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Unique determination of the tensor spin-spin part of the nucleon-nucleon forward scattering amplitude

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An *n-p* scattering observable $\Delta\sigma_{\text{tensor}}$ is presented which uniquely determines the tensor spin-spin part of the nucleon-nucleon forward scattering amplitude.

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In spite of its importance, the strength of the nucleon-nucleon (*NN*) tensor force remains controversial. An analysis by Henneck [1] of scattering data between 15 and 160 MeV concludes that the tensor force is stronger than *NN* potential model predictions. Until recently, all data reported below 20 MeV [2,3] pointed to a weaker tensor force. In contrast to both of these, recent measurements of the transverse spin-dependent difference in total cross section $\Delta\sigma_T$ below 20 MeV made at TUNL [4] as well as the new Nijmegen partial-wave analysis [5] report no disagreement with potential model predictions. New measurements are needed to resolve this apparent discrepancy.

In a phase-shift description of nucleon-nucleon scattering, the strength of the tensor force is characterized by the mixing parameters ϵ_J. Below 100 MeV, the first such mixing parameter ϵ_1, which gives the mixing between $^3S_1$ and $^3D_1$, dominates. Measurements of scattering observables which have large sensitivities to the value of ϵ_1 are thus valuable in determining the strength of the *NN* tensor interaction. Two such observables are the longitudinal and transverse spin-dependent differences in the total *n-p* cross section $\Delta\sigma_L$ and $\Delta\sigma_T$. The expressions for $\Delta\sigma_L$ and $\Delta\sigma_T$ in terms of the phase shifts are obtained by generalizing Eqs. (8) and (9) of Ref. [4]:

$$\Delta\sigma_L = -\frac{2\pi}{k^2}\sum_{J=0}^{\infty}\{(2J+1)(\sin^2\delta_{J,J}-\sin^2\delta_J) + \cos 2\epsilon_J(\sin^2\delta_{J,J-1}-\sin^2\delta_{J,J+1}) - 2\sqrt{J(J+1)}\sin 2\epsilon_J \sin(\delta_{J,J-1}+\delta_{J,J+1})\} \quad (1)$$

and

$$\Delta\sigma_T = -\frac{2\pi}{k^2}\sum_{J=0}^{\infty}\{(2J+1)(\sin^2\epsilon_J-\sin^2\delta_J) + \cos 2\epsilon_J[J\sin^2\delta_{J,J-1}+(J+1)\sin^2\delta_{J,J+1}] + \sqrt{J(J+1)}\sin 2\epsilon_J \sin(\delta_{J,J-1}+\delta_{J,J+1})\}, \quad (2)$$

where k is the wave number, δ_J are the singlet phase shifts with total angular momentum J, and $\delta_{J,l}$ are the triplet phase shifts with total angular momentum J and orbital angular momentum l.[1] Unfortunately, in addition to large sensitivities to ϵ_1, these observables also are very sensitive to the singlet phase shifts, in particular $^1S_0$.

As has been pointed out by Tornow *et al.* [6], the sensitivity to the singlet components is due to the central spin-dependent part of $\Delta\sigma_L$ and $\Delta\sigma_T$ and not due to the tensor force and thus it subtracts out completely in the difference $\Delta = \Delta\sigma_L - \Delta\sigma_T$:

$$\Delta = -\frac{2\pi}{k^2}\sum_{J=0}^{\infty}\{(2J+1)(\sin^2\delta_{J,J}-\sin^2\epsilon_J) - \cos 2\epsilon_J[(J-1)\sin^2\delta_{J,J-1}+(J+2)\sin^2\delta_{J,J+1}] - 3\sqrt{J(J+1)}\sin 2\epsilon_J \sin(\delta_{J,J-1}+\delta_{J,J+1})\}. \quad (3)$$

As can be seen, this difference depends only on the triplet waves and the mixing parameters.[2] This method, however, requires measuring two observables and therefore involves two sets of systematic uncertainties and normalizations. A single measurement is certainly preferable. In what follows I discuss a single observable which is equivalent to $\Delta\sigma_L - \Delta\sigma_T$ and is thus sensitive only to the tensor spin-spin part of the *NN* forward scattering amplitude.

