s\text{-hole})$  is fixed to  $E_x=20.0$  MeV.] Then the proton, neutron, deuteron, and triton fragmentation widths amount, respectively, to 19%, 38%, 20%, and 23% of the calculated escape width ( $\Gamma^\dagger \approx 7$  MeV). It should be noted that the experimental excitation energy ( $E_x$ ) of the  $^{11}\text{B}(s\text{-hole})$  state is slightly different for each experimental group because of the very large width; for example,  $E_x=18.0 \pm 2.0$  MeV [1],  $E_x=19.6 \pm 1.0$  MeV [2],  $E_x=20.0 \pm 0.3$  MeV [19], and  $E_x=22.2 \pm 1.0$  MeV [20]. When the experimental excitation energy of  $^{11}\text{B}(s\text{-hole})$  is



$E_x=21.5$  MeV, the calculated escape width ( $\Gamma^\dagger$ ) becomes about 9 MeV. However, the percentage of the proton, neutron, deuteron, and triton fragmentation widths does not change seriously in the two cases of  $E_x=20.0$  and 21.5 MeV.

In the present study, the  $^{11}\text{B}(s\text{-hole})$  doorway state was assumed to be given by the  $\text{SU}(3)(04)$  state, since the ground-state wave function of  $^{12}\text{C}$  has mainly the  $\text{SU}(3)(04)$  spatial symmetry. However, in the realistic  $^{11}\text{B}(s\text{-hole})$  wave function ( $L^\pi=0^+$ ), spatial symmetry states other than  $[443](04)$  and  $[4421](04)$  may be mixed by nucleon-nucleon interaction; for example,  $[443](42)$ ,  $[4421](42)$ ,  $[443](20)$ ,  $[4421](20)$ , and  $[4421](50)$  (see Table II). Even if this occurs, the above-mentioned selection rule for fragmentation of doorway  $s$ -hole states in light nuclei should persist for the following reasons. According to the shell model calculation, the main component of the  $[443](42)$  and  $[4421](42)$  eigenstates contributes to the low-lying positive parity states around  $E_x \approx 7\text{--}8$  MeV and therefore the admixture of  $[443](42)$  and  $[4421](42)$  in the doorway  $s$ -hole state becomes small, though the spatial symmetry states of  $[443](42)$  and  $[4421](42)$  have a large  $\alpha$ -decay width. Concerning the  $[443](20)$ ,  $[4421](20)$ , and  $[4421](50)$  eigenstates, the  $\alpha$ -cluster fragmentation ( $^7\text{Li}+\alpha$ ) is forbidden, because the  $(\lambda\mu)=(04)$  representation of  $^{11}\text{B}$  cannot be constructed from the three  $\text{SU}(3)$  representations,  $(\lambda_1\mu_1)=(30)$  for  $^7\text{Li}$ ,  $(\lambda_2\mu_2)=(00)$  for  $\alpha$ , and  $(\lambda_r\mu_r)=(50)$  for  $^7\text{Li}-\alpha$  [ $(\lambda_1\mu_1) \otimes (\lambda_2\mu_2) \otimes (\lambda_r\mu_r) = (30) \otimes (00) \otimes (50) \neq (20), (50)$ ]. Therefore, the selection rule persists in the actual situation, namely, that the  $\alpha$ -cluster fragmentation is suppressed in comparison with the nucleon, deuteron, and triton fragmentations.

Finally let us discuss the three- and four-body fragmentation channels. The three-body channel with the largest  $Q$  value is the  $\alpha+\alpha+t$  channel ( $Q=8.6$  MeV), which appears around the  $^{10}\text{Be}+p$ ,  $^{10}\text{B}+n$ , and  $^8\text{Be}+t$  channels (see Fig. 2). Therefore, the three-body channel might make the  $\alpha$ -cluster fragmentation width large. However, the contribution is expected not to be very large for the following reason. In Fig. 2, the  $^7\text{Li}(1^-)+\alpha$  and  $^7\text{Li}(3^-)+\alpha$  channels appear at  $E_x=8.7$  and 13.3 MeV ( $Q=11.0$  and 6.4 MeV), respectively, and the  $^8\text{Be}(0^+)+t$ ,  $^8\text{Be}(2^+)+t$ , and  $^8\text{Be}(4^+)+t$  channels appear at  $E_x=11.2$ , 14.1, and 22.6 MeV ( $Q=8.5$ , 5.6, and  $-2.9$  MeV), respectively. According to our selection rule, the two-body fragmentations of the  $^{11}\text{B}(s\text{-hole})$  states to the  $^7\text{Li}+\alpha$  channel are forbidden, while those to the  $^8\text{Be}+t$  channels are allowed. Since the  $^7\text{Li}(1^-3^-)$  and  $^8\text{Be}(0^+-2^+-4^+)$  states are described by the microscopic  $\alpha+t$  and  $\alpha+\alpha$  cluster models, respectively, a considerable amount of the  $\alpha+\alpha+t$  configurational space should be exhausted by the  $^7\text{Li}+\alpha$  and  $^8\text{Be}+t$  configurational spaces. Consequently, the contribution of the  $\alpha$ -cluster fragmentation from the  $\alpha+\alpha+t$  channel seems to be small. Moreover, it is possible to separate experimentally whether the  $\alpha$  cluster comes from the two-body fragmentation or the three-body fragmentation. On the other hand, the  $Q$  values for the three-body channels other than  $\alpha+\alpha+t$  and four-body channels are small; for example, the three-body channel with the second largest  $Q$  value is the  $^6\text{Li}+\alpha+n$  channel with  $Q=4.0$  MeV, and the four-body channel with the largest  $Q$  value is the  $2\alpha+d+n$  channel with  $Q=2.6$  MeV. Thus the phase spaces for these channels become small. Therefore their contributions to the fragmentation of the  $^{11}\text{B}(s\text{-hole})$  state should be small.

