Fermion dynamical symmetry model for the even-even and even-odd nuclei in the Xe-Ba region

Xing-Wang Pan,[1] Jia-Lun Ping,[1,2] Da Hsuan Feng,[1] Jin-Quan Chen,[3] Cheng-Li Wu,[4] and Michael W. Guidry[5,6]
[1]Department of Physics, Drexel University, Philadelphia, Pennsylvania 19104
[2]Physics Department, Nanjing Normal University, Nanjing, 210097, People's Republic of China
[3]Physics Department, Nanjing University, Nanjing, 210008, People's Republic of China
[4]Department of Physics, Chung Yuan Christian University, Chung-Li, Taiwan 32023, Republic of China
[5]Department of Physics, University of Tennessee, Knoxville, Tennessee 37996-1200
[6]Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831

(Received 27 July 1995)

The even-even and even-odd nuclei ^{126}Xe–^{132}Xe and ^{131}Ba–^{137}Ba are shown to have a well-realized $SO_8 \supset SO_6 \supset SO_3$ fermion dynamical symmetry. Their low-lying energy levels can be described by a unified analytical expression with two (three) adjustable parameters for even-odd (even-even) nuclei that is derived by using the vector and spinor representations of the fermion model. Analytical expressions are given for wave functions and for $E2$ transition rates that agree well with data. The distinction between the fermion dynamical symmetry model (FDSM) and the interacting boson model (IBM) SO_6 limits is discussed. The experimentally observed suppression of the energy levels with increasing SO_5 quantum number τ can be explained as a perturbation of the pairing interaction on the SO_6 symmetry, which leads to an SO_5 pairing effect for SO_6 nuclei.

PACS number(s): 21.60.Ev, 27.60.+j

I. INTRODUCTION

In the past few decades, extremely rich experimental data have accumulated for low-lying nuclear spectroscopy. The observed levels are interwoven in a rich and complicated manner, and understanding them is a challenging problem. The low-energy spectroscopy of even-even, medium-heavy, and heavy nuclei can now be explained rather well by the interacting boson model (IBM) [1]. The first attempt to describe on the same footing the spectroscopy of both even-even and even-odd nuclei in an analytical way is due to Iachello [2], through the concept of supersymmetry.

In this paper, we shall use the terminology NSUSY to denote nuclear dynamical supersymmetry. NSUSY is an outgrowth of the phenomenological IBM that treats fermions and bosons as basic building blocks and identifies the even-even and even-odd collective nuclear states as multiplets of a higher symmetry described by a supergroup U(6/4), U(6/12), or U(6/20) [2–5]. The decomposition of U(6/4) [U(6/20)] contains $U^B(6)\times U^F(4)$ [$U^B(6)\times U^F(20)$] with $U^B(6)$ referring to the six bosons and $U^F(4)$ [$U^F(20)$] to the odd fermion moving in a single-j level of $j=3/2$ (multi-j levels with $j=1/2$, $j=3/2$, $j=5/2$, and $j=7/2$). Later, Jolie *et al.* [6] proposed a new reduction scheme for the supergroup U(6/20) and applied it to the $A=130$ mass region. They considered the single-particle orbits 1/2, 3/2, 5/2, and 7/2 resulting from the coupling of a pseudo-orbital $l=2$ part and a pseudospin $s=3/2$. Instead of the group chain $U(6/20) \supset U^B(6)\times U^F(4)$ [5], they suggested use of the group chain

$$U(6/20)\supset U^B(6)\times U^F(20)\supset O^B(6)\times U^F(4)\times U^F(5)\supset \mathrm{spin}(6)\times U^F(5)\supset \mathrm{spin}(5)\times O^F(5)\supset \overline{\mathrm{spin}}(5)\supset \mathrm{spin}(3). \quad (1)$$

NSUSY has had some successes (generally up to 15–30 % accuracy for spectroscopic fitting for a few nuclei). But the basic building blocks of nuclei are fermions, and the s and d bosons in the IBM are supposedly simulations of coherent nucleon pairs with angular momenta 0 and 2. Therefore, it is a simplification of the real situation to introduce $U^B(6)$ and $U^F(4)$ [or $U^F(5)$, and $U^F(20)$] as a direct product $U^B(6)\times U^F(4)$ [or $U^B(6)\times U^F(20)$, or $U^B(6)\times U^F(4)\times U^F(5)$].

The fermion dynamical symmetry model (FDSM) [7,8] is defined by a fermionic Lie algebra. It has symmetry limits analogous to the IBM limits and takes Pauli principle into account [9,10]. Furthermore, the states for even and odd systems in the FDSM belong to vector and spinor representations, respectively, of the same SO_8 or Sp_6 group; thus, one can describe even-even and even-odd nuclei in the FDSM without the concept of supersymmetry or additional degrees of freedom.

Two general regions are thought to exhibit some level of supersymmetry in the properties of low-lying nuclear states: the Pt region [5,11] and the $A=130$ (Xe-Ba) region [12]. In the Pt region, the normal-parity valence protons and neutrons are in the sixth and seventh shells, respectively, and according to the FDSM [8] they have $SO_8^\pi\times Sp_6^\nu$ symmetry, which does not permit an analytical solution for the proton-neutron coupled system. However it could have effective SO_6-like symmetry as we have shown in [13]. On the other hand, nuclei in the Xe-Ba region have both their neutrons and their protons in the sixth shell with FDSM pseudo-orbital angular momentum $k=2$ and pseudospin $i=3/2$ for the normal-parity levels. Thus they are expected to possess $SO_8^{\pi}\times SO_8^{\nu}$ symmetry, which contains coupled SO_8 symmetry (the FDSM analog of IBM-1), and have analytic solutions for the $SO_5\times SU_2$, SO_6 and SO_7 dynamical symmetries. In fact, there is now empirical evidence [12] that nuclei in the $A=130$ region are better portrayed by an SO_6 limit than the Pt region. For these reasons, we have chosen the Xe-Ba region to discuss the possibility of a simplified and unified description for even-even and even-odd nuclei by the FDSM as an alternative to the IBM and to NSUSY.

The organization of the paper is as follows: energy formulas for both even-even and even-odd nuclei and a comparison with data are given in Sec. II, the respective wave functions are constructed in Sec. III, the electromagnetic transitions are discussed in Sec. IV, and conclusions are presented in Sec. V.

II. THE ENERGY SPECTRA

In the simplest version of the FDSM, the numbers of nucleons in the normal and abnormal orbits are fixed for a given nucleus and therefore the quasispin group SU_2 for the abnormal levels plays no explicit dynamical role for low-lying states (it enters implicitly through the conservation of particle number and through effective interaction parameters). The wave functions for both even and odd nuclei are given by the following group chain:

$$\begin{array}{cccccc} (SO_8^i\supset & SO_6^i\supset & SO_5^i\times & SO_5^k\supset & SO_5^{i+k}\supset & SO_3^{k+i} \\ [l_1l_2l_3l_4] & [\sigma_1\sigma_2\sigma_3] & [\tau_1\tau_2] & [\tau 0] & [\omega_1\omega_2] & J, \end{array} \tag{2}$$

where $[l_1l_2l_3l_4]$, $[\sigma_1\sigma_2\sigma_3]$, and $[\tau_1\tau_2]$ are the Cartan-Weyl labels for the groups SO_8, SO_6, and SO_5, respectively, $\tau=0(1)$ for even (odd) nuclei, and the superscript indices k and i indicate pseudo-orbital and pseudospin parts of the groups, respectively. The pseudo-orbital $\vec{k}$ and pseudospin $\vec{i}$ compose single-particle angular momentum $\vec{j}$ [9,8]. We note the resemblance between Eq. (2) and the NSUSY group chain Eq. (1). The FDSM Hamiltonian is

$$H_{\rm FDSM}=\varepsilon_1 n_1+G_0S^{\dagger}S+G_2D^{\dagger}\cdot D+\sum_{r=1}^{3}B_rP^r(i)\cdot P^r(i)$$
$$+\sum_{r=1,3}[\mathscr{B}_rP^r(k)\cdot P^r(k)+2b_rP^r(i)\cdot P^r(k)], \tag{3}$$

where ε_1 is the energy for the normal-parity orbits (assumed degenerate) and n_1 is the number of nucleons in the normal-parity orbits,

$$S^{\dagger}=A^{0\dagger},\quad D_{\mu}^{\dagger}=A_{\mu}^{2\dagger},$$
$$A_{\mu}^{r\dagger}=\sqrt{\Omega_1/2}[b_{ki}^{\dagger}b_{ki}^{\dagger}]^{0r}_{0\mu},\quad r=0,2, \tag{4}$$

where $k=2$, $i=3/2$, and $\Omega_1\equiv\Omega_{ki}=(2k+1)(2i+1)/2$. Similarly,

$$P_{\mu}^r(i)=\sqrt{\Omega_1/2}[b_{ki}^{\dagger}\tilde{b}_{ki}]^{0r}_{0\mu},\quad r=0,1,2,3, \tag{5}$$

$$\bar{P}_{\mu}^r(k)=(-)^{[r/2]}\sqrt{8/5}P_{\mu}^r(k),$$
$$P_{\mu}^r(k)=\sqrt{\Omega_1/2}[b_{ki}^{\dagger}\tilde{b}_{ki}]^{r0}_{\mu 0},\quad r=0,1,2,3, \tag{6}$$

where $[r/2]$ is the integer part of $r/2$. The operators $P_{\mu}^r(i)$ and $\bar{P}_{\mu}^r(k)$ for $r=1$ and 3 form the Lie algebras SO_5^i and SO_5^k, respectively. The commutators among the $P_{\mu}^r(i)$ are given by Eq. (3.12) of [7] for the i-active case and those for $\sqrt{\Omega_1/2}[b_{ki}^{\dagger}\tilde{b}_{ki}]^{r0}_{\mu 0}$ can be obtained from Eq. (3.12) of [7] for the k-active case with

$$\sqrt{3}\begin{Bmatrix} r & s & t\\ 1 & 1 & 1\end{Bmatrix}\to\sqrt{5}\begin{Bmatrix} r & s & t\\ 2 & 2 & 2\end{Bmatrix}.$$

In Eq. (6) we have renormalized the multipole operators $\bar{P}_{\mu}^r(k)$ so that they are isomorphic with $P_{\mu}^r(i)$. Furthermore, $P_{\mu}^1(i)$ and $\bar{P}_{\mu}^1(k)$ are related to the total pseudo-orbital angular momentum and pseudospin by

$$P_{\mu}^1(i)=\frac{1}{\sqrt{5}}I_{\mu},\quad \bar{P}_{\mu}^1(k)=\frac{1}{\sqrt{5}}L_{\mu}. \tag{7}$$

