Neutrinoless double beta decay within the quasiparticle random-phase approximation with proton-neutron pairing

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We have investigated the role of proton-neutron pairing in the context of the quasiparticle random phase approximation formalism. This way the neutrinoless double beta decay matrix elements of the experimentally interesting $A=48$, 76, 82, 96, 100, 116, 128, 130, and 136 systems have been calculated. We have found that the inclusion of proton-neutron pairing influences the neutrinoless double beta decay rates significantly, in all cases allowing for larger values of the expectation value of light neutrino masses. Using the best presently available experimental limits on the half lifetime of neutrinoless double beta decay we have extracted the limits on lepton number violating parameters.

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I. INTRODUCTION

Among the exotic processes the neutrinoless double β decay ($0\nu\beta\beta$ decay),

$$(A,Z)\rightarrow(A,Z\pm2)+e^{\mp}+e^{\mp}, \tag{1}$$

has been sought experimentally for about half a century but it has not yet been observed [1]. Its observation will undoubtedly be a signal of interesting physics beyond the standard model of electroweak interactions. First of all it will demonstrate the breakdown of lepton number conservation which, being associated with a global, not gauge, symmetry is expected to be broken at some level. It will also give us useful information about the neutrino mass if it is in the region $\leqslant 1$ eV. Finally, and most importantly, it is the best process, if not the only one, to settle the issue of whether the neutrino mass eigenstates are of the Majorana type, i.e., whether the particle coincides with its own antiparticle (π^0), or of the Dirac type (K^0).

It is, therefore, not surprising that the experimental searches [2–15] for the above process have persisted with great devotion up to the present day and have lead to the unbelievably long lifetime limit [2]

$$T_{1/2}^{0\nu}>5.6\times10^{24}\ \text{y}.$$

From this limit, in conjunction with calculations of the nuclear matrix elements involved, the limit $|\langle m_\nu\rangle|\leqslant 0.8$ eV has been extracted for the average light neutrino mass. It is interesting to remark that neutrino masses in this neighborhood can constitute candidates for the hot dark matter (HDM) component required for the understanding of the large scale structure of the universe as indicated by the KOBE data [16,17].

The analysis of the $0\nu\beta\beta$ decay data, if and when they become available, is unfortunately not going to be simple. Depending upon the extension of the standard model assumed there are many mechanisms which can lead to process (1) which can interfere with one another (see [18–22] for reviews). In those mechanisms in which the exchanged particle is light (e.g., light Majorana neutrino) the effective transition operator is of long range (essentially Coulombic) with or without spin dependence. If on the other hand the exchanged particle is heavy (e.g., heavy Majorana neutrino, exotic Higgs scalars, etc.) the effective operator is of short range and in the presence of short range correlations, one must be careful not to ignore [18] the finite size of the nucleon ($\sim$0.8 fm). In this case the size of the operator is set by the size of the nucleon. It is obvious that the two cases involve different nuclear physics and as we shall see later they lead to the extraction of different parameters of the gauge theory.

It is clear from the above discussion that all nuclear matrix elements must be computed reliably. This is easier said than done, however, since the nuclear systems which can undergo double β decay, with the possible exception of the $A=48$ system, are far away from closed shells and as a result have complicated structure [18–21]. In the shell model approach it is clearly impossible to construct all the needed intermediate states of the nucleus which is one unit of charge away. Since, however, the energy denominators are dominated by the virtual neutrino momentum, rather than the nuclear excitation energy, the construction of all these states can be avoided using a version of the closure approximation [18–21]. But even then it is quite hard to construct the wave functions of the initial and final nuclei employing a full shell model basis. Thus a weak coupling scheme has been employed, starting from products of neutron and proton wave functions and employing truncations according to the energies of the unperturbed proton and neutron states. In any case it is quite clear that it is very hard to substantially improve these old calculations [23–25].

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It is thus not surprising that other approximation schemes have been employed. The most prominent among them has been the quasiparticle random-phase approximation (QRPA) [26–34]. In this scheme the construction of the intermediate states is unavoidable, but not extremely hard. In the first step one constructs the intermediate states $|mJM\rangle$ as proton-particle neutron-hole excitations built on the ground state of initial nucleus. In the second step one views the intermediate states $|\bar{m}JM\rangle$ as neutron-particle proton-hole, built on the ground state of the final nucleus. These intermediate states are expressed as two quasiparticle states and one makes proper adjustments for the fact that the two sets of states $|mJ_m\rangle$ and $|\bar{m}J_m\rangle$ are not orthogonal.

In the QRPA approximation there is no need to invoke the closure approximation [26–28]. In fact it was possible to use QRPA to explicitly check how well the closure approximation works. It was rewarding to find that it works quite well except in those situations when the matrix elements are unusually suppressed.

As we have previously mentioned a number of QRPA calculations [24–38] for almost all nuclei of practical interest in $0\nu\beta\beta$ decay have been performed. In all such calculations the two quasiparticle states were of the proton-neutron variety. Proton-neutron $(p\text{-}n)$ pairing correlations [39–43] had been neglected. Such correlations were recently found, however, to be important [44–46] in the evaluation of the nuclear matrix elements entering $2\nu\beta\beta$ decay:

$$(A,Z)\rightarrow(A,Z+2)+e^{-}+e^{-}+\tilde{\nu}_e+\tilde{\nu}_e, \tag{2}$$

$$(A,Z)\rightarrow(A,Z-2)+e^{+}+e^{+}+\nu_e+\nu_e, \tag{3}$$

which proceeds only via the 1^+ intermediate nuclear states.

It is the purpose of this paper to investigate what effect, if any, the $p\text{-}n$ pairing has on the $0\nu\beta\beta$ decay matrix elements. To this end we will repeat and extend our previous calculations [26–28] to cover most of the nuclear targets of experimental interest (^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{136}Xe).

II. LEPTON VIOLATING PARAMETERS AND ASSOCIATED NUCLEAR MATRIX ELEMENTS

As we have mentioned in the Introduction there exist many mechanisms which can lead to $0\nu\beta\beta$ decay some of which are exotic (exotic Higgs scalars, supersymmetric R-parity violating interactions, etc., [18]). The most popular scenario is one which involves intermediate massive Majorana neutrinos. We will concentrate on the last mechanism in this work even though some of the nuclear matrix elements computed may be used for the more exotic mechanisms as well.

We will distinguish the following two cases.

(i) Both leptonic currents are of the same chirality, i.e., both left handed or both right handed. Then out of the neutrino propagator one picks the part

$$\frac{m_j}{q^2-m_j^2}, \tag{4}$$

which for light neutrinos yields an amplitude proportional to

$$\langle m_\nu\rangle=\sum_j |U_{ej}^{(11)}|^2 m_j e^{i\alpha_j}, \tag{5}$$

$\exp(i\alpha_j)$ is the CP eigenvalue of the neutrino mass eigenstate $|\nu_j\rangle$. For light neutrinos the contribution of the right-handed currents is negligible. For heavy neutrinos the amplitude becomes proportional to

$$\left\langle \frac{1}{M_N}\right\rangle_L=\sum_j |U_{ej}^{(12)}|^2\frac{1}{M_j}e^{i\phi_j}, \tag{6}$$

$$\left\langle \frac{1}{M_N}\right\rangle_R=\sum_j |U_{ej}^{(22)}|^2\frac{1}{M_j}e^{i\phi_j}, \tag{7}$$

for left- and right-handed currents, respectively. $\exp(i\phi_j)$ and M_j are the CP eigenvalue and the mass of the heavy neutrino mass eigenstate $|N_j\rangle$. The submatrices $U^{(11)}$, $U^{(12)}$, $U^{(21)}$, $U^{(22)}$ are the parts of the unitary matrix which connect the weak eigenstates $(\nu_\alpha^0,\nu_\alpha^{0c}),\alpha=e,\mu,\tau,$ with the mass eigenstates (ν_i,N_i), i = 1, 2, 3 (see, e.g., [18]). The upper index "1" stands for the light and "2" for the heavy neutrinos of left-right symmetric models. For each case one encounters a spin independent operator (vector$\times$vector or Fermi) and one which is a spin dependent scalar (axial$\times$axial or Gamow-Teller), yielding the matrix elements M_F and $M_{\rm GT}$, respectively.

(ii) Leptonic currents of opposite chirality. Then the relevant part of the intermediate neutrino propagator is

$$\frac{q_\lambda}{q^2-m_j^2}. \tag{8}$$

Since the right-handed currents are expected to be suppressed, this contribution is expected to be significant for light neutrinos. Thus the amplitude does not explicitly depend on the neutrino mass. One can now extract from the data two dimensionless parameters [18]

$$\lambda\cong\kappa\,\eta_{RL}, \tag{9}$$

$$\eta\cong\epsilon\,\eta_{RL}, \tag{10}$$

where

$$\kappa=(M_L/M_R)^2, \tag{11}$$

$$\epsilon=tg\zeta\quad(\text{mixing}). \tag{12}$$

M_L and M_R are, respectively, the masses of the vector bosons W_L and W_R associated with the left- and right-handed interactions. ζ is the W_L-W_R mixing angle. η_{RL} is given by [18]

$$\eta_{RL}=\sum_j U_{ej}^{(11)}U_{ej}^{(21)}e^{i\alpha_j}. \tag{13}$$

The λ term arises when the chiralities of the hadronic currents match those of the leptonic currents, i.e., they are of the J_L-J_R combination. The η term arises when the two hadronic currents are of the same chirality, i.e., of the J_L-J_L type (this is possible due to the W-boson mixing). For the extraction of λ one should know five matrix elements, i.e., $(M_{F\omega}, M_{\mathrm{GT}\omega})$ which arise from the time component of the propagator of Eq. (8) and $[M'_F, M'_{\mathrm{GT}}, M'_T$ (tensor)$]$ arising from the space component of that propagator. Due to the different energy dependence $M_{F\omega}, M_{\mathrm{GT}\omega}, M'_F$, and M'_{GT} are different from the two matrix elements M_F and M_{GT} encountered in the light neutrino mass mechanism. We will see, however, that to a good approximation

$$M_F = M'_F = M_{F\omega}, \tag{14}$$

$$M_{\mathrm{GT}} = M'_{\mathrm{GT}} = M_{\mathrm{GT}\omega}. \tag{15}$$

The situation is a bit more complicated in the case of the η term since one encounters two additional matrix elements M'_P and M_R. The first arises from an operator which is antisymmetric both in the spin and angular momentum indices [18–21]. The second matrix element arises from the momentum dependent terms of the hadronic current [25] (weak magnetism, etc.). The contribution of the momentum dependent term is normally a small correction. This is not true in this case, however, since due to the structure of the propagator of Eq. (8), the standard leading nonvanishing term is proportional to the average lepton momenta [1].

