Hadron masses at finite density from the Zimanyi-Moskowski model

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The density dependence of hadron masses has been calculated from different versions of the Zimanyi-Moskowski (ZM) model and the results have been compared with the Walecka model. The ZM model has been extended to include pions. The meson masses have been calculated in the random phase approximation self-consistently. The σ, ω, and π masses are found to increase with density, as also in the Walecka model; interestingly, it has been found that the abnormal increase of pion mass with density, which is found in the Walecka model, can be avoided in the ZM models.

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The description of the strongly interacting hadronic matter at high energy should be based on a reliable relativistic model. It is required in such a model that the properties of normal nuclear matter be well reproduced. A large number of models has been proposed until now, but there does not yet exist a consensus preferring any one model to the others. The behavior of different physical quantities coming out of these models is found to vary widely, not only in magnitude but also qualitatively [1]. Such a physical quantity is the density dependence of hadron masses.

In this Brief Report we would like to calculate the density dependence of hadron masses from different versions of a newly proposed model: the Zimanyi-Moskowski (ZM) [2] model. The model differs from the usual Walecka [3,4] model only in the form of the coupling of the nucleon to the scalar meson. As a result it yields the incompressibility $K=224.49$ MeV, which is much closer to the experimental value ($K=210\pm 30$ MeV) [5] compared to the Walecka model ($K=534$ MeV), and a nucleon effective mass $M^*=797.64$ at the nuclear matter saturation density ($k_F=1.42$ fm^{-1}). The model is no longer renormalizable due to the derivative coupling. But, the price is not too severe, since it is believed that the description of hadronic matter need only be valid up to temperatures $T\leqslant 200$ MeV and densities $\leqslant 5-6$ times the normal nuclear matter beyond which a new state of matter, the quark-gluon plasma (QGP), should form [6–10].

On the other hand, a better description of the ground-state properties of nuclear matter is most desirable. Recently the modified versions of the ZM models have been extensively used to describe the nuclear matter properties [5]. We will extend the study to the mesonic sector. First, we shall briefly recapitulate the different variants of the ZM model [2,5] and extend it to include the pseudoscalar meson: pion. Then we shall calculate the density dependence of hadron masses from this model.

The Lagrangian densities for the Walecka, ZM, ZM2, and ZM3 models, respectively, can be written as (following the nomenclature of Delfino *et al.* [5])

$$\begin{aligned}\mathscr{L}_W=&\bar{\psi}i\gamma_\mu\partial^\mu\psi-\bar{\psi}M_N\psi+1/2(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)\\&+g_\sigma\bar{\psi}\sigma\psi-(1/4)G_{\mu\nu}G^{\mu\nu}+(1/2)m_v^2V_\mu V^\mu\\&-g_\omega\bar{\psi}\gamma_\mu\psi V^\mu,\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{\rm ZM}=&-\bar{\psi}M_N\psi+m^{*-1}(\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_v\bar{\psi}\gamma_\mu\psi V^\mu)\\&+1/2(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)-(1/4)G_{\mu\nu}G^{\mu\nu}\\&+(1/2)m_v^2V_\mu V^\mu,\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{\rm ZM2}=&-\bar{\psi}M_N\psi+m^{*-1}(\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_v\bar{\psi}\gamma_\mu\psi V^\mu\\&-(1/4)G_{\mu\nu}G^{\mu\nu}+(1/2)m_v^2V_\mu V^\mu)\\&+1/2(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2),\end{aligned}$$

and

$$\begin{aligned}\mathscr{L}_{\rm ZM3}=&-\bar{\psi}M_N\psi+m^{*-1}\bar{\psi}i\gamma_\mu\partial^\mu\psi-g_v\bar{\psi}\gamma_\mu\psi V^\mu\\&+1/2(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)-(1/4)G_{\mu\nu}G^{\mu\nu}\\&+(1/2)m_v^2V_\mu V^\mu,\end{aligned}\tag{1}$$

where ψ, σ, and V^μ are, respectively, the nucleon, the scalar meson, and the vector meson fields; M_N, m_s, and m_v are the corresponding masses; g_σ and g_v are the couplings of nucleon to scalar and vector mesons, respectively; $G_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$ and m^* is given as

$$m^*=(1+g_\sigma\sigma/M_N)^{-1}.\tag{2}$$

If we now rescale the nucleon field as $\psi\rightarrow m^{*1/2}\psi$ for all ZM models and $V_\mu\rightarrow m^*V_\mu$ for ZM2 and ZM3 models, we can write the Lagrangian densities for all these models (Walecka, ZM, ZM2, and ZM3) in a unified form [2,5]:

$$\begin{aligned}\mathscr{L}_R=&\bar{\psi}i\gamma_\mu\partial^\mu\psi-\bar{\psi}(M_N-m^{*\beta}g_\sigma\sigma)\psi-m^{*\alpha}[g_v\bar{\psi}\gamma_\mu\psi V^\mu\\&-(1/4)G_{\mu\nu}G^{\mu\nu}+(1/2)m_v^2V_\mu V^\mu]\\&+1/2(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2),\end{aligned}\tag{3}$$