By using the Casimir operators of SO_8, SO_6, and SO_5, the Hamiltonian of Eq. (3) can be rewritten as

$$H_{\rm FDSM}=H_0+\epsilon_1n_1+g_SS^{\dagger}\cdot S+g_6C_{SO_6^i}+g_5C_{SO_5^{k+i}}+g_5^iC_{SO_5^i}$$
$$+g_5^kC_{SO_5^k}+g_I\mathbf{I}^2+g_L\mathbf{L}^2+g_J\mathbf{J}^2, \tag{8}$$

where the total angular momentum is $\mathbf{J}=\mathbf{I}+\mathbf{L}$, and

$$H_0=\tfrac{1}{4}(n_1)^2+G_2[C_{SO_8^i}-S_0(S_0-6)],$$
$$S_0=\tfrac{1}{2}(n_1-\Omega_1),$$
$$g_S=(G_0-G_2),\quad g_6=(B_2-G_2),\quad g_5=b_3,$$
$$g_5^i=g_5'-g_5,\quad g_5'=B_3-B_2,\quad g_5^k=\mathscr{B}_3-b_3,$$
$$g_I=g_I'-g_J,\quad g_I'=\tfrac{1}{5}(B_1-B_3),$$
$$g_J=\tfrac{1}{5}(b_1+b_3),\quad g_L=\tfrac{1}{5}(\mathscr{B}_1-\mathscr{B}_3)-\tfrac{1}{8}g_J. \tag{9}$$

The eigenvalue of C_{SO_8} is

$$C_{SO_8}=\sum_{i=1}^{4}l_i(l_i+8-2i), \tag{10}$$

and the condition for the realization of the symmetry is $g_S=0$, implying that

$$G_0=G_2. \tag{11}$$

The low-lying states of even-even nuclei belong to the SO_8 irrep $[\Omega_1/4\ \Omega_1/4\ \Omega_1/4\ \Omega_1/4]$, i.e., the irrep with heritage $u=0$, u being the number of valence nucleons not contained in S and D pairs [8,9]. By letting $G_0=G_2$ and $\mathscr{B}_i=b_i=0$ for $i=1,3$, from Eqs. (8)–(10) we have

$$H^{\rm even}=E_0^{(e)}+g_6C_{SO_6^i}+g_5'C_{SO_5^i}+g_I'\mathbf{I}^2, \tag{12}$$

where

$$E_0^{(e)}=\tfrac{1}{4}(1-G_2)(n_1)^2+[\tfrac{1}{2}G_2(\Omega_1+6)+\varepsilon_1]n_1. \quad (13)$$

For odd nuclei the low-lying states are expected to belong to the SO_8 irrep $[\Omega_1/4\ \ \Omega_1/4\ \ \Omega_1/4\ \ \Omega_1/4-1]$, corresponding to heritage $u=1$. The conditions for realizing the symmetry of Eq. (2) are Eq. (11) and $g_I=0$, which implies that

$$B_1-B_3=b_1+b_3. \quad (14)$$

Under the conditions of Eqs. (11) and (14), and noting that for $u=1$ the pseudo-orbital angular momentum is $\mathbf{L}^2=2(2+1)=6$, we have

$$H^{\text{odd}}=E_0^{(o)}+g_6C_{SO_6^i}+(g_5'-g_5)C_{SO_5^i}+g_5C_{SO_5^{i+k}}+g_J\mathbf{J}^2, \quad (15)$$

$$E_0^{(o)}=E_0^{(e)}+4g_5^k+6g_L+\tfrac{1}{2}G_2(2-\Omega_1). \quad (16)$$

Using the eigenvalue formulas for the SO_6 and SO_5 Casimir operators, the energies for even and odd systems are

$$E^{\text{even}}=E_0^{(e)}+g_6\sigma(\sigma+4)+g_5'\tau(\tau+3)+g_I'J(J+1), \quad (17)$$

where $J(J+1)$ is used instead of $I(I+1)$ (since $L=0$, and thus $J=I$), and

$$E^{\text{odd}}=E_0^{(o)}+g_6[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+(\sigma_3)^2]+g_JJ(J+1)+(g_5'-g_5)[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+g_5[\omega_1(\omega_1+3)+\omega_2(\omega_2+1)]. \quad (18)$$

The reduction rules are as follows [2]:

$$\begin{array}{lllll} \text{even system:} & SO_8\supset & SO_6 & \supset SO_5 & \supset SO_3 \\ & [wwww] & [\sigma 00] & [\tau 0] & J, \end{array} \quad (19)$$

$$w=\frac{\Omega_1}{4}, \quad \sigma=N_1,N_1-2,\ldots,1 \text{ or } 0,$$

$$\tau=\sigma,\sigma-1,\ldots,0, \quad \tau=3n_\Delta+\lambda,$$

$$n_\Delta=0,1,2,\ldots, \quad J=\lambda,\lambda+1,\ldots,2\lambda-2,2\lambda, \quad (20)$$

with $N_1=n_1/2$, where n_1 is the number of the nucleons in the normal parity levels, and

$$\begin{array}{lllllll} \text{odd system:} & (SO_8^i\supset & SO_6^i\supset & SO_5^i\times & SO_5^k\supset & SO_5^{k+i}\supset & SO_3^{k+i} \\ & [www,w-1] & [\sigma+\frac{1}{2},\frac{1}{2}\ \frac{1}{2}] & [\tau+\frac{1}{2},\frac{1}{2}] & [10] & [\omega_1\omega_2] & J, \end{array} \quad (21)$$

$$\sigma=N_1,N_1-1,\ldots,1,0, \quad \tau=\sigma,\sigma-1,\ldots,0,$$

where $N_1=[n_1/2]$. The relevant Clebsch-Gordan series for SO_5 are [14]

$$[\tau_1\tfrac{1}{2}]\times[10]=[\tau_1+1,\tfrac{1}{2}]+[\tau_1\tfrac{3}{2}]+[\tau_1\tfrac{1}{2}]+[\tau_1-1,\tfrac{1}{2}],$$

$$[\tfrac{1}{2}\tfrac{1}{2}][10]=[\tfrac{3}{2}\tfrac{1}{2}]+[\tfrac{1}{2}\tfrac{1}{2}] \quad (22)$$

and the SO_3 content of the SO_5 irreps $[\omega_1\omega_2]$ is [2]

$$J=[2(\omega_1-\omega_2)-6\nu_\Delta+\tfrac{3}{2}],[2(\omega_1-\omega_2)-6\nu_\Delta+\tfrac{1}{2}],\ldots,$$

$$[(\omega_1-\omega_2)-3\nu_\Delta-\tfrac{1}{4}[1-(-)^{2\nu_\Delta}+\tfrac{3}{2}], \quad (23)$$

$$\nu_\Delta=0,\tfrac{1}{2},1,\tfrac{3}{2},\ldots. \quad (24)$$

The above discussions adopt as a simplification that the numbers of valence nucleon pairs in the normal- and abnormal-parity levels, N_1 and N_0, are fixed. In reality N_1 or N_0 have a distribution and the semiempirical formula of [8] has been used to obtain N_1, which may generally take noninteger values to simulate an average behavior of the nuclear states with different values of N_1. For computing the spectra and the $B(E2)$ values, we have taken the nearest integer to the noninteger number.

The low-lying energy spectra for $^{120-132}$Xe isotopes predicted by Eq. (17) are compared with data in Figs. 1 and 2, and the parameters used in the calculations are given in Table I. The experimental spectra indicate that the SO_3 parameter g_I' is not sensitive to the neutron number in fitting the spectra of a chain of isotopes (including both even-even and even-odd nuclei). Therefore, in fitting the even-even nuclear spectra we fix the parameter g_I' to be 11.9 keV, and the adjustable parameters were taken to be g_6 and g_5', which will be used for the neighboring even-odd isotopes as well.

From Table I, we find that $-g_6$ and g_5' are nearly equal. It is interesting to note that if the quadrupole-quadrupole interaction is dominant over the pairing, $|B_2|\gg|G_0|$ ($=|G_2|$ in the symmetry limit), and $|B_2|\gg|B_3|$ (cf. Eq. (5.3c) in [15]), from Eq. (9) we obtain the relation

$$g_5'\cong-g_6, \quad (25)$$

which is precisely the empirical relation $A/4\cong B$ for the parameters in the IBM SO_6 limit [12]. The parameter g_6 can be determined through the 0_3^+ (i.e., $\sigma=N_1-2$) level. With the parameters g_6 given in Table I, the calculated 2_4^+ levels ($\sigma=N_1-2,\tau=1$) are in good agreement with the experimental results.

Comparing Eq. (17) with Eqs. (6.2) and (6.3) of [3], or Eqs. (2.9) of [6], we see that the spectral formulas in the U(6/4) or U(6/20) NSUSY and in the FDSM are essentially identical (except for the replacement of N by N_1) for even

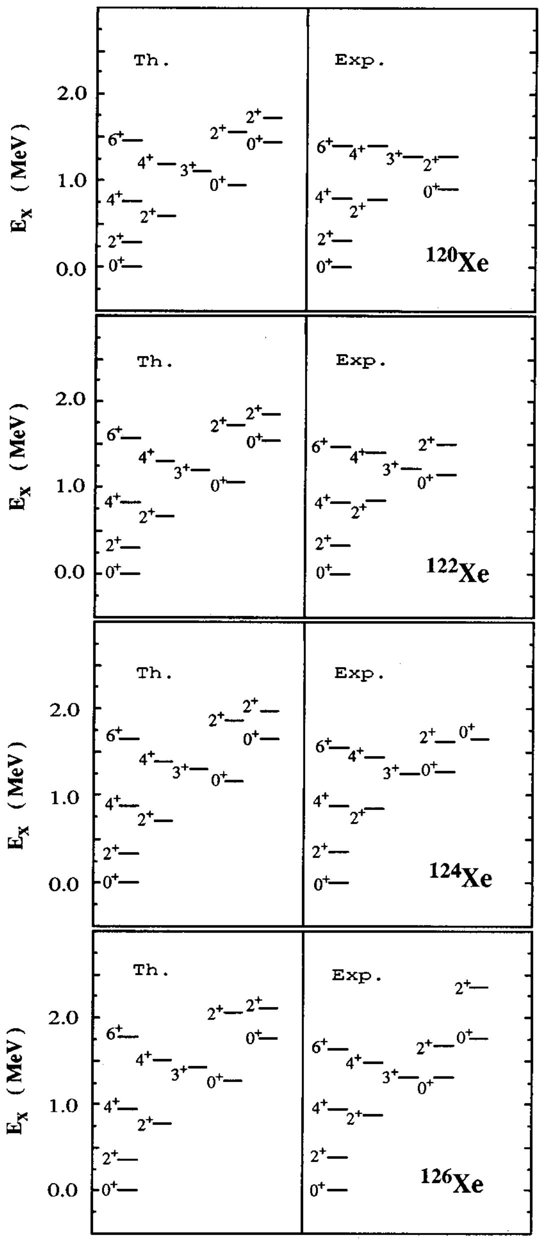

FIG. 1. Comparison between calculated levels using Eq. (17) and experimental energy levels for the even-even $^{120-126}$Xe isotopes.

