Distorted wave analyses of the ^{7}Li(α,2α)^{3}H reaction

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The distorted wave–peripheral three-body coupling model has been applied for knockout of particles bound in $\ell = 1$ state. As an example the ^{7}Li(α,2α)^{3}H reaction at 77, 99, and 119 MeV has been analyzed. The distorted wave impulse approximation peripheral three-body coupling model calculations are compared with the conventional kinematic coupling approximation and with the data.

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Knockout reactions have always been expected to yield reliable nuclear structure information. However, the distorted wave impulse approximation (DWIA) analyses of the large amount of the knockout data have mostly resulted in unphysical and thus unreliable and inconsistent information about nuclear structure [1–3]. Most of the large inconsistencies arose in the cases of cluster knockout such as (α,2α) and (p,$p\alpha$) where large optical distortion effects were present [3–6]. Checks on various approximations have resulted in some improvements in the analyses of knockout data. In one such attempt it was demonstrated [7] that in the DWIA formalism peripheral three-body coupling model (PTBCM) is a significant improvement over the conventional kinematic coupling approximation (KCA) treatment. The examples treated by using the PTBCM formalism have, however, been confined to the knockout reactions where the knocked out particle was bound in the target nucleus in a state of zero orbital angular momentum, $\ell=0$. It is well known that when the knocked out particle comes from an initial $\ell=0$ bound state the coincidence spectrum normally has a broad distribution, peaked close to the zero recoil momentum position. This lack of detailed structure in the spectrum of $\ell=0$ knockout data hinders the extraction of detailed and specific information about the various input parameter values [8]. Moreover, it has been found that with reasonable bound state wave functions, while the plane wave impulse approximation (PWIA) overpredicts the peak cross sections (corresponding to zero recoil momentum) the use of conventional DWIA-KCA drastically reduces them so much so that spectroscopic factors take absurdly large values [3]. Larger recoil momentum components in the spectrum have been found to be even more suppressed due to optical distortions [9]. Therefore the $\ell=0$ knockout DWIA predictions of energy sharing spectra are usually much sharper compared to the data. This behavior may arise from the incorrect treatment of large optical distortions in the initial and final scattering states [10]. A feeling of the amount of distortions is sometimes helpful. One may perceive it theoretically from the ratio of the plane wave to distorted wave predictions. Experimentally one can look for special kinematic conditions where some limiting value is known such as the ratio of cross sections at the dip around the zero recoil momentum position and at a peak position in the case of knockout from $\ell\neq0$ bound state. Knockout from a $\ell\neq0$ bound state in the PWIA gives null cross section at zero recoil momentum position. Due to optical distortions this dip gets filled up and the amount of filling up measures the influence of distorting optical potentials. Special kinematic conditions of noncoplanar geometry, however, indicate [11,12] somewhat reduced influence of optical distortions while spanning the large recoil momentum distributions in knockout reactions.

In the ^{7}Li(α,2α)^{3}H reaction the knockout of alpha cluster mostly occurs from an $\ell=1$ bound state of ^{4}He and ^{3}H in the ground state of ^{7}Li. In order to check on the effects of optical distortion in the $\ell\neq0$ case, recently reported data on ^{7}Li(α,2α)^{3}H reaction at 77, 99, and 119 MeV has been analyzed using the KCA and PTBCM formalisms [13]. Expression for the differential cross section in the impulse approximation is written in the usual form:

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=KS\left(\frac{d\sigma}{d\Omega}\right)_{\alpha\alpha}\sum_{\Lambda}|T_L^{\Lambda}(\vec{Q})|^2, \qquad (1)$$

where K is the kinematic factor and S is the clustering probability. The term $(d\sigma/d\Omega)_{\alpha\alpha}$ is the α-α scattering cross section at the relevant center of mass scattering angle and final state relative energy. The distorted momentum distribution, $\Sigma|T_L^{\Lambda}(\vec{Q})|^2$ at a recoil momentum $\vec{Q}$ is obtained by a distorted wave computer code which can be run with options of KCA as well as PTBCM formalisms. For the distortions of the initial scattering state a factor of $\frac{3}{7}$ has been used to reduce the α-^{7}Li optical potential so as to suppress the double counting of the α-α interaction accounted for by the impulse approximation. Use has been made of the α-t intercluster wave function which gives 3.54 fm as its rms radius (the other intercluster wave function with 2.4 fm rms radius gives much lower prediction for the absolute cross sections).