Following the notation of Hnizdo and Gould [7], the total cross section σ_t for the scattering of neutrons of spin s from nuclei of spin I can be written

$$\sigma_t = \sum_{kK}\tilde{t}_{k0}(s)\tilde{t}_{K0}(I)\sigma_{kK}, \quad (4)$$

where the $\tilde{t}_{kq}$ are statistical polarization tensors[3] and

[1] The sensitivity, for example, of $\Delta\sigma_L$ to ϵ_1 can be seen numerically by using the VPI SM94 *n-p* phase-shift solution [10] at 10 MeV to obtain $\Delta\sigma_L = (\pi/k^2)(5.536\sin 2\epsilon_1 - 0.390)$, for $J \leqslant 1$.

[2] Using the VPI SM94 solution at 10 MeV in this case gives $\Delta\sigma_L - \Delta\sigma_T = (\pi/k^2)(-8.304\sin 2\epsilon_1 - 0.011)$, for $J \leqslant 1$.

[3] The statistical tensors are chosen to be diagonal along the spin direction such that $\tilde{t}_{kq}(s) = \tilde{t}_{k0}(s)\delta_{q0}$.

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$$\sigma_{kK}=4\pi\lambda\!\!\!^{-2}\frac{\hat{k}\hat{K}}{\hat{s}\hat{I}}\mathrm{Im}\sum_{\Lambda}\hat{\Lambda}C_{kK\Lambda}(\hat{\mathbf{s}}\hat{\mathbf{I}}\hat{\mathbf{p}})\sum_{Jljl'j'}(2J+1)\hat{l}\hat{j}\hat{j}'$$
$$\times\langle l\Lambda 00|l'0\rangle W(JjIK;Ij')\begin{Bmatrix} l & s & j\\ \Lambda & k & K\\ l' & s & j'\end{Bmatrix}T^{J}_{l'j',lj} \qquad (5)$$

is the cross section of rank kK. Here $\lambda\!\!\!^{-}$ is the reduced wavelength, $\hat{k}=(2k+1)^{1/2}$, etc., and $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$ are unit vectors along the projectile spin, target spin, and beam momentum directions. Angular braces denote a Clebsch-Gordon coefficient, braces indicate a 9-j symbol, and W is a Racah coefficient. The T-matrix elements are given in terms of the elastic-scattering S-matrix elements by

$$T^{J}_{l'j',lj}=\frac{1}{2i}(S^{J}_{l'j',lj}-\delta_{ll'}\delta_{jj'}). \qquad (6)$$

The correlation terms $C_{kK\Lambda}$ contain all the angular dependence and are given by

$$C_{kK\Lambda}(\hat{\mathbf{s}}\hat{\mathbf{I}}\hat{\mathbf{p}})=\frac{(4\pi)^{3/2}}{\hat{k}\hat{K}}\{[Y_k(\hat{\mathbf{s}}),Y_K(\hat{\mathbf{I}})]_\Lambda,Y_\Lambda(\hat{\mathbf{p}})\}_0. \qquad (7)$$

The notation $[,]_k$ is used to indicate a spherical-tensor product of rank k.