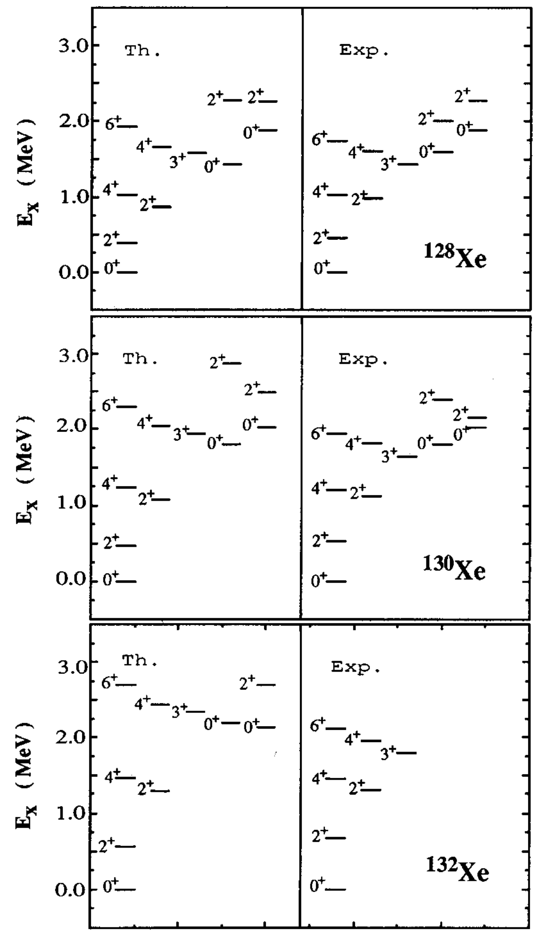

FIG. 2. Comparison between calculated levels using Eq. (17) and experimental energy levels for the even-even $^{128-132}$Xe isotopes.

nuclei, while for the odd nuclei they are similar in appearance but differ in two ways: (1) the parameters of the SO(3) group for the even and odd systems are different here, but the same in U(6/20) or U(6/4) NSUSY. This difference comes from the coupling term $\mathbf{I}\cdot\mathbf{L}$ in our case. (2) There is one SO_6 Casimir operator and two SO_5 Casimir operators for the FDSM, in contrast to two SO_6 Casimir operators and one SO_5 Casimir operator in the U(6/4) NSUSY. These differences have a significant effect on the spectrum. In the U(6/4) NSUSY, the five lowest-energy irreps of SO_5 are [1/2 1/2], [3/2 1/2], [5/2 1/2], [7/2 1/2] and [9/2 1/2]; therefore the states 3/2, 5/2, 7/2, 9/2, and 11/2 belonging to the irrep [5/2 1/2] of SO_5 lie quite low in energy, while in the FDSM the lowest six SO_5 irreps are $[1/2\ 1/2]^2$, $[3/2\ 1/2]^2$, [3/2 3/2], and [5/2 1/2/]. Consequently, for the FDSM in the low-energy region there are more low-spin states (there are two 1/2's and four 3/2's, while the 7/ 2, 9/2, and 11/2 states are pushed up). This shows the same pattern as in U(6/20).

Using Table I and Eq. (18), the spectra of the neighboring even-odd Xe isotopes can be calculated. For the odd-mass Xe and Ba isotopes, we take g_J to be 35.3 keV except for ^{127}Xe . The difference between g_J and g'_I comes from the coupling of SO_3^i and SO_3^k. We present the calculated and experimental results for $^{127-133}$Xe in Figs. 3 and 4, and for $^{131-135}$Ba in Fig. 5, with the parameters given in Table II.

TABLE I. Parameters for the even Xe isotopes.

Nuclei	g_6 (keV)	g'_5 (keV)	g'_I (keV)
^{120}Xe	-60	53	11.9
^{122}Xe	-64	59	11.9
^{124}Xe	-68.8	64	11.9
^{126}Xe	-73.3	71	11.9
^{128}Xe	-78.2	79	11.9
^{130}Xe	-100.9	100	11.9
^{132}Xe	-106.5	122	11.9

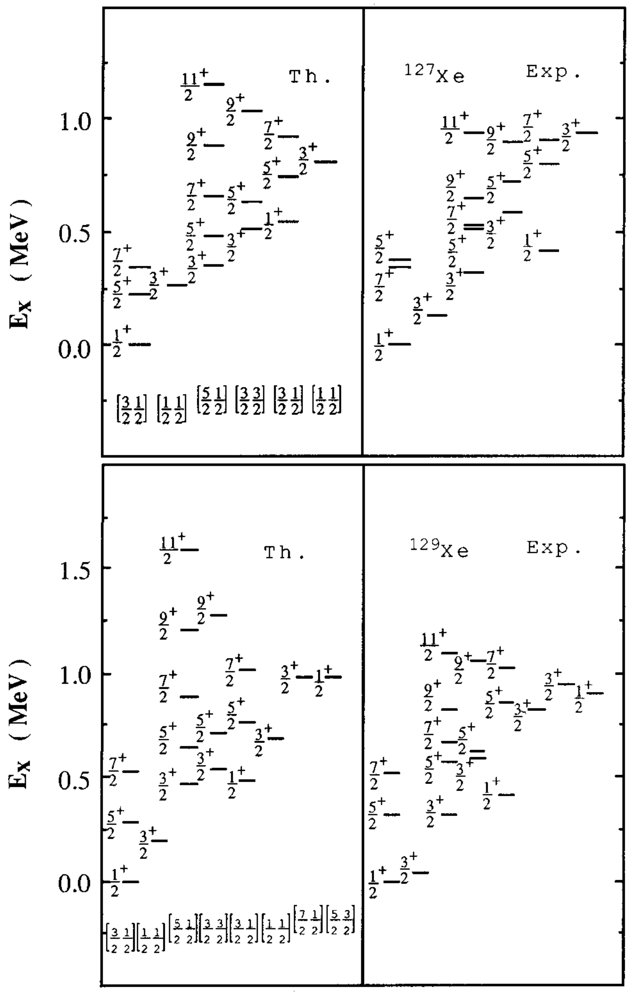

FIG. 3. Comparison between calculated levels using Eq. (18) and experimental energy levels for the even-odd $^{127-129}$Xe isotopes. The even-odd nuclei are constructed by coupling the neighboring even-even core to a valence neutron.

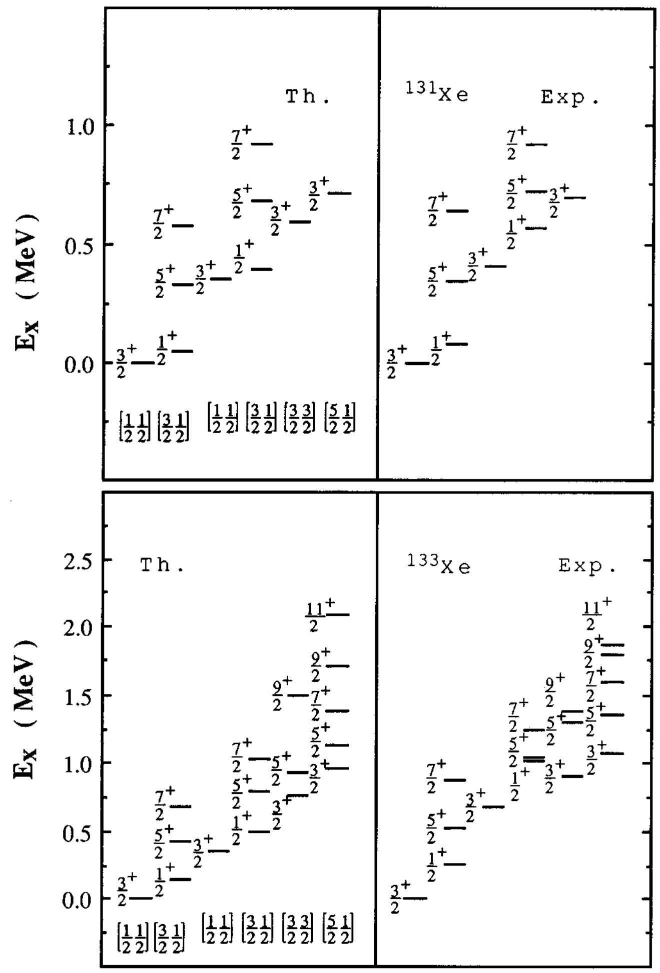

FIG. 4. Comparison between calculated levels using Eq. (18) and experimental energy levels for the even-odd $^{131-133}$Xe isotopes. The even-odd nuclei are constructed by coupling the neighboring even-even core to a valence neutron.

Apart from the constant term, the formula for E^{even} contains three parameters, and that for E^{odd} contains two parameters beyond the three parameters that are determined by fitting the spectra of neighboring even-even nuclei. With two extra parameters (g_J is kept constant in the region discussed, except for ^{127}Xe), Eq. (18) reproduces the spectral patterns for the nuclei $^{127-133}$Xe and $^{131-135}$Ba with about 15 levels each.

Experimentally, the odd nuclei in this region have both 1/2 and 3/2 as the ground-state spin. The second SO_5 Casimir operator in Eq. (18) provides this possibility. For alternative signs of the parameter g_5, the ground-state spin can take the values 1/2 or 3/2 . What is more, by allowing g_5 to change smoothly from positive to negative, we can reproduce the systematic shift of the ground band of the Xe and Ba isotopes, as shown in Figs. 6 and 7. From Figs. 3–5, we see that just as for U(6/20) NSUSY [6], for ^{129}Xe, ^{131}Ba, and ^{133}Ba, the FDSM predicts a natural occurrence of $1/2^+$ as the ground state and four low-energy $3/2^+$ states as the excited states. We note that the major aim of this present FDSM description is to give a simple and unified description of spectral pattern of even and even-odd nuclei. For more quantitative agreement, additional physics should be taken into account; for instance, the mixing of particles in normal and unique parity levels and the single-particle energy contribution.

Due to scarcity of data, we are constrained to discuss only the low-lying levels ($\sigma_1 = N_1 + 1/2$). We could in principle compare them with levels belonging to $\sigma_1 = N_1 - 1/2$. If more data for higher levels were available, this would allow us to determine the parameter g_6 for even-odd nuclei. Clearly, a comparison of the g_6 values obtained from fitting even and odd nuclear spectra is a meaningful test for the validity of the SO_6 symmetry. Here we have only considered fitting the levels with $\tau = 1,2,3$. If the levels with $\tau > 3$ were taken into account, the g_5' value would have to be smaller in order to fit the high-lying states; as a price, the unified good fit of the low-$[\tau_1 \tau_2]$ states for the even-even nuclei and even-odd nuclei would be spoiled.