Results obtained from the two formalisms, PTBCM and KCA, are compared for the three energies [Figs. 1(a), 1(b),1(c)]. In these figures it is seen that the absolute cross sections in the two formalisms do not differ significantly. The shape of the energy sharing spectra are not drastically different in the two formalisms but there are some interesting points to be observed. The PTBCM formalism gives somewhat sharper distributions as compared to the KCA formalism. This pruning of the wings of the spectra by the PTBCM as compared to the KCA formalism indicates, as expected, that the nuclear interior contributions to the matrix element (which are opposite in sign to the exterior contributions) have been partially suppressed by the KCA. Enhancement of lower momentum components (the region around the dip po-

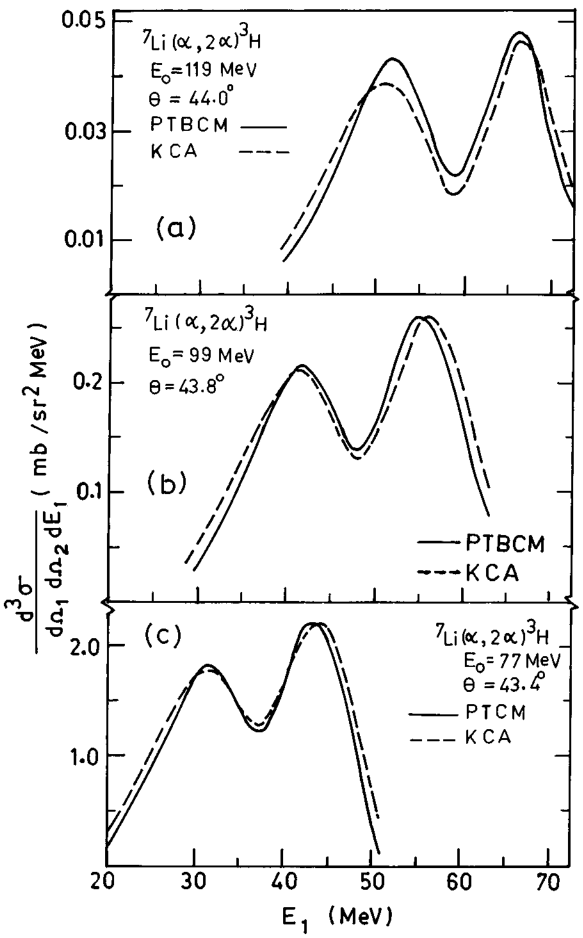

FIG. 1. Comparison of the DWIA calculations using PTBCM (——) and KCA (- - - -) formalisms for the ^{7}Li(α,2α)^{3}H at (a) 119 MeV, (b) 99 MeV, and (c) 77 MeV incident energy.