TABLE VI.  $\text{SU}(3)[4443](\lambda\mu)$  and  $[44421](\lambda\mu)$  states with quanta  $N_Q=12$  obtained by the microscopic  $^{12}\text{C}+2N+N$  cluster model technique, together with  $\text{SU}(3)(\lambda\mu)$  states with quanta  $N_Q=12$  obtained by the microscopic  $^{12}\text{C}+t$  cluster model technique.

$^{12}\text{C}+2N+N$		$^{12}\text{C}+t$
$\text{SU}(3)[4443](\lambda\mu)$	$\text{SU}(3)[44421](\lambda\mu)$	$\text{SU}(3)(\lambda\mu)$
(00)	(00)	(00)
(11)	(11)	(11)
(22)	(22)	(22)

### B. Fragmentation of $^{15}\text{N}(s\text{-hole})$ doorway state

The  $^{15}\text{N}(s\text{-hole})$  state has the total quanta  $N_Q=12$  and therefore the norm kernel on the basis given in Eq. (19) is diagonalized within the framework of the microscopic  $^{12}\text{C}+2N+N$  cluster model in the condition of  $N_Q=12$ . Table VI shows the resultant eigenstates which are classified into three  $\text{SU}(3)[4443](\lambda\mu)$  and three  $\text{SU}(3)[44421](\lambda\mu)$  labels, together with the results within the framework of the microscopic  $^{12}\text{C}+t$  cluster model. It should be noted that the  $[4443](\lambda\mu)$  state within the framework of  $^{12}\text{C}+2N+N$  is exactly the same as the  $(\lambda\mu)$  state within the framework of  $^{12}\text{C}+t$ , and is orthogonal with the  $[44421](\lambda\mu)$  state. The number of the resultant  $(\lambda\mu)$  representations within the framework of  $^{12}\text{C}+2N+N$  is smaller than that within the framework of  $\alpha+\alpha+2N+N$  (see Tables II and VI). This is due to the fact that the  $^{12}\text{C}$  nuclear part is fixed to the  $\text{SU}(3)(\lambda\mu)=(04)$  representation. Since the doorway  $s$ -hole state of  $^{15}\text{N}$  is produced by quasifree knockout reactions such as  $^{16}\text{O}(p,2p)$ , the  $s$ -hole state produced should have the same spatial symmetry as the ground state of  $^{16}\text{O}$ ,  $\text{SU}(3)(\lambda\mu)=(00)$ . Therefore the two  $(\lambda\mu)=(00)$  representations in Table VI, namely,  $[4443](00)$  and  $[44421](00)$ , which are degenerate in energy, are mainly responsible for the  $^{15}\text{N}(s\text{-hole})$  doorway state.

The calculated spectroscopic factors are shown in Table VII for the fragmentation of the  $\text{SU}(3)[f](\lambda\mu)L^\pi=[4443](00)0^+$  and  $[44421](00)0^+$  eigenstates into the  $^{14}\text{C}+p$ ,  $^{14}\text{N}+n$ ,  $^{13}\text{C}+d$ ,  $^{12}\text{C}+t$ ,  $^{11}\text{B}+\alpha$ ,  $^{10}\text{B}+^5\text{He}$ ,  $^9\text{Be}+^6\text{Li}$ , and  $^8\text{Be}+^7\text{Li}$ , together with their excited channels. They are summarized in Table VIII. One sees the surprising result that the spectroscopic factors for the fragmentation of the two  $(\lambda\mu)=(00)$  eigenstates into the  $^{11}\text{B}+\alpha$ ,  $^{10}\text{B}+^5\text{He}$ ,  $^9\text{Be}+^6\text{Li}$ , and  $^8\text{Be}+^7\text{Li}$  channels are exactly zero, while those into the  $^{14}\text{C}+p$ ,  $^{14}\text{N}+n$ ,  $^{13}\text{C}+d$ , and  $^{12}\text{C}+t$  channels are nonzero. This means that the  $^{15}\text{N}(s\text{-hole})$  doorway state can be fragmented into the less-than- $\alpha$ -cluster channels, but fragmentation into the more-than- $\alpha$ -cluster channels is forbidden. In particular, the result that the  $\alpha$ -cluster fragmentation is forbidden is striking, if we take into account the fact that the  $Q$  value for the  $\alpha$ -cluster fragmentation channel is almost the same as for the proton and neutron channels (see Fig. 3). This is the same result as in the case of the  $^{11}\text{B}(s\text{-hole})$  doorway state (see Sec. III B). Thus the selection rule for fragmentation of doorway  $s$ -hole states in light nuclei is also realized in  $^{15}\text{N}(s\text{ hole})$ .

