COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Role of heavy meson exchange in near threshold $NN\to d\pi$"

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In a recent paper by Horowitz [Phys. Rev. C **48**, 2920 (1993)] a heavy meson exchange is incorporated into threshold $NN\to d\pi$ to enhance the underestimated cross section. However, that calculation uses an unjustified assumption on the initial and final momenta, which causes an overestimate of this effect by a factor of 3–4. Further, I point out that the inclusion of the $\Delta(1232)$ isobar increases the cross section significantly even at threshold.

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A recent paper [1] proposes that heavy meson exchange (HME) involving a nucleon-antinucleon pair may be important in threshold pion production in the reaction $np\to d\pi^0$ and $pp\to d\pi^+$ (here generically included in the first reaction). This mechanism contributes to the two-nucleon axial charge [2,3], and so far has been the only way to explain the surprisingly large $pp\to pp\pi^0$ cross section at threshold [4]. The importance of this effect in this reaction is partly due to the absence of charge-exchange pion s-wave rescattering, dominant in the present $np\to d\pi^0$. A motivation for the inclusion of the HME mechanism to the deuteron reaction in Ref. [1] is the stated underestimation of the cross section [5] by theory almost by a factor of 2. This addition to the conventional one-nucleon axial charge and s-wave pion rescattering doubles the s-wave cross section bringing the calculated results close to the data.

One aim of this Comment is to criticize an approximation used in Ref. [1], which exaggerates the HME effect in this reaction. The σ meson exchange leads to the operator

$$\mathscr{M}_{fi}\propto\frac{\boldsymbol{\sigma}_i\cdot(\mathbf{p}'+\mathbf{p})}{2M}\frac{1}{M}\frac{g_\sigma^2}{m_\sigma^2+\mathbf{k}^2}\tau_{i0} \tag{1}$$

for each nucleon i. Except for the momentum transfer dependent σ propagator this is similar to the Galilean invariance (axial charge) part of the direct production operator. Exchange of the other important ω meson has an additional part $\propto\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2$, which changes the spin and does not contribute to s-wave production here. Eventually this operator leads to radial integrals (with an opposite sign convention from Ref. [1])

$$J_\sigma=\frac{g_\sigma^2}{4\pi}\int_0^\infty\left[\left(\frac{d}{dr}-\frac{1}{r}\right)v(r)j_0\left(\frac{qr}{2}\right)\left(\frac{e^{-m_\sigma r}}{2Mr}\right)u_{11}(r)\right.$$
$$\left.-v(r)j_0\left(\frac{qr}{2}\right)\left(\frac{e^{-m_\sigma r}}{2Mr}\right)\left(\frac{d}{dr}+\frac{1}{r}\right)u_{11}(r)\right]dr \tag{2}$$

for the deuteron S-state part $v(r)$ and

$$J_\sigma=\frac{g_\sigma^2}{4\pi}\int_0^\infty\left[\left(\frac{d}{dr}+\frac{2}{r}\right)w(r)j_0\left(\frac{qr}{2}\right)\left(\frac{e^{-m_\sigma r}}{2Mr}\right)u_{11}(r)\right.$$
$$\left.-w(r)j_0\left(\frac{qr}{2}\right)\left(\frac{e^{-m_\sigma r}}{2Mr}\right)\left(\frac{d}{dr}-\frac{2}{r}\right)u_{11}(r)\right]dr \tag{3}$$

for the D-state $w(r)$, with the derivatives acting only on the nearest wave function. Similar equations are valid also for ω exchange.

Following Koltun and Reitan [6], Eqs. (25)–(30) of Ref. [1] seem to replace the momentum operator $(\mathbf{p}'+\mathbf{p})$ operating on both the initial and the final state wave functions with $2\mathbf{p}$, because the pion momentum does not significantly affect s-wave production at threshold. Although valid for the direct production part, this is no more allowed in the presence of the momentum transferring HME potential, which does not commute with the momentum operator. This assumption only picks (double) the latter terms in the above equations. Elimination of the derivative in the final state by integration by parts does not help, since a derivative of the potential emerges. The first line in Table I shows these integrals using $2\mathbf{p}$ for the momentum operator and agrees well with Ref. [1]. Correcting this approximation essentially halves the contribution from the deuteron S state, since there the final state

TABLE I. Integrals of Eqs. (2) and (3) (in fm$^{-1/2}$) for σ and ω exchanges and S and D final states for $\eta=q_\pi/m_\pi=0.1424$. The total has also a factor $1/\sqrt{2}$ multiplying the D state as required by angular momentum algebra [6].