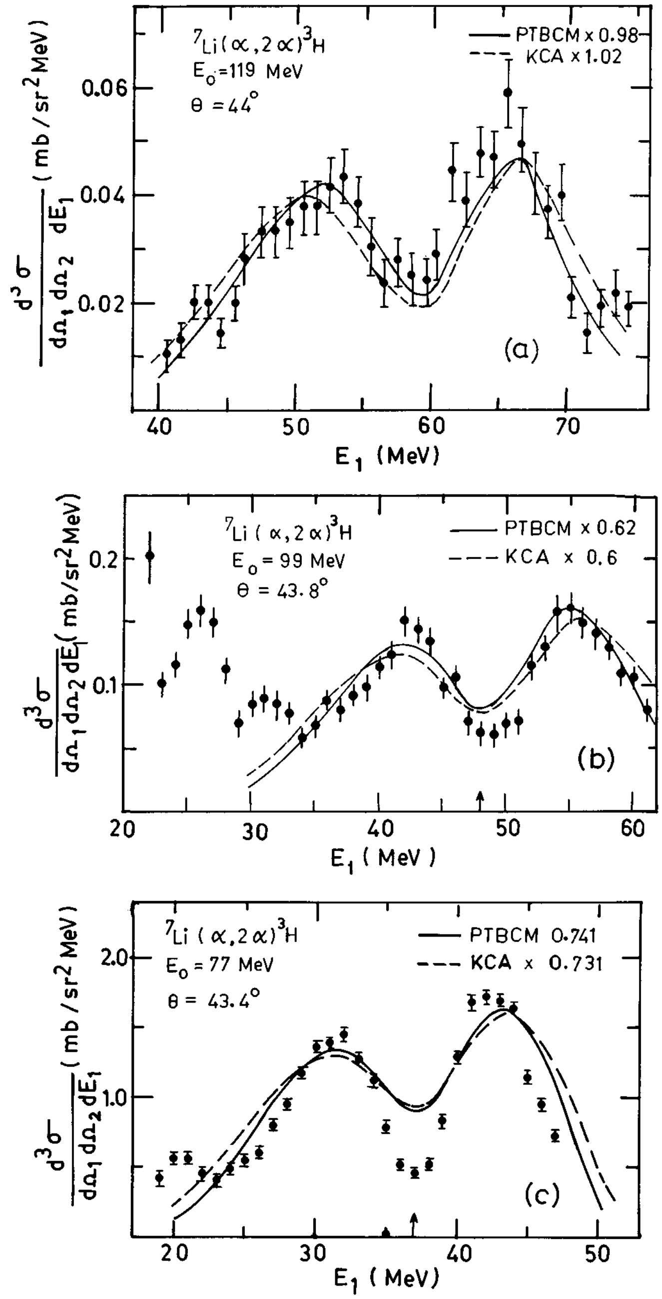

FIG. 2. (a) Comparison of the DWIA calculations at 119 MeV, normalized at the higher E_1 peak with the ^{7}Li(α,2α)^{3}H data [13] using PTBCM (——) and KCA (- - - -) formalisms. The ^{7}Li intercluster bound wave function used has a rms radius of 3.54 fm. (b) Same as (a) except at 99 MeV. (c) Same as (a) except at 77 MeV.

sition and in between the peaks) by the PTBCM indicates that the KCA was suppressing the nuclear interior as well as the surface contributions to the matrix element. Peak to valley ratio, however, is seen to increase with incident energy in both treatments of the DWIA calculations. This is to be expected from the prevailing wisdom that with increased incident energy the filling up of the dip in the $\ell \neq 0$ distributions should be less.

The distorted wave calculations using KCA and PTBCM formalisms are compared with the experimental data at 119, 99, and 77 MeV in Figs. 2(a,b,c), respectively. The calculated results, however, are normalized to the experimental data so as to match the cross sections for the higher energy E_1 peak of the spectrum. A closer look at the energy sharing data on the reaction indicates that the ratio of the cross sections of peak to dip increases as one goes down in energy from 119 to 77 MeV. This means that distortion effects effectively decrease as one goes from higher energy to lower energy because the dip filling is more in the higher energy data. Besides this the humps on either side of the dip tend to broaden as one goes to higher energies. Both of these observations go against the common understanding (viz. an increase of the asymptotic energy relative to the depth of the optical potential should lead to reduced distortions in the wave function). The distorting optical potentials are known to fudge the scattering wave function. The higher momentum region gets more contribution from the nuclear interior whose attenuation should lead to a sharper spectrum. To ascertain this, an improved treatment of the nuclear surface and interior is thus warranted through the PTBCM formalism. It has been seen earlier that with reasonable bound α-t intercluster wave function ($R_{\rm rms}^{\alpha-t} \simeq 2.4$ fm) while the PWIA overpredicts the absolute cross section , the DWIA-KCA underpredicts it drastically (by factors of $\sim 5-8$) [9]. With this α-t bound intercluster wave function the ratio of the peak to dip cross sections are well described by the DWIA-KCA at the three incident energies but the wings of the distributions (the higher recoil momentum components) are overpredicted indicative of higher absorption than theoritically predicted. On the other hand with a bound wave function having $R_{\rm rms}^{\alpha-t} \simeq 3.54$ fm one gets reasonable predictions of peak absolute cross sections at the three energies [13]. Large disagreement has, however, been noticed in the ratios of the peak to dip cross sections between predictions and observations especially at lower energies.

It is seen from the figures that the normalization factors do not differ significantly between KCA and PTBCM results. The normalization factors, however, differ slightly with the incident energy (they are 0.74 for 77 MeV, 0.62 for 99 MeV, and 0.98 for 119 MeV). The reason for the slightly higher normalization constants at 77 and 119 MeV may be due to the increased α-α reaction cross section around these energies [14]. The larger reaction cross section takes away the flux from the free α-α elastic channel and hence it reduces α-α scattering cross section leading to lower ^{7}Li$(\alpha,2\alpha)^3$H cross section in the impulse approximation. The opening up of the reaction channels is apparently modified in the off-shell effects present in the $(\alpha,2\alpha)$ reactions.

Regarding the shape of the distribution it is seen that because of the sharper distribution of PTBCM as compared to KCA, the former is much closer to the experimental results. In fact the 119 MeV spectrum using PTBCM fits the data far better than the corresponding KCA distribution. For 99 MeV spectrum (where the two predictions differ less) one notices the PTBCM prediction to be much closer to the data than the KCA prediction, even though the error bars on the data are smaller. The dip is, however, slightly more filled up than in the 119 MeV prediction. On the other hand, for the lowest energy (77 MeV) spectrum one sees that the distribution of the observed data is much sharper than the predictions of the two formalisms. The PTBCM prediction is somewhat sharper than the KCA , but the dip at the zero recoil momentum is drastically filled up in both the formalisms. Although the deeper dip in the experimental data at 77 MeV indicates a smaller optical distortion effect, the use of unusually large radius for α-t binding potential is necessitated so that larger absolute cross sections are predicted. However, even the use of a larger radius for the bound state potential does not constrain the result of higher cross sections at large recoil momenta.