The reason why the  $^{15}\text{N}(s\text{-hole})$  doorway state can (cannot) be fragmented into the less-than- $\alpha$ -cluster (more-than- $\alpha$ -cluster) channels is as follows. Let us consider the frag-

TABLE VII. Calculated spectroscopic factors ( $S^2$ ) and partial fragmentation widths ( $\Gamma$ ) of  $^{15}\text{N}$  ( $\lambda\mu$ ) $_L$  = (00) $_0$  states.  $T$  denotes the isospin of  $^{14}\text{N}$  or  $^{14}\text{C}$  and  $a$  expresses the channel radius (see the text).

	$Q$ (MeV)	[4443](00)		[44421](00)		$a$ (fm)
		$S^2$	$\Gamma$ (MeV)	$S^2$	$\Gamma$ (MeV)	
$^{14}\text{N}^{T=0}(0^+) + n$	19.2	0.019	0.22	0.096	1.09	4.3
$^{14}\text{N}^{T=0}(2^+) + n$	16.2	0.096	0.78	0.478	3.88	4.0
$^{14}\text{N}^{T=1}(0^+) + n$	16.9	0.006	0.06	0.032	0.34	4.3
$^{14}\text{N}^{T=1}(2^+) + n$	13.6	0.032	0.23	0.159	1.12	4.0
$^{14}\text{C}^{T=0}(0^+) + p$	19.8	0.013	0.14	0.064	1.30	4.3
$^{14}\text{C}^{T=1}(2^+) + p$	16.8	0.064	0.48	0.319	0.20	4.0
$^{13}\text{C}(1^-) + d$	13.8	0.115	0.88	0.144	1.11	3.6
$^{13}\text{C}(3^-) + d$	10.8	0.269	0.74	0.336	0.93	3.2
$^{12}\text{C}(0^+) + t$	15.2	0.036	0.28	0.000	0.00	3.5
$^{12}\text{C}(2^+) + t$	10.7	0.181	1.01	0.000	0.00	3.3
$^{12}\text{C}(4^+) + t$	1.1	0.326	0.00	0.000	0.00	3.0
$^{11}\text{B}(1^-) + \alpha$	19.1	0.000	0.00	0.000	0.00	
$^{11}\text{B}(3^-) + \alpha$	14.4	0.000	0.00	0.000	0.00	
$^{10}\text{B}^{T=0}(0^+) + ^5\text{He}$	4.4	0.000	0.00	0.000	0.00	
$^{10}\text{B}^{T=0}(2^+) + ^5\text{He}$	2.3	0.000	0.00	0.000	0.00	
$^{10}\text{B}^{T=1}(0^+) + ^5\text{He}$	2.7	0.000	0.00	0.000	0.00	
$^9\text{Be}(1^-) + ^6\text{Li}(0^+)$	2.4	0.000	0.00	0.000	0.00	
$^8\text{Be}(0^+) + ^7\text{Li}(1^-)$	7.6	0.000	0.00	0.000	0.00	
$^8\text{Be}(2^+) + ^7\text{Li}(1^-)$	4.7	0.000	0.00	0.000	0.00	
$^8\text{Be}(0^+) + ^7\text{Li}(3^-)$	1.8	0.000	0.00	0.000	0.00	
Total sum			4.83		9.96	

mentation of the  $s$ -hole state into the  $^A_1Z_1 + ^A_2Z_2$  channel, where the  $^A_1Z_1$  and  $^A_2Z_2$  nuclei are assumed to be given by the  $\text{SU}(3)(\lambda_1\mu_1)$  and  $(\lambda_2\mu_2)$  representations, respectively, and the relative part between  $^A_1Z_1$  and  $^A_2Z_2$  is assumed to be given by the  $\text{SU}(3)(\lambda_r\mu_r)$  representation. According to the  $\text{SU}(3)$  group theory, if the  $(\lambda\mu)=(00)$  representation of  $^{15}\text{N}$  is (not) able to be constructed from the three  $(\lambda_1\mu_1)$ ,  $(\lambda_2\mu_2)$ , and  $(\lambda_r\mu_r)$  representations, the fragmentation of the  $s$ -hole state into the  $^A_1Z_1 + ^A_2Z_2$  channel is possible (forbidden). As shown in Table IX, the  $(\lambda\mu)=(00)$  state of  $^{15}\text{N}$  is (not) able to be constructed in the case of the  $^{14}\text{C}+p$ ,  $^{14}\text{N}+n$ ,  $^{13}\text{C}+d$ , and  $^{12}\text{C}+t$  ( $^{11}\text{B}+\alpha$ ,  $^{10}\text{B}+^5\text{He}$ ,  $^9\text{Be}+^6\text{Li}$ , and  $^8\text{Be}+^7\text{Li}$ ) channels, so that the fragmentations of the doorway  $^{15}\text{N}(s\text{-hole})$  state into the  $^{14}\text{C}+p$ ,  $^{14}\text{N}+n$ ,  $^{13}\text{C}+d$ , and  $^{12}\text{C}+t$  ( $^{11}\text{B}+\alpha$ ,

$^{10}\text{B}+^5\text{He}$ ,  $^9\text{Be}+^6\text{Li}$ , and  $^8\text{Be}+^7\text{Li}$ ) channels are allowed (forbidden). It should be noted that the selection rule does not depend on the partition  $[f]$  and therefore the rule is realized in both the [4443] and [44421] cases (see Tables VII and VIII).