One must note, furthermore, that the kinematics are different, reflecting the difference between (4) and (8). Thus the coefficients entering the various combinations of nuclear matrix elements in the decay rate are energy dependent. As a result the relative importance of the various nuclear matrix elements may vary from nucleus to nucleus (depending on the available energy).

In the case of heavy neutrino intermediate states one encounters two matrix elements M_{HF} and $M_{H\mathrm{GT}}$ which differ from the above matrix elements M_F and M_{GT} due to the fact that the radial part of the relevant operator is short ranged.

The nuclear matrix elements mentioned above are associated with a set of transition operators which in momentum space can be cast in the general form (see [26–28,39] for details)

$$\Omega = \sum_{i\neq j} \tau^+(i)\tau^+(j)\omega(i,j)g_m(\mathbf{q}_i,\mathbf{q}_j), \tag{16}$$

with

$$g_m(\mathbf{q}_i,\mathbf{q}_j) = \frac{4\pi R_0}{(2\pi)^3}\frac{1}{2\sqrt{2}}\frac{\delta(\mathbf{q}_i+\mathbf{q}_j)}{\Delta_m(q_i\sqrt{2})}, \tag{17}$$

$$R_0 = r_0 A^{1/3} (r_0 \simeq 1.1 \text{ fm}) \quad \text{(nuclear radius)}, \tag{18}$$

$$\Delta_m(q) = \sqrt{q^2+m_\nu^2}[\epsilon_m + \sqrt{q^2+m_\nu^2}], \tag{19}$$

where m_ν is the mass of the virtual neutrino and ϵ_m a suitable energy denominator [see Eq. (46) below].

The most important matrix element M_{GT} is associated with $\vec{\sigma}_1\cdot\vec{\sigma}_2$, i.e.,

$$\omega_{\mathrm{GT}}(1,2) = \vec{\sigma}_1\cdot\vec{\sigma}_2 \leftrightarrow M_{\mathrm{GT}}. \tag{20}$$

Similarly

$$\omega_F(1,2) = 1 \leftrightarrow M_F. \tag{21}$$

The matrix elements $M_{FH}, M_{\mathrm{GT}H}$ associated with heavy neutrino are related to the operators ω_{FH} and $\omega_{\mathrm{GT}H}$ where

$$\omega_{FH}(1,2) = \frac{m_\nu^2}{m_e m_p} \leftrightarrow M_{FH}, \tag{22}$$

$$\omega_{\mathrm{GT}H}(1,2) = \frac{m_\nu^2}{m_e m_p}\vec{\sigma}_1\cdot\vec{\sigma}_2 \leftrightarrow M_{\mathrm{GT}H}. \tag{23}$$

Notice that for heavy neutrino

$$\frac{m_\nu^2}{m_e m_p}\frac{1}{\Delta_m(q)} \approx \frac{1}{m_e m_p} \tag{24}$$

(independent of momentum) to be compared with

$$\frac{1}{\Delta_m(q)} \approx \frac{1}{q^2} \quad \text{(for light neutrino)}. \tag{25}$$

For processes which do not explicitly depend on the neutrino mass ($j_L - j_R$ interference) we encounter the operators $\omega_{F\omega}$ and $\omega_{\mathrm{GT}\omega}$ which differ from ω_F and ω_{GT} by the inclusion of the extra kinematical factor $\delta_m(q_1\sqrt{2})$ with

$$\delta_m(q) = \frac{\sqrt{q^2+m_\nu^2}}{\epsilon_m + \sqrt{q^2+m_\nu^2}}. \tag{26}$$

One also encounters the matrix elements M'_F and M'_{GT} associated with the operators

$$\omega'_F(1,2) = -2\vec{q}_1\cdot\vec{\nabla}_1 \leftrightarrow M'_F, \tag{27}$$

$$\omega'_{\mathrm{GT}}(1,2) = -2\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{q}_1\cdot\vec{\nabla}_1 \leftrightarrow M'_{\mathrm{GT}}, \tag{28}$$

and the matrix element M'_T associated with

$$\omega'_T(1,2) = \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{q}_1\cdot\vec{\nabla}_1 - \vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_2\cdot\vec{\nabla}_1 - \vec{\sigma}_1\cdot\vec{\nabla}_1\vec{\sigma}_2\cdot\vec{q}_1 \leftrightarrow M'_T. \tag{29}$$

As it has been mentioned above in the case of $j_L - j_R$ but $J_L - J_L$ combination we encounter two additional matrix elements namely M'_P and M_R. M'_P is associated with the operator $\omega'_P(1,2)$:

$$\omega'_P(1,2) = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)(\ell_1 - \ell_2) \leftrightarrow M'_P, \tag{30}$$

while M_R is associated with the operator ω_R:

$$\omega_R \simeq \frac{g_V}{g_A}\frac{2\mu}{m_e m_p a^2}\left(\frac{2}{3}\omega_S^R - \omega_T^R\right) \leftrightarrow M_R, \tag{31}$$

with $\mu = \mu_p - \mu_{n+1} = 4.7$ (a is the oscillator length) and

$$\omega_S^R(1,2) = (\vec{\sigma}_1\cdot\vec{\sigma}_2)q_1^2 a^2, \tag{32}$$

$$\omega_T^R(1,2) = (\vec{\sigma}_1\cdot\hat{q}_1\vec{\sigma}_2\cdot\hat{q}_1 - \tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2)q_1^2 a^2. \tag{33}$$

In the expression for the $0\nu\beta\beta$ decay lifetime various combinations of the above nuclear matrix elements appear. These will be given in units of M_{GT} and be denoted [19] by χ. First those associated with the mass mechanism

$$X_L=\frac{\langle m_\nu\rangle}{m_e}(\chi_F-1)+\left\langle\frac{m_p}{M_N}\right\rangle_L\chi_H, \tag{34}$$

$$X_R=(\kappa^2+\epsilon^2)\left\langle\frac{m_p}{M_N}\right\rangle_R\chi_H, \tag{35}$$

with

$$\chi_F=\left(\frac{g_V}{g_A}\right)^2\frac{M_F}{M_{\mathrm{GT}}}, \tag{36}$$

$$\chi_H=\left[\left(\frac{g_V}{g_A}\right)^2 M_{FH}-M_{\mathrm{GT}H}\right]\Big/M_{\mathrm{GT}}. \tag{37}$$

Second those not connected with the mass mechanism: $\chi_{F\omega}$, $\chi_{\mathrm{GT}\omega}$, χ_R, $\chi_{1^\pm}$, $\chi_{2^\pm}$ χ'_F, χ'_{GT}, χ'_T, χ'_P where

$$\chi_{F\omega}=\left(\frac{g_V}{g_A}\right)^2\frac{M_{F\omega}}{M_{\mathrm{GT}}}, \tag{38}$$

$$\chi_{\mathrm{GT}\omega}=\frac{M_{\mathrm{GT}\omega}}{M_{\mathrm{GT}}}, \tag{39}$$

$$\chi_R=\frac{M_R}{M_{\mathrm{GT}}}, \tag{40}$$

and

$$\chi_{1^\pm}=\pm 3\chi'_F+\chi'_{\mathrm{GT}}-6\chi'_T, \tag{41}$$

$$\chi_{2^\pm}=\pm\chi_{F\omega}+\chi_{\mathrm{GT}\omega}-\frac{1}{9}\chi_{1^\pm} \tag{42}$$

in an obvious notation ($\chi'_F=M'_F/M_{\mathrm{GT}}$, etc.). In the limit in which the energy denominator ϵ_m can be neglected we obtain

$$\chi_F=\chi'_F=\chi_{F\omega}, \tag{43}$$

$$\chi_{\mathrm{GT}}=\chi'_{\mathrm{GT}}=\chi_{\mathrm{GT}\omega}=1. \tag{44}$$

As we have already mentioned the closure approximation was not employed in our approximation. In writing, however, the expression for ϵ_m we made the standard approximation of replacing the electron energy by an average value. Furthermore the lepton wave functions were taken out of the nuclear integral incorporating their effect via a distortion factor (see [18,19]).

In avoiding the closure approximation [26–28,39] the momentum space representation was found extremely useful. As it was shown in [39] by exploiting the properties of the harmonic oscillator wave functions it was possible to express the energy dependent radial integrals of each type of operator in addition to the energy in terms of only two parameters n and ℓ. The parameter ℓ takes values ($\ell=0,1,2$ for scalar, vector, and tensor rank, respectively). n also takes few values limited by the number $\tilde{N}$ of oscillator quanta involved in the two-particle wave function of the interacting nucleons ($n\leqslant 2\tilde{N}$) regardless of the number of single-particle configurations employed [39]. In the systems we studied $n\leqslant 10$.

III. THE QRPA FORMALISM WITH PROTON-NEUTRON PAIRING

The QRPA formalism employed in the $\beta\beta$ decay, a modification of the usual RPA formalism that involves a change of charge by two units, is fairly well known [26–37]. So we will briefly mention its main features here. $\beta\beta$ decay is viewed as a two-step process. In the first step a proton particle neutron hole is created on the initial state $|0_i^+\rangle$ and associated with the intermediate states $|mJM\rangle$ of the $(A,Z+1)$ nuclear system. In the second step, rather than considering an additional proton particle-neutron hole acting on the excited intermediate states, we consider a neutron particle-proton hole on the final state $|0_f^+\rangle$ leading also to states of the $(A,Z+1)$ nucleus labeled $|\bar{m}JM\rangle$. The states $|mJM\rangle$ and $|\bar{m}JM\rangle$ are obtained by solving the well-known QRPA equations. Unfortunately they are not orthogonal to each other. The nuclear physics dependence of $\beta\beta$ decay is essentially contained in the matrix element

$$\langle 0_f^+\|\tilde{u}^k(a_{pc}^+a_{nd})\|\bar{m}J\rangle\langle\bar{m}J|mJ\rangle\langle mJ\|u^k(a_{pa}^+a_{nb})\|0_i^+\rangle, \tag{45}$$

where $u_\lambda^k(a^+a)$ are the usual tensors of rank k ($k=J$ here) built of protons and neutrons as indicated. $\tilde{u}_\lambda^k(a^+a)=(-1)^{k-\lambda}u_{-\lambda}^k(a^+a)$ is the time reversed operator of $u_\lambda^k(a^+a)$. a,b,c,d designate all the single-particle quantum numbers except for the angular momentum projection m_a, i.e., $a\Leftrightarrow(n_a\ell_a j_a)$, etc. The overlap is necessary since, as we have mentioned above, these intermediate states are not orthogonal to each other. Hence the energy denominator ϵ_m encountered in the previous section is given by the prescription

$$\begin{aligned}\epsilon_m&=E_m-\langle E_e\rangle-E_f=E_m-\frac{1}{2}(E_i+E_f)\\&\rightarrow\frac{1}{2}(E_m+E_{\bar{m}})-\frac{1}{2}(E_i+E_f)\\&=\frac{1}{2}(\Omega_m+\Omega_{\bar{m}}).\end{aligned} \tag{46}$$

Here, Ω_m and $\Omega_{\bar{m}}$ are the QRPA energies of the excited states $|mJM\rangle$ and $|\bar{m}JM\rangle$ calculated from the ground state energy of the initial and final nucleus, respectively.