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where α and β have the following values for different models: Walecka: $\alpha=0$, $\beta=0$; ZM; $\alpha=0$, $\beta=1$; ZM2; $\alpha=1$, $\beta=1$; ZM3; $\alpha=2$, $\beta=1$.

If we now extend the model to include pion, which is a pseudoscalar meson, the total Lagrangian becomes

$$\mathscr{L}=\mathscr{L}_R+\mathscr{L}_\pi,$$

where

$$\mathscr{L}_\pi=1/2(\partial_\mu\pi\partial^\mu\pi-m_\pi^2\pi^2)+m^{*\beta}\left[\frac{f_{\pi NN}}{m_\pi}\right]\bar{\psi}\gamma^\mu\gamma_5\tau\cdot\psi\partial_\mu\pi \tag{4}$$

with π the pion field and $f_{\pi NN}$ the pion-nucleon coupling constant. We have used the pion-nucleon pseudovector coupling [11].

In the mean-field approximation (MFA) we replace the meson fields by their ground-state expectation values. In this limit the Lagrangian density is given as [5]

$$\mathscr{L}_0=\bar{\psi}[i\gamma\cdot\partial-(M_N-g_\sigma m^{*\beta}\sigma_0)-g_v\gamma_0V^0]\psi-\tfrac{1}{2}m_s^2s_0^2+\tfrac{1}{2}m_v^2V_0^2.$$

Then the equation of motion for the baryon field is [5]

$$[i\gamma\cdot\partial-M_N^*-g_v\gamma_0V^0]\psi=0, \tag{5}$$

where $M_M^*=M_N-g_\sigma m^{*\beta}\sigma_0$ is the nucleon effective mass and

$$V_0=\frac{g_v}{m_v^2}\langle\psi^+\psi\rangle=\frac{g_v}{m_v^2}\rho_B,$$

$$\sigma_0=\frac{g_\sigma}{m_\sigma^2}m^{*2\beta}\langle\bar{\psi}\psi\rangle+\frac{\alpha}{2}\left[\frac{m_v}{m_\sigma}\right]^2\frac{g_\sigma}{M}m^{*(\alpha+1)}V_0^2.$$

The baryon propagator can be given by

$$G^*_{\alpha\beta}(p)=(\not{p}+M_N^*)\left[\frac{1}{(p^2-M_N^{*2}+i\varepsilon)}+2i\pi\delta(p^2-M_N^{*2})\theta(p-p_F)\right]. \tag{6}$$

The energy momentum tensor, from $\mathscr{L}_0$, may be determined in the canonical fashion

$$T^0_{\mu\nu}=-g_{\mu\nu}\mathscr{L}_0+\Sigma\partial_\nu\psi\frac{\delta\mathscr{L}_0}{\delta(\partial^\mu\psi)}$$

which, on using $\mathscr{L}_0$, gives

$$\mathscr{E}=\frac{g_v^2}{2m_v^2}m^{*\alpha}\rho_B^2+\frac{m_\sigma^2}{2g_\sigma^2}\left[\frac{1-m^*}{m^{*\beta}}\right]^2+\frac{2\gamma}{\pi^2}\int_0^{k_F}p^2(p^2+m^{*2})^{1/2}dp. \tag{7}$$

The energy density can be fitted to the nuclear ground-state energy at zero temperature and nuclear saturation density to obtain the different coupling constants. They are given by $C_s^2=g_\sigma^2(M_N/m_s)^2=357.4$ and $C_v^2=g_v^2(M_N/m_v)^2=273.8$ for the W model; $C_s^2=169.2$ and $C_v^2=59.1$ for the ZM model; $C_s^2=219.3$ and $C_v^2=100.5$ for the ZM2 model, and $C_s^2=443.3$ and $C_v^2=305.5$ for the ZM3 model [5]. The pion-nucleon coupling constant is determined from the experimental data on pion-nucleon scattering: $f_{\pi NN}=0.988$ [12].