The spectroscopic factor of the [44421](00) state for the  $t$ -cluster fragmentation channel is exactly zero, which is in contrast to the case of [4443](00) (see Tables VII and VIII). This is a characteristic of the [44421](00) state which is orthogonal to the [4443](00) one. Reflecting the fact that the [44421] spatial symmetry state has more freedom for the nucleon part outside the  $^{12}\text{C}$  core than does [4443], the values of the spectroscopic factors for the  $^{14}\text{B}+p$  and  $^{14}\text{N}+n$  channels in the case of [44421](00) are larger than those in the case of [4443](00), while those for the  $^{12}\text{C}+t$  ( $^{13}\text{C}+d$ ) channel in the former are smaller than (nearly equal to) those

TABLE VIII. Summary of the calculated partial fragmentation widths of  $^{15}\text{N}$  ( $\lambda\mu$ ) $_L$  = (00) $_0$  states given in Table VII. See the text.

	[4443](00)		[44421](00)	
	$S^2$	$\Gamma$ (MeV)	$S^2$	$\Gamma$ (MeV)
$^{14}\text{N}+n$	0.153	1.29	0.765	6.43
$^{14}\text{C}+p$	0.077	0.62	0.383	1.50
$^{13}\text{C}+d$	0.384	1.62	0.480	2.04
$^{12}\text{C}+t$	0.543	1.29	0.000	0.00
$^{11}\text{B}+\alpha$	0.000	0.00	0.000	0.00
$^{10}\text{B}+^5\text{He}$	0.000	0.00	0.000	0.00
$^9\text{Be}+^6\text{Li}$	0.000	0.00	0.000	0.00
$^8\text{Be}+^7\text{Li}$	0.000	0.00	0.000	0.00
Total sum		4.83		9.96

TABLE IX.  $\text{SU}(3)$ -algebra analysis of whether the  $(\lambda\mu)=(00)$  representation can be constructed from the three  $\text{SU}(3)$  representations  $(\lambda_1\mu_1)$ ,  $(\lambda_2\mu_2)$ , and  $(\lambda_r\mu_r)$ .

$^A_1Z_1 + ^A_2Z_2$	$(\lambda_1\mu_1)$	$(\lambda_2\mu_2)$	$(\lambda_r\mu_r)$	$(\lambda\mu)=(00)$
$^{14}\text{N}+n$	(02)	(00)	(20)	allowed
$^{14}\text{C}+p$	(02)	(00)	(20)	allowed
$^{13}\text{C}+d$	(03)	(00)	(30)	allowed
$^{12}\text{C}+t$	(04)	(00)	(40)	allowed
$^{11}\text{B}+\alpha$	(13)	(00)	(40)	forbidden
$^{10}\text{B}+^5\text{He}$	(22)	(10)	(50)	forbidden
$^9\text{Be}+^6\text{Li}$	(31)	(20)	(50)	forbidden
$^8\text{Be}+^7\text{Li}$	(40)	(30)	(50)	forbidden

in the latter. Thus the  $[44421](00)$  and  $[4443](00)$  eigenstates are responsible mainly for the nucleon and triton fragmentations of the doorway  $s$ -hole state, respectively, and for the deuteron fragmentation both the  $[44421](00)$  and  $[4443](00)$  eigenstates contribute rather equally. This situation is similar to the case of  $^{11}\text{B}$  as mentioned above.

The calculated partial fragmentation widths of the  $[4443](00)$  and  $[44421](00)$  eigenstates are also given in Tables VII and VIII. Reflecting the fact that the spectroscopic factor for the  $^{11}\text{B} + \alpha$  channel ( $Q = 19.1$  MeV) is zero, the  $\alpha$ -cluster fragmentation width becomes exactly zero, and one sees also zero partial widths for the  $^{10}\text{B} + ^5\text{He}$  ( $Q = 4.4$  MeV),  $^9\text{Be} + ^6\text{Li}$  (2.4 MeV), and  $^8\text{Be} + ^7\text{Li}$  (7.6 MeV) channels. This result is greatly in contrast to that of the statistical model, because the model predicts a larger fragmentation width for a fragmentation channel with a larger  $Q$  value. The total sums of the partial widths for  $[4443](00)$  and  $[44421](00)$  are  $\Gamma([4443]) = 4.83$  MeV and  $\Gamma([44421]) = 9.96$  MeV, respectively. Since both the  $[4443](00)$  and  $[44421](00)$  eigenstates contribute to the doorway  $s$ -hole state, the escape width ( $\Gamma^\uparrow$ ) of the  $s$ -hole state of  $^{15}\text{N}$  corresponds to the sum of  $\Gamma([4443])$  and  $\Gamma([44421])$  and thus  $\Gamma^\uparrow \approx 15$  MeV, which is nearly equal to the experimental result ( $\approx 14$  MeV). Then, the proton, neutron, deuteron, and triton fragmentation widths amount, respectively, to 14%, 52%, 25%, and 9% of the calculated escape width ( $\Gamma^\uparrow \approx 15$  MeV), which results are almost the same for the  $^{11}\text{B}(s \text{ hole})$  qualitatively. The calculated spreading width of  $^{15}\text{N}(s \text{ hole})$  ( $\Gamma^\uparrow \approx 15$  MeV) is larger than that of  $^{11}\text{B}(s \text{ hole})$  ( $\Gamma^\uparrow \approx 7$  MeV). This is due to the fact that the  $Q$  values for the proton, neutron, deuteron, and triton channels of  $^{15}\text{N}(s \text{ hole})$  are about twice larger than those of  $^{11}\text{B}(s \text{ hole})$ .