The next step consists in expressing the tensor operators $u_\lambda^k(a^+a)$ in terms of quasiparticles [26], i.e.,

$$\begin{pmatrix}a_{pam_a}^+\\ a_{pa\tilde{m}_a}\end{pmatrix}=\begin{pmatrix}u_p(a) & -v_p(a)\\ v_p(a) & u_p(a)\end{pmatrix}\begin{pmatrix}b_{1am_a}^+\\ b_{1a\tilde{m}_a}\end{pmatrix}, \tag{47}$$

$$\begin{pmatrix}a_{nam_a}^+\\ a_{na\tilde{m}_a}\end{pmatrix}=\begin{pmatrix}u_n(a) & -v_n(a)\\ v_n(a) & u_n(a)\end{pmatrix}\begin{pmatrix}b_{2am_a}^+\\ b_{2a\tilde{m}_a}\end{pmatrix}, \tag{48}$$

where b^+ and b are the construction and destruction operators for quasiparticles. The tilde indicates the time reversed states $a_{a\tilde{m}_a} = (-)^{j_a - m_a} a_{a-m_a}$, etc. Clearly since protons and neutrons do not mix the indices 1 and 2 can be identified with protons and neutrons, respectively. The parameters u and v are the occupation probabilities obtained by solving the standard BCS equations for the initial state. A similar set of equations for the final states $(A,Z+2)$ yields the occupation probabilities $\bar{u}$ and $\bar{v}$ entering the matrix element on the left of Eq. (45).

The QRPA states $|mJM\rangle$ are of the form [29–31]

$$|mJM\rangle = Q^{m}_{JM}{}^{+}|0^+_i\rangle_{\mathrm{RPA}} \equiv \sum_{a,b}[X^m_{12}(a,b,J)B^+_{12}(a,b,J,M) - Y^m_{12}(a,b,J)\tilde{B}_{12}(a,b,J,M)]|0^+_i\rangle, \tag{49}$$

where

$$B^+_{\mu\nu}(a,b,J,M) = n(\mu a,\nu b)\sum_{m_a,m_b} C^{JM}_{j_a m_a j_b m_b} b^+_{\mu a m_a} \times b^+_{\nu b m_b}(\mu,\nu=1,2), \tag{50}$$

$$\tilde{B}_{\mu\nu}(a,b,JM) = (-1)^{J-M}B_{\mu\nu}(a,b,J,-M), \tag{51}$$

and

$$n(\mu a,\nu b) = \frac{1+(-1)^J\delta_{\mu\nu}\delta_{ab}}{(1+\delta_{\mu\nu}\delta_{ab})^{3/2}}. \tag{52}$$

The forward- and backward-going amplitudes X^m_{12} and Y^m_{12} and the energies of the excited states Ω_m are obtained by solving the QRPA equation for the initial nucleus (A,Z) [27–31]. By performing the QRPA diagonalization for the final nucleus $(A,Z+2)$ we obtain the amplitudes $X^{\bar{m}}_{12}$, $Y^{\bar{m}}_{12}$ and eigenenergies $\Omega_{\bar{m}}$ of the QRPA state $|\bar{m}JM\rangle$.

One can show that

$$\langle mJ\|u^k(a_{pa^+}a_{nb}\|0_{i^+} = \delta_{kJ}\sqrt{2J+1}[u_p(a)v_n(b)X^m_{12}(a,b,J) + v_p(a)u_n(b)Y^m_{12}(a,b,J)], \tag{53}$$

$$\langle 0^+_f\|\tilde{u}^k(a^+_{pc}a_{nd})\|\bar{m}J\rangle = \delta_{kJ}\sqrt{2J+1}[\bar{v}_p(c)\bar{u}_n(d)X^{\bar{m}}_{12}(c,d,J) + \bar{u}_p(c)\bar{v}_n(d)Y^{\bar{m}}_{12}(c,d,J)]. \tag{54}$$

The overlap integral takes the form

$$\langle \bar{m}J|mJ\rangle = \sum_{a,b}[X^m_{12}(a,b,J)X^{\bar{m}}_{12}(a,b,J) - Y^m_{12}(a,b,J)Y^{\bar{m}}_{12}(a,b,J)]. \tag{55}$$

Once proton-neutron correlations are introduced the quasiparticle labels can no longer be identified with protons and neutrons, but they become mere labels. The transformation matrix [40–46] is generalized to the 4×4 matrix

$$\begin{pmatrix} a^+_{pam_a} \\ a^+_{nam_a} \\ a_{pa\tilde{m}_a} \\ a_{na\tilde{m}_a}\end{pmatrix} = \begin{pmatrix} u_{1p}(a) & u_{2p}(a) & -v_{1p}(a) & -v_{2p}(a) \\ u_{1n}(a) & u_{2n}(a) & -v_{1n}(a) & -v_{2n}(a) \\ v_{1p}(a) & v_{2p}(a) & u_{1p}(a) & u_{2p}(a) \\ v_{1n}(a) & v_{2n}(a) & u_{1n}(a) & u_{2n}(a)\end{pmatrix} \times \begin{pmatrix} b^+_{1am_a} \\ b^+_{2am_a} \\ b_{1a\tilde{m}_a} \\ b_{2a\tilde{m}_a}\end{pmatrix} \tag{56}$$

independently of the angular momentum projection quantum number m_a.

The columns of the above matrix are the eigenvectors of the generalized Hartree-Fock-Bogoliubov (HFB) equations (see [44–46] for details). As such are, of course, determined only up to an overall phase (for each one).

If the p-n pairing interaction is switched on, the angular momentum coupled phonon operator takes the form [44–46]

$$Q^{m+}_{JM} = \sum\nolimits_{a,b}\{X^m_{12}(a,b,J)B^+_{12}(a,b,J,M) + Y^m_{12}(a,b,J)\tilde{B}_{12}(a,b,J,M)\} + \sum_{\substack{a\leq b\\ \mu=1,2}}\{X^m_{\mu\mu}(a,b,J)B^+_{\mu\mu}(a,b,J,M) + Y_{\mu\mu^m}(a,b,J)\tilde{B}_{\mu\mu}(a,b,J,M), \tag{57}$$

The amplitudes X^m, Y^m are obtained by solving the QRPA eigenproblem

$$\begin{pmatrix} A & B \\ B & A\end{pmatrix}\begin{pmatrix} X_m \\ Y_m\end{pmatrix} = \Omega_m\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix} X_m \\ Y_m\end{pmatrix}, \tag{58}$$

with

$$A = \begin{pmatrix} A^{11,11} & A^{11,22} & A^{11,12} \\ A^{22,11} & A^{22,22} & A^{22,12} \\ A^{12,11} & A^{12,22} & A^{12,12}\end{pmatrix}, \tag{59}$$

$$B = \begin{pmatrix} B^{11,11} & B^{11,22} & B^{11,12} \\ B^{22,11} & B^{22,22} & B^{22,12} \\ B^{12,11} & B^{12,22} & B^{12,12}\end{pmatrix}, \tag{60}$$

and

$$X^m = \begin{pmatrix} X^m_{11} \\ X^m_{22} \\ X^m_{12}\end{pmatrix}, Y^m = \begin{pmatrix} Y^m_{11} \\ Y^m_{22} \\ Y^m_{12}\end{pmatrix}. \tag{61}$$

The QRPA equation in Eq. (58) represents a general equation for all excited states of a given even-even nucleus. The explicit form of the A and B submatrices has been reviewed in

TABLE I. The single-particle orbitals used in the present calculations.

Nuclear orbitals	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
1	$0s_{1/2}$	$1s_{1/2}$	$1s_{1/2}$	$0f_{7/2}$	$0f_{7/2}$	$0f_{5/2}$	$1p_{3/2}$	$1p_{3/2}$	$1p_{3/2}$
2	$0p_{3/2}$	$0f_{7/2}$	$0f_{7/2}$	$0f_{5/2}$	$0f_{5/2}$	$1p_{3/2}$	$1p_{1/2}$	$1p_{1/2}$	$1p_{1/2}$
3	$0p_{1/2}$	$0f_{5/2}$	$0f_{5/2}$	$1p_{3/2}$	$1p_{3/2}$	$1p_{1/2}$	$0g_{7/2}$	$0g_{7/2}$	$0g_{7/2}$
4	$0d_{5/2}$	$1p_{3/2}$	$1p_{3/2}$	$1p_{1/2}$	$1p_{1/2}$	$0g_{9/2}$	$0g_{7/2}$	$0g_{7/2}$	$0g_{7/2}$
5	$0d_{3/2}$	$1p_{1/2}$	$1p_{1/2}$	$0g_{9/2}$	$0g_{9/2}$	$0g_{7/2}$	$1d_{5/2}$	$1d_{5/2}$	$1d_{5/2}$
6	$1s_{1/2}$	$0g_{9/2}$	$0g_{9/2}$	$0g_{7/2}$	$0g_{7/2}$	$1d_{5/2}$	$1d_{3/2}$	$1d_{3/2}$	$1d_{3/2}$
7	$0f_{7/2}$	$0g_{7/2}$	$0g_{7/2}$	$1d_{5/2}$	$1d_{5/2}$	$1d_{3/2}$	$2s_{1/2}$	$2s_{1/2}$	$2s_{1/2}$
8	$0f_{5/2}$	$1d_{5/2}$	$1d_{5/2}$	$1d_{3/2}$	$1d_{3/2}$	$2s_{1/2}$	$0h_{11/2}$	$0h_{11/2}$	$0h_{11/2}$
9	$1p_{3/2}$	$1d_{3/2}$	$1d_{3/2}$	$2s_{1/2}$	$2s_{1/2}$	$0h_{11/2}$	$0h_{9/2}$	$0h_{9/2}$	$0h_{9/2}$
10	$1p_{1/2}$	$2s_{1/2}$	$2s_{1/2}$	$0h_{11/2}$	$0h_{11/2}$	$0h_{9/2}$	$1f_{7/2}$	$1f_{7/2}$	$1f_{7/2}$
11				$0h_{9/2}$	$0h_{9/2}$	$1f_{7/2}$			
12						$1f_{5/2}$			

our previous publications [44–46]. We note that if proton-neutron pairing interaction is neglected, the QRPA submatrices $A^{12,11}$, $A^{11,12}$, $A^{12,22}$ $A^{22,12}$ and $B^{12,11}$, $B^{11,12}$, $B^{12,22}$, $B^{22,12}$ are equal to zero and the QRPA equation possesses two types of eigenstates: The eigenstates of the first type (I) with wave functions $X^m=(X^m_{11},X^m_{22},0)$, $Y^m=(Y^m_{11},Y^m_{22},0)$ having the origin in proton-proton and neutron-neutron quasiparticle excitations. The eigenstate of the second type (II) with the wave functions $X^m=(0,0,X^m_{12})$, $Y^m=(0,0,Y^m_{12})$ generated by the phonon operator in Eq. (49).