Comparison of pure SO_6 spectra with the data for $^{120-132}$Xe (see Figs. 1 and 2) gives reasonable agreement for $\tau \leqslant 3$ states for $^{120-126}$Xe. However, the experimental energies for the high τ values in the nuclei $^{128-132}$Xe are much lower than predicted, with the discrepancies increasing for the higher τ values. This is strongly reminiscent of the

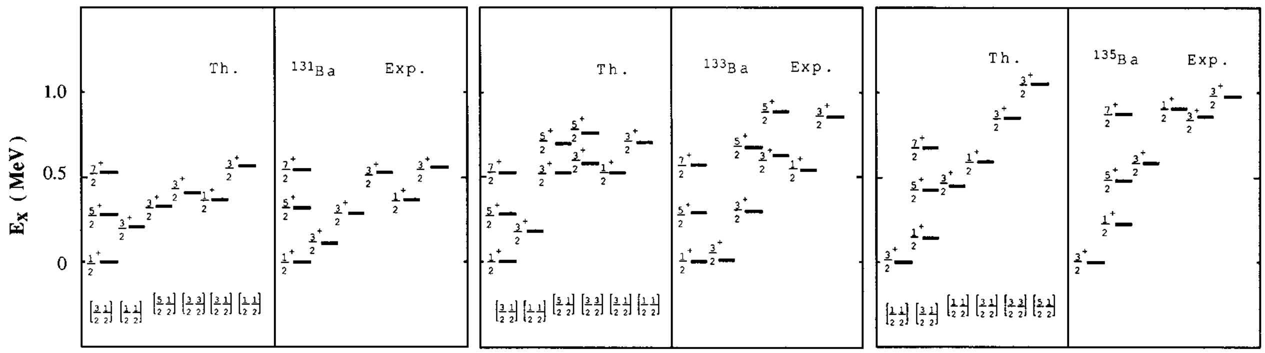

FIG. 5. Comparison between calculated levels and experimental energy levels for the even-odd $^{131-135}$Ba isotopes. For the excited $[\tau_1\tau_2]$ states, the bandhead states are compared with experimental ones.

stretching effect in nuclear rotational spectra. However, a more careful comparison shows that the energy levels within the same SO_5 irrep (same τ) follow the $J(J+1)$ rule rather well. Therefore the aforementioned discrepancy cannot be due to the usual stretching effect in which the deformation should increase with angular momentum. In [16], it was pointed out that it is in fact an SO_5 τ-compression effect. The driving force for this effect is the reduction of pairing correlation with increasing τ. Allowing for $g_S=G_0-G_2\neq 0$, thereby deviating from the SO_6 limit, and treating the $g_S S^\dagger\cdot S$ term as a perturbation, leads to the following energy formula:

$$E'^{\,\mathrm{even}}\cong E_0^{(e)}+g_6\sigma(\sigma+4)+A'\tau(\tau+3)-B'[\tau(\tau+3)]^2 +g_I'J(J+1). \qquad (26)$$

Figure 8 shows the spectrum for ^{126}Xe predicted by Eqs. (17) and (26), respectively. Inclusion of the SO_5 stretching effect improves the agreement significantly. More examples can be found in [16].

For even-odd nuclei, the levels calculated in this investigation are limited to those for $[\tau_1\tau_2]=[1/2\ 1/2],[3/2\ 1/2]$ bands, which corresponds to the $\tau=0,1$ states of the even-even core. Experimental energy levels for even-even nuclei show that the τ-compression effect is negligible for $\tau\leqslant 2$ states. Therefore, the effect in the even-odd nuclei is not so conspicuous as in the even-even nuclei.

TABLE II. (a) Parameters for the odd Xe isotopes. (b) Parameters for the odd Ba isotopes.

(a)

Nuclei	g_6(keV)	g_5' (keV)	g_5 (keV)	g_J (keV)
^{127}Xe	-73.3	71.0	-38.0	25.0
^{129}Xe	-78.2	79.0	-18.0	35.3
^{131}Xe	-100.9	100.0	30.0	35.3
^{133}Xe	-106.5	122.0	50.5	35.3
^{135}Xe		142.1	70.6	35.3

(b)

Nuclei	g_5^i (keV)	g_5 (keV)	g_J (keV)
^{131}Ba	72.5	-20	35.3
^{133}Ba	105	-15	35.3
^{135}Ba	90	50	35.3
^{137}Ba	72	70	35.3

III. WAVE FUNCTIONS

A. Even-even nuclei

According to Eq. (4.3a) in [15] the FDSM wave function in the SO_6 limit for $u=0$ is

$$|N_1\sigma\tau n_\Delta IM\rangle=\mathscr{P}_{N_1\sigma\tau}|N_1\sigma\tau n_\Delta IM)^{\mathrm{IBM}}_{b\to f}, \qquad (27)$$

where $\mathscr{P}_{N_1\sigma\tau}$ is a Pauli factor,

$$\mathscr{P}_{N_1\sigma\tau}=\left[\frac{(\Omega_1-N_1-\sigma)!!(\Omega_1-N_1+\sigma+4)!!}{\Omega_1!!(\Omega_1+4)!!}\right]^{1/2}, \qquad (28)$$

and $|N_1\sigma\tau n_\Delta IM)^{\mathrm{IBM}}_{b\to f}$ denotes a wave function resulting from replacing the boson operators $s^\dagger$ and $d^\dagger_\mu$ by the fermion operators $S^\dagger$ and $D^\dagger_\mu$, in the $U_6\supset O_6\supset O_5\supset O_3$ IBM wave function $|N_1\sigma\tau n_\Delta IM)^{\mathrm{IBM}}$,

$$|N_1\sigma\tau n_\Delta IM)^{\mathrm{IBM}}_{b\to f} =\xi_{N_1\sigma}(I^\dagger)^{(N_1-\sigma)/2}f_{\sigma\tau}(S^\dagger,I^\dagger)|\tau\tau\tau n_\Delta IM)^{\mathrm{IBM}}_{b\to f}, \qquad (29)$$

where $|\tau\tau\tau n_\Delta IM)^{\mathrm{IBM}}$ denotes the IBM $U_6\supset U_5\supset SO_5\supset SO_3$ wave function and

$$\xi_{N_1\sigma}=\left[\frac{(2\sigma+4)!!}{(N+\sigma+4)!!(N-\sigma)!!}\right]^{1/2}, \qquad (30)$$

$$I^\dagger=D^\dagger\cdot D^\dagger-S^\dagger\cdot S^\dagger, \qquad (31)$$

where $I^\dagger$ is a generalized pair and $f_{\sigma\tau}(S^\dagger,I^\dagger)$ is a polynomial in $S^\dagger$ and $D^\dagger$ of order $(\sigma-\tau)$,

$$f_{\sigma\tau}(S^\dagger,I^\dagger)=\sum_{p=0}^{[(\sigma-\tau)/2]}D_p(\sigma\tau)(S^\dagger)^{\sigma-\tau-2p}(I^\dagger)^p, \qquad (32)$$

$$D_p(\sigma\tau)=\left[\frac{2^{\sigma+1}(\sigma-\tau)!(2\tau+3)!!}{(\sigma+1)!(\sigma+\tau+3)!}\right]^{1/2}\frac{(\sigma+1-p)!}{4^p(\sigma-\tau-2p)!p!}. \qquad (33)$$

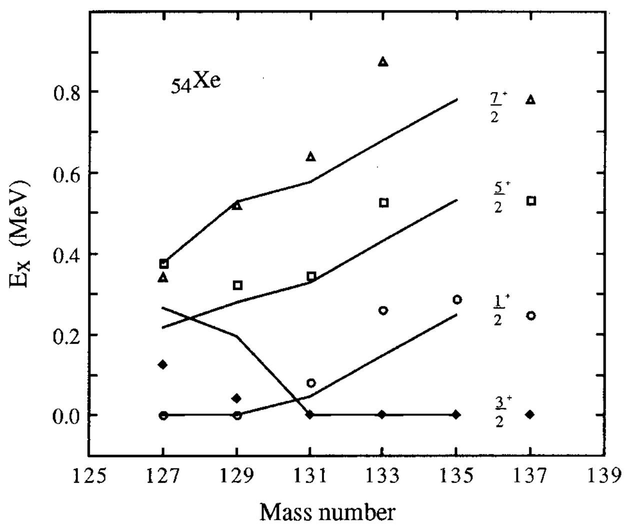

FIG. 6. The systematic shift of the ground band as a function of mass number for Xe isotopes.

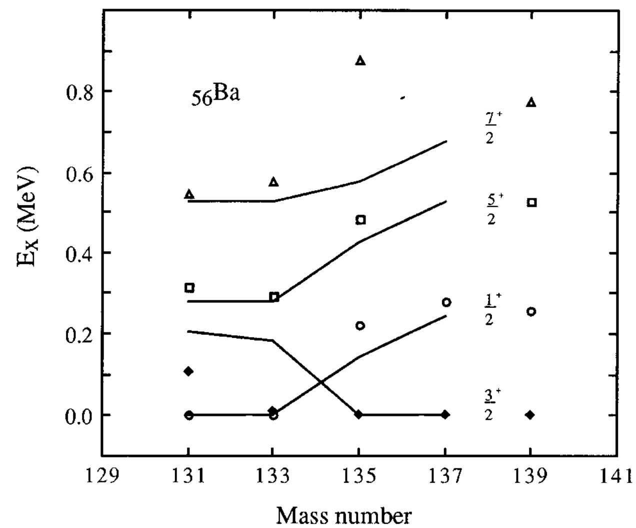

FIG. 7. The systematic shift of the ground band as a function of mass number for Ba isotopes.

Here the notation for the wave functions is the same as in [15].

B. Even-odd nuclei

In this section we construct the wave functions for states in odd-mass nuclei that corresponds to heritage $u=1$. According to the vector coherent state technique [17], the wave function for the i-active part can be constructed by coupling the "collective wave function" $|N_1\sigma\tau n_\Delta I'M'\rangle^{\rm IBM}_{b\to f}$ and the "intrinsic state," now the one-fermion state $|u=1\rangle$, by means of the $SO_6\supset SO_5\supset SO_3$ *CG* coefficients,

$$|2N_1+1,[\sigma_1\sigma_2\sigma_3][\tau_1\tau_2]n_\Delta IM\rangle$$

$$=K_l^{-1}(N_1,[\sigma_1\sigma_2\sigma_3])[|N_1[\sigma])^{\rm IBM}_{b\to f}|u=1\rangle]^{[\sigma_1\sigma_2\sigma_3]}_{[\tau_1\tau_2]n_\Delta IM}, \tag{34}$$

where N_1 is the total number of S,D pairs in the "collective wave function," l is the number of generalized pairs $I^\dagger, N_1=\sigma+2l$, and $K_l^{-1}(N_1,[\sigma_1\sigma_2\sigma_3])$ is the diagonal matrix element of the inverse of the K matrix [17].

Written out explicitly, Eq. (34) becomes

$$|2N_1+1,\{\sigma_1+\tfrac{1}{2},\tau_1\}n_\Delta IM,[10]k=2,m\rangle=\frac{1}{\sqrt{2^{N_1}}}K_l^{-1}(N_1,\langle\sigma+\tfrac{1}{2}\rangle)\sum_{\tau I' n'_\Delta}\xi^{\sigma\tau n'_\Delta I'}_{\sigma+\frac{1}{2}\tau_1 n_\Delta I}[b^\dagger_{2m,3/2}|N_1\sigma\tau n'_\Delta I')^{\rm IBM}_{b\to f}]^I_M, \tag{35}$$

where the factor $1/\sqrt{2_1^N}$ is due to the different definitions of A^+_{LM} in [8,17], the shorthand notation stands for

$$\{\sigma+\tfrac{1}{2},\tau_1\}\equiv\{\langle\sigma+\tfrac{1}{2}\rangle,[\tau_1\,\tfrac{1}{2}]\},\equiv[\sigma+\tfrac{1}{2}\rangle,\tfrac{1}{2}\;\tfrac{1}{2}], \tag{36}$$

and $\xi^{\sigma\tau n'_\Delta I'}_{\sigma+(1/2)\tau_1 n_\Delta I}$ is the isoscalar factor for the group chain $SO_6\supset SO_5\supset SO_3$ [18], which is a product of the $SO_6\supset SO_5$ and $SO_5\supset SO_3$ isoscalar factors,

$$\xi^{\sigma\tau n'_\Delta I'}_{\sigma+(1/2)\tau_1 n_\Delta I}=\left(\begin{array}{cc|c}[\sigma 00]&\langle\tfrac{1}{2}\rangle&\langle\sigma+\tfrac{1}{2}\rangle\\ [\tau 0]&[\tfrac{1}{2}\;\tfrac{1}{2}]&[\tau_1\,\tfrac{1}{2}]\end{array}\right)\left(\begin{array}{cc|c}[\tau 0]&[\tfrac{1}{2}\;\tfrac{1}{2}]&[\tau_1\,\tfrac{1}{2}]\\ n'_\Delta I'&\tfrac{3}{2}&n_\Delta I\end{array}\right). \tag{37}$$

