Leading-order nuclear π-γ exchange force

J. L. Friar
Theoretical Division, MS B283, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
and Institute for Nuclear Theory, University of Washington, Seattle, Washington 98105

S. A. Coon
Department of Physics, New Mexico State University, Las Cruces, New Mexico 88003
and Institute for Nuclear Theory, University of Washington, Seattle, Washington 98105
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The leading-order isospin-violating force from simultaneous π and γ exchange is calculated. The charge-symmetric, but charge-dependent, force is calculated for static nucleons and in Coulomb gauge. Infrared divergences and other technical problems are discussed. The resulting force is roughly 3 orders of magnitude smaller than OPEP and roughly the same size as the Breit correction to single-photon exchange. The present calculation corresponds to a subset of one-loop diagrams in chiral perturbation theory for isospin violation in the nucleon-nucleon force.

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Isospin violation in the nuclear force is a topic of recurring interest [1,2]. Charge dependence or charge-independence breaking (CIB) (differences between n-p $T=1$ scattering and nn or pp scattering) is large and well established. Charge-symmetry breaking (CSB) (the difference between nn and pp scattering after long-range electromagnetic forces are removed) is smaller and fairly well established.

The presently accepted value [3] of the nn scattering length is more attractive than the corresponding pp quantity (this was not always true). This difference in the short-range nuclear forces can be coupled with long-range electromagnetic forces to explain the $^3\mathrm{He}-^3\mathrm{H}$ mass difference [4]. Although the bulk of the 764-keV binding-energy difference is due to the well-understood [5] Coulomb force (648 keV), much of the rest is explained by the stronger nn (than pp) force [6–8]. This understanding is one of the most important accomplishments in the few-nucleon field, where there have been many recent successes [9].

Charge dependence is fairly large (~few % of the strong force) and also plays a role in the few-nucleon problem. It has long been known that nuclear forces fit to the $T=1$ np data produce ~200 keV too much binding in the triton, while those fit to the (weaker) pp force give ~100 keV too little binding [10]. This CIB is important as we struggle to reconcile the ~900-keV discrepancy between experiment and nonrelativistic triton calculations using the best local potential models. These "best" models are recent ones [11,4] that fit *all* nucleon-nucleon scattering data very well and *require* a charge-dependent force. The latter is a combination of using the (different) physical masses for the charged and neutral pions in the one-pion exchange potential (OPEP) (the larger charged-pion mass generates a stronger np force) and less-well-understood shorter-range components.

There are other forces of pion range, however, that are rarely considered. Indeed, there are widely varying estimates of their sizes, and their status, even their viability, remains murky. These forces are the (simultaneous) π-γ exchange forces between two nucleons [12–18], and three-nucleons [19–22]. Because the photon is massless, the π-γ force has a nominal one-pion range. We wish to calculate the leading-order part of these forces and simultaneously estimate the size. We will find that this part of the force is charge symmetric, but does break charge independence.

Typical mechanisms for this type of force are shown in Fig. 1. These graphs can be considered either as conventional Feynman diagrams or as time-ordered diagrams. Our rules for the calculation are: (1) work in the static limit for the nucleon ($M\rightarrow\infty$, when M is the nucleon mass), and (2) work in the Coulomb gauge for the photon exchange. The reason for the static limit is tractability and hindsight. In addition, it corresponds to the leading order for such forces in chiral perturbation theory. We will see that the force is not large and the leading order probably suffices. The tractability argument is both obvious and subtle.

The subtlety arises from ambiguities in defining nucleon fields. Many years ago [23] it was shown that PS and PV forms of pion-nucleon coupling in chiral Lagrangians were basically the same (if one ignores nonlinearities). The (unitary) transformation that accomplishes the transmogrification has been much used in nuclear physics calculations [24,25] and involves an unphysical chiral rotation parameter μ. In addition, there is another parameter ν that determines the quasipotential (three-dimensional) representation of the four-dimensional amplitude that defines the potential.

Consistent calculations of observables must be independent of μ and ν, although the explicit forms of various operators and wave functions will depend on the values. The appropriate transformations U are of order V_π/M, where V_π is OPEP. Performing the transformation on the Coulomb interaction part of the nuclear Hamiltonian leads to new interactions of order $(\alpha V_\pi/M)$. In order to avoid treating the ambiguity problem, we take $M\rightarrow\infty$ and calculate only static terms. Our operators are therefore independent of those problems, although the full problem appears in next-to-leading order.