In this paper, we assumed the main component of the  $^{15}\text{N}(s\text{-hole})$  doorway state to be given by the  $\text{SU}(3)(\lambda\mu) = (00)$  state. However, the spatial symmetry states such as  $[4443](22)$  and  $[44421](22)$  might be mixed in the doorway  $s$ -hole state ( $L^\pi = 0^+$ ) of  $^{15}\text{N}$  (see Table VI). Even if this occurs, the above-mentioned selection rule for fragmentation of doorway  $s$ -hole states in light nuclei persists for the following reasons. According to the shell model calculation, the component of the  $[4443](22)$  and  $[44421](22)$  eigenstates contributes mainly to the low-lying positive parity states around  $E_x \approx 5\text{--}8$  MeV. Therefore the admixture of  $[4443](22)$  and  $[44421](22)$  in the doorway  $s$ -hole state becomes small, although the spatial symmetry states,  $[4443](22)$  and  $[44421](22)$ , have a large  $\alpha$ -decay width. This fact suggests that the  $\alpha$ -cluster fragmentation of the  $^{15}\text{N}(s\text{-hole})$  state is suppressed considerably in the actual situation.

Concerning the three-body fragmentation channels, the largest and second largest  $Q$  value channels are the  $^{13}\text{C} + p + n$  ( $Q = 12.0$  MeV) and  $^7\text{Li} + \alpha + \alpha$  (10.6 MeV) channels (see Fig. 3). Moreover, the four-body fragmentation channel with the largest  $Q$  value is the  $\alpha + \alpha + \alpha + t$  channel ( $Q = 8.3$  MeV). Therefore, the three-body and four-body fragmentation channels appear above the  $^{14}\text{C} + p$  (19.8 MeV),  $^{14}\text{B} + n$  (19.2 MeV),  $^{13}\text{C} + d$  (13.8 MeV),  $^{12}\text{C} + t$  (15.2 MeV), and  $^{11}\text{B} + \alpha$  (19.1 MeV) channels. This is in contrast to the case of  $^{11}\text{B}$  in which the  $\alpha + \alpha + t$  channel which has the largest  $Q$  value among three-body channels appears

around the two-body fragmentation channels of  $^{10}\text{Be} + p$ ,  $^{10}\text{B} + n$ ,  $^9\text{Be} + d$ ,  $^8\text{Be} + t$ , and  $^7\text{Li} + \alpha$ . Since the phase space of the three-body and four-body fragmentation channels is not very large, it seems that the contribution from the three-body and four-body fragmentation channels to the  $\alpha$  fragmentation width of  $^{15}\text{N}(s \text{ hole})$  does not become very large.

### C. Application of the selection rule to hypernuclei

A characteristic of the (in-flight  $K^-, \pi^-$ ) hypernuclear production reaction is the selective population of the substitutional states such as  $(s_N^{-1}s_\Lambda)$  and  $(p_N^{-1}p_\Lambda)$  because of the small recoil momentum transfer for the produced  $\Lambda$  particle ( $q < 50$  MeV/c) [7,21,22]. According to the experimental data for the  $^{12}\text{C}(K^-, \pi^-)^{12}_\Lambda\text{C}$  reaction performed by Grace *et al.* [23], the  $^{12}_\Lambda\text{C}(s_N^{-1}s_\Lambda)$  state which appears at  $E_\Lambda \approx 10$  MeV with respect to the  $\Lambda + ^{11}\text{C}$  threshold emits one light hypernuclear fragment from which weak decay occurs. The emitted light hyperfragment is not identified experimentally. However, it is considered that the substitutional state is fragmented into  $^9_\Lambda\text{Be}$  and  $^3\text{He}$ , since the experimental weak decay lifetime of the emitted unknown hypernucleus suggests that it may be the  $^9_\Lambda\text{Be}$  hypernucleus.