Equations (53) and (54) are generalized as follows:

$$\langle mJ\|u^k(a^+_{pa}a_{nb})\|0^+_i\rangle=\delta_{kJ}\sqrt{2J+1}\sum_{\mu,\nu=1,2} m(\mu a,\nu b)\times[u_{\mu p}(a)v_{\nu n}(b)X^m_{\mu\nu}(a,b,J)+v_{\mu p}(a)u_{\nu n}(b)Y^m_{\mu\nu}(a,b,J)], \tag{62}$$

$$\langle 0^+_f\|\tilde{u}^k(a^+_{pc}a_{nd})\|\bar{m}J\rangle=\delta_{kJ}\sqrt{2J+1}\sum_{\mu,\nu=1,2} m(\mu a,\nu b)\times[\bar{v}_{\mu p}(c)\bar{u}_{\nu n}(d)X^{\bar{m}}_{\mu\nu}(c,d,J)+\bar{u}_{\mu p}(c)\bar{v}_{\nu n}(d)Y^{\bar{m}}_{\mu\nu}(c,d,J)], \tag{63}$$

with

$$m(\mu a,\nu b)=[1+(-1)^J\delta_{\mu\nu}\delta_{ab}]/(1+\delta_{\mu\nu}\delta_{ab})^{1/2}. \tag{64}$$

We note that the $X^m_{\mu\nu}(a,b,J)$ and $Y^m_{\mu\nu}(a,b,J)$ amplitudes are calculated by the QRPA equation in Eq. (58) only for the configurations $\mu a\leqslant\nu b$ (i.e., $\mu=\nu$ and the orbitals are ordered $a\leqslant b$ and $\mu=1$, $\nu=2$ and the orbitals are not ordered). For different configurations $X^m_{\mu\nu}(a,b,J)$ and $Y^m_{\mu\nu}(a,b,J)$ in Eqs. (62), and (63) are given as follows:

$$X^m_{\mu\nu}(a,b,J)=-(-1)^{j_a+j_b-J}X^m_{\nu\mu}(b,a,J), \tag{65}$$

$$Y^m_{\mu\nu}(a,b,J)=-(-1)^{j_a+j_b-J}Y^m_{\nu\mu}(b,a,J). \tag{66}$$

Note that in the limit in which there is no proton-neutron pairing, i.e., $u_{2p}=v_{2p}=u_{1n}=v_{1n}=0$, Eqs. (62), and (63) reduce to Eqs. (53), and (54), respectively, by setting $u_{1p}=u_p, v_{1p}=v_p, u_{2n}=u_n$, and $v_{2n}=v_n$. Clearly, in the case without proton-neutron pairing the eigenstates of type I do not contribute to the beta decay transition matrix elements in Eqs. (62) and (63).

The overlap integral becomes

$$\langle\bar{m}J|mJ\rangle=\sum_{\mu a\leqslant\nu b}[X^m_{\mu\nu}(a,b,J)X^{\bar{m}}_{\mu\nu}(a,b,J)-Y^m_{\mu\nu}(a,b,J)Y^{\bar{m}}_{\mu\nu}(a,b,J)]. \tag{67}$$

By setting $X^m_{\mu\mu}=Y^m_{\mu\mu}=X^{\bar{m}}_{\mu\mu}=Y^{\bar{m}}_{\mu\mu}=0$ the above overlap is reduced to that of Eq. (55).

To complete the discussion we mention that the single-particle wave functions and energies were obtained by using a Coulomb corrected Woods-Saxon potential. The interaction employed was the Brueckner G matrix which is a solution of the Bethe-Goldstone equation employed using the Bonn one-boson exchange potential (OBEP) [47]. Proton and neutron number conservation in the initial and final state was respected on the average with

$$(N_{np}-\langle N_{np}\rangle)/N_{np}\leqslant 10^{-4}. \tag{68}$$

The BCS pp and nn parameters d_{pp} and d_{nn} were obtained by fits to the experimental proton and neutron gaps as in [44–46]. The np strength parameter d_{np} is fixed by a renormalization of the $T=1$ $J=0$ pairing force as in [44–46]. In the QRPA calculations it is necessary to introduce renormalization parameters g_{pp} and g_{ph} for the particle-particle and particle-hole interactions, which in principle should be close to unity. Our adopted values were $g_{pp}=1.0$ and $g_{ph}=0.8$. For higher value of g_{ph} the particle-hole interaction for some multipolarities is too strong. The lowest eigenvalue becomes imaginary and leads to a collapse of the correlated ground state.

TABLE II. The matrix elements of $0\nu\beta\beta$ decay for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, and ^{136}Xe calculated in the framework of QRPA with and without p-n pairing.

Nucleus	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
				QRPA without p-n pairing					
$M^{0\nu}_{\mathrm{GT}}$	-0.785	2.929	-2.212	2.097	0.615	0.449	2.437	2.327	1.598
χ_F	-0.468	-0.038	-0.008	-0.149	-0.766	-1.103	-0.0179	-0.004	0.028
$\lvert M^{0\nu}_{\mathrm{GT}}(1-\chi_F)\rvert$	1.152	3.040	2.230	2.409	1.086	0.944	2.480	2.335	1.553
χ_H	-134.9	-68.37	-44.27	-47.24	-124.8	-47.06	-41.54	-39.82	-21.92
χ'_F	-0.504	-0.035	-0.004	-0.168	-0.817	-1.173	-0.022	-0.007	0.022
χ'_{GT}	0.975	1.077	1.050	1.143	1.174	1.074	1.097	1.097	1.123
χ'_T	-0.212	0.244	0.079	0.121	-0.477	-0.812	0.307	0.282	0.349
$\chi_{F\omega}$	-0.437	-0.038	-0.013	-0.130	-0.709	-1.032	-0.012	0.001	0.036
$\chi_{\mathrm{GT}\omega}$	1.057	0.916	0.960	0.845	0.683	0.859	0.894	0.895	0.875
χ'_P	0.168	-1.147	-0.049	-0.836	-3.843	-3.891	-1.400	-1.451	-1.627
χ_R	172.1	193.0	124.2	113.8	105.1	-151.5	157.1	149.0	124.8
				QRPA with p-n pairing					
$M^{0\nu}_{\mathrm{GT}}$	-0.405	1.846	-1.153	0.280	-0.584	0.119	1.270	1.833	1.346
χ_F	0.158	0.274	-0.416	2.282	0.939	-6.784	0.308	0.184	0.066
$\lvert M^{0\nu}_{\mathrm{GT}}(1-\chi_F)\rvert$	0.341	1.340	1.633	0.358	0.036	0.926	0.879	1.495	1.257
χ_H	6.075	-32.75	-57.20	-41.64	-14.22	-453.8	-34.02	-55.72	-35.37
χ'_F	0.184	0.322	-0.467	2.601	1.067	-7.400	0.370	0.218	0.082
χ'_{GT}	1.226	1.124	1.082	1.587	0.934	0.927	1.159	1.115	1.167
χ'_T	0.130	0.214	0.179	0.209	0.853	-3.991	0.343	0.411	0.332
$\chi_{F\omega}$	0.131	0.235	-0.379	2.069	0.812	-6.170	0.260	0.159	0.052
$\chi_{\mathrm{GT}\omega}$	0.775	0.876	0.927	0.335	1.142	0.938	0.831	0.879	0.832
χ'_P	-0.009	-0.479	-1.621	-4.802	2.519	-7.592	-2.907	-0.993	-2.441
χ_R	57.32	129.3	131.1	157.3	162.2	-333.6	158.6	192.6	138.4

By the method outlined above we obtained the matrix elements $M^{0\nu}_{\mathrm{GT}}$, χ_F, χ_H, χ'_F, χ'_{GT}, χ'_T, $\chi_{F\omega}$, $\chi_{\mathrm{GT}\omega}$, χ'_P, and χ_R for the nuclei ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, and ^{136}Xe for the orbitals shown in Table I. These matrix elements are given in Table II. For comparison we also present in the same Table II the values obtained without p-n pairing. These last results differ slightly from those of our earlier calculations [26–28] due to the different model space employed.

By glancing at the Table II we see that the effect of the inclusion of the p-n pairing is significant. Perhaps the most important matrix element is $|M^{0\nu}_{\mathrm{GT}}(1-\chi_F)|$, which connect us directly with the effective neutrino mass $|\langle m_\nu\rangle|$. We see that the inclusion of the p-n pairing reduces the value of this matrix element. The largest reductions of $|M^{0\nu}_{\mathrm{GT}}(1-\chi_F)|$ by factors 30.2, 6.7, 3.4, 2.8, and 2.3 are associated with $A=100$, 96, 48, 128, and 76 systems. By these factors also the lower limits on $|\langle m_\nu\rangle|$ are enhanced in respect to the calculations without p-n pairing. To study the influence of p-n pairing on the evaluation of the limits on lepton number nonconserving parameters of right-handed currents it is necessary to calculate the integrated kinematical factors G_{0k} [19,20]. They are listed in Table III. A small difference with the values of [19] has origin in a different adopted value of nuclear radius R_0 [see Eq. (18)]. The problem of the extraction of the lepton number nonconserving parameters we shall study in Sec. IV.