The choice of Coulomb gauge is highly appropriate to bound-state problems mediated by a potential [26]. The dominant part of one-photon exchange is the *static* Coulomb

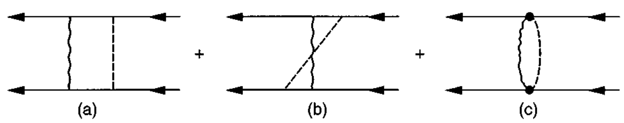

FIG. 1. Sequential interactions of the OPEP and the Coulomb potential (contained in any solution of the Schrödinger equation) is shown in (a). The overlapping retarded pion-Coulomb interaction is depicted in (b), while the double seagull is illustrated in (c). In the Coulomb gauge in the static limit, only (b) and (c) contribute to the π-γ exchange force. Nucleons are shown as solid lines, pions shown as dashed lines, and photons are depicted by wavy lines.

interaction between two nucleons and can be immediately incorporated into the nuclear force, with the retarded transverse components treated perturbatively. This has several advantages. Coulomb photons in any other gauge do not propagate instantaneously and can overlap the exchanges of an arbitrary number of mesons, which leads to a challenging calculation. The second reason is that the transverse photons couple to the nucleon currents, which are explicitly of order $(1/M)$ and by our rules can be neglected. The third reason is that there are infrared divergences in Feynman gauge [$\sim\int d^3q/q^3 \sim \ln(q)$ for small momentum transfers q]. These cancel in a complete and consistent treatment, but will not otherwise [27]. At least to order $(1/M^2)$, there are no infrared problems in the Coulomb gauge for our process.

Viewed as an exercise in heavy-baryon chiral perturbation theory (χPT), our calculation below will treat a subset of leading-order graphs. This approach [28] (χPT) relies on an expansion in powers of $1/\Lambda$, where $\Lambda\sim 1$ GeV is the large-mass scale of QCD. For the purposes of this and similar calculations, Λ subsumes heavy meson and baryon masses as well as the nucleon mass M. Especially important is the expansion in powers of $1/M$. By leading order we mean order $(1/\Lambda)^0$. To this order only short-range (δ-function) counterterms arise, and these can be ignored. All of the vertices we require are contained in the lowest-order Lagrangian

$$L^{(0)} = f\bar{N}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}(\boldsymbol{\tau}\cdot\boldsymbol{\pi})N - ef\bar{N}\boldsymbol{\sigma}\cdot\mathbf{A}(\boldsymbol{\tau}\times\boldsymbol{\pi})_z N - e\bar{N}\hat{e}A_0 N + \cdots , \tag{1}$$

where e is the fundamental (proton) charge, f is the π-nucleon coupling constant, and $\hat{e}$ has the value 1 (0) for protons (neutrons). The symbols N, $\boldsymbol{\pi}$, and A^μ refer to nucleon, pion, and photon fields. The ellipsis refers to multipion interactions that we will not need. In addition to $L^{(0)}$, $L^{(n)}$ (for $n>0$) represents higher-order in $(1/\Lambda)$ Lagrangians that we explicitly neglect. We expect these terms to generate corrections of order $(m_\pi/\Lambda\sim 10-20\,\%)^n$. The chiral expansion in the nuclear physics case is discussed in great detail in [29], where extensive references can also be found.

Thus our calculational task is enormously simplified. We will throw away all operators and contributions that are nonstatic, and we will calculate all diagrams for which the photon (in the Coulomb gauge) traverses from one nucleon to another. There are three such processes. Figure 1(a) (and a corresponding one where the pion and photon are interchanged) are merely the iteration of a *static* Coulomb interaction and a *static* pion exchange. Since this is already implicitly contained in the Schrödinger equation, we ignore it. Figure 1(b) corresponds to a static Coulomb interaction overlapped by a nonstatic pion exchange. The corresponding retarded potential operator for this time ordering was developed in Eqs. (5)–(8) of Ref. [25] and can be obtained by replacing H in those equations with V_c,

$$V^R_{\pi\gamma} = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{[J^\alpha_\pi(\mathbf{q}),[V_C, J^\alpha_\pi(-\mathbf{q})]]}{2E^3_\pi}, \tag{2}$$

where $E_\pi = (q^2+m_\pi^2)^{1/2}$, V_C is the Coulomb interaction,

$$V_C = \frac{\alpha}{2}\sum_{i\neq j}\frac{\hat{e}_i\hat{e}_j}{x_{ij}}, \tag{3}$$