By following the steps given in [17], the matrix K_l^{-1} is found to be

$$K_l^{-1}(N_1,\langle\sigma+\tfrac{1}{2}\rangle)=\left[\frac{(\Omega_1-N_1+\sigma+4)!!(\Omega_1-N_1-\sigma-2)!!}{2^{-N_1}(\Omega_1-2)!!(\Omega_1+4)!!}\right]^{1/2}=\left[\frac{2^{-N_1}\Omega_1}{\Omega_1-N_1-\sigma}\right]^{1/2}\mathscr{P}_{N_1\sigma\tau}. \tag{38}$$

Inserting (38) into (35)

$$|2N_1+1,\{\sigma+\tfrac{1}{2},\tau_1\}n_\Delta IM,[10]k=2,m\rangle=\left[\frac{\Omega_1}{\Omega_1-N_1+\sigma}\right]^{1/2}\xi^{\sigma\tau n' I'}_{\sigma+\frac{1}{2}\tau_1 n_\Delta I}[b^\dagger_{2m\,\frac{3}{2}}|N_1\sigma\tau n'_\Delta I'\rangle]^I_M. \tag{39}$$

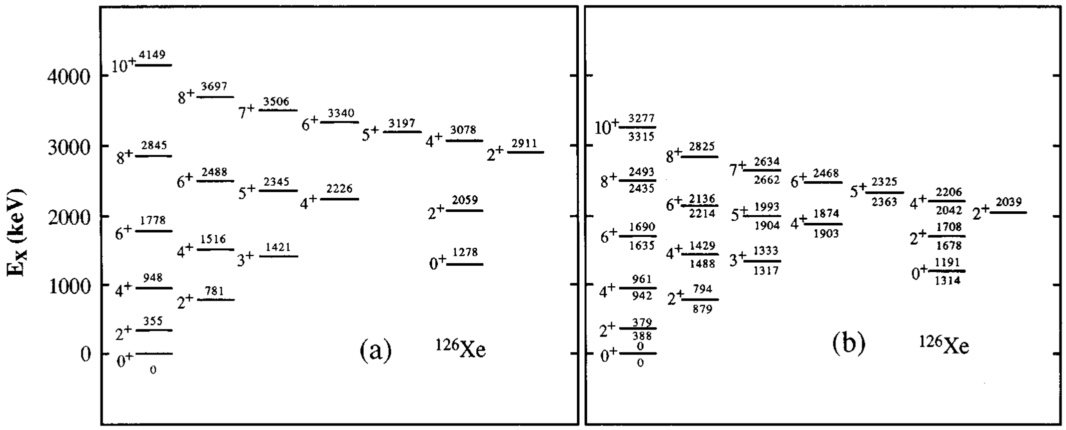

FIG. 8. (a) The SO_6 spectrum calculated with the parameters are $g_6=-73.3$ (keV), $g_5=$ 71 (keV), and $g_I=11.9$ (keV); (b) the comparison of the experimental spectrum of ^{126}Xe (lower numbers) and the spectrum of SO(6) plus a perturbative pairing term (upper numbers) [i.e., Eq. (26)]. The parameters here are $g_6=-73.3$ (keV), $A'=80$ (keV). $B'=0.77$ (keV) and $g_I=11.9$ (keV).

Coupling the i-active and k-active parts gives the total wave function of the SO_6 FDSM for even-odd nuclei

$$|2N_1+1,\{\sigma+\tfrac{1}{2},\tau_1\}[\omega_1\omega_2]n''_\Delta JM\rangle=\sum_{In_\Delta}\left(\begin{array}{c|c}[\tau_1\,\tfrac{1}{2}][10] & [\omega_1\omega_2]\\ n_\Delta I\ \ 2 & n''_\Delta J\end{array}\right)[|2N_1+1,\{\sigma+\tfrac{1}{2},\tau_1\}n_\Delta I,[10]k=2\rangle]^J_M. \qquad (40)$$

Combining (34)–(40) we have

$$|2N_1+1,\{\sigma+\tfrac{1}{2},\tau_1\}[\omega_1\omega_2]n''_\Delta JM\rangle=\sum_{In_\Delta}\sum_{\tau I'n'_\Delta}\left(\begin{array}{c|c}[\tau_1\,\tfrac{1}{2}][10] & [\omega_1\omega_2]\\ n_\Delta I\ \ 2 & n''_\Delta J\end{array}\right)\left(\begin{array}{c|c}[\sigma 00]\langle\tfrac{1}{2}\rangle & \langle\sigma+\tfrac{1}{2}\\ [\tau 0]\ \ [\tfrac{1}{2}\ \tfrac{1}{2}] & [\tau_1\,\tfrac{1}{2}]\end{array}\right)\left(\begin{array}{c|c}[\tau 0][\tfrac{1}{3}\ \tfrac{1}{2}] & [\tau_1\,\tfrac{1}{2}]\\ n'_\Delta I'\ \tfrac{3}{2} & n_\Delta I\end{array}\right)$$

$$\times\sum_j\ (-)^{I'+3/2+J}\left\{\begin{array}{ccc}I' & \tfrac{3}{2} & I\\ 2 & J & j\end{array}\right\}\hat{I}\hat{J}[|N_1\sigma\tau n'_\Delta I'\rangle a^{\dagger J}_{jM}, \qquad (41)$$

where $\hat{J}=\sqrt{2J+1}$. It is interesting to note that when the fermion state for the core, $|N_1\sigma\tau n'_\Delta I'\rangle$, is replaced by the boson state $|N_1\sigma\tau n'_\Delta I')$ and the Pauli factor is ignored, Eq. (41) goes over to the U(6/20) NSUSY wave function (2.14) of [6]. Thus, NSUSY can be obtained as an approximation to the FDSM for odd-mass SO_6 nuclei.

IV. ELECTROMAGNETIC TRANSITIONS

A. The $E2$ transition rate for even-even nuclei

In [8], the $E2$ transition operator in the FDSM is defined as

$$T(E2)^2_\mu=qP^2_\mu(i) \qquad (42)$$

while the $E2$ transition operator for the IBM SO_6 limit is [18]

$$T(E2)^2_\mu=qB^2_\mu,\quad B^2_\mu=(d^\dagger\tilde{s}+s^\dagger\tilde{d})^2_\mu. \qquad (43)$$

Owing to the isomorphism between the commutators for the FDSM and IBM:

$$[P^2_\mu(i),S^\dagger]\leftrightarrow[B^2_\mu,s^\dagger], \qquad (44)$$

$$[P^2_\mu(i),D^\dagger_\nu]\leftrightarrow[B^2_\mu,d^\dagger_\nu], \qquad (45)$$

the formula for the reduced matrix elements of the $E2$ transition operator in the FDSM is identical to that in the IBM,

$$\langle N\sigma\tau' n'_\Delta J'\|P^2(i)\|N\sigma\tau n_\Delta J\rangle^{\mathrm{FDSM}}$$
$$=(N\sigma\tau' n'_\Delta J'\|B^2\|N\sigma\tau n_\Delta J)^{\mathrm{IBM}}. \qquad (46)$$

Although it is well known now, it is nevertheless a remarkable fact that the FDSM and IBM have the same selection rules, $\Delta\sigma=0$ and $\Delta\tau=\pm1$, and the same closed expression for the $E2$ transition rates [1]. The commonly needed results are given in Table III . It should be noted that the O_6 limit of the IBM and the SO_6 limit of the FDSM share the same analytical form for the spectra and the $E2$ transitions, but the accounting of the collective pair number is different in these two models: the collective pair number is half of the total valence nucleons (N) in the IBM, whereas in FDSM it is taken as half of the total valence nucleons in the normal-parity levels (N_1).

TABLE III. $B(E2)$ formulas for even-even nuclei in the SO_6 limit.

$\|N_1\sigma\tau J_i\rangle$	$\rightarrow$	$\|N_1\sigma'\tau' J_f\rangle$	$B(E2;J_i\rightarrow J_f)$
$\|N_1N_1 1+L/2 L+2\rangle$	$\rightarrow$	$\|N_1N_1 L/2 L\rangle$	$q^2\frac{(L+2)(2N_1+L+8)}{8(L+5)}(2N_1-L)$
$\|N_1N_1 12\rangle$	$\rightarrow$	$\|N_1N_1 00\rangle$	$\frac{1}{5}q^2N_1(N_1+4)$
$\|N_1N_1 22\rangle$	$\rightarrow$	$\|N_1N_1 12\rangle$	$\frac{2}{7}q^2(N_1-1)(N_1+5)$
$\|N_1N_1 34\rangle$	$\rightarrow$	$\|N_1N_1 24\rangle$	$\frac{10}{63}q^2(N_1-2)(N_1+6)$
$\|N_1N_1 33\rangle$	$\rightarrow$	$\|N_1N_1 24\rangle$	$\frac{2}{21}q^2(N_1-2)(N_1+6)$
$\|N_1N_1 33\rangle$	$\rightarrow$	$\|N_1N_1 22\rangle$	$\frac{5}{21}q^2(N_1-2)(N_1+6)$
$\|N_1N_1 34\rangle$	$\rightarrow$	$\|N_1N_1 22\rangle$	$\frac{11}{63}q^2(N_1-2)(N_1+6)$
$\|N_1N_1 30\rangle$	$\rightarrow$	$\|N_1N_1 22\rangle$	$\frac{7}{21}q^2(N_1-2)(N_1+6)$

As pointed out in [19], when we define the $E2$ transition operators as Eq. (42) or Eq. (43), the $\Delta\sigma=0$ and $\Delta\tau=\pm1$ selection rules prohibit some transitions that are observed in many nuclei. As a particular case of these selection rules, the quadrupole moments are predicted to be zero in the SO_6 limit, but most of the observed quadrupole moments of the transitional nuclei differ from zero. This deviation from zero may be due to two causes: one is the breaking of the SO_6 symmetry; the other is that the $E2$ transition operator may require a more general definition. We have chosen the latter to study this problem. It is straightforward to define a new $E2$ transition operator that relaxes the selection rule while assuming that the wave functions still have good $SO_6\supset SO_5$ symmetry. The new operator takes the form

$$T(E2)^2_\mu = qP^2_\mu(i) + q'(D^\dagger\tilde{D})^2_\mu . \tag{47}$$

The $(D^\dagger\tilde{D})^2_\mu$ term makes the following transition possible:

$$\Delta\sigma=\pm2, \quad \Delta\tau=0,\pm2. \tag{48}$$

The reduced matrix element of $(D^\dagger\tilde{D})^2_\mu$ can be calculated by inserting a complete set of intermediate states (the reduced matrix elements used here are defined according to the Rose convention)

$$\langle N_1\sigma\tau' n'_\Delta L'\|(D^\dagger\tilde{D})^2\|N_1\sigma\tau n_\Delta L\rangle=\sqrt{5(2L+1)}\sum_{\sigma''\tau''L''}(-)^{L''+L'}\langle N_1\sigma'\tau' n'_\Delta L'\|D^\dagger\|N_1-1\sigma''\tau'' n''_\Delta L''\rangle \times\langle N_1\sigma\tau n_\Delta L\|D^\dagger\|N_1-1\sigma''\tau'' n''_\Delta L''\rangle\begin{Bmatrix}2&2&2\\L&L'&L''\end{Bmatrix}. \tag{49}$$

The SO_3 reduced matrix element of $D^\dagger$ in Eq. (49) is

$$\langle N_1+1\sigma'\tau' n'_\Delta L'\|D^\dagger\|N_1\sigma\tau n_\Delta L\rangle=\langle N_1+1\sigma'\|D^\dagger\|N_1\sigma\rangle\left(\begin{array}{cc|c}\sigma&1&\sigma'\\\tau&1&\tau'\end{array}\right)\left(\begin{array}{cc|c}\tau&1&\tau'\\L&2&L'\end{array}\right), \tag{50}$$

where $\langle N_1+1\sigma'\|D^\dagger\|N_1\sigma\rangle$ is the SO_6 reduced matrix element and the last two factors are the $SO_6\supset SO_5$ and $SO_5\supset SO_3$ isoscalar factors, respectively, which have been given in [21] for some simple cases. The SO_5 reduced matrix elements of $S^\dagger$ and $D^\dagger$ are given in the Appendix.