Here, an interesting question arises as to why  $^{12}_\Lambda\text{C}(s_N^{-1}s_\Lambda)$  with the  $[^{11}\text{C}(s \text{ hole}) \otimes s_\Lambda]$  configuration is fragmented into the  $^9_\Lambda\text{Be} + ^3\text{He}$  channel, in spite of the fact that the  $Q$  value of  $^8_\Lambda\text{Be} + \alpha$  is larger than that of  $^9_\Lambda\text{Be} + ^3\text{He}$  [11,24]. According to the statistical model, the  $^8_\Lambda\text{Be} + \alpha$  fragmentation rate should become larger than the  $^9_\Lambda\text{Be} + ^3\text{He}$  one because the larger  $Q$  value gives a larger fragmentation rate. However, the experimental result is reverse. This problem was discussed by Bando, one of the current authors (T.Y.), and Žofka [24]. They showed that the nuclear-core  $^{11}\text{C}(s\text{-hole})$  state plays an important role in the fragmentation of the  $^{12}_\Lambda\text{C}(s_N^{-1}s_\Lambda)$  state. The  $^{11}\text{C}(s\text{-hole})$  state has the same spatial symmetry as the  $^{11}\text{B}(s\text{-hole})$  state, since the two nuclei are mirror nuclei of each other. Therefore, the selection rule for fragmentation of  $^{11}\text{B}(s \text{ hole})$  as discussed in Sec. III B should be realized also for the  $^{11}\text{C}(s\text{-hole})$  state, namely, that the  $^{11}\text{C}(s\text{-hole})$  state can be fragmented into  $^8\text{Be} + ^3\text{He}$  but the fragmentation of the  $^7\text{Be} + \alpha$  channel is largely suppressed, reflecting the characteristic of the  $s$ -hole state which has mainly  $\text{SU}(3)(\lambda\mu) = (04)$  spatial symmetry. Thus, the fragmentation of the  $^{12}_\Lambda\text{C}(s_N^{-1}s_\Lambda)$  state into the  $^9_\Lambda\text{Be} + ^3\text{He}$  channel is allowed, while that into the  $^8_\Lambda\text{Be} + \alpha$  channel is largely hindered. This result is consistent with Grace *et al.*'s experimental data. It might suggest evidence for our selection rule for fragmentation of  $s$ -hole states in light nuclei.

Further evidence for our selection rule may be the twin- $\Lambda$  hypernuclear production in a  $\Xi^-$  atomic capture reaction into  $(^{12}\text{C}, \Xi^-)_{\text{atom}} \rightarrow ^9_\Lambda\text{Be} + ^4_\Lambda\text{H}$  [25,26]. The number of twin- $\Lambda$  hypernuclear production channels below the  $^{12}\text{C} + \Xi^-$  threshold is only 3,  $^8_\Lambda\text{Li} + ^5_\Lambda\text{He}$  ( $Q = 13.6$  MeV),  $^9_\Lambda\text{Be} + ^4_\Lambda\text{H}$  ( $Q = 10.0$  MeV), and  $^{10}_\Lambda\text{Be} + ^3_\Lambda\text{H}$  ( $Q = 6.0$  MeV), where the  $Q$  value is measured from the  $^{12}\text{C} + \Xi^-$  threshold appearing at  $E_x = 39.5$  MeV from the ground state of the double- $\Lambda$  hypernucleus  $^{13}_{\Lambda\Lambda}\text{B}$ . According to the hypernuclear compound model based on the statistical picture, a twin- $\Lambda$  hypernuclear channel with

a larger (smaller)  $Q$  value has a larger (smaller) production rate. Therefore the production rate of  ${}^8_\Lambda\text{Li} + {}^5_\Lambda\text{He}$  should be larger than that of  ${}^9_\Lambda\text{Be} + {}^4_\Lambda\text{H}$ , since the  $Q$  value of the former channel is larger than that of the latter. However, the experimental result is the reverse. This problem was discussed by two of the present authors (T.Y. and K.I.) [11]. Our scenario is the following. The binding energy of an  $s$ -state proton in  ${}^{12}\text{C}$  is approximately equal to the  $Q$  value of the elementary process,  $p + \Xi^- \rightarrow \Lambda + \Lambda$ . Therefore, a  $\Xi^-$  particle in an atomic orbit interacts with an  $s$ -state proton in  ${}^{12}\text{C}$ , so that both the  $\Xi^-$  particle and proton convert to two  $\Lambda$  particles to create the proton  $s$ -hole in the nuclear core [ ${}^{11}\text{B}(s \text{ hole})$ ] and then the highly excited  ${}^{13}_{\Lambda\Lambda}\text{B}$  with the [ ${}^{11}\text{B}(s \text{ hole}) \otimes \Lambda \otimes \Lambda$ ] configuration is produced as an intermediate state. Reflecting the selection rule for fragmentation of the  ${}^{11}\text{B}(s\text{-hole})$  state, the intermediate state of  ${}^{13}_{\Lambda\Lambda}\text{B}$  is fragmented mainly into the  ${}^9_\Lambda\text{Be} + {}^4_\Lambda\text{H}$  channel, while the  ${}^8_\Lambda\text{Li} + {}^5_\Lambda\text{He}$  channel is largely suppressed. For the fragmentation into the  ${}^{10}_\Lambda\text{Be} + {}^3_\Lambda\text{H}$  channel, it is possible. However, the production rate of  ${}^{10}_\Lambda\text{Be} + {}^3_\Lambda\text{H}$  becomes smaller because of the smaller phase space which corresponds to the smaller  $Q$  value. Therefore, the twin- $\Lambda$  hypernuclear production events found in the  $\Xi^-$  atomic capture experiment might suggest further evidence of the selection rule for fragmentation of  $s$ -hole states in light nuclei.