For the case of a proton target and assuming parity conservation, Eq. (4) can be written

$$\sigma_t=\sigma_{00}+P_nP_p\sigma_{11}, \qquad (8)$$

where P_n and P_p are the magnitudes of the neutron beam and proton target polarizations. The relevant correlation terms are given by

$$C_{000}=1,$$

$$C_{11\Lambda}=\begin{cases}-\sqrt{\frac{1}{3}}\,\hat{\mathbf{s}}\cdot\hat{\mathbf{I}}, & \Lambda=0,\\ \sqrt{\frac{1}{6}}\,[3(\hat{\mathbf{s}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}})-\hat{\mathbf{s}}\cdot\hat{\mathbf{I}}], & \Lambda=2,\end{cases} \qquad (9)$$

where $\Lambda=0$ corresponds to the central spin-spin part of the forward scattering amplitude and $\Lambda=2$ corresponds to the tensor part. C_{000} is invariant under the reversal of neutron spin, while C_{110} and C_{112} change sign. Taking the difference in total cross section $\Delta\sigma$ and normalizing to P_n and P_p then selects only the spin-spin cross section:

$$\Delta\sigma=\frac{\sigma_t(\hat{\mathbf{s}}\hat{\mathbf{I}}\hat{\mathbf{p}})-\sigma_t(-\hat{\mathbf{s}}\hat{\mathbf{I}}\hat{\mathbf{p}})}{P_nP_p}=2\sigma_{11}. \qquad (10)$$

This cross section can be divided into scalar and tensor components and written as

$$\sigma_{11}=C_{110}\sigma_{\mathrm{scalar}}+C_{112}\sigma_{\mathrm{tensor}}, \qquad (11)$$

with σ_{tensor} being the quantity of interest for the present work.

The familiar longitudinal and transverse [8] differences in total cross section, $\Delta\sigma_L$ and $\Delta\sigma_T$, defined by[4]

$$\Delta\sigma_L=\sigma\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}-\sigma\begin{pmatrix}\rightarrow\\ \rightarrow\end{pmatrix}, \qquad (12)$$

$$\Delta\sigma_T=\sigma(\uparrow\downarrow)-\sigma(\uparrow\uparrow), \qquad (13)$$

where the arrows indicate the relative orientation of target and beam spin directions, can then be written as

$$\Delta\sigma_L=2\left(\sqrt{\frac{1}{3}}\sigma_{\mathrm{scalar}}-2\sqrt{\frac{1}{6}}\sigma_{\mathrm{tensor}}\right), \qquad (14)$$

$$\Delta\sigma_T=2\left(\sqrt{\frac{1}{3}}\sigma_{\mathrm{scalar}}+\sqrt{\frac{1}{6}}\sigma_{\mathrm{tensor}}\right). \qquad (15)$$

Predictions of $\Delta\sigma_L$ and $\Delta\sigma_T$ from phase-shift analyses are shown in Fig. 1. A unique determination of σ_{tensor} requires the measurement of both $\Delta\sigma_L$ and $\Delta\sigma_T$ [9]. This is not ideal as there will be different normalizations and systematic uncertainties for the two measurements. It is possible, however, to construct an observable which only contains σ_{tensor}. Using this observable, only one measurement is required to uniquely determine the strength of the tensor spin-spin part of the forward scattering amplitude.

For arbitrary center-of-mass angles θ_s between $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$, and θ_I between $\hat{\mathbf{I}}$ and $\hat{\mathbf{p}}$, and additionally assuming $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$ are coplanar, the correlation terms are

$$C_{11\Lambda}=\begin{cases}-\sqrt{\frac{1}{3}}(\cos\theta_s\cos\theta_I+\sin\theta_s\sin\theta_I), & \Lambda=0,\\ \sqrt{\frac{1}{6}}(2\cos\theta_s\cos\theta_I-\sin\theta_s\sin\theta_I), & \Lambda=2.\end{cases} \qquad (16)$$

The scalar term C_{110} can be made zero by choosing $\theta_s-\theta_I=\pm\pi/2$, giving for the tensor term

$$C_{112}=\mp\sqrt{\frac{3}{8}}\sin 2\theta_I, \qquad (17)$$

depending on the sign of $\theta_s-\theta_I$. This term is a maximum for $\theta_s=\pm\pi/4$, $\pm 3\pi/4$. Choosing $\theta_I=+\pi/4$ and $\theta_s=-\pi/4$ gives the plus sign, defining a difference in total cross section which I will call $\Delta\sigma_{\mathrm{tensor}}$:

$$\Delta\sigma_{\mathrm{tensor}}=\sqrt{\frac{3}{2}}\sigma_{\mathrm{tensor}}. \qquad (18)$$