Using vector coherent state techniques [17] for the $u=0$ case, the SO_6^i reduced matrix element of $D^\dagger$ can be expressed as

$$\langle p+1\sigma'\|D^\dagger\|p\sigma\rangle=\sqrt{2}(\Lambda_{p+1\sigma'}-\Lambda_{p\sigma})^{1/2}\langle p+1\sigma'\|z\|p\sigma\rangle, \tag{51}$$

where $\Lambda_{p\sigma}$ is the eigenvalue of the Toronto auxiliary operator

$$\Lambda_{p\sigma}=-\frac{1}{4}[p(p-2\Omega_1-6)-\sigma(\sigma+4)], \tag{52}$$

$$\langle p+1\sigma+1\|z\|p\sigma\rangle=\left[\frac{(\sigma+1)(l+\sigma+3)}{(\sigma+3)}\right]^{1/2}, \tag{53}$$

$$\langle p+1\sigma-1\|z\|p\sigma\rangle=\left[\frac{(l+1)(\sigma+3)}{(\sigma+3)}\right]^{1/2}, \tag{54}$$

TABLE IV. Some quadrupole moments and $B(E2)$ values for the transitions $\Delta\sigma=0,\pm 2$, $\Delta\tau=0,\pm 2$.

$B(E2;N_1N_1 22\to N_1N_1 00)$	$=q'^2\dfrac{N_1(N_1-1)(N_1+4)(N_1+5)}{70(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1 33\to N_1N_1 12)$	$=q'^2\dfrac{5(N_1-1)(N_1-2)(N_1+5)(N_1+6)}{294(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1 34\to N_1N_1 12)$	$=q'^2\dfrac{11(N_1-1)(N_1-2)(N_1+5)(N_1+6)}{882(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1 34\to N_1N_1 33)$	$=q'^2\dfrac{2(N_1^2+4N_1+23)^2}{231(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1 30\to N_1N_1 12)$	$=q'^2\dfrac{(N_1-1)(N_1-2)(N_1+5)(N_1+6)}{42(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1 22\to N_1N_1 24)$	$=q'^2\dfrac{16(N_1^2+4N_1+15)^2}{2205(N_1+1)^2}(\Omega_1-2N_1+2)^2$
$B(E2;N_1N_1-200\to N_1N_1 12)$	$=q'^2\dfrac{(N_1+2)(N_1+3)(N_1+4)(N_1+5)}{14N_1(N_1+1)^2}(\Omega_1-2N_1+2)(\Omega_1+4)$
$B(E2;N_1N_1-212\to N_1N_1 12)$	$=q'^2\dfrac{(N_1-1)(N_1-2)(N_1+3)(N_1+4)}{49N_1(N_1+1)^2}(\Omega_1-2N_1+2)(\Omega_1+4)$
$Q(N_1N_1 12)$	$=q'\sqrt{\dfrac{2\pi}{35}}\dfrac{4(N_1^2+4N_1+9)}{7(N_1+1)}(\Omega_1-2N_1+2)$

with $p=N_1=\sigma+2l=(1/2)(n_1-1)$. For the special cases that will be used below, we have

$$\langle p+1\,\sigma+1\|D^\dagger\|p\,\sigma\rangle=\left[\frac{(\Omega_1-2\sigma-2l)(\sigma+1)(l+\sigma+3)}{(\sigma+3)}\right]^{1/2}, \tag{55}$$

$$\langle p+1\,\sigma-1\|D^\dagger\|p\,\sigma\rangle=\left[\frac{(\Omega_1+4-2l)(l+1)(\sigma+3)}{(\sigma+1)}\right]^{1/2}. \tag{56}$$

In Table IV we summarize some expressions for $B(E2)$ values and quadrupole moments in the FDSM SO_6 limit for transitions with $\Delta\tau=0,\pm 2$ and $\Delta\sigma=\pm 2$. The contribution to the $B(E2)$ from the second term $q'(D^\dagger\tilde{D})^2_\mu$ differs from the corresponding term $q'(d^\dagger\tilde{d})^2_\mu$ in the IBM [19] by the Pauli factors.

B. The transition rates for even-odd nuclei

For the $u=1$ case, the $E2$ transition operator can be defined as

$$T(E2)^2_\mu=qP^2_\mu(i)+q''P^2_\mu(k). \tag{57}$$

The reduced matrix element is

$$\langle\{N_1+\tfrac{1}{2},\tau_1'\}[\omega_1'\omega_2']J'\|qP^2(i)+q''P^2(k)\|\{N_1+\tfrac{1}{2},\tau_1\}[\omega_1\omega_2]J\rangle$$
$$=\sum_{II'}\left(\begin{array}{cc|c}[\tau'\,\tfrac{1}{2}] & [10] & [\omega_1'\omega_2']\\ I' & 2 & J'\end{array}\right)\left(\begin{array}{cc|c}[\tau_1\,\tfrac{1}{2}] & [10] & [\omega_1\omega_2]\\ I & 2 & J\end{array}\right)M, \tag{58}$$

$$M=\langle\{N_1+\tfrac{1}{2},\tau_1'\}(I',[10]k=2)J'\|qP^2(i)+q''P^2(k)\|\{N_1+\tfrac{1}{2},\tau_1\}(I,[10]k=2)J\rangle. \tag{59}$$

According to Eq. (6.8) in Judd [20], we have

$$[b^\dagger_{ki}b_{ki}]^{K0}_{\mu 0}=\frac{1}{\sqrt{2i+1}}\sum_{p=1}^{n}[b^\dagger_k(p)b_k(p)]^K_\mu. \tag{60}$$

TABLE V. Relative $B(E2)$ values for the even Xe isotopes.

	^{120}Xe		^{124}Xe		^{126}Xe		^{128}Xe		^{130}Xe	
$J_i \to J_f$	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.
$2_2^+ \to 2_1^+$	100	100	100	100	100	100	100	100	100	100
$\to 0_1^+$	5.6	5.6	3.9	3.9	1.5	1.4	1.2	1.2	0.6	0.6
$3_1^+ \to 2_2^+$	100	100	100	100	100	100	100	100	100	100
$\to 4_1^+$	50	40	46	40	34	40	37	40	25	40
$\to 2_1^+$	2.7	7.1	1.6	4.9	2.0	1.85	1.0	1.5	1.4	0.72
$4_2^+ \to 2_2^+$	100	100	100	100	100	100	100	100	100	100
$\to 4_1^+$	62	91	91	91	76	91	133	91	107	91
$\to 2_1^+$	—	7.11	0.4	4.91	1.0	1.83	1.7	1.49	3.2	0.97
$0_2^+ \to 2_2^+$	100	100	100	100	100	100	100	100	100	100
$\to 2_1^+$	—	7.11	1	4.91	7.7	1.83	14	1.49	26	0.97

Therefore, in computing the matrix elements of $P_\mu^2(k)$ the operator can be replaced by

$$P_\mu^2(k) = \sqrt{\frac{\Omega_1}{8}} \sum_{p=1}^{n} [b_k^\dagger(p) b_k(p)]_\mu^2 . \tag{61}$$

Using (61) we have

$$M = \hat{J}\hat{J}'(-)^{I'+J} \begin{Bmatrix} J' & J & 2 \\ I & I' & 2 \end{Bmatrix} \langle \{N_1 + \tfrac{1}{2}, \tau_1'\} I' \| q P^2(i) \| \{N + \tfrac{1}{2}, \tau_1\} I \rangle$$
$$+ q'' \delta_{\tau_1 \tau_1'} \delta_{II'} (-)^{I+J'} \hat{J}\hat{J}' \sqrt{\frac{5\Omega_1}{2}} \frac{\hat{I}}{\hat{i}} \frac{(\Omega_1 - 2N_1 - 1)}{(\Omega_1 - 1)} \begin{Bmatrix} J' & J & 2 \\ 2 & 2 & I \end{Bmatrix} . \tag{62}$$

Now only the matrix elements of $P_\mu^2(i)$ remain to be calculated. The generators of spin(6) for the IBFM are

$$G_\mu^2 = B_\mu^2 + F_\mu^2 , \quad F_\mu^2 = (a_{3/2}^\dagger \tilde{a}_{3/2})_\mu^2 \tag{63}$$

corresponding to the commutator for the IBFM

$$[F_\mu^2, a_{(3/2)m_i}^\dagger] = (-1)^{3/2 - m_i} \langle i m_i + \mu, i - m_i | 2 \mu \rangle a_{(3/2)m_i + \mu}^\dagger . \tag{64}$$

There is a similar commutator in the FDSM

$$[P_\mu^2(i), b_{22(3/2)m_i}^\dagger] = \sqrt{\frac{\Omega_1}{2(2k+1)}} (-1)^{3/2 - m_i} \langle i m_i + \mu, i - m_i | 2\mu \rangle b_{22(3/2)2m_i + \mu}^\dagger , \tag{65}$$

where $\sigma = \pi$ or ν, and the factor $\sqrt{\Omega_1 / 2(2k+1)}$ is always equal to 1 for the sixth shell.

Because of (44) and (45) and (64) and (65), we have the following isomorphism between the commutators in the FDSM and IBFM:

$$[P_\mu^2, S^\dagger] \leftrightarrow [B_\mu^2, s^\dagger] = [G_\mu^2, s^\dagger], \tag{66}$$

$$[P_\mu^2(i), D_\nu^\dagger] \leftrightarrow [B_\mu^2, d_\nu^\dagger] = [G_\mu^2, d_\nu^\dagger], \tag{67}$$

$$[P_\mu^2(i), b_{22(3/2)m_i}^\dagger] \leftrightarrow [F_\mu^2, a_{(3/2)m_i}^\dagger] = [G_\mu^2, a_{(3/2)\, m_i}^\dagger]. \tag{68}$$

Therefore we establish the following identity:

$$\langle \{N + \tfrac{1}{2}, \tau_1'\} I' \| P^2(i) \| \{N + \tfrac{1}{2}, \tau_1\} I \rangle^{\mathrm{FDSM}}$$
$$= (\{N + \tfrac{1}{2}, \tau_1'\} I' \| G^2 \| \{N + \tfrac{1}{2}, \tau_1\} I)^{\mathrm{IBFM}}. \tag{69}$$

The reduced matrix element of G_μ^2 is derived in [18]. With these results we can calculate the $B(E2)$ values and the quadrupole moments for odd-mass nuclei. The selection rules for U(6/4) are [18]

$$\Delta \tau_1 = 0, \pm 1, \quad \Delta \tau_2 = 0. \tag{70}$$

For the $u = 1$ case in the FDSM, owing to the Kronecker product (22) the corresponding selection rules are

$$\Delta \omega_1 = 0, \pm 1, \pm 2, \pm 3, \quad \Delta \omega_2 = 0, \pm 1. \tag{71}$$

TABLE VI. Relative $B(E2)$ values for the even Ba isotopes.