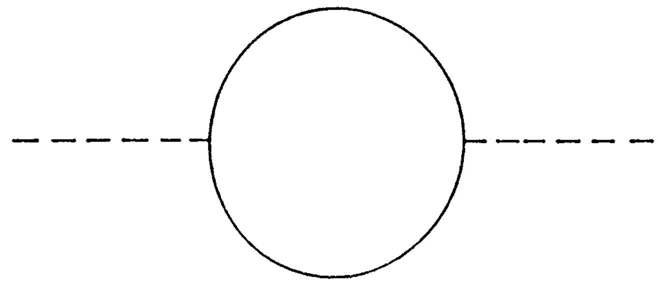

FIG. 1. One-loop Feynman diagram contributing to meson polarization function. The internal solid lines represent nucleons and the external dashed lines represent the mesons, which can be any one of the π, σ, or ω mesons.

The nucleon effective mass at finite density, in the MFA, is

$$M_N^*=M_N-g_\sigma m^{*\beta}\sigma_0=M_N-\frac{4M_N^*g_\sigma^2}{\pi^2m_\sigma^2}\int_0^{p_F}\frac{p^2dp}{E_p^*}, \tag{8}$$

where $E_p^*=\sqrt{p^2+M_N^{*2}}$ is the effective energy of the nucleon. A self-consistent solution of Eq. (8) gives the variation of the effective nucleon mass with density. The density variation of the effective nucleon mass has recently been reported in the literature [5]. In the present work we look at the density variation of the effective masses of π, σ, and ω mesons, using the density dependence of the effective nucleon mass in the random-phase approximation (RPA).

The calculation of meson masses is done using the following prescription: Dyson's equation relates the total Green's function $D(p)$ to the free Green's function $D_0(p)$ as

$$D^{-1}(p)=D_0^{-1}(p)+\Pi(p),$$

where $\Pi(p)$ is the polarization function. The pole of the full propagator then gives the effective mass. We restrict ourselves to only the pole mass and ignore the screening mass [12] or the Landau mass. In the RPA, the Feynman graph that arises is given in Fig. 1.

So the polarization functions can be given by

$$\Pi_{v\mu\nu}=-ig_v^2\gamma\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\gamma_\mu G^*(p)\gamma_\nu G^*(p+q)]$$

for vector mesons,

$$\Pi_{ps}=-ig_\pi^2\gamma m^{*2\beta}\left[\frac{f_{\pi NN}}{m_\pi}\right]^2\int\frac{d^4p}{(2\pi)^4}\times\mathrm{Tr}[q\gamma_5G^*(p)g\gamma_5G^*(p+q)]$$

for pseudoscalar mesons,

$$\Pi_s = -ig_\sigma^2 \gamma m^{*2\beta} \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}[G^*(p)G^*(p+q)]$$

for scalar mesons,

where the value of γ, the degeneracy factor, is 2 for neutron matter and 4 for symmetric nuclear matter.

The real part contributes to the effective mass. The real part also contains two parts, a density-dependent and a density-independent part. The density-dependent real parts for different mesons are

$$\mathrm{Re}\Pi_{\mathrm{DL}} = -16\gamma g_v^2 \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{q^2(E_p^{*2} - |p|^2\cos^2\vartheta)}{E_p^*[q^4 - 4(p\cdot q)^2]}\Bigg|_{p_0=E_p^*},$$

$$\mathrm{Re}\Pi_{\mathrm{DT}} = -16\gamma g_v^2 \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \times \frac{(p\cdot q)^2 - q^2|p|^2(1-\cos^2\vartheta)/2}{E_p^*[q^4 - 4(p\cdot q)^2]}\Bigg|_{p_0=E_p^*}$$

for vector mesons,

$$\mathrm{Re}\Pi_{\mathrm{PS}} = -16\gamma \left[\frac{f_{\pi N}}{m_\pi}\right]^2 \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{4q^4 M_N^{*2}}{E_p^*[q^4 - 4(p\cdot q)^2]}\Bigg|_{p_0=E_p^*}$$

for a pseudoscalar meson, and

$$\mathrm{Re}\Pi_S = -16\gamma g_\sigma^2 m^{*2\beta} \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{4(p\cdot q)^2}{q^4 - 4(p\cdot q)^2}\Bigg|_{p_0=E_p^*} \tag{9}$$

for scalar mesons.

Here DL and DT represent the density-dependent longitudinal and transverse parts, respectively. The density-independent parts have been neglected which is consistent with the MFA of the ground state [13].