$\hat{e}_i$ has the value 1(0) for the ith proton (neutron), and $J^\alpha_\pi(\mathbf{q})$ is the *static* operator for absorbing an incoming pion with momentum $\mathbf{q}$ in charge state α obtained from Eq. (1),

$$J^\alpha_\pi(\mathbf{q}) = -if\sum_i \tau^\alpha(i)\,\boldsymbol{\sigma}(i)\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{x}_i}, \tag{4}$$

where $f = g_A/2\,f_\pi$, g_A is the axial-vector coupling constant (1.26), and f_π is the pion decay constant (92.4 MeV). The quantities $\boldsymbol{\sigma}(i)$, $\boldsymbol{\tau}(i)$, and $\mathbf{x}_i$ are the (Pauli) spin and isospin operators and the coordinates of nucleon i.

The remaining static diagram that contributes is Fig. 1(c), which is generated by the static Kroll-Ruderman interaction [i.e., the gauge term in PV coupling and the second term in Eq. (1)]. This contributes an amplitude for absorbing both a pion (with isospin component α and momentum $\mathbf{q}$) and a photon (with momentum $\mathbf{k}$). The latter couples only via the vector potential and this leads to a seagull operator with the form

$$\mathbf{S}^\alpha = -ef\sum_i \boldsymbol{\sigma}(i)\,\tau^\beta(i)\,\epsilon^{3\alpha\beta}e^{i(\mathbf{q}+\mathbf{k})\cdot\mathbf{x}_i}. \tag{5}$$

Note that any coupling of the vector potential from this interaction to a nucleon line is $O(1/M)$ and can be ignored. Just as Eq. (2) follows immediately from second-order (old-fashioned) perturbation theory applied to Fig. 1(b), the same process applied to Fig. 1(c) yields

$$V^{\rm SG}_{\pi\gamma} = -\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{\mathbf{S}^\alpha_\perp\cdot\mathbf{S}^\alpha}{(2E_\pi)(2E_\gamma)(E_\pi+E_\gamma)}. \tag{6}$$

The factors $(2E_\pi)$ and $(2E_\gamma)$ are the wave function normalization factors for the pion and photon, respectively, while $(E_\pi+E_\gamma)$ is the (static) energy denominator. The notation $\mathbf{S}^\alpha_\perp\cdot\mathbf{S}^{\,\alpha}$ recalls that in the Coulomb gauge only the transverse (to $\hat{k}$, the photon direction) components of $\mathbf{S}^\alpha$ contribute. Substituting Eq. (5) for $\mathbf{S}^\alpha$ and extracting the potential gives immediately

$$V_{\pi\gamma}^{\mathrm{SG}}=-e^2 f^2 \sum_{i\neq j}[\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)-\tau^z(i)\tau^z(j)]\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{e^{i(\mathbf{q}+\mathbf{k})\cdot\mathbf{x}_{ij}}\boldsymbol{\sigma}(i)_\perp\cdot\boldsymbol{\sigma}(j)}{(2E_\pi)(2E_\gamma)(E_\pi+E_\gamma)},\tag{7}$$

where $\mathbf{x}_{ij}\equiv\mathbf{x}_i-\mathbf{x}_j$. The isospin factor vanishes for π^0 exchange and is charge symmetric (but CIB). There is no CSB contribution in this order.

Expression (6) is simple in momentum space, but involves a complicated convolution in configuration space. We use the identity [30]

$$\frac{1}{E_1E_2(E_1+E_2)}=\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{(E_1^2+\lambda^2)(E_2^2+\lambda^2)},\tag{8}$$

which leads to

$$V_{\pi\gamma}^{\mathrm{SG}}=-2\alpha f^2\sum_{i\neq j}T_{ij}\int_0^\infty d\lambda\int\frac{d^3q}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{x}_{ij}}\boldsymbol{\sigma}(i)_\perp\cdot\boldsymbol{\sigma}(j)}{(E_\pi^2+\lambda^2)(E_\gamma^2+\lambda^2)},\tag{9}$$

where

$$T_{ij}=\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)-\tau^z(i)\tau^z(j).\tag{10}$$