	^{126}Ba		^{128}Ba		^{130}Ba		^{132}Ba		^{134}Ba		
$J_i \rightarrow J_f$	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.	Expt.	Theo.a	Theo.b
$2_2^+ \rightarrow 2_1^+$	100	100	100	100	100	100	100	100	100	100	100
$\rightarrow 0_1^+$	11	11	9.2	9.2	5.7	5.7	0.2	0.2	1.0	1.1	3.06
$3_1^+ \rightarrow 2_2^+$	100	100	100	100	100	100	100	100	100	100	100
$\rightarrow 4_1^+$	13	40	—	40	30	40	73	40	40	40	40
$\rightarrow 2_1^+$	5.8	14.4	—	12	1.5	7.46	0.2	0.24	1.0	1.26	3.89
$4_2^+ \rightarrow 2_2^+$	100	100	100	100	100	100	100	100	100	100	100
$\rightarrow 3_1^+$	—	26.1	—	21.8	—	13.5	—	0.44	14.5	3.06	9.45
$\rightarrow 4_1^+$	28	91	42	91	89	91	75	91	77	91	91
$\rightarrow 2_1^+$	1.1	14.4	1.7	12	3.9	7.46	2.2	0.24	2.5	1.26	3.89
$0_2^+ \rightarrow 2_2^+$	100	100	100	100	100	100	100	100	100	100	100
$\rightarrow 2_1^+$	—	14.4	—	12	—	7.46	0	0.24	4	1.26	3.89

With these rules, the restrictions for the $B(E2)$ values will be less severe than that of the IBFM [18]. This enables us to explain some data that cannot be explained by the U(6/4) IBFM.

From Table III and Table IV, we can calculate the $B(E2)$ values for the transitions $\Delta\omega_1=0,\pm1,\pm2$. In order to make a direct comparison between the calculated and experimental results without a knowledge of q and q', we compute the relative $B(E2)$ value rather than the absolute values. The FDSM prediction and the experimental results [12,21] for Xe and Ba isotopes are listed in Tables V and VI. For the transitions with $\Delta\sigma=0$ and $\Delta\tau=\pm1$, the formulas for the $B(E2)$ in the IBM and FDSM are of the same form, but the numerical values differ for a given nucleus because in the IBM the $B(E2)$ is a function of N, while in the FDSM it is a function of N_1. For the transitions with $\Delta\sigma=\pm2$ and $\Delta\tau=0,\pm2$, they also differ by the Pauli factors $(\Omega_1-2N_1+2)^2$ or $(\Omega_1+4)(\Omega_1-2N_1+2)$, as shown in Table IV.

From Tables V and VI, we can see that the $B(E2)$ transitions for Xe and Ba isotopes exhibit an SO_6 symmetry, especially for the $\Delta\tau=\pm1$ transitions. There are two possible reasons to expect less accuracy for the $\Delta\tau=\pm2$ transitions: one is the definition of the new $T(E2)$ operator and the other is the fitting of the parameter $[q'/q]^2$. In fact, the determination of $[q'/q]^2$ from the rate $B(E2,2_2^+\rightarrow0_1^+)/B(E2,2_2^+\rightarrow2_1^+)$ is very inaccurate. A possible way to obtain q and q' is, as in [17], through fitting the $B(E2,2_1^+\rightarrow0_1^+)$ and the quadrupole moment $Q(2_1^+)$, respectively. For example, from $Q(2_1^+)=-0.16$ e b and $B(E2,2_1^+\rightarrow0_1^+)$ = 0.146 $(e\text{ b})^2$ for ^{134}Ba, we can determine $(q'/q)^2$ to be equal to 0.34, which in turn gives the $B(E2)$ value listed in the last column (theo.b) of Table VI. By comparing the last two columns in Table VI, we see that the last column gives a better fit.

In [22] the ratio R_4 between two $B(E2)$ values is introduced to distinguish the SO_6 limit from the U_5 limit of the IBM,

$$R_4=\frac{B(E2,4_1^+\rightarrow2_1^+)}{B(E2,4_1^+\rightarrow0_1^+)}. \tag{72}$$

The explicit expression for R_4 predicted by the IBM is

$$R_4=\begin{cases}\dfrac{2(N-1)}{N} & \text{for the } U_5 \text{ limit,}\\[2ex] \dfrac{10(N-1)(N+5)}{7N(N+4)} & \text{for the } SO_6 \text{ limit,}\end{cases} \tag{73}$$

where N is the boson number. The R_4 value derived from the FDSM has the same form as above, but with N replaced by N_1,

$$R_4=\frac{10(N_1-1)(N_1+5)}{7N_1(N_1+4)}. \tag{74}$$

The N_1 values can be estimated from shell model configurations of protons and neutrons in the odd-A nuclei, and are shown in the Table 7.1 of [23]. In Table VII we list the FDSM prediction for R_4 along with the experimental results of [22]. It can be seen that the SO_6 limit of the FDSM seems to explain the experimental data better than the IBM. Alternatively, we note that if accurate R_4 values are available, we may be used to obtain the empirical N_1 value from Eq. (74).

It should be mentioned again that apart from the Pauli effect, the FDSM differs from the IBM [21] in the value of the number of the collective pairs (N_1 vs N). The spectrum of the $\sigma=N_1$ band is not sensitive to the value of N_1, but the observation that the parameters g_6 and g_5' in Table I change smoothly between nuclei, and that the experimental spectra for the even and odd nuclei can be fitted with the same g_6 and g_5' values, suggest that the choice of N_1 taken in this paper is reasonable.

The difference between N and N_1 does affect the energies for the bands with $\sigma=N_1-2,N_1-4,\ldots$. In [12], it is

TABLE VII. The value of R_4.

Nuclei	N	N_1	R_4^{exp}	R_4^{FDSM} (SO_6)	R_4^{IBM} (SO_6)	R_4^{IBM} (U_5)
^{120}Xe	10	7	1.46(20)	1.34	1.38	1.80
^{124}Xe	8	6	1.29(15)	1.31	1.35	1.75
^{126}Ba	9	6	1.12(20)	1.34	1.37	1.75
^{128}Ba	8	6	1.03(14)	1.31	1.35	1.75
^{130}Ba	7	5	0.90(13)	1.27	1.34	1.71
^{130}Xe	5	4	1.35(18)	1.21	1.27	1.60

TABLE VIII. The $E(0_3^+)/[E(2_2^+)-E(2_1^+)]$ ratio.

Nuclei	N	N_1	Expt.	IBM	FDSM
^{118}Xe	9	6	2.912	6.67	4.67
^{122}Xe	9	6	3.68	6.67	4.67
^{124}Xe	8	6	3.44	6.00	4.67
^{126}Xe	7	5	3.58	5.33	4.00
^{128}Xe	6	5	3.56	4.67	4.00
^{130}Xe	5	4	3.44	4.00	3.33
^{134}Ba	5	5	3.84	4.00	4.00
^{136}Ba	4	4	2.918	3.33	3.33
^{138}Ce	5	5	3.25	4.00	4.00

pointed out that the parameters for the SO_6 nuclei in both the $A=130$ and Pt regions have a common characteristic that $g_5' \cong -g_6$ [see Eq. (28)]. With such an empirical relation, the following energy ratio has a simple form in both the IBM and the FDSM

$$\frac{E^+_{O_3}(\sigma=N-2)}{E^+_{2_2}-E^+_{2_1}}=\frac{2(N+1)}{3}, \tag{75}$$

$$\frac{E^+_{O_3}(\sigma=N_1-2)}{E^+_{2_2}-E^+_{2_1}}=\frac{2(N_1+1)}{3}. \tag{76}$$

A comparison of the ratios calculated using Eqs. (75) and (76) and the experimental data is shown in Table VIII. This example indicates also that the FDSM SO_6 model reproduces this ratio better than the IBM SO_6 for nuclei in the Xe-Ba region. This suggests that in this region an empirical effective boson number may be needed to give a better agreement with data in IBM calculations.

The $E2$ transition rate is generally more sensitive to the parameter N_1 than the spectra. The reasonableness of the chosen N_1 value can also be seen from the good agreement between the calculated and experimental values of the $B(E2)$ values for the isotopes of Ba and Xe, as shown in Tables V and VI. Finally, we reiterate that as N_1 increases in the shell, the spins for the ground states of odd nuclei swap naturally between 1/2 and 3/2 (this transition occurs at ^{131}Xe and ^{135}Ba for the isotopes of Xe and Ba, respectively).

In Table IX, both experimental and theoretical $B(E2)$ values for the even-odd nuclei of ^{129}Xe and ^{131}Xe are given, and compared with the calculated results of the NSUSY case. Here the effective charges (i.e., q) are the same as the neighboring even-even nuclei, and determined by the experimental $B(E2,2_1^+\to 0_1^+)$ values. While effective charges for k-active part (i.e., q'') are fitted by $E2$ transitions of even-odd nuclei. In this work, $(q,q'')=(0.129,0.075)$ e b, $(0.143,0.093)$ e b for ^{129}Xe and ^{131}Xe, respectively. The agreement with data is comparable in the two cases, although the NSUSY calculations give a somewhat better agreement of the weaker transitions. Finally, we reiterate that the group chain (2) is very similar to the NSUSY group chain (1). However, the pseudo-orbital angular momentum 2 in group chain (1) is introduced as one of several possible group reductions of U(6/20) in the NSUSY, while in the FDSM it is a result of the reclassification (in terms of the k-i basis) of the shell model single-particle states for the sixth shell (see Table 2.1 in [8]).

V. CONCLUSIONS

In this work, we provide simple but unified analytic solutions of even and even-odd nuclei within the frameworkof the fermion dynamical symmetry model. The good agreement of both level pattern and $E2$ transitions with our simplified solutions indicates a good SO(6) symmetry for both even and even-odd nuclei in $A=130$ region. We find that generally the FDSM results provide a unified description of

TABLE IX. Transition probabilities in ^{129}Xe ($N_1=5$) and ^{131}Xe ($N_1=4$), $\Omega_1=20$.