Model	σ,S	ω,S	σ,D	ω,D	Total
$2\mathbf{p}$	−0.0700	−0.0519	−0.0004	−0.0002	−0.1223
$\mathbf{p}'+\mathbf{p}$	−0.0284	−0.0202	0.0162	0.0119	−0.0287
$N\Delta$	−0.0260	−0.0181	0.0059	0.0037	−0.0373

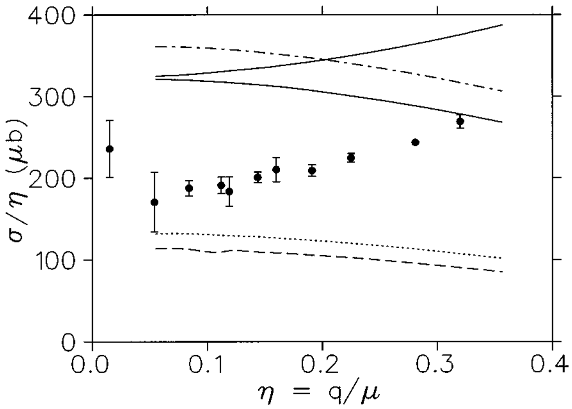

FIG. 1. Low-energy $pp \to d\pi^+$ cross section divided by $\eta = q_\pi / m_\pi$. The solid curves show the starting point before the addition of HME with the Δ included in all partial waves (the lower one is the s-wave contribution), while the dashed curve is the s-wave contribution without the Δ. The dotted and dash-dotted curves also have the HME added to these calculations of the s wave. The data are from Ref. [5].

momentum contribution is small. However, for the D state with its higher momentum components this is significant and in destructive interference with the main s-wave part, as can be seen from Table I. Overall the HME contribution to the amplitude is decreased by a factor of 4 and cannot account for the missing cross section. Instead of an increase of the conventional α by 86 μb reported in Ref. [1], the change is now 18 μb. In these calculations the Bonn potential $A(R)$ σ and ω couplings and form factors are used with Reid soft core wave functions as in Ref. [1]. This smallish effect is shown by the dashed and dotted curves in Fig. 1 for σ/η which in the limit of small η gives the parameter α.

The same approximation for $(\mathbf{p}+\mathbf{p}')$ is used later also for $pp \to pp\pi^\circ$ in Ref. [3]. Therefore, as a check, it was established that in $pp \to pp\pi^0$ the σ and ω exchanges alone give a good description of the low-energy data [4], so that the overall success of the theory in this reaction is not corrupted by this correction. With the more precise treatment of the final state momentum, however, the σ contribution decreases to nearly a half, while the ω effect is enhanced enough to compensate this loss. It may be noted that the σ and ω mesons were by far the most important in Ref. [3].

Another purpose of this Comment is to remind of the little known fact that the Δ(1232) isobar contributes significantly to s-wave pion production even at threshold. This is a well established mechanism in p-wave production requiring the inclusion of explicit ΔN admixtures to the NN wave functions [in particular ${}^1D_2(NN) \to {}^5S_2(\Delta N)$] or the use of a corresponding two-body operator acting in the NN space. Without these the cross section in the Δ region would be underestimated by a factor of 10 as compared with data. There is no other known mechanism to bring the cross section up by this factor. The threshold description of Koltun and Reitan [6], even as employed in Ref. [1] with modern two-nucleon potentials and deuteron wave functions, does not include the role of an explicit virtual Δ(1232) isobar excitation in producing *external* pions. Only various direct and crossed box diagrams in elastic scattering, involving isobars but always starting and ending with just two nucleons, are simulated by standard NN potentials, even if they are fitted to experimental phase shifts.

The ${}^3P_1$ pp state, relevant in the s-wave pion case, is coupled to such ΔN admixtures in the ${}^3P_1$, ${}^5P_1$, and ${}^5F_1$ states. It may be surprising that these components persist also to lower energies, to pion threshold and as a virtual off-shell effect even below. As shown, e.g., in Ref. [7], the decay of the Δ in these states gives rise to s- and d-wave pions. The contribution from the direct decay is very small, since parity and angular momentum conservation requires the second term $j_1(qr/2)$ of the plane wave expansion of the pion. However, if the pion from the Δ decay suffers an s-wave rescattering from the second nucleon (similarly to s-wave rescattering in the case of pure NN waves), then the effect is magnified, because the parity and angular momentum can be taken care of by the internal momentum transfer by the pion, and the first term $j_0(qr/2)$ of the expansion appears in the overlap integrals shown in Eq. (A.11) of Ref. [7]. Therefore, with rescattering the p-wave nature of the resonance is not reflected in the external momentum dependence.