The total cross section for this geometry is then

$$\sigma_t=\sigma_{00}+\tfrac{1}{2}P_nP_p\Delta\sigma_{\mathrm{tensor}}. \qquad (19)$$

In terms of the usual $\Delta\sigma_L$ and $\Delta\sigma_T$, $\Delta\sigma_{\mathrm{tensor}}$ is given by

$$\Delta\sigma_{\mathrm{tensor}}=\tfrac{1}{2}(\Delta\sigma_T-\Delta\sigma_L), \qquad (20)$$

[4]Although it is conventional to define $\Delta\sigma_L$ and $\Delta\sigma_T$ as antiparallel minus parallel, this is opposite from the definition of the general $\Delta\sigma$.

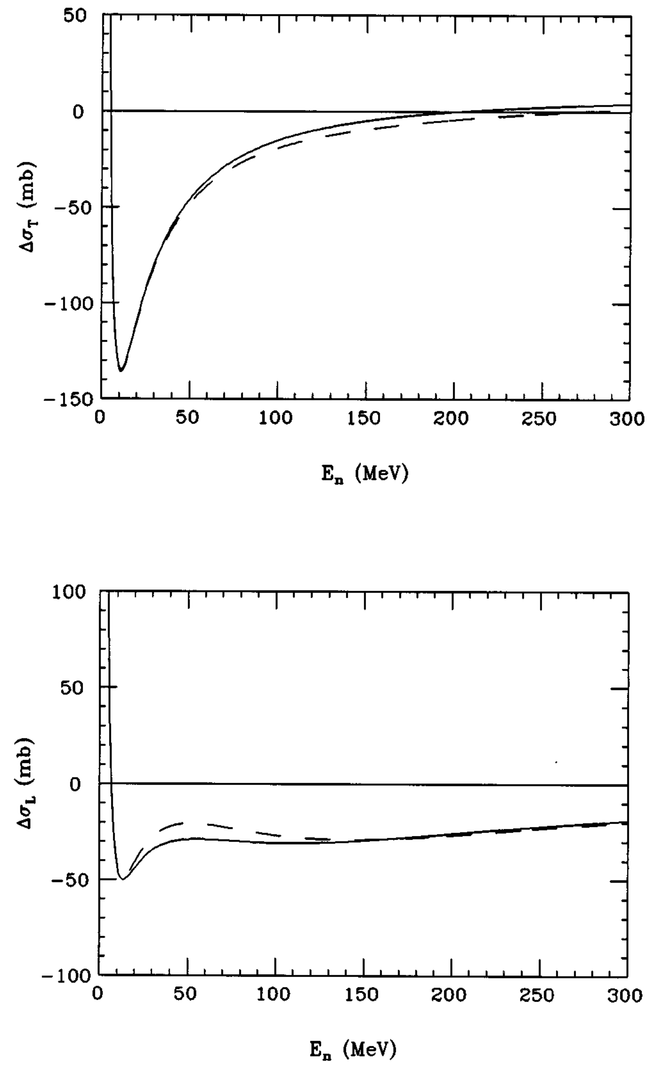

FIG. 1. Predictions of $\Delta\sigma_T$ and $\Delta\sigma_L$ from the Nijmegen PWA93 [5] (solid curves) and the SM94 [10] (dashed curves) phase-shift analyses.