	^{129}Xe $B(E2)(e^2\,b^2)$				^{131}Xe $B(E2)(e^2\,b^2)$		
$J_i\to J_f$	FDSM	Expt.	Ref. [5]	$J_i\to J_f$	FDSM	Expt.	Ref. [5]
$\frac{3}{2}^+_1\to\frac{1}{2}^+_1$	0.036		0.007	$\frac{1}{2}^+_1\to\frac{3}{2}^+_1$	0.0953	0.0039	0.0012
$\frac{3}{2}^+_2\to\frac{3}{2}^+_1$	0.0186	<0.0005	0.013	$\frac{5}{2}^+_1\to\frac{1}{2}^+_1$	0.075	0.030	0.016
$\frac{3}{2}^+_2\to\frac{1}{2}^+_1$	0.084	0.12	0.12	$\frac{5}{2}^+_1\to\frac{3}{2}^+_1$	0.004	0.10	0.10
$\frac{5}{2}^+_1\to\frac{3}{2}^+_1$	0.011	0.22	0.10	$\frac{3}{2}^+_2\to\frac{3}{2}^+_1$	0.053	0.057	0.058
$\frac{5}{2}^+_1\to\frac{1}{2}^+_1$	0.070	0.077	0.039	$\frac{1}{2}^+_2\to\frac{1}{2}^+_1$	0.0000		
$\frac{1}{2}^+_2\to\frac{3}{2}^+_2$	0.028			$\frac{1}{2}^+_2\to\frac{3}{2}^+_1$	0.124	0.048	0.115
$\frac{1}{2}^+_2\to\frac{3}{2}^+_1$	0.14	0.044	0.12	$\frac{7}{2}^+_1\to\frac{5}{2}^+_1$	0.028	0.005	0.0013
$\frac{1}{2}^+_2\to\frac{1}{2}^+_1$	0.0000			$\frac{7}{2}^+_1\to\frac{3}{2}^+_1$	0.043	0.081	0.082
$\frac{5}{2}^+_2\to\frac{1}{2}^+_1$	0.004	0.057	0.071	$\frac{3}{2}^+_3\to\frac{3}{2}^+_1$	0.025	0.027	0.017
$\frac{3}{2}^+_3\to\frac{1}{2}^+_1$	0.056	0.0032	0.0056	$\frac{5}{2}^+_2\to\frac{3}{2}^+_2$	0.004	<0.031	0.0011
$\frac{3}{2}^+_4\to\frac{1}{2}^+_1$	0.0133	0.0030	0.0004	$\frac{5}{2}^+_2\to\frac{1}{2}^+_1$	0.071	0.068	0.056
				$\frac{5}{2}^+_2\to\frac{3}{2}^+_1$	0.014	0.013	0.043
				$\frac{7}{2}^+_2\to\frac{3}{2}^+_1$	0.124	0.005	0.026

the even and odd nuclei in this region that is comparable to or even somewhat better than NSUSY approach, but to a lesser degree in choosing one limit from NSUSY multigroup chains.

ACKNOWLEDGMENTS

This work was supported by the NSF (Drexel). The research in CYCU was supported by the National Science Council of ROC. Theoretical nuclear physics research at the University of Tennessee is supported by the U.S. Department of Energy through Contract Nos. DE-FG05-93ER40770 and DE-FG05-87ER40461. Oak Ridge National Laboratory is managed by Lockheed Martin Energy Systems, Inc. for the U. S. Department of Energy under Contract No. DE-AC05-84OR21400. X.W.P. and D.H.F. are grateful to Dr. Sun Yang for typesetting part of the manuscript.

APPENDIX

The SO_5 reduced matrix elements:

$$\langle N+1,\sigma+1,\tau\|S^{\dagger}\|N\sigma\tau\rangle=\left[\frac{(\Omega_1-\sigma-N)(\sigma-\tau+1)(\sigma+\tau+4)(N+\sigma+6)}{4(\sigma+2)(\sigma+3)}\right]^{1/2}, \quad \text{(A1)}$$

$$\langle N+1,\sigma-1,\tau\|S^{\dagger}\|N\sigma\tau\rangle=-\left[\frac{(\Omega_1+\sigma-N+4)(\sigma-\tau)(\sigma+\tau+3)(N-\sigma+2)}{4(\sigma+1)(\sigma+2)}\right]^{1/2}, \quad \text{(A2)}$$

$$\langle N+1,\sigma+1,\tau'\|D^{\dagger}\|N\sigma\tau\rangle=\left[\frac{(\Omega_1-\sigma-N)(N+\sigma+6)}{4(\sigma+2)(\sigma+3)}\right]^{1/2}\times\begin{cases}\left[\frac{(\sigma+\tau+4)(\sigma+\tau+5)(\tau+1)}{(2\tau+5)}\right]^{1/2}, & \tau'=\tau+1,\\ -\left[\frac{(\sigma-\tau+1)(\sigma-\tau+2)(\tau+2)}{(2\tau+1)}\right]^{1/2}, & \tau'=\tau-1,\end{cases} \quad \text{(A3)}$$

$$\langle N+1,\sigma-1,\tau'\|D^{\dagger}\|N\sigma\tau\rangle=\left[\frac{(\Omega_1+\sigma-N+4)(N-\sigma+2)}{4(\sigma+1)(\sigma+2)}\right]^{1/2}\times\begin{cases}-\left[\frac{(\sigma+\tau+2)(\sigma+\tau+3)(\tau+2)}{(2\tau+1)}\right]^{1/2}, & \tau'=\tau+1,\\ \left[\frac{(\sigma-\tau-1)(\sigma-\tau)(\tau+1)}{(2\tau+5)}\right]^{1/2}, & \tau'=\tau-1.\end{cases} \quad \text{(A4)}$$

The $SO_6 \supset SO_5$ isoscalar factors:

$$\left(\begin{array}{cc|c}[\sigma+\frac{1}{2},\frac{1}{2}\ \frac{1}{2}] & [100] & [\sigma'+\frac{1}{2},\frac{1}{2}\ \frac{1}{2}]\\ [\tau+\frac{1}{2},\frac{1}{2}] & [10] & [\tau+\frac{1}{2},\frac{1}{2}]\end{array}\right),$$

$$\sigma'=\sigma+1,\quad \sigma'=\sigma-1,$$

$$\left[\frac{(\sigma-\tau+1)(\sigma+\tau+5)}{2(\sigma+1)(\sigma+3)}\right]^{1/2}\quad \left[\frac{(\sigma-\tau)(\sigma+\tau+4)}{2(\sigma+2)(\sigma+4)}\right]^{1/2},$$

$$\left(\begin{array}{cc|c}[\sigma+\frac{1}{2},\frac{1}{2}\ \frac{1}{2}] & [100] & [\sigma'+\frac{1}{2},\frac{1}{2}\ \frac{1}{2}]\\ [\tau+\frac{1}{2},\frac{1}{2}] & [10] & [\tau_1\tau_2]\end{array}\right),$$

$[\tau_1\tau_2]$	$\sigma'=\sigma+1$	$\sigma'=\sigma-1$
$[\tau+\frac{3}{2},\frac{1}{2}]$	$\left[\frac{(\sigma+\tau+5)(\sigma+\tau+6)(\tau+1)}{2(\sigma+1)(\sigma+3)(2\tau+5)}\right]^{1/2}$	$-\left[\frac{(\sigma-\tau)(\sigma-\tau-1)(\tau+1)}{2(\sigma+2)(\sigma+4)(2\tau+5)}\right]^{1/2}$
$[\tau+\frac{1}{2},\frac{3}{2}]$	0	0
$[\tau+\frac{1}{2},\frac{1}{2}]$	$-\left[\frac{(\sigma+\tau+5)(\sigma-\tau+1)(\tau+3)}{2(\sigma+1)(\sigma+3)(3\tau+2)(2\tau+5)}\right]^{1/2}$	$\left[\frac{(\sigma+t+4)(\sigma-\tau)(\tau+2)}{2(\sigma+2)(\sigma+4)(3\tau+2)(2\tau+5)}\right]^{1/2}$
$[\tau-\frac{3}{2},\frac{1}{2}]$	$-\left[\frac{(\sigma-\tau+2)(\sigma-\tau+1)(\tau+3)}{2(\sigma+1)(\sigma+3)(2\tau+3)}\right]^{1/2}$	$\left[\frac{(\sigma+\tau+4)(\sigma+\tau+3)(\tau+3)}{2(\sigma+2)(\sigma+4)(2\tau+3)}\right]^{1/2}$

[1] A. Arima and F. Iachello, Annu. Rev. Nucl. Part. Sci. **31**, 75 (1981); Ann. Phys. (N.Y.) **99**, 253 (1976); **111**, 201 (1976); **115**, 325 (1978); **123**, 468 (1979).
[2] F. Iachello, Phys. Rev. Lett. **44**, 772 (1980).
[3] A.B. Balentekin, I. Bars, and F. Iachello, Nucl. Phys. **A370**, 284 (1981).
[4] A.B. Balentekin, I. Bars, R. Bijker, and F. Iachello, Phys. Rev. **C 27**, 1761 (1983); P. Van Isacker, J. Jolie, K. Heyde, and A. Frank, Phys. Rev. Lett. **54**, 653 (1985).
[5] Y.S. Ling, M. Zhang, J.M. Wu, M. Vallieres, R. Gilmore, and D.H. Feng, Phys. Lett. **148B**, 13 (1984).
[6] J. Jolie, K. Heyde, P. Van Isacker, and A. Frank, Nucl. Phys. **A466**, 1 (1987).
[7] C.L. Wu, D.H. Feng, X.G. Chen, J.Q. Chen, and M. Guidry, Phys. Rev. **C 36**, 1157 (1987).
[8] C.-L. Wu, D.H. Feng, and M.W. Guidry, Adv. Nucl. Phys. **21**, 227 (1994).
[9] J.N. Ginocchio, Ann. Phys. **126**, 234 (1980).
[10] J.Q. Chen, D.H. Feng, and C.L. Wu, Phys. Rev. **C 34**, 2269 (1986).
[11] J.A. Cizewski, R.F. Casten, G.J. Smith, M.L. Stelts, and W.R. Kane, Phys. Rev. Lett. **40**, 167 (1978).
[12] R.F. Casten and P. von Brentano, Phys. Lett. **152B**, 22 (1985); R.F. Casten, J.N. Ginocchio, D.H. Feng, and C.L. Wu, Mod. Phys. Lett. A **1**, 161 (1986).
[13] D.H. Feng *et al.*, Phys. Rev. C **48**, R1488 (1993).
[14] G.R. Black, R.C. King, and B.G. Wybourne, J. Phys. A **16**, 1555 (1983).
[15] Z.M. Lu, X.W. Pan, J.Q. Chen, X.G. Chen, and D.H. Feng, Phys. Rev. C **37**, 3789 (1988).
[16] X.W. Pan, T. Otsuka, J.Q. Chen, and A. Arima, Phys. Lett. B **287**, 1 (1992); X.W. Pan, D.H. Feng, J.Q. Chen, and M. Guidry, Phys. Rev. C **49**, 2493 (1994).
[17] K.T. Hecht, Nucl. Phys. **A475**, 276 (1987).
[18] F. Iachello and S. Kuyucak, Ann. Phys. **136**, 19 (1981).
[19] P. van Isacker, Nucl. Phys. **A465**, 497 (1987).
[20] B.R. Judd, *Operator Techniques In Atomic Spectroscopy* (McGraw-Hill, New York, 1963).
[21] R. Reinhardt, A. Dewald, A. Gelberg, W. Lieberz, K. Schiffer, K.P. Schmittgen, K.O. Zell, and P. von Brentano, Z. Phys. A **329**, 507 (1988).
[22] P. von Brentano, A. Gelberg, S. Harissopulos, and R.F. Casten, Phys. Rev. **C 38**, 2386 (1988).
[23] M.A. Preston and R.K. Bhaduri, *Structure of the Nucleus* (Addison-Wesley, Reading, MA, 1975).