In the detailed analysis of knockout reactions, the localization of the overlap function indicates that the difference between the low and high recoil momentum components comes mainly from the reduction of contribution from the extreme surface region. (See Fig. 2 of Ref. [15] and Fig. 1 of Ref. [16].) Whenever the optical distortions are large, the DWIA predictions for $\ell=0$ knockout spectra are normally sharper whereas $\ell\neq 0$ are invariably broad. This feature may be understood in terms of the localization of the contributions to the matrix element. The $\ell=0$ wave function normally has a peak around $R=0$ whereas the wave function vanishes at this point for $\ell\neq 0$. Contributions to the matrix element in the case of $\ell=0$, both from the nuclear interior as well as the exterior regions therefore add on to give the largest value at around the zero recoil momentum position. It is seen in the localization analyses of knockout reactions that as one approaches higher recoil momenta, the contribution is reduced mostly from the exterior region. The use of unrealistic distorting optical potentials introduces some imbalance between external and internal contributions to the knockout reaction overlap function. For $\ell=0$ knockout the prediction of a sharper distribution compared to the experimental data indicates that there is too much reduction (in the overlap function) as one approaches higher recoil momenta. It is well known that the extreme surface region essentially gets contributions from large partial waves which are primarily undistorted plane wave component of the scattering states. One can therefore visualize that the internal contribution is reduced greatly because of the optical distortions. Thus a relatively large contribution to the overlap function is cut off in going to higher recoil momenta resulting in a sharper correlation spectrum.

Applying the same logic to the $\ell\neq 0$ knockout spectrum it is well known that the minimum close to the zero recoil momentum position arises from the cancellation of the scattering state parts of the wave functions [in the plane wave case $j_{l\neq 0}(0)=0$] so as to yield a smaller value of the overlap function. The interplay of the optical distortions will lead to inadequate cancellations and hence a higher value of the scattering state part leading to the filling of the dip close to the zero recoil momentum position. Moreover, the peaks close to and on either side of the $\vec{Q}=0$ position will correspond to rising contributions from the external region of the bound wave function. Beyond the peak, however, there is a decline in the external contributions to the overlap function as the bound wave function itself starts declining inwards so as to go through the node (see Fig. 4 of Ref. [13]). For a compact bound wave function, however, the node is much deeper inside the nuclear surface and hence the peak position occurs at a larger recoil momentum . With increasing $\vec{Q}$ not only is there a decline in the surface contributions but there is an increasing internal contribution of the opposite sign. For strong optical distortion the internal contribution of the opposite sign is suppressed and hence a higher cross section is predicted. If, however, the optical distortions were weak a sharper decline on the sides of the peaks would result. The observation of sharp peaks compared to DWIA predictions in the $\ell=1$ spectra therefore requires a weaker optical distortion. The disagreement between DWIA predictions in shape as well as in magnitude with the experimental data is thus understandable for $\ell=0$ as well as $\ell\neq 0$. Though the two look different, both indicate weaker optical distortions than present in the corresponding two-body optical scattering results.

Lack of detailed structure in the $\ell=0$ knockout spectrum does not allow one to obtain much information from the DWIA analysis. These analyses yield large clustering probabilities and rather sharp spectra. However, for the $\ell\neq 0$ spectrum (having a dip at the zero recoil momentum position) the filling up of the dip is a measure of the influence of the distorting optical potentials. The data on ^{7}Li$(\alpha,2\alpha)^3$H reaction at 77, 99, and 119 MeV, surprisingly, shows more filling up of the dip and broadening of the humps at higher energies. Therefore, an improved treatment of optical distortions was required through the peripheral three-body coupling model (PTBCM) of DWIA. The PTBCM treats the nuclear surface and interior contributions to the transition matrix element more accurately than the conventional kinematic coupling approximation (KCA). The PTBCM formalism gives somewhat sharper distributions as compared to the KCA, thus fitting the data better. However, for a better fit to the data both the formalisms require weaker optical distortions than are available from the optical model analyses of two body scattering. This conclusion from DWIA analyses of coplanar cluster knockout data about reduction in optical distortions therefore supports the analyses of noncoplanar results in Refs. [11] and [12].

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