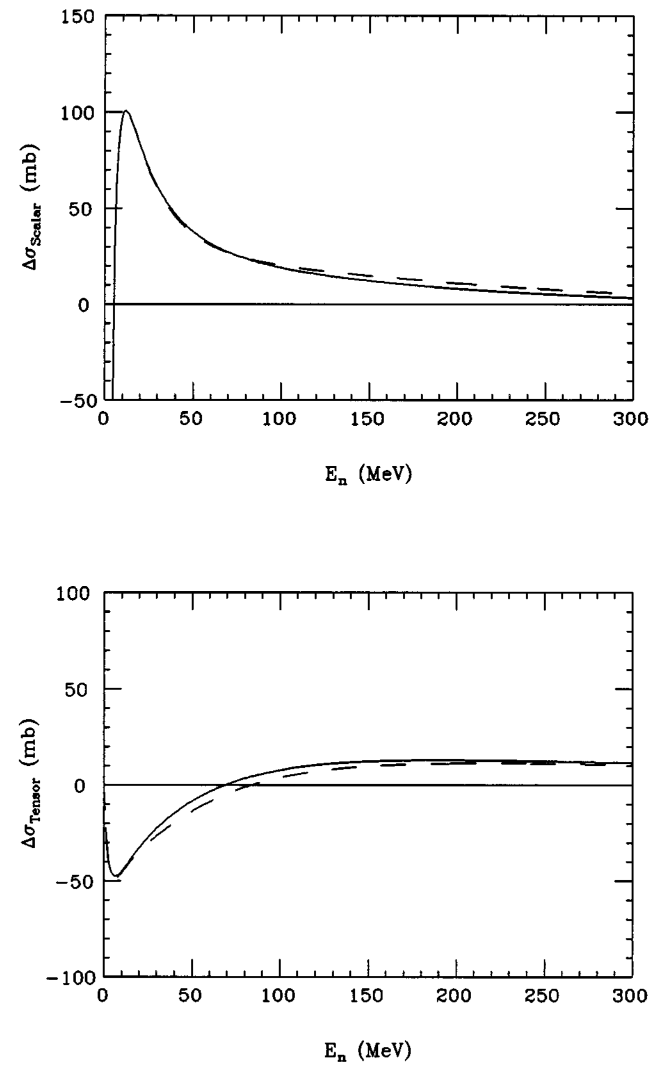

FIG. 2. Predictions of $\Delta\sigma_{\mathrm{scalar}}$ and $\Delta\sigma_{\mathrm{tensor}}$ from the Nijmegen PWA93 [5] (solid curves) and the SM94 [10] (dashed curves) phase-shift analyses.

and in terms of the phase shifts,

$$\Delta\sigma_{\mathrm{tensor}} = -\frac{\pi}{k^2}\sum_{J=0}^{\infty}\{(2J+1)(\sin^2\epsilon_J - \sin^2\delta_{J,J}) + \cos 2\epsilon_J[(J-1)\sin^2\delta_{J,J-1} + (J+2)\sin^2\delta_{J,J+1}] + 3\sqrt{J(J+1)}\sin 2\epsilon_J \sin(\delta_{J,J-1}+\delta_{J,J+1})\}. \quad (21)$$

There is a corresponding geometry for selecting the scalar component. This requires $\tan\theta_s \tan\theta_I = 2$ which results in $C_{112}=0$ and

$$C_{110} = \frac{\mp\frac{\sqrt{3}}{2}\sin 2\theta_I}{\sqrt{1+3\cos^2\theta_I}}, \quad (22)$$

where the upper sign applies for $-\pi/2 \leq \theta_s \leq +\pi/2$. This defines a difference in total cross section $\Delta\sigma_{\mathrm{scalar}}$ given by

$$\Delta\sigma_{\mathrm{scalar}} = -\sqrt{\frac{6}{5}}\sigma_{\mathrm{scalar}}, \quad (23)$$

where again $\theta_I = +\pi/4$ is chosen. Predictions of $\Delta\sigma_{\mathrm{scalar}}$ and $\Delta\sigma_{\mathrm{tensor}}$ from phase-shift analyses are shown in Fig. 2. In terms of the longitudinal and transverse differences in cross section,