The Fourier transforms are elementary. Using the definition of $\perp$ ($\delta^{\alpha\beta}-\hat{k}^\alpha\hat{k}^\beta=\delta^{\alpha\beta}-k^\alpha k^\beta/k^2$) leads to the form

$$V_{\pi\gamma}^{\mathrm{SG}}=\frac{-2\alpha f^2}{(4\tau)^2}\sum_{i\neq j}T_{ij}\left\{\frac{\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j)}{x_{ij}^2}\int_0^\infty d\lambda\; e^{-\lambda x_{ij}}e^{-\sqrt{\lambda^2+m_\pi^2}x_{ij}}+\int_0^\infty\frac{d\lambda\; e^{-\sqrt{\lambda^2+m_\pi^2}x_{ij}}}{\lambda^2x_{ij}}\boldsymbol{\sigma}(i)\cdot\boldsymbol{\nabla}_{ij}\boldsymbol{\sigma}(j)\cdot\boldsymbol{\nabla}_{ij}\left[\frac{1-e^{-\lambda x_{ij}}}{x_{ij}}\right]\right\}.\tag{11}$$

Note that form factors could easily be inserted in Eq. (9) to regulate the small-x_{ij} behavior, which would lead to the Yukawa functions in Eq. (11) being replaced by a more complicated sum of terms. Note also that the Feynman-gauge result (the first term) leads to a central force, while the Coulomb-gauge corrector (second term) produces a tensor force, as well.

Performing the tedious derivatives and evaluating the integrals leads to the final result

$$V_{\pi\gamma}^{\mathrm{SG}}=-\alpha\left(\frac{g_A}{4\pi f_\pi}\right)^2\frac{m_\pi^3}{2}\sum_{i\neq j}T_{ij}\{\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j)I_c^{\mathrm{SG}}(m_\pi x_{ij})+S_{ij}I_t^{\mathrm{SG}}(m_\pi x_{ij})\},\tag{12}$$

where S_{ij} is the usual tensor operator. In Feynman gauge we obtain $I_t^{\mathrm{SG}}=0$ and

$$I_c^{\mathrm{SG}}(z)=\frac{1}{2}\left[\frac{E_i(-z)}{z}+\frac{e^{-z}(1+z)}{z^3}\right],\tag{13a}$$

whereas in Coulomb gauge one has

$$I_c^{\mathrm{SG}}=\frac{1}{3}\left[\frac{E_i(-z)}{z}+\frac{e^{-z}(1+z)}{z^3}\right]\tag{13b}$$

and

$$I_t^{\mathrm{SG}}=-\frac{1}{z^3}\left[\left(1+\frac{z^2}{6}\right)E_i(-z)+K_0(z)-\frac{e^{-z}}{6}(5-z)\right].\tag{13c}$$

The factor of ($\frac{2}{3}$) in going from (13a) to (13b) reflects the loss of the longitudinal photon direction in Coulomb gauge. This result is quite singular ($\sim 1/z^3$) and requires regularization.

The retarded pion-exchange contribution in Eq. (2) can be easily evaluated to give

$$\Delta V_{\pi\gamma}^{R}=-\alpha\left(\frac{g_A}{4\pi f_\pi}\right)^2\frac{m_\pi^3}{2}\sum_{i\neq j}T_{ij}\{\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j)I_c^{R}(m_\pi x_{ij})+S_{ij}I_t^{R}(m_\pi x_{ij})\},\tag{14}$$

where

$$I_c^R(z)=\frac{1}{3z}\left[K_0''(z)+\frac{2}{z}K_0'(z)\right]=\frac{1}{6z}[3K_0(z)-K_2(z)]\tag{15a}$$

and

$$I_t^R(z)=\frac{1}{3z}\left[K_0''(z)-\frac{1}{z}K_0'(z)\right]=\frac{1}{3z}K_2(z). \qquad (15b)$$

The factor of $1/z$ in front of the square brackets arises from the Coulomb potential, while the modified Bessel functions arise from the integral in Eq. (2). This completes the derivation of the static π-γ exchange potential in the Coulomb gauge. Note that both forces are two body in nature. To the best of our knowledge, these explicit results are new.