#### IV. CONCLUSION

The fragmentation characteristics of the doorway  $s$ -hole states in light nuclei such as  ${}^{11}\text{B}$  and  ${}^{15}\text{N}$  have been investigated by calculating the spectroscopic factors and partial widths for the fragmentation of the doorway  $s$ -hole state into two-body cluster systems. In the present paper, the  ${}^{11}\text{B}(s\text{-hole})$  and  ${}^{15}\text{N}(s\text{-hole})$  doorway states were given by the  $\text{SU}(3)(\lambda\mu)=(04)$  and  $(00)$  representations, respectively, since the main components of the  ${}^{12}\text{C}$  and  ${}^{16}\text{O}$  ground-state wave functions are given by the  $\text{SU}(3)(\lambda\mu)=(04)$  and  $(00)$  representations, respectively, and the  $s$ -hole states produced by quasifree knockout reactions such as  $(p, 2p)$  and  $(e, e'p)$ , etc., for the targets of  ${}^{12}\text{C}$  and  ${}^{16}\text{O}$  should have the same spatial symmetry as that of the target nucleus.

A notable selection rule for fragmentations of doorway  $s$ -hole states in light nuclei was found, namely, that in the two-body fragmentation process of the doorway  $s$ -hole states, smaller fragments than the  $\alpha$  particle ( $p$ ,  $n$ ,  $d$ ,  $t$ , and  ${}^3\text{He}$ ) are allowed, while the fragments such as the  $\alpha$  particle and the larger particles are forbidden, in spite of the fact that the  $Q$  value for the  $\alpha$  fragment is similar to or even larger than those for the  $p$ ,  $n$ , and  $d$  fragments. For example, in the  ${}^{11}\text{B}(s\text{-hole})$  doorway state, the fragmentations to the  ${}^{10}\text{Be} + p$  ( $Q=8.5$  MeV),  ${}^{10}\text{B} + n$  (8.2 MeV),  ${}^9\text{Be} + d$  (3.9 MeV), and  ${}^8\text{Be} + t$  (8.5 MeV) channels are possible, while the fragmen-

tations to  ${}^7\text{Li} + \alpha$  (11.0 MeV) and  ${}^6\text{Li} + {}^5\text{He}$  (3.0 MeV) are largely suppressed. These results are in contrast to the results of the statistical model, in which a fragmentation channel with larger  $Q$  value has a larger fragmentation width. The selection rule comes from the spatial symmetry of the doorway  $s$ -hole states.

The calculated escape widths for the  ${}^{11}\text{B}(s\text{-hole})$  and  ${}^{15}\text{N}(s\text{-hole})$  states are about 7 and 15 MeV, respectively, which are in good correspondence with the experimental widths [ $\Gamma^{\text{expt}}({}^{11}\text{B}) \approx 9$  MeV and  $\Gamma^{\text{expt}}({}^{15}\text{N}) \approx 14$  MeV]. This result may support our fragmentation scenario for the  $s$ -hole states in light nuclei, that the fragmentation process of the  $s$ -hole state in the doorway stage is superior to or competitive with that in the compound nuclear stage, as shown in the analysis of the ratios of the nuclear radius ( $R$ ) to the mean free path ( $\lambda$ ) of the  $s$ -hole state,  $(R/\lambda)$ . The partial fragmentation widths of the calculated escape width  $\Gamma^\uparrow$  are as follows: in the  ${}^{11}\text{B}(s\text{-hole})$  [ ${}^{15}\text{N}(s\text{-hole})$ ] state,  $\Gamma_p/\Gamma^\uparrow=19\%$  (14%),  $\Gamma_n/\Gamma^\uparrow=38\%$  (52%),  $\Gamma_d/\Gamma^\uparrow=20\%$  (25%),  $\Gamma_t/\Gamma^\uparrow=23\%$  (9%), and  $\Gamma_\alpha/\Gamma^\uparrow=0\%$  (0%).

The realistic wave functions of the  ${}^{11}\text{B}(s\text{-hole})$  and  ${}^{15}\text{N}(s\text{-hole})$  states may deviate considerably from the pure  $\text{SU}(3)$  eigenstate. However, the selection rule is clear enough to persist in reality, since the zeroth wave function of the  ${}^{11}\text{B}(s\text{-hole})$  and  ${}^{15}\text{N}(s\text{-hole})$  states should be given by the  $\text{SU}(3)$  eigenstates.