In the nuclear systems $A=96$, 100, and 116 we have noticed a sensitivity of this matrix element with respect to the number of orbitals employed. Since including in our present calculation with p-n pairing all the 15 orbitals employed in the earlier calculations was prohibitive in term of computer time, we decided to employ 12 orbitals. This is admissible since we are interested in comparing the results with and without p-n pairing in the same model space. Furthermore our present results without p-n pairing for ^{100}Mo agree with those of [2] which used the same model space. The results so obtained for this nucleus are comparable with those of the other nuclei, which makes our choice reasonable. The other matrix elements depend a bit more strongly on the p-n pairing correlations. The effect is even stronger for some individual multipoles, especially if the corresponding matrix element is suppressed.

TABLE III. The integrated kinematical factors G_{0k} for $0^+ \to 0^+$ transition of $(\beta\beta)_{0\nu}$ decay. The definition of G_{0k} is given in [19] in Eqs. (3.5.17)–(3.5.21).

	$(\beta\beta)_{0\nu}$ decay: $0^+ \to 0^+$ transition								
	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
$(E_i - E_f)$ (MeV)	5.294	3.067	4.027	4.372	4.055	3.830	1.891	3.555	3.503
G_{01} [10^{-14} y^{-1}]	8.031	0.7928	3.524	7.362	5.731	6.233	2.207×10^{-1}	5.543	5.914
G_{02} [10^{-13} y^{-1}]	5.235	0.1296	1.221	3.173	2.056	1.957	6.309×10^{-3}	1.441	1.483
G_{03} [10^{-14} y^{-1}]	6.037	0.4376	2.413	5.380	4.036	4.305	6.177×10^{-2}	3.669	3.890
G_{04} [10^{-14} y^{-1}]	1.705	0.1538	0.724	1.530	1.178	1.269	3.368×10^{-2}	1.113	1.183
G_{05} [10^{-12} y^{-1}]	1.265	0.253	0.931	2.009	1.718	2.118	1.390×10^{-1}	2.083	2.298
G_{06} [10^{-11} y^{-1}]	1.398	0.196	0.665	1.226	1.009	1.103	6.969×10^{-2}	1.011	1.077
G_{07} [10^{-10} y^{-1}]	11.46	1.248	5.523	12.07	9.563	10.69	4.363×10^{-1}	9.544	10.25
G_{08} [10^{-11} y^{-1}]	5.247	0.793	3.852	9.886	8.109	10.20	4.227×10^{-1}	9.749	10.84
G_{09} [10^{-9} y^{-1}]	6.262	0.491	1.980	3.686	2.819	2.800	1.125×10^{-1}	2.335	2.424
G_{01,χ^0} [10^{-14} y^{-1}]	2.425	0.0763	0.6202	1.5315	1.0230	0.9879	5.206×10^{-3}	0.7487	0.7734

IV. DECAY RATES

The $0\nu\beta\beta$ decay can be expressed in terms of the lepton violating parameter $\langle m_\nu \rangle / m_e$, etc., defined in Sec. II as follows [27], namely

$$
\begin{aligned}
[T_{1/2}^{0\nu}]^{-1} = G_{01}^{0\nu} |M_{\rm GT}^{0\nu}|^2 \{ & |X_L|^2 + |X_R|^2 - \tilde{C}_1' X_L X_R \\
& + \tilde{C}_2 |\lambda| X_L \cos\psi_1 + \tilde{C}_3 |\eta| X_L \cos\psi_2 + \tilde{C}_4 |\lambda|^2 \\
& + \tilde{C}_5 |\eta|^2 + \tilde{C}^6 |\lambda| |\eta| \cos(\psi_1 - \psi_2) \\
& + \mathrm{Re}(\tilde{C}_2 \lambda X_R + \tilde{C}_3 \eta X_R) \},
\end{aligned} \tag{69}
$$

where X_L and X_R are defined in Eqs. (34) and (35). ψ_1 and ψ_2 are the relative phases between X_L and λ and X_L and η, respectively. The ellipses $\{\cdots\}$ indicate contributions arising from other particles, e.g., intermediate SUSY particles [48] or unusual particles which are predicted by superstring models [49] or exotic Higgs scalars [50], etc.

The quantities $G_{01}^{0\nu}$ are calculated using the prescription of [4]. The coefficients C_1', C_i, $i = 2-6$ are combinations of kinematical functions and the nuclear matrix elements discussed in the previous section. They are defined as follows:

$$\tilde{C}_2 = -(1-\chi_F)(\chi_{2-}\tilde{G}_{03} - \chi_{1+}\tilde{G}_{04}),$$

$$\tilde{C}_3 = -(1-\chi_F)(\chi_{2+}\tilde{G}_{03} - \chi_{1-}\tilde{G}_{04} - \chi_P'\tilde{G}_{05} + \chi_R\tilde{G}_{06}),$$

$$\tilde{C}_4 = \chi_{2-}^2\tilde{G}_{02} + \frac{1}{9}\chi_{1+}^2\tilde{G}_{04} - \frac{2}{9}\chi_{1+}\chi_{2-}\tilde{G}_{03},$$

$$
\begin{aligned}
\tilde{C}_5 = & \chi_{2+}^2\tilde{G}_{02} + \frac{1}{9}\chi_{1-}^2\tilde{G}_{04} - \frac{2}{9}\chi_{1-}\chi_{2+}\tilde{G}_{03} + (\chi_P')^2\tilde{G}_{08} \\
& - \chi_P'\chi_R\tilde{G}_{07} + \chi_R^2\tilde{G}_{09},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{C}_6 = & -2[\chi_{2-}\chi_{2+}\tilde{G}_{02} - \tfrac{1}{9}(\chi_{1+}\chi_{2+} + \chi_{2-}\chi_{1-})\tilde{G}_{03} \\
& + \tfrac{1}{9}\chi_{1+}\chi_{1-}\tilde{G}_{04}].
\end{aligned} \tag{70}
$$

Here $\tilde{C}_1' \cong 10(\epsilon_0^2 + 6\epsilon_0 + 6)/(\epsilon_0^4 + 10\epsilon_0^3 + 10\epsilon_0^2 + 60\epsilon_0 + 30)$, ϵ_0 is the available energy in electron mass units. C_1' is less than 10% and it can be safely neglected.

The quantities $\tilde{G}_{0i}$ are defined as follows:

$$
\begin{aligned}
\tilde{G}_{0i} &= G_{0i}/G_{01} \quad (i = 2,3,4), \\
\tilde{G}_{05} &= 2G_{05}/G_{01}, \\
\tilde{G}_{06} &= \frac{1}{4} m_e R_0 G_{06}/G_{01}, \\
\tilde{G}_{07} &= 2(\tfrac{1}{4} m_e R_0) G_{07}/G_{01}, \\
\tilde{G}_{08} &= 4G_{08}/G_{01}, \\
\tilde{G}_{09} &= (\tfrac{1}{4} m_e R_0)^2 G_{09}/G_{01}.
\end{aligned} \tag{71}
$$

The values of the parameters $\tilde{G}_{0i}$, $i = 2, \ldots, 6$ are presented in Table IV. The coefficients $\tilde{C}_i$, $i = 2, \ldots, 6$ with and without p-n pairing are shown in Table V.

The most stringent experimental limits are

$$A = 48: \; T_{1/2}^{0\nu} \geqslant 9.5\times10^{21} \text{ y (76\% CL) [5]},$$

$$A = 76: \; T_{1/2}^{0\nu} \geqslant 5.6\times10^{24} \text{ y (90\% CL) [2]},$$

$$A = 82: \; T_{1/2}^{0\nu} \geqslant 2.7\times10^{22} \text{ y (68\% CL) [3]},$$

$$A = 96: \; T_{1/2}^{\rm all} \geqslant 3.9\times10^{19} \text{ y (geochem.) [11]},$$

$$A = 100: \; T_{1/2}^{0\nu} \geqslant 4.4\times10^{22} \text{ y (68\% CL) [4]},$$

$$A = 116: \; T_{1/2}^{0\nu} \geqslant 2.9\times10^{22} \text{ y (90\% CL) [6]},$$

TABLE IV. The kinematical functions $\tilde{G}_{0i}$, $i=2-9$. They are given in the notation of Pantis *et al.* [27].

Nuclear transition	$\tilde{G}_{02}$	$\tilde{G}_{03}$	$\tilde{G}_{04}$	$\tilde{G}_{05}$	$\tilde{G}_{06}$	$\tilde{G}_{07}$	$\tilde{G}_{08}$	$\tilde{G}_{09}$
$^{48}Ca\rightarrow^{48}Ti$	6.518	0.752	0.212	31.50	0.450	73.87	2613.	0.522
$^{76}Ge\rightarrow^{76}Se$	1.635	0.552	0.194	63.93	0.745	95.01	4001.	0.563
$^{82}Se\rightarrow^{82}Kr$	3.465	0.685	0.205	52.9	0.584	96.99	4372.	0.538
$^{96}Zr\rightarrow^{96}Mo$	4.310	0.731	0.208	54.58	0.543	107.0	5371.	0.5324
$^{100}Mo\rightarrow^{100}Ru$	3.588	0.704	0.206	59.97	0.582	110.3	5660.	0.5375
$^{116}Cd\rightarrow^{116}Sn$	3.140	0.691	0.204	67.94	0.614	119.1	6547.	0.5419
$^{128}Te\rightarrow^{128}Xe$	0.286	0.280	0.153	126.0	1.133	141.9	7662.	0.6565
$^{130}Te\rightarrow^{130}Xe$	2.599	0.662	0.201	75.15	0.658	124.2	7035.	0.5483
$^{136}Xe\rightarrow^{136}Ba$	2.507	0.658	0.200	77.72	0.667	127.0	7331.	0.5497

$$A=128:\quad T_{1/2}^{\rm all}\geqslant 7.7\times 10^{24}\ \text{y (geochem.) [7]},$$

$$A=130:\quad T_{1/2}^{0\nu}\geqslant 2.3\times 10^{22}\ \text{y (68\% CL) [8]},$$

$$A=136:\quad T_{1/2}^{0\nu}\geqslant 3.4\times 10^{23}\ \text{y (68\% CL) [9]}.$$

Using Eq. (69), the functions $\tilde{G}_{01}$ of Table IV and the matrix elements $M_{\rm GT}^{0\nu}$ of Table II with p-n pairing we get the constrains:

$$A=48:\quad \{\cdots\}\leqslant 8.0\times 10^{-9},$$

$$A=76:\quad \{\cdots\}\leqslant 6.6\times 10^{-12},$$

$$A=82:\quad \{\cdots\}\leqslant 7.9\times 10^{-10},$$

$$A=96:\quad \{\cdots\}\leqslant 4.4\times 10^{-6},$$

$$A=100:\quad \{\cdots\}\leqslant 1.2\times 10^{-9},$$

$$A=116:\quad \{\cdots\}\leqslant 3.9\times 10^{-8},$$

$$A=128:\quad \{\cdots\}\leqslant 3.8\times 10^{-11},$$

$$A=130:\quad \{\cdots\}\leqslant 2.3\times 10^{-10},$$

$$A=136:\quad \{\cdots\}\leqslant 1.4\times 10^{-11}.$$

Here $\{\cdots\}$ indicates the quantity which is enclosed in the curly bracket of Eq. (69). Clearly the nucleus with the small-

TABLE V. The coefficients $\tilde{C}_i, i=1,2,3,4,5,6$ which are combinations of nuclear matrix elements and phase space factors [see Eq. (55) of the text] needed for the extraction of $|\langle m_\nu\rangle|$, λ, η, etc., from the data.