Ignoring the dimensionless radial factors in Eqs. (12) and (14), the potentials have a "size" of $\alpha(g_A/4\pi f_\pi)^2 m_\pi^3=(\alpha m_\pi)(g_A m_\pi/4\pi f_\pi)^2$. This result could have been anticipated using the fact that a potential carries with it a factor of $(1/4\pi)$ (which allowed e^2 to be converted to α), and a loop integral [such as those in Figs. 1(b) and 1(c)] produces a factor of $(1/4\pi)^2$. Numerically, the size factor is 23 keV. If one also renders the OPEP into the product of a dimensionful parameter, $(g_A m_\pi/2 f_\pi)^2 m_\pi/4\pi$, and a dimensionless radial factor, the ratio of dimensionful parameters for the two potentials is just α/π, a not unexpected result from an electromagnetic loop integral. We can also compare the size of the π-γ force to the Breit (relativistic-correction) terms in one-photon exchange [4,31]. These have a size $\sim(\alpha m_\pi^3/M^2)$, a factor of $(m_\pi/M)^2$ times the Coulomb potential, implying a ratio of strengths $\sim(g_A M/4\pi f_\pi)^2\sim 1$.

We expect [1,32] a dominant (or extremely important) contribution to CIB to be given by the pion-mass difference Δm_π in the OPEP. This scales as $(g_A m_\pi/2 f_\pi)^2(\Delta m_\pi/4\pi)$, and the π-γ force is a fraction of this force, $\alpha m_\pi/\pi\Delta m_\pi\sim 1/15$, which we characterize as small, but not entirely negligible. Explicit calculation of the effect of these potentials on the $^1S_0$ scattering length difference $\Delta a=|a_{np}|-|a_{nn}|$ is consistent with this dimensional argument. The value of Δa from each CIB potential was obtained in two ways: (i) Add $\Delta V=V_{pn}-V_{nn}$ from Eqs. (12) and (14) to a model for the charge-independent interaction and interpret the change in the scattering length as the Δa desired, or (ii) use the familiar perturbative formula [33] that has been shown to be essentially exact [34]. The second method isolates an integral over the dimensionless radial factors in Eqs. (12) and (14), closer to the spirit of the dimensional estimates above, and agrees quite well numerically with the first method. Because we need only a rough estimate, we choose the Reid soft core potential as the dominant charge-independent interaction. Our results (without any regularization) are

$$\Delta a_{\rm SG}\simeq -0.35 \ \text{fm}, \quad \Delta a_R\simeq +0.18 \ \text{fm},$$

$$\Delta a_{\Delta m_\pi}\simeq +2.62 \ \text{fm},$$

where the potential due to the pion mass difference Δm_π is that of Ref. [32]. The total effect on the scattering lengths of the static CIB π-γ exchange potential derived here,

$$\Delta a_{\pi-\gamma}\simeq -0.18 \ \text{fm},$$

is largely unchanged by changing to the Reid hard core potential or by regularizing the singularities at small r by Gaussian cutoffs of the pion exchange and by the expression F_C of Ref. [4], which represents the the finite size of the nucleon charge distribution. That is, a variety of permutations of short-range cutoffs and model charge-independent interactions yield a range of $\Delta a_{\pi\text{-}\gamma}\simeq -0.15\pm 0.03$ fm, although the individual terms $\Delta a_{\rm SG}$ and Δa_R are more sensitive to these short-range effects. In any event, the total static CIB π-γ exchange potential makes a very small contribution to the empirical $\Delta a=|a_{np}|-|a_{nn}|$, which is about $+5$ fm. The leading-order CSB terms from π-γ exchange are expected to be $\sim(m_\pi/M)$ smaller than the results above, and this is the order where contributions to the pp interaction and to three-nucleon forces [19–22] will arise.

Finally, we remark that the quantitative role of isobars in this process must be clarified. Although the (single) isobar contributions can be expected to vanish in the static *nucleon* limit (the isobar excitation is predominantly magnetic and couples to a nucleon through the vector potential), their small excitation energy nevertheless suggests an important role [15,17]. Processes where the energy denominator that characterizes (virtual) Δ propagation in a nucleus is small would compete very favorably with the processes that we have calculated herein. Conversely, those with a large denominator would only be corrections. Recent work on isobar contributions to binding energies [35] and meson-exchange currents [36] suggests that the "effective" energy needed to excite the isobar in those cases is at least twice the nominal mass difference and that the isobars may therefore play a minor role. Whether this speculation is valid for π-γ exchange should be studied in detail.

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