We write the effective masses as

$$D_i(q_0)|_{q_0=m_i^*} = q_0^2 - m_i^2 - \Pi(q)|_{|q|=0,\, q_0=m_i^*} = 0,$$

$$i = \omega,\ \sigma,\ \text{or}\ \pi.$$

Figures 2–4 show these effective masses as functions of density.

To conclude, the density dependence of meson masses has been calculated from the modified Zimany-Moskowski model in the RPA. The model has been extended to include the pseudoscalar pion. In the symmetric matter approximation, the nuclear energy density at the nuclear saturation density has been calculated to obtain the coupling constants at the MFA. The density dependence of the nucleon mass at MFA has been used to calculate the density dependence of the meson masses, in the RPA, self-consistently. The σ, ω, and π masses are found to increase with density for both the models but, the change in the case of the ZM model is comparatively much less in magnitude. Special attention may be paid to the pion mass. The main problem of treating pions in the Walecka model is that its mass increases with density abnormally [11,14]. On the other hand one expects the pion mass not to change much, as the pion, being a Goldstone boson, is protected by chiral symmetry [15]. As seen from Fig. 4 here, the pion mass in the ZM models does not change much with density and one can expect that the problem of

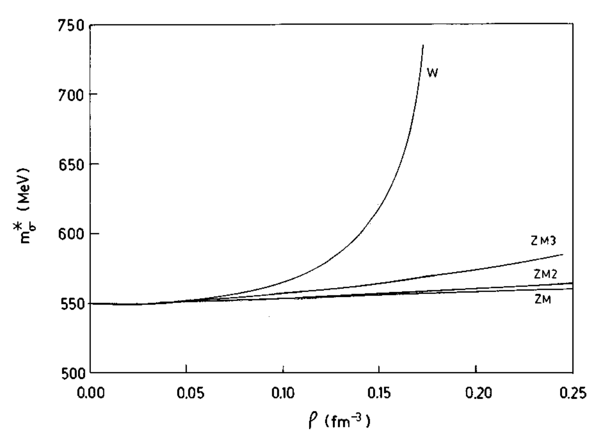

FIG. 2. Density dependence of the σ-meson mass. *W* refers to the Walecka model and ZM, ZM2, and ZM3 refer to the different versions of the Zimanyi model.

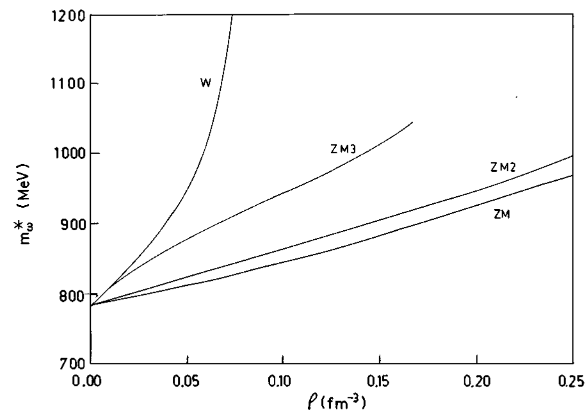

FIG. 3. Density dependence of the ω-meson mass. The nomenclature is same as that of Fig. 2.

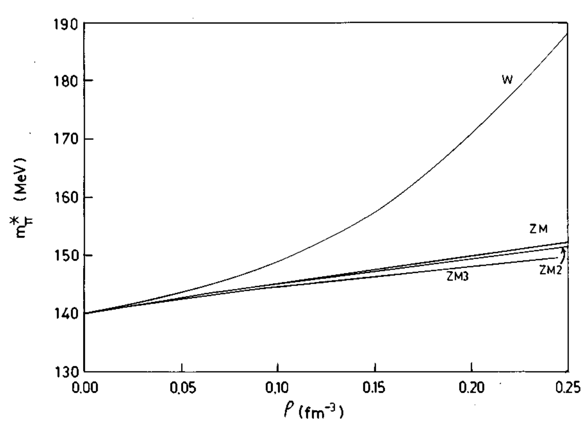

FIG. 4. Density dependence of the π-meson mass. The nomenclature is same as that of Fig. 2.

abnormal change in pion mass, which is observed in the Walecka model, can be overcome in the ZM models. It should of course be mentioned in this context that neither the Walecka model nor the ZM models incorporate chiral symmetry and thus the preceding argument is somewhat extraneous. Nevertheless, a model of nuclear matter at supernuclear density with ''correct'' behavior of the pion mass is quite appealing and in this respect the ZM models appear to perform better. The model can be extended to include the meson-meson couplings. Such calculations will be reported shortly.

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