Nuclear transition	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
			QRPA without p-n pairing			
$^{48}Ca\rightarrow^{48}Ti$	2.154	-0.96	-1.05×10^2	7.43	1.34×10^4	-6.92
$^{76}Ge\rightarrow^{76}Se$	1.078	-0.66	-2.26×10^2	1.65	4.73×10^4	-3.10
$^{82}Se\rightarrow^{82}Kr$	1.015	-0.51	-7.61×10	2.78	8.90×10^3	-5.42
$^{96}Zr\rightarrow^{96}Mo$	1.321	-0.75	-1.23×10^2	3.29	2.08×10^4	-5.32
$^{100}Mo\rightarrow^{100}Ru$	3.118	-0.26	-5.12×10^2	1.51	1.34×10^5	1.13
$^{116}Cd\rightarrow^{116}Sn$	4.421	-0.18	-3.55×10^2	2.03	4.13×10^4	2.35
$^{128}Te\rightarrow^{128}Xe$	1.036	-0.40	-3.61×10^2	0.33	6.24×10^4	-0.65
$^{130}Te\rightarrow^{130}Xe$	1.007	-0.76	-2.08×10^2	2.48	5.38×10^4	-4.98
$^{136}Xe\rightarrow^{136}Ba$	0.944	-0.78	-2.04×10^2	2.43	5.37×10^4	-5.16
			QRPA with p-n pairing			
$^{48}Ca\rightarrow^{48}Ti$	0.708	-0.24	-2.25×10	2.72	1.76×10^3	-6.67
$^{76}Ge\rightarrow^{76}Se$	0.526	-0.19	-9.27×10	0.90	1.62×10^4	-2.53
$^{82}Se\rightarrow^{82}Kr$	2.005	-1.52	-2.30×10^2	4.86	4.13×10^4	-5.41
$^{96}Zr\rightarrow^{96}Mo$	1.643	-3.01	4.48×10^2	6.24	2.18×10^5	17.6
$^{100}Mo\rightarrow^{100}Ru$	0.0037	-0.062	3.27	4.95	4.99×10^3	-19.0
$^{116}Cd\rightarrow^{116}Sn$	60.59	-5.86	-2.35×10^3	10.4	1.36×10^5	70.8
$^{128}Te\rightarrow^{128}Xe$	0.479	-0.13	-3.78×10^2	0.17	1.47×10^5	-0.56
$^{130}Te\rightarrow^{130}Xe$	0.665	-0.62	-1.65×10^2	2.41	5.11×10^4	-5.91
$^{136}Xe\rightarrow^{136}Ba$	0.872	-0.66	-2.64×10^2	2.11	9.71×10^4	-4.53

TABLE VI. The limits on lepton number nonconserving parameters $|\langle m_\nu\rangle|$, $|\langle 1/M_N\rangle_L|^{-1}$, and $|\langle 1/M_N\rangle_R|^{-1}$ deduced from the experimental limits of $0\nu\beta\beta$ decay lifetimes for the nuclei studied in this work. For the extraction of the parameter $|\langle 1/M_N\rangle_R|^{-1}$ the value of $\varepsilon^2+\kappa^2=10^{-2}$ was assumed. Only one term was assumed dominant. $T_{1/2}^{0\nu\ 1\ \mathrm{eV}}$ is the calculated $0\nu\beta\beta$ half lifetimes assuming $|\langle m_\nu\rangle|=1$ eV.

Nucleus	$\|\langle m_\nu\rangle\|$ (eV)	$\left\|\left\langle \frac{1}{M_N}\right\rangle_L\right\|^{-1}$ (GeV)	$\left\|\left\langle \frac{1}{M_N}\right\rangle_R\right\|^{-1}$ (GeV)	$T_{1/2}^{0\nu\ 1\ \mathrm{eV}}$ (years)	$T_{1/2}^{0\nu\ \mathrm{expt}}$ (years) Ref.
			QRPA without p-n pairing		
^{48}Ca	$\leqslant 16$	$\geqslant 1.9\times10^{6}$	$\geqslant 1.9\times10^{4}$	2.5×10^{24}	$\geqslant 9.5\times10^{21}$ [5]
^{76}Ge	$\leqslant 0.8$	$\geqslant 3.8\times10^{7}$	$\geqslant 3.8\times10^{5}$	3.6×10^{24}	$\geqslant 5.6\times10^{24}$ [2]
^{82}Se	$\leqslant 7.4$	$\geqslant 2.8\times10^{6}$	$\geqslant 2.8\times10^{4}$	1.5×10^{24}	$\geqslant 2.7\times10^{22}$ [3]
^{96}Zr	$\leqslant 125$	$\geqslant 1.4\times10^{5}$	$\geqslant 1.4\times10^{3}$	6.1×10^{23}	$\geqslant 3.9\times10^{19}$ [11]
^{100}Mo	$\leqslant 9.4$	$\geqslant 2.0\times10^{6}$	$\geqslant 2.0\times10^{4}$	3.9×10^{24}	$\geqslant 4.4\times10^{22}$ [4]
^{116}Cd	$\leqslant 13$	$\geqslant 4.0\times10^{5}$	$\geqslant 4.0\times10^{3}$	4.7×10^{24}	$\geqslant 2.9\times10^{22}$ [6]
^{128}Te	$\leqslant 1.6$	$\geqslant 1.2\times10^{7}$	$\geqslant 1.2\times10^{5}$	1.9×10^{25}	$\geqslant 7.3\times10^{24}$ [7]
^{130}Te	$\leqslant 6.1$	$\geqslant 3.1\times10^{6}$	$\geqslant 3.1\times10^{4}$	8.6×10^{23}	$\geqslant 2.3\times10^{22}$ [10]
^{136}Xe	$\leqslant 1.7$	$\geqslant 6.6\times10^{6}$	$\geqslant 6.6\times10^{4}$	1.8×10^{24}	$\geqslant 6.4\times10^{23}$ [9]
			QRPA with p-n pairing		
^{48}Ca	$\leqslant 54$	$\geqslant 7.6\times10^{4}$	$\geqslant 7.6\times10^{2}$	2.8×10^{25}	$\geqslant 9.5\times10^{21}$ [5]
^{76}Ge	$\leqslant 1.8$	$\geqslant 1.6\times10^{7}$	$\geqslant 1.6\times10^{5}$	1.8×10^{25}	$\geqslant 5.6\times10^{24}$ [2]
^{82}Se	$\leqslant 10$	$\geqslant 1.3\times10^{6}$	$\geqslant 1.3\times10^{4}$	2.8×10^{24}	$\geqslant 2.7\times10^{22}$ [3]
^{96}Zr	$\leqslant 841$	$\geqslant 1.4\times10^{4}$	$\geqslant 1.4\times10^{2}$	2.7×10^{25}	$\geqslant 3.9\times10^{19}$ [11]
^{100}Mo	$\leqslant 285$	$\geqslant 6.4\times10^{6}$	$\geqslant 6.4\times10^{4}$	3.6×10^{27}	$\geqslant 4.4\times10^{22}$ [4]
^{116}Cd	$\leqslant 13$	$\geqslant 2.8\times10^{5}$	$\geqslant 2.8\times10^{3}$	4.9×10^{24}	$\geqslant 2.9\times10^{22}$ [6]
^{128}Te	$\leqslant 4.6$	$\geqslant 7.4\times10^{6}$	$\geqslant 7.4\times10^{4}$	1.5×10^{26}	$\geqslant 7.3\times10^{24}$ [7]
^{130}Te	$\leqslant 9.6$	$\geqslant 4.2\times10^{6}$	$\geqslant 4.2\times10^{4}$	2.1×10^{24}	$\geqslant 2.3\times10^{22}$ [10]
^{136}Xe	$\leqslant 2.1$	$\geqslant 9.3\times10^{6}$	$\geqslant 9.3\times10^{4}$	2.8×10^{24}	$\geqslant 6.4\times10^{23}$ [9]

est value of $\{\cdots\}$ is going to provide the most stringent limit on the lepton violating parameters. The lifetime itself is not a clear indicator since the function G_{01} varies from nucleus to nucleus. Large G_{01}, i.e., large phase space, leads to short lifetimes for a given lepton violation parameter. Thus the most stringent limits are expected from the $A=76$ system.

To impose limits on X_L, X_R, λ, η one must make four-dimensional plots making some assumptions about ψ_1, ψ_2 and the relative signs of λ and η with χ_R. Then for a given value of X_L one can extract limits on $\langle m_\nu\rangle + m_e\langle m_p/M_N\rangle_L \chi_H/(\chi_F-1)$. To extract limits on $\langle m_\nu\rangle$ and $\langle 1/M_N\rangle$ one must make further plots (knowing χ_H, χ_F from the calculations). This is really a complicated procedure to be worth doing only if and when $0\nu\beta\beta$ decay is definitely seen. At present we will constrain the above parameters by assuming that one mechanism at a time dominates. The limits thus obtained appear in Table VI. We must mention that in the case of heavy neutrino only the parameters $[\langle 1/M_N\rangle_R(\epsilon^2+\kappa^2)]^{-1}=\langle 1/M_N\rangle_L^{-1}$ can be extracted this way. The parameter $\langle 1/M_N\rangle_R^{-1}$ shown in Table VI was obtained by taking [18] $\epsilon^2+\kappa^2=10^{-2}$. In line with what we mentioned above the extraction of the parameter η depends on $\tilde{C}_5$ alone. In all cases $\tilde{C}_5$ is dominated by χ'_P and /or χ_R. So in Table VII we present two values of η, one with nuclear recoil included and one without recoil. With the possible exception of the $A=100$ and $A=128{,}130$ system, the recoil contribution is dominant. In the Te isotope χ'_P and χ_R compete with each other.