The generation of the ΔN components is based on exchange of isovector mesons $\pi + \rho$ on which details can be found, e.g., in Ref. [7]. This coupled channels method treats the Δ isobar on the same basis as the nucleons via a system of Schrödinger equations. Since solving this system automatically generates attractive ΔN box diagrams, the phenomenologically fitted Reid soft core NN potential used here must be modified to avoid doubly counting this effect. This causes some further short range changes in the NN wave function reflected in HME as seen in the third line of Table I, decreasing each individual contribution but increasing the total result. In this work HME is included only in the nucleon sector. Furthermore, energy dependence is allowed for s-wave pion rescattering to fit on-shell πN scattering [8], but except for the virtual $N\Delta$ admixtures, at threshold the model reduces to the formalism of Koltun and Reitan. (A monopole form factor with $\Lambda = 700$ MeV is included to account for off-shell rescattering.)

Although the centrifugal barrier in the P-wave baryon states suppresses the Δ components to some extent, it can be

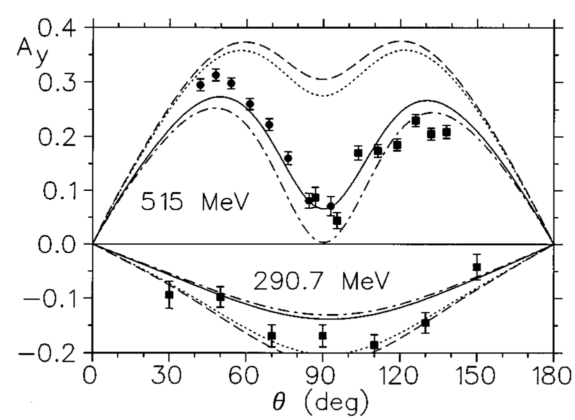

FIG. 2. The analyzing power A_y in $pp \to d\pi^+$ at two energies. The different curves differ in their treatment of the s-wave pion production amplitude as in Fig. 1. The other partial waves have always the same full model as the solid curve also has for the s wave. The data are from Refs. [10,11].

seen from the solid curves in Fig. 1 that even for threshold s-wave pions the isobar effect is by no means negligible and its inclusion triples the cross section. Therefore, in a more complete model the threshold cross section may be actually *overestimated* even without HME. As discussed above, by far most of this increase in s-wave pion production comes from the normal elementary p-wave emission of the pion from the Δ followed by s-wave rescattering from the second nucleon. The addition of HME slightly increases the overestimation as shown by the dash-dotted curve in Fig. 1. It may be further noted that in the threshold cross section for $pp \rightarrow pp\pi^0$ the isobar effect was only an increase by 30% [9] reflecting in part the strongly suppressed s-wave rescattering in this reaction.

A further aspect for caution in adding new mechanisms to threshold amplitudes is the changes caused in observables at higher energies, where there are much more data available to basically fix the amplitudes. The analyzing power A_y between 500 and 600 MeV is particularly sensitive to the s-wave pion amplitude. In this region the coupled-channels model used above to produce the solid curves has been successful. The use of a smaller s-wave amplitude to fit the threshold cross section would produce too high an analyzing power, whereas a larger one would yield too deep a minimum in it. Again it is fortunate that the HME effect is small in this reaction, as can be seen comparing with the 515 MeV data [10] in Fig. 2. Further, since the low-energy analyzing power data [11], due to the s- and p-wave interference, can be easily fitted by simply scaling with the factor $\sqrt{\sigma(\mathrm{theor})/\sigma(\mathrm{expt.})}$, which compensates for the overestimation of the s-wave amplitude in the cross section, one may conclude that apparently the p-wave amplitude is under control also close to threshold. Of course, the quality of the A_y data at 290.7 MeV only allows a qualitative statement.

The overestimation of the threshold cross section by the full model, which agrees well with the data in the Δ region, poses a problem of detail indicating that either the energy dependence of off-shell pion rescattering is not properly incorporated or some physical mechanism is still missing in the present models of pion production. However, HME does not appear as important here as indicated in Ref. [1], whereas a significant contribution from the Δ is likely to survive improvements of the model.

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[1] C.J. Horowitz, Phys. Rev. C **48**, 2920 (1993).
[2] T.S.H. Lee and D.O. Riska, Phys. Rev. Lett. **70**, 2237 (1993).
[3] C.J. Horowitz, H.O. Meyer, and D.K. Griegel, Phys. Rev. C **49**, 1337 (1994).
[4] H.O. Meyer *et al.*, Nucl. Phys. **A539**, 633 (1992).
[5] D.A. Hutcheon *et al.*, Nucl. Phys. **A535**, 618 (1991).
[6] D.S. Koltun and A. Reitan, Phys. Rev. **141**, 1413 (1966).
[7] J.A. Niskanen, Nucl. Phys. **A298**, 417 (1978); Phys. Lett. **141B**, 301 (1984).
[8] J.A. Niskanen, Phys. Rev. C **49**, 1285 (1994).
[9] J.A. Niskanen, Phys. Lett. B **289**, 227 (1992).
[10] E. Aprile *et al.*, Nucl. Phys. **A379**, 369 (1982).
[11] E. Korkmaz *et al.*, Nucl. Phys. **A535**, 637 (1991).