$$\Delta\sigma_{\mathrm{scalar}} = -\frac{1}{\sqrt{10}}(2\Delta\sigma_T + \Delta\sigma_L), \quad (24)$$

and in terms of the phase shifts,

$$\Delta\sigma_{\mathrm{scalar}} = -\sqrt{\frac{2}{5}}\frac{\pi}{k^2}\sum_{J=0}^{\infty}(2J+1) \times [2\sin^2\epsilon_J + \sin^2\delta_{J,J} - 3\sin^2\delta_J + \cos 2\epsilon_J(\sin^2\delta_{J,J-1} + \sin^2\delta_{J,J+1})]. \quad (25)$$

Thus a measurement of the observable $\Delta\sigma_{\rm tensor}$ serves to uniquely determine the tensor spin-spin part of the forward scattering amplitude and a measurement of $\Delta\sigma_{\rm scalar}$ uniquely determines the central part. Our interest is in the tensor part.

A measurement of $\Delta\sigma_{\rm tensor}$ can be performed using a polarized proton target with its spin direction oriented at a center-of-mass angle of $+45°$ with respect to the beam direction and a polarized neutron beam with its spin at $-45°$. Reversing the neutron spin allows the measurement of the asymmetry in transmitted flux ε, given by

$$\varepsilon=\frac{N_+-N_-}{N_++N_-}, \tag{26}$$

where N_+ and N_- are the numbers of counts for the two spin states. The transmitted flux is given by the expression

$$N_\pm=N_0e^{-x(\sigma_{00}\pm P_nP_p\Delta\sigma_{\rm tensor}/2)}, \tag{27}$$

where N_0 is the incident flux and x is the target thickness. The asymmetry, in the linear approximation, is then

$$\varepsilon=-\frac{1}{2}xP_nP_p\Delta\sigma_{\rm tensor}. \tag{28}$$

Such a measurement is no more difficult than a single $\Delta\sigma_L$ or $\Delta\sigma_T$ measurement, with two exceptions. First, the requirement that the target be polarized at a 45° center-of-mass angle makes access for the beam difficult. If high fields are required [4], a specially constructed magnet may be needed. The second and more serious problem is related to misalignments in the angles of the target and beam polarization and to neutron spin precession. Errors in the spin angles of the beam and target, $\delta\theta_s$ and $\delta\theta_I$, will add a term containing $\sigma_{\rm scalar}$. Since $\sigma_{\rm scalar}$ is of the order of $\sigma_{\rm tensor}$ and the effect goes as $\sin\delta\theta$, this is not a large effect in practice. However, precession of the neutron spin about the magnetic field used to polarize the target can be large. As this precession is about the axis of target polarization, the relation $|\theta_s-\theta_I|=\pi/2$ still holds, keeping the scalar contribution equal to zero. The effect of a precession α is then to reduce the measured asymmetry by a factor $\cos\alpha$. This precession can be reliably calculated, allowing the initial spin direction to be adjusted to compensate for the precession. An error of 5° in the compensation would only affect the measured asymmetry by 0.4%. Measurements of $\Delta\sigma_{\rm tensor}$ are therefore possible and practical. In addition, since θ_I is the same for $\Delta\sigma_{\rm tensor}$ and $\Delta\sigma_{\rm scalar}$, both terms of the spin-spin forward scattering amplitude can be determined without disturbing the target polarization by simply changing θ_s. This is not possible with $\Delta\sigma_L$ and $\Delta\sigma_T$.

In summary, there is interest in accurately determining the strength of the *NN* tensor interaction. Observables measured in the past are sensitive to both the tensor and scalar spin-spin parts of the *NN* forward scattering amplitude, requiring additional measurements or information to isolate the tensor component. I have presented an *n-p* scattering observable $\Delta\sigma_{\rm tensor}$ which uniquely determines the tensor spin-spin part of the forward scattering amplitude. Measurements of this observable are practical and should be considered for future experiments.

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