Another lepton violating process is the $0\nu\beta\beta$ decay with Majoron emission. The corresponding expression is

$$(T_{1/2}^{0\nu,\chi^0})^{-1}=|\eta_{\chi^0}|^2 G_{0\nu,\chi^0}|M_{\mathrm{GT}}^{0\nu}(1-\chi_F)|^2, \tag{72}$$

where

$$G_{0\nu,\chi^0}=\tilde{G}_{0\chi^0}G_{01},$$

$$\tilde{G}_{0\chi^0}=\frac{1}{(2\pi)^2}\epsilon_0^2\frac{g_1(\epsilon_0)}{g_0(\epsilon_0)},$$

$$g_0(\epsilon_0)=\epsilon_0^4+10\epsilon_0^3+40\epsilon_0^2+60\epsilon_0+30,$$

$$g_1(\epsilon_0)=\epsilon_0^4+14\epsilon_0^3+84\epsilon_0^2+210\epsilon_0+210,$$

$$\eta_{\chi^0}=\sum_{i,j} U_{ei}^{(11)}U_{ej}^{(11)}\frac{1}{\sqrt{2}}e^{i(\alpha_i-\alpha_j)}g_{ij}. \tag{73}$$

TABLE VII. The limits on lepton number nonconserving parameters $|\langle\lambda\rangle|$ and $|\langle\eta\rangle|$ deduced from the experimental limits of $0\nu\beta\beta$ decay lifetimes for the nuclei studied in this work. For $\langle\eta\rangle$ our results with and without inclusion of the recoil nuclear matrix elements are presented. Only one term was assumed dominant.

Nucleus	$\|\langle\lambda\rangle\|$	$\|\langle\eta\rangle\|$ Without recoil	$\|\langle\eta\rangle\|$ With recoil	$T_{1/2}^{0\nu\ \mathrm{expt}}(0\nu)$ (years) Ref.
		QRPA without p-n pairing		
^{48}Ca	$\leqslant 1.7\times10^{-5}$	$\leqslant 5.3\times10^{-6}$	$\leqslant 4.0\times10^{-7}$	$\geqslant 9.5\times10^{21}$ [5]
^{76}Ge	$\leqslant 1.3\times10^{-6}$	$\leqslant 2.2\times10^{-8}$	$\leqslant 7.4\times10^{-9}$	$\geqslant 5.6\times10^{24}$ [2]
^{82}Se	$\leqslant 8.8\times10^{-6}$	$\leqslant 4.1\times10^{-6}$	$\leqslant 1.5\times10^{-7}$	$\geqslant 2.7\times10^{22}$ [3]
^{96}Zr	$\leqslant 1.6\times10^{-4}$	$\leqslant 4.6\times10^{-6}$	$\leqslant 1.9\times10^{-6}$	$\geqslant 3.9\times10^{19}$ [11]
^{100}Mo	$\leqslant 2.6\times10^{-5}$	$\leqslant 1.1\times10^{-7}$	$\leqslant 8.8\times10^{-8}$	$\geqslant 4.4\times10^{22}$ [4]
^{116}Cd	$\leqslant 3.7\times10^{-5}$	$\leqslant 1.7\times10^{-7}$	$\leqslant 2.6\times10^{-7}$	$\geqslant 2.9\times10^{22}$ [6]
^{128}Te	$\leqslant 5.6\times10^{-6}$	$\leqslant 2.6\times10^{-8}$	$\leqslant 1.3\times10^{-8}$	$\geqslant 7.3\times10^{24}$ [7]
^{130}Te	$\leqslant 7.6\times10^{-6}$	$\leqslant 9.9\times10^{-8}$	$\leqslant 5.2\times10^{-8}$	$\geqslant 2.3\times10^{22}$ [10]
^{136}Xe	$\leqslant 2.1\times10^{-6}$	$\leqslant 2.3\times10^{-8}$	$\leqslant 1.4\times10^{-8}$	$\geqslant 6.4\times10^{23}$ [9]
		QRPA with p-n pairing		
^{48}Ca	$\leqslant 5.4\times10^{-5}$	$\leqslant 4.3\times10^{-5}$	$\leqslant 2.1\times10^{-6}$	$\geqslant 9.5\times10^{21}$ [5]
^{76}Ge	$\leqslant 2.7\times10^{-6}$	$\leqslant 8.5\times10^{-8}$	$\leqslant 2.0\times10^{-8}$	$\geqslant 5.6\times10^{24}$ [2]
^{82}Se	$\leqslant 1.3\times10^{-5}$	$\leqslant 2.6\times10^{-7}$	$\leqslant 1.4\times10^{-7}$	$\geqslant 2.7\times10^{22}$ [3]
^{96}Zr	$\leqslant 8.4\times10^{-4}$	$\leqslant 6.0\times10^{-6}$	$\leqslant 4.5\times10^{-6}$	$\geqslant 3.9\times10^{19}$ [11]
^{100}Mo	$\leqslant 1.5\times10^{-5}$	$\leqslant 1.8\times10^{-7}$	$\leqslant 4.8\times10^{-7}$	$\geqslant 4.4\times10^{22}$ [4]
^{116}Cd	$\leqslant 6.1\times10^{-5}$	$\leqslant 3.2\times10^{-7}$	$\leqslant 5.4\times10^{-7}$	$\geqslant 2.9\times10^{22}$ [6]
^{128}Te	$\leqslant 1.5\times10^{-5}$	$\leqslant 2.4\times10^{-8}$	$\leqslant 1.6\times10^{-8}$	$\geqslant 7.3\times10^{24}$ [7]
^{130}Te	$\leqslant 9.8\times10^{-6}$	$\leqslant 1.8\times10^{-7}$	$\leqslant 6.8\times10^{-8}$	$\geqslant 2.3\times10^{22}$ [10]
^{136}Xe	$\leqslant 2.6\times10^{-6}$	$\leqslant 1.8\times10^{-8}$	$\leqslant 1.2\times10^{-8}$	$\geqslant 6.4\times10^{23}$ [9]

ϵ_0 is the available energy in units of the electron mass. g_{ij} is the coupling of the Majoron to the neutrino mass eigenstates, i.e.,

$$\mathscr{L}=\frac{g_{ij}}{\sqrt{2}}\bar{\nu}_{iL}\nu_{jR}\chi^0+\text{H.c.} \tag{74}$$

It can also be written as

$$\frac{g_{ee}}{\sqrt{2}}\cong\eta_{\chi^0}, \quad \mathscr{L}=\frac{g_{ee}}{\sqrt{2}}\nu_{eL}\nu_{eR}^c\chi^0+\text{H.c.} \tag{75}$$

The corresponding experimental limits are

$$A=48:\quad T_{1/2^{0\nu,\chi}}{}^{0}>7.2\times10^{20}\ \ (90\%\ \ \text{CL})\ \ [14],$$
$$A=76:\quad T_{1/2}^{0\nu,\chi^0}>3.9\times10^{22}\ \ (90\%\ \ \text{CL})\ \ [2],$$
$$A=82:\quad T_{1/2}^{0\nu,\chi^0}>1.6\times10^{21}\ \ (68\%\ \ \text{CL})\ \ [13],$$
$$A=96:\quad T_{1/2}^{\text{all}}>3.9\times10^{19}\ \ (\text{geochem.})\ \ [11],$$
$$A=100:\quad T_{1/2}^{0\nu,\chi^0}>7.9\times10^{20}\ \ (68\%\ \ \text{CL})\ \ [15],$$
$$A=116:\quad T_{1/2}^{0\nu,\chi^0}>1.8\times10^{19}\ \ (99\%\ \ \text{CL})\ \ [6],$$
$$A=128:\quad T_{1/2}^{\text{all}}>7.7\times10^{24}\ \ (\text{geochem.})\ \ [7],$$
$$A=130:\quad T_{1/2}^{\text{all}}>2.7\times10^{21}\ \ (\text{geochem.})\ \ [7],$$
$$A=136:\quad T_{1/2}^{0\nu,\chi^0}>4.9\times10^{21}\ \ (90\%\ \ \text{CL})\ \ [9].$$

From the above experimental limits and the values of G_{01,χ^0} (Table IV) and $|M_{\mathrm{GT}}^{0\nu}(1-\chi_F)|$ (Table II) we obtain the limits of $|\eta_{\chi^0}|$ listed below:

$$A=48:\quad |\eta_{\chi^0}|<7.0\times10^{-4},$$
$$A=76:\quad |\eta_{\chi^0}|<1.4\times10^{-4},$$
$$A=82:\quad |\eta_{\chi^0}|<1.9\times10^{-4},$$
$$A=96:\quad |\eta_{\chi^0}|<3.6\times10^{-3},$$
$$A=100:\quad |\eta_{\chi^0}|<9.9\times10^{-3},$$
$$A=116:\quad |\eta_{\chi^0}|<2.6\times10^{-3},$$
$$A=128:\quad |\eta_{\chi^0}|<5.7\times10^{-5},$$
$$A=130:\quad |\eta_{\chi^0}|<1.5\times10^{-4},$$
$$A=136:\quad |\eta_{\chi^0}|<1.3\times10^{-4}.$$

V. SUMMARY AND CONCLUSIONS

In the present work we have evaluated the nuclear matrix elements entering the double beta decay of the experimentally most interesting nuclear systems. We have employed the quasiparticle random-phase approximation which seems to be the most practical method for nuclear structure calculation of nuclear systems which are far away from closed shells. In these calculations we have included the proton-neutron pairing correlations which have been neglected in the previous calculations. We have found that such correlations have important effects on all the needed matrix elements and should not be neglected. The magnitude of the effect depends, of course, on the type of operator employed, i.e., on the mechanism for the $0\nu\beta\beta$ decay. We will concentrate our discussion on those matrix elements which were not unusually suppressed. We will give their value both without *p-n* pairing correlations and after such correlations are turned on. Since we have assumed that one mechanism at a time is important we can summarize our results as follows.

(i) Light neutrino mass mechanism. The relevant nuclear matrix element is $|M^{0\nu}_{\rm GT}(1-\chi_F)|^2$. The five largest matrix elements are 9.2, 6.2, 5.8, 5.6, and 4.9 associated with $A=76$, 128, 96, 130, and 82, respectively. Once *p-n* pairing correlations are turned on they become 1.8, 0.77, 0.13, 2.2, and 2.7, respectively, i.e., we have a reduction factor ranging between 2 and 8. In the most interesting case of the $A=76$ system we have a reduction of about 5. The effect on the extraction on the neutrino mass is less pronounced. (The square root of the above factor.)