We applied the selection rule to hypernuclei. Two experimental observations, (1) the fragmentation of the  ${}^{12}\text{C}(s_N^{-1}s_\Lambda)$  state into the  ${}^9_\Lambda\text{Be} + t$  channel and (2) the fragmentation of the  $({}^{12}\text{C}, \Xi^-)_{\text{atom}}$  system into the  ${}^9_\Lambda\text{Be} + {}^4_\Lambda\text{He}$  channel, might show evidence of the selection rule for fragmentation of  $s$ -hole states in light nuclei.

Unfortunately, at the present stage, there is no experimental information on the selection rule and also on how the deep hole state is fragmented in light nuclei, statistically or nonstatistically. Therefore, it is important and attractive to study experimentally the fragmentation process of  $s$ -hole states. Such an experiment will open one of the new frontiers of physics, the fragmentation of  $s$ -hole states, and also give fruitful information to develop hypernuclear and other physics. It is greatly hoped that a  $(p, 2px)$  triple coincident experiment ( $x$  denotes  $p$ ,  $n$ ,  $d$ ,  $t$ ,  ${}^3\text{He}$ , and  $\alpha$ , etc.) will be done in the near future, in order to clarify the fragmentation mechanism of  $s$ -hole states in light nuclei.

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- [1] H. Tyrén, S. Kullander, O. Sundberg, R. Ramachandran, P. Isacsson, and T. Berggren, Nucl. Phys. **79**, 321 (1966).  
 [2] G. Landaud, J. Yonnet, S. Kullander, F. Lemeilleur, P. U. Renberg, B. Fagerström, A. Johansson, and G. Tibell, Nucl. Phys. **A173**, 337 (1971); S. Kullander, F. Lemeilleur, P. U. Renberg,

- G. Landaud, J. Yonnet, B. Fagerström, A. Johansson, and G. Tibell, *ibid.* **A173**, 357 (1971).  
 [3] G. Jacob and T. A. J. Maris, Rev. Mod. Phys. **45**, 6 (1973).  
 [4] J. Mougey, J. Phys. Soc. Jpn. Suppl. **44**, 420 (1978).  
 [5] S. Fullani and J. Mougey, Adv. Nucl. Phys. **19**, 67 (1989).

- [6] B. Fagerström, J. Källne, O. Sundberg, and G. Tibell, Phys. Scr. **13**, 101 (1976).
- [7] R. Bertini, O. Bing, P. Birien, K. Braune, W. Brückner, A. Chaumeaux, M. A. Faessler, R. W. Frey, D. Garreta, T. J. Ketel, K. Kilian, B. Mayer, J. Niewisch, B. Pietrzyk, B. Povh, H. G. Ritter, and M. Uhrmacher, Nucl. Phys. **A368**, 365 (1981).
- [8] W. Fritsch, R. Lipperheide, and U. Wille, Nucl. Phys. **A241**, 79 (1975).
- [9] H. Orland and R. Schaeffer, Nucl. Phys. **A229**, 442 (1978); J. Phys. (Paris) Colloq. **39**, C3 115 (1978).
- [10] R. Sartor and C. Mahaux, Phys. Rev. C **21**, 2613 (1980).
- [11] T. Yamada and K. Ikeda, Prog. Theor. Phys. Suppl. No. 117, 445 (1994).
- [12] J. P. Elliot, Proc. R. Soc. London **245**, 128 (1958); **245**, 562 (1958); J. P. Elliot and M. Harvey, *ibid.* **272**, 557 (1963).
- [13] M. Harvey, Adv. Nucl. Phys, **1**, 67 (1973).
- [14] W. Hornyak, *Nuclear Structure* (Academic, New York, 1975), p. 1.
- [15] H. S. Köhler, Nucl. Phys. **88**, 529 (1966).
- [16] B. F. Bayman and A. Bohr, Nucl. Phys. **9**, 596 (1958/59).
- [17] H. Horiuchi, Prog. Theor. Phys. Suppl. No. 68, Chap. III (1980).
- [18] For example, N. K. Glendenning, Phys. Rev. **137**, B102 (1965).
- [19] K. Nakamura, S. Hiramatsu, T. Kamae, H. Muramatsu, N. Izutsu, and Y. Watase, Nucl. Phys. **A268**, 381 (1976).
- [20] J. Mougey, M. Bernheim, A. Bussiere De Nercy, A. Gillebert, Phan Xuan Ho, M. Priou, D. Royer, I. Sick, and G. J. Wagner, Nucl. Phys. **A262**, 461 (1976).
- [21] W. Brückner *et al.*, Phys. Lett. **55B**, 107 (1975); **62B**, 481 (1976); **79B**, 157 (1978).
- [22] R. Bertini *et al.*, Nucl. Phys. **A360**, 315 (1980).
- [23] R. Grace *et al.*, Phys. Rev. Lett. **55**, 1055 (1985); P. D. Barnes, Nucl. Phys. **A450**, 43c (1986).
- [24] H. Bandō, T. Yamada, and J. Žofka, Phys. Rev. C **36**, 1640 (1987).
- [25] S. Aoki *et al.*, Prog. Theor. Phys. **89**, 493 (1993).
- [26] K. Nakazawa, Nucl. Phys. **A585**, 75c (1995); S. Aoki *et al.*, Phys. Lett. B **355**, 45 (1995).