(ii) Heavy intermediate neutrino mass mechanism. The relevant nuclear matrix element is $|\chi_H M^{0\nu}_{\rm GT}|^2$. The five largest matrix elements are 4.0×10^4, 1.1×10^4, 1.0×10^4, 0.98×10^4, and 0.96×10^4 for $A=76$, 48, 128, 96, and 82, respectively. With *p-n* pairing correlations they become 0.37×10^4, 6.0, 0.19×10^4, 1.4×10^2, and 0.43×10^4. Notice the almost complete suppression for $A=48$ and 96 and the large reduction factor for $A=76$ (about 10).

(iii) The mass independent λ mechanism. The relevant matrix element is $\tilde{C}_4|M^{0\nu}_{\rm GT}|^2$. The largest matrix elements 14.5, 14.1, 13.6, 13.4, and 6.2 are associated with $A=96$, 76, 82, 130, and 136, respectively. They become 0.49, 3.07, 6.5, 8.1, and 3.8, respectively. Notice that quite unexpectedly the matrix element of the $A=128$ system is much smaller than that for the $A=130$ system. In the case of the $A=76$ we have a reduction factor of about 5.

(iv) The neutrino mass independent η mechanism. The relevant matrix element is $\tilde{C}_5|M^{0\nu}_{\rm GT}|^2$. The largest values are 4.1×10^5, 3.7×10^5, 2.9×10^5, 1.4×10^5, and 9.1×10^4 associated with $A=76$, 128, 130, 136, and 96. They are reduced to 5.5×10^4, 2.4×10^5, 1.7×10^5, 1.8×10^5, and 1.7×10^4. In the case of the $A=76$ system the reduction factor is about 7.

We do not fully understand why the effect of the *p-n* pairing correlations should be so large. In the case of $A=100$ this effect is much more dramatic. On the contrary for the $A=136$ system the matrix elements are fairly large and not much affected by such correlations. Finally we do not understand why the effect of *p-n* correlations is so different on the two tellurium isotopes. From Eq. (69) it is clear that the $A=76$ system provides the most stringent limits on the lepton violating parameters. This is partly due to the large matrix elements obtained for this system but mainly due to the fact that the experimental life-time limit is the best. Unfortunately, the introduction of *p-n* pairing correlations makes the extracted limits of the lepton violating parameters less stringent. In fact we find

$$|\langle m_\nu\rangle|<\begin{cases}0.8\ \text{eV},\\ 1.8\ \text{eV},\end{cases}$$

$$\left|\left\langle\frac{1}{M_N}\right\rangle_L\right|^{-1}>\begin{cases}3.8\times10^7\ \text{GeV},\\ 1.6\times10^7\ \text{GeV},\end{cases}$$

$$\left|\left\langle\frac{1}{M_N}\right\rangle_R\right|^{-1}>\begin{cases}3.8\times10^5\ \text{GeV},\\ 1.6\times10^5\ \text{GeV},\end{cases}$$

$$|\langle\lambda\rangle|<\begin{cases}1.3\times10^{-6},\\ 2.7\times10^{-6},\end{cases}$$

$$|\langle\eta\rangle|<\begin{cases}7.4\times10^{-9},\\ 2.0\times10^{-8},\end{cases}$$

$$|\eta_{\chi^0}|<\begin{cases}8.4\times10^{-5},\\ 1.9\times10^{-4}.\end{cases}$$

In the above expressions the upper (lower) values correspond to the case without (with) *p-n* pairing correlations.

It is interesting to note that the $A=128$ system, in spite of the fact that its $0\nu\beta\beta$ decay width is kinematically suppressed, provides quite stringent limits on the lepton violating parameters with the possible exception of λ. This is quite surprising since the nuclear matrix elements involved are not favored compared to the $A=76$ system and in any case they should not be very different from those of the $A=130$ system. Furthermore the extracted limits on the lepton violating parameters will become even more stringent if the 0ν lifetime is used since $T^{0\nu}_{1/2}\geqslant T^{\rm all}_{1/2}$. We are, therefore, inclined to suspect that the lifetime of $T^{0\nu}_{1/2}\geqslant5.6\times10^{24}$ y is quite a bit exaggerated. We will not, however, elaborate further on this controversial point concerning the lifetime of this long lived isotope.

Finally we should mention that the above extracted limits still suffer from uncertainties of nuclear origin. We should not forget that the effect of the interference between the various mechanisms has not been taken into account.

[1] For a recent review of the experimental situation see, e.g., V. I. Tretyak and Yu. G. Zdesenko, ''Tables of double beta decay data,'' Universite Louis Pasteur Report No. IN2P3, CRN, 1995.
[2] A. Balysh *et al.*, Heidelberg-Moscow Collaboration, Phys. Lett. B **356**, 450 (1995).
[3] S. E. Elliott *et al.*, Phys. Rev. C **46**, 1535 (1992).
[4] M. Alston-Garnjost *et al.*, Phys. Rev. Lett. **71**, 831 (1993).
[5] You Ke *et al.*, Phys. Lett. B **265**, 53 (1991).
[6] F. N. Danevich *et al.*, in *Proceedings of the Third International Symposium on Weak and Electromagnetic Interaction in Nuclei*, Dubna, 1992, edited by Ts. Vylov (World Scientific, Sin-

gapore, 1993); Phys. Lett. B **344**, 72 (1995).

[7] T. Bernatowicz *et al.*, Phys. Rev. Lett. **69**, 2341 (1992).

[8] A. Alessandrello *et al.*, Milano Group, Nucl. Phys. B (Proc. Suppl.) **35**, 394 (1994); Phys. Lett. B **335**, 519 (1994).

[9] J. C. Vuilleumier *et al.*, Caltech-Neuchattel-Sherrer Group, Phys. Rev. D **48**, 1009 (1993).

[10] M. Moe and P. Vogel, Annu. Rev. Nucl. Part. Sci. **44**, 247 (1994); M. K. Moe *et al.*, Prog. Part. Nucl. Phys. **32**, 247 (1994).

[11] A. Kawashima, K. Takahashi, and A. Masuda, Phys. Rev. C **47**, 2452 (1993).

[12] J. Tanaka and H. Ejiri, Phys. Rev. D **48**, 5412 (1993).

[13] Yu. G. Zdesenko, I. A. Mytsyk, and A. S. Nikolaiko, Sov. J. Nucl. Phys. **32**, 312 (1984).

[14] A. S. Barabash *et al.*, Phys. Lett. B **216**, 257 (1989).

[15] R. K. Bardin, P. S. Collon, J. D. Ullman, and C.S. Wu, Nucl. Phys. A **158**, 337 (1970).

[16] G. Smott *et al.*, KOBE Data, Astrophys. J. **396**, L1 (1992).

[17] G. K. Leontaris *et al.*, University of Ioannina report, 1994.

[18] J. D. Vergados, Phys. Rep. **133**, 1 (1986).

[19] M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys. (Suppl.) **83**, 1 (1985).

[20] W. C. Haxton and G. J. Stephenson, Jr., Prog. Part. Nucl. Phys. **12**, 409 (1984).

[21] T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991).

[22] M. G. Schepkin, Sov. Phys. Usp. **27**, 555 (1984).

[23] W. C. Haxton, G. J. Stephenson, Jr., and D. Strottman, Phys. Rev. Lett. **47**, 153 (1981); Phys. Rev. D **25**, 2360 (1982).

[24] J. Sinatkas, L. D. Skouras, and J. D. Vergados, Phys. Rev. C **37**, 229 (1988).

[25] T. Tomoda, A. Faessler, K. W. Schmidt, and F. Grümmer, Phys. Lett. **157B**, 4 (1985).

[26] A. Faessler, W. A. Kaminski, G. Pantis, and J. D. Vergados, Phys. Rev. C **43**, R21 (1991).

[27] G. Pantis, A. Faessler, W. A. Kaminski, and J. D. Vergados, J. Phys. G **18**, 605 (1992).

[28] G. Pantis and J. D. Vergados, Phys. Rep. **242**, 285 (1994); Phys. Lett. B **242**, 1 (1990).

[29] T. Tomoda and A. Faessler, Phys. Lett. B **199**, 475 (1987).

[30] J. Engel, P. Vogel, and M. R. Zirnbauer, Phys. Rev. C **37**, 731 (1988).

[31] K. Muto, E. Bender, and H. V. Klapdor, Z. Phys. A **334**, 187 (1989).

[32] J. Engel, P. Vogel, O. Civitarese, and M. R. Zirnbauer, Phys. Lett. B **208**, 187 (1988).

[33] J. Suhonen, O. Civitarese, and A. Faessler, Nucl. Phys. A **543**, 645 (1992).

[34] J. Suhonen, T. Taigel, and A. Faessler, Nucl. Phys. A **486**, 91 (1988).

[35] A. Staudt, T. T. S. Kuo, and H. V. Klapdor-Kleingrothaus, Phys. Lett. B **242**, 17 (1990).

[36] M. Hirsch *et al.*, Z. Phys. A **345**, 163 (1993).

[37] F. Krmpotić, J. Hirsch, and H. Dias, Nucl. Phys. A **542**, 85 (1992).

[38] J. Suhonen, S. B. Khadkikar, and A. Faessler, Nucl. Phys. A **535**, 509 (1991).

[39] J. D. Vergados, Nucl. Phys. A **506**, 482 (1990).

[40] H. T. Chen and A. Goswami, Nucl. Phys. **88**, 208 (1966).

[41] H. H. Wolter, A. Faessler, and P. U. Sauer, Nucl. Phys. A **167**, 108 (1971).

[42] K. Goeke, J. Garcia, and A. Faessler, Nucl. Phys. A **208**, 477 (1973).

[43] A. L. Goodman, Adv. Nucl. Phys. **11**, 263 (1979).

[44] M. K. Cheoun, A. Bobyk, Amand Faessler, F. Šimkovic, and G. Teneva, Nucl. Phys. A **561**, 74 (1993).

[45] M. K. Cheoun, A. Bobyk, Amand Faessler, F. Šimkovic, and G. Teneva, Nucl. Phys. A **564**, 329 (1993).

[46] M. K. Cheoun, A. Faessler, F. Šimkovic, G. Teneva, and A. Bobyk, Nucl. Phys. A **587**, 301 (1995).

[47] For a review see, e.g., K. Holinde, Phys. Rep. **68**, 121 (1981).

[48] J. D. Vergados, Phys. Lett. B **188**, 327 (1987).

[49] G. K. Leontaris and J. D. Vergados, Z. Phys. C **41**, 623 (1989).

[50] R. N. Mohapatra and J. D. Vergados, Phys. Rev. Lett. **47**, 